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Prediction of the Lorentz Force Detuning and Pressure Sensitivity for a Pillbox Cavity

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Abstract: The Lorentz Force Detuning (LFD) and the pressure sensitivity are two critical concerns during the design of a Superconducting Radio Frequency (SRF) cavity resonator. The mechanical deformation of the bare Niobium cavity walls, due to the electromagnetic fields and fluctuation of the external pressure in the Helium bath, can dynamically and statically detune the frequency of the cavity and can cause beam phase errors. The frequency shift can be compensated by additional RF power, that is required to maintain the accelerating gradient, or by sophisticated tuning mechanisms and control-compensation algorithms. Passive stiffening is one of the simplest and most effective tools that can be used during the early design phase, capable of satisfying the Radio Frequency (RF) requisites. This approach requires several multiphysics simulations as well as a deep mechanical and RF knowledge of the phenomena involved. In this paper, is presented a new numerical model for a pillbox cavity that can predict the frequency shifts caused by the LFD and external pressure. This method allows to greatly reduce the computational effort, which is necessary to meet the RF requirements and to keep track of the frequency shifts without using the time consuming multiphysics simulations.

Keywords: Acceleration cavities, Overall mechanics design, Simulation methods and programs

Contents

1 Introduction

The pursuit of high quality factors (Q) and high gradients for SRF cavities, which will be used in future superconducting based accelerators, will allow to reduce the operating costs of the cryogenic plants and the overall accelerator costs, which is almost inversely proportional to the average accelerating gradient for a linear accelerator operating in pulsed mode [1]. If the quality factor is raised as the gradient is raised, thus keeping the dynamic heat load to the cryogenics identical, for a Continuous Wave (CW) mode of operation, the same accelerator cost dependency on the accelerating gradient can be applied.

Small mechanical deformations due to, for example, the surronding vibration noise (microphonics) or Lorentz forces, especially for pulsed operation, can cause a shift in the electromagnetic resonance frequency (detuning). Since the generator power required for a given beam loading scales quadratically with the frequency shifts, it is crucial to develop a computationally efficient model that allows the optimization of the cavity shape in order to enhance the passive compensation of detuning sources rather than sophisticated tuning mechanisms and control - compensation algorithms.

The RF electromagnetic fields produce a radiation pressure, which is known as Lorentz force, on the surrounding cavity resonator walls. It can be calculated as:

$$P = \frac{1}{4}(\mu_0|\vec{H}|^2 - \varepsilon_0|\vec{E}|^2) \tag{1.1}$$

where:

- P is the Lorentz pressure;
- $\vec{H}$ and $\vec{E}$ are, respectively, the magnetic field and the electric field .

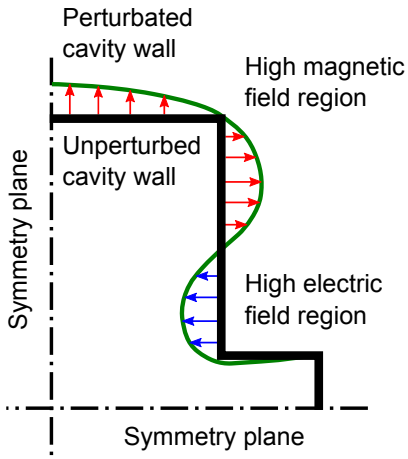


Figure 1. Qualitative Lorentz pressure for TM010 mode in a pillbox cavity.

This pressure deforms the cavity walls and tends to push outwards the areas where there is an high magnetic field (positive volume change) and inwards the areas of high electric field (negative volume change). For the fundamental Transverse Magnetic (TM010) mode in a Pillbox cavity, it means that the walls near the iris tend to get closer while they tend to move away on the external cylindrical shell (Fig. 1). The change of shape of the resonator due to these displacements may change the inner volume of the cavity by a quantity ΔV. The *Slater perturbation theorem* [2] provides the resonant frequency shift of a cavity, which is subjected to a shape perturbation, according to:

$$\frac{\omega_0 - \omega}{\omega_0} = \frac{\int_{\Delta V}(\mu_0|\vec{H}_0|^2 - \varepsilon_0|\vec{E}_0|^2)dV}{\int_V(\mu_0|\vec{H}_0|^2 + \varepsilon_0|\vec{E}_0|^2)dV} \tag{1.2}$$

where:

- ω_0 and ω are, respectively, the angular resonant frequency of the unperturbed and perturbed cavity shape after the application of the mechanical load;
- V and ΔV are, respectively, the volume inscribed by the cavity walls and the volume perturbation;
- $\vec{H}_0$ and $\vec{E}_0$ are the unperturbed magnetic and electric fields.

The above equation can also be written in terms of time-averaged stored energy; the numerator represents the variation of this quantity while the denominator is the total electromagnetic energy of the unperturbed resonator.

The pressure fluctuations in the Helium bath are an additional concern responsible for producing elastic displacements and small oscillation of the resonator walls. As can therefore be inferred from the Slater theorem, the resonator frequency drops if the areas with high magnetic field are pushed in and, on the contrary, it increases if the same areas are subjected to an high electric field. Traditionally this effect is evaluated as $\frac{df}{dp}$ which gives the resonant frequency shift for a given applied external pressure.

The state of the art for the evaluation of LFD and $\frac{df}{dp}$ involves the use of several combined RF and structural simulations which are possible through software like Comsol [3] and Ansys multiphysics [4]. A different approach that takes advantage of the Helium jacket and cavity geometry was specifically developed and succeflully implemented for low beta cavities [5].

The use of stiffeners placed on the external walls of the cavity can greatly reduce the resonant frequency shifts due to the mechanical perturbation of the resonator shape. Therefore, during the mechanical design of an SRF cavity, several iterations, thus several multiphysics simulations are usually required to optimize the shape and position of the stiffeners. Thus, the purpose is to obtain a numerical method with which it is possible to avoid the multiphysics simulations and have a real-time feedback of the RF characteristics of the resonator.

2 The numerical model

If Eq. 1.1, describing the radiation Lorentz pressure, and Eq. 1.2, describing the angular frequency shift, are considered, one can notice that the integral: $\int_{\Delta V}(\mu_0|\vec{H}_0|^2 - \varepsilon_0|\vec{E}_0|^2)dV$, contains an integrand, which is exactly the radiation pressure P up to a multiplicative constant. Conversely, the integral $\int_V(\mu_0|\vec{H}_0|^2 + \varepsilon_0|\vec{E}_0|^2)dV$, represents the time-averaged electromagnetic energy of the unperturbed resonator, which can be written as $W_{tot} = W_m + W_e$.

In the design phase in which the cavity shell is optimized to minimize the Lorentz force detuning and $\frac{df}{dp}$, it is reasonable to accept the fact that the RF volume, which is the volume inscribed in the cavity walls, is not subject to further changes of shape or volume. Hence, the two quantities P and W_{tot} are not subjected to further changes and they can be evaluated with one simple RF simulation. Thus, to calculate the frequency shift, according to Eq. 1.2, only one more term needs to be evaluated: ΔV. The ΔV represents the local change in volume at every point of the cavity resonator inner surface of the walls.

2.1 Calculation of ΔV

SRF cavities may differ in geometry depending on the particular application. A Finite Element Method (FEM) analysis is usually carried out to evaluate the displacement at each point of the structure. In FEM software codes such as Ansys, the solid structure is discretized by elements and each element is described by nodes. If small enough elements are used, it is possible to obtain a good estimation of the displacement at each point due to the applied load. Using quadratic tetrahedrons to build the mesh in the simulation, the inner surfaces of the cavity walls are described by curved triangles with three nodes on each side. Fig. 2 shows the triangular face of a tetrahedron element before and after the load has been applied.

Nodal coordinates in a user defined Cartesian coordinate system, for the initial element face and the final element face, that is the face after the application of the load, can be extracted for all the nodes of the structure and exported to external files. Only the nodes describing the inner surface of the cavity are selected to be exported. Indeed, only these nodes are describing the RF volume and they are required for the calculation of the frequency shift. This also allows reducing the computational time requested by the FEM software with respect to the case where all the node coordinates are exported. Nodal coordinates can then be imported to any third party programming language software (for this work Matlab [6] has been used) to calculate several quantities (see Fig.

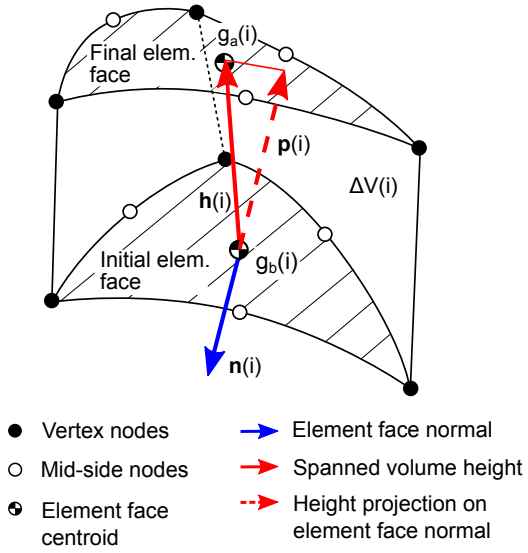


Figure 2. ΔV evaluation in a FEM environment.

2), which are needed to eventually obtain ΔV. Only the vertex nodes are used for these estimations requiring:

- Centroid coordinates for both the initial and final element face
- Normal unit vector of the initial element face
- Height vector of the volume spanned by the element face
- Projection vector of the height on the initial element face unit normal vector.

The normal unit vector of the initial element face and the height vector have the origin in the centroid. If only the six vertex nodes describing the initial and final element face are used for the computation, a simple estimation of the spanned volume can be evaluated dividing the prism in three tetrahedrons and using the Heron's closed-form formula [7] to compute the volume of each tetrahedron. It could have been possible to consider also the mid-side nodes, which could have led to a better estimation of ΔV, although, if the mesh elements in the FEM software are small with respect to the cavity shell curvature at any given point, the improvement of the estimation of the spanned volume is totally negligible, as it is shown in the results given in section 3.

2.2 Sign calculation for ΔV

Since during the application of the the Lorentz pressure some of the element faces are displaced inward and some outward, it is necessary to attribute a sign to the calculated value of ΔV. Element faces which move inward with respect to the undeformed surface represent a negative ΔV while the faces moving outward represent a positive ΔV. To establish the sign, the height vector of the spanned element volume is used. The height vector is defined with the origin in the centroid of the initial element face and the end in the centroid of the final element face. If the projection of this vector on the normal unit vector of the initial element face is calculated, it is possible to compare the directions and calculate the sign of ΔV.

2.3 Calculation of the angular frequency shift

At this point, knowing both the Lorentz pressure P and the change in volume ΔV at any element face of the cavity inner surface, it is possible to evaluate the angular frequency shift. The angular frequency shift according to Eq. 1.2 can be written as:

$$\omega_0 - \omega = \omega_0 \frac{\sum_{i=1}^{N} P_i \Delta V_i}{W_{tot}} \tag{2.1}$$

where

- N is the total number of elements with at least one face on the cavity shell inner surface
- P_i is the average Lorentz pressure on the i-th element face
- ΔV_i is the spanned volume by the element for the i-th element face

W_{tot} and P_i are obtained from one RF simulation. For this reason, it is not necessary to run multiple analysis for the different cavity models, provided that all the shapes under study inscribe the same RF volume. It is important to notice that for the estimation of $\frac{df}{dp}$ the Lorentz pressure is used in Eq. 2.1 and not the applied external pressure, which is used instead for the evaluation of the node displacements using the FEM software, thus for the calculation of ΔV_i.

2.4 Algorithm

The following algorithm can be implemented into any third party programming language software to evaluate the frequency shifts.

Algorithm 1: Prediction of Lorentz Force Detuning and Pressure sensitivity

Data: Node coordinates before load, node coordinates after load, Lorentz pressure
Result: Frequency shift
n = number of element faces;
for $i = 1$ **to** n **do**
  Compute centroid $g_b(i)$ for each element face before load;
  Compute centroid $g_a(i)$ for each element face after load;
  Compute normal versor $\mathbf{n}(i)$ for each element face before load;
  Compute height vector $\mathbf{h}(i)$ for $\Delta V(i)$;
  Compute projection vector $\mathbf{p}(i)$ of $\mathbf{h}(i)$ onto $\mathbf{n}(i)$;
  Compute $\Delta V(i)$;
  Assign sign to $\Delta V(i) \rightarrow \Delta V(i) = \Delta V(i) \cdot sign(\mathbf{p}(i))$;
  Compute frequency shift $f(i)$ associated with each $\Delta V(i)$;
end
$ftot = \sum_{i=1}^{n} f(i)$;

3 A case study: the Pillbox cavity

3.1 Pillbox RF simulation

To validate the numerical model, a simple Pillbox cavity is considered. In Table 1 are listed the cavity figures of merit calculated using the formulas provided in [8]. An arbitrary value of 4 MV is chosen for the effective accelerating voltage.

Table 1. Pillbox figures of merit

Symbol	Description	Value
r	pillbox radius	0.115 m
L	pillbox length without beam pipes	0.1 m
f_0	frequency	$9.978 \cdot 10^8$ Hz
G	geometric factor	210.7 Ω
R/Q	shunt impedance over quality factor	220.3 Ω

The Pillbox RF volume and shell 3D models are built using a CAD software. Taking advantage of the symmetry, only one eighth of the whole geometry is modeled to reduce the computational effort. Fig. 3 shows the 3D models used. Four different models, with different stiffener shapes and positions, are analyzed to assess the soundness of the numerical model. These are:

(a) Simple pillbox cavity;

(b) Pillbox with stiffeners on the side walls (high electric RF field region);

(c) Pillbox with stiffeners on the perimeter (high magnetic RF field region);

(d) Combination of cases b and c.

The model *a* is then imported into Comsol multiphsics and one RF simulation is carried out. As mentioned earlier, only one RF simulation is necessary if the RF volume is not affected by the modifications of the cavity wall.

All the required RF quantities that depend on the electromagnetic stored energy must be normalized at this point. The Comsol eigenfrequency solver computes the solution at a nominally small amount of stored energy by default (W_{comsol}), and this value must be used to scale the cavity figures of merit. The total energy at any specified accelerating voltage can be directly calculated as:

$$W_{tot} = \frac{V_{acc}}{\omega \cdot \frac{R}{Q}} \tag{3.1}$$

Herein ω and R/Q were computed with Comsol for the un-deformed cavity. The results are listed in Table 2:

To evaluate the Lorentz pressure of the cavity at 4 MV accelerating field, it is necessary to multiply P in Eq. 1.1 by $\frac{W_{tot}}{W_{comsol}}$.

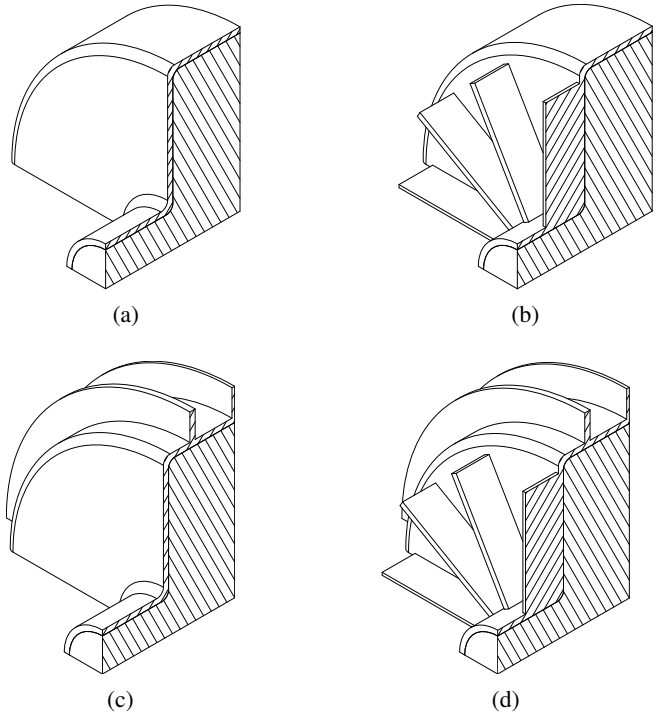


Figure 3. 3D models depicting the RF volume and cavity wall. Adding stiffeners provide greater rigidity in either the high electric field area (b), high magnetic field area (c) or both areas (d).

Table 2. Pillbox figures of merit: data from Comsol

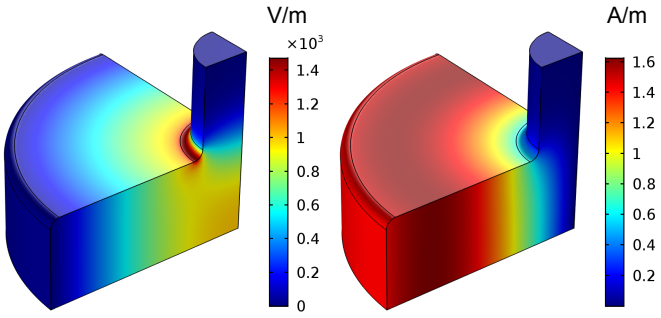
Symbol	Description	Value
V_{acc}	effective accelerating voltage	4.0001 MV
f_0	frequency	$1.0078 \cdot 10^9$ Hz
G	geometric factor	216.9 Ω
R/Q	shunt impedance over quality factor	271.2 Ω
W_{tot}	total time-averaged stored energy	9.31 J

Fig. 4(a) shows the electric and magnetic field contour lines for the undeformed pillbox cavity under study. Fig. 4(b) shows the contour lines representing the Lorentz pressure acting on the cavity walls.

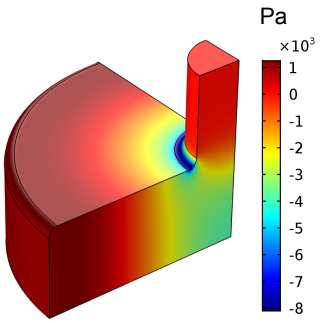
3.2 Implementation of the numerical model

The Lorentz pressure at any point of the surface can be extracted from Comsol and imported in Ansys to evaluate the displacement fields and extract the nodal coordinates before and after the load is applied. Two structural simulations are necessary to evaluate the frequency shifts due to the Lorentz and a uniform external pressure, respectively:

- The Lorentz pressure is applied to the cavity walls



(a) Electric and magnetic field contour lines



(b) Lorentz pressure

Figure 4. RF simulation results from Comsol for the RF volume of the pillbox cavity. Since the RF model is shared between the four different cavity shell models, only one RF simulation is required for the computation of the Lorentz pressure.

- An outside pressure of 1 bar is applied to the cavity walls.

Because of the linear elastic behavior of the shell material, an arbitrary external pressure could have been chosen to evaluate the pressure sensitivity.

An APDL script allows to select the element faces on the shell inner surface and to export the nodal coordinates and nodal displacements.

To implement the numerical model described in section II and to evaluate the frequency shifts, two inputs are required: the Lorentz pressure and the nodal data from the structural simulations. Matlab is used to carry out this evaluation for this paper, but any other third party programming language software could be used instead. Fig. 5 shows the work-flow that leads to the frequency shift evaluation.

Since the Lorentz pressure exported from Comsol is evaluated as a discrete function of the Cartesian coordinates which may not correspond to the nodal coordinated if the mesh is altered, a scattered 3D interpolation is carried out. The interpolation returns a surface of the form $P = F(x, y, z)$, which passes through the sample values, the Lorentz pressure values in this case, at the point locations. The Lorentz pressure is then evaluated at the nodal coordinates imported from the structural simulations.

From the nodal coordinates, the centroid of the element faces before and after the application

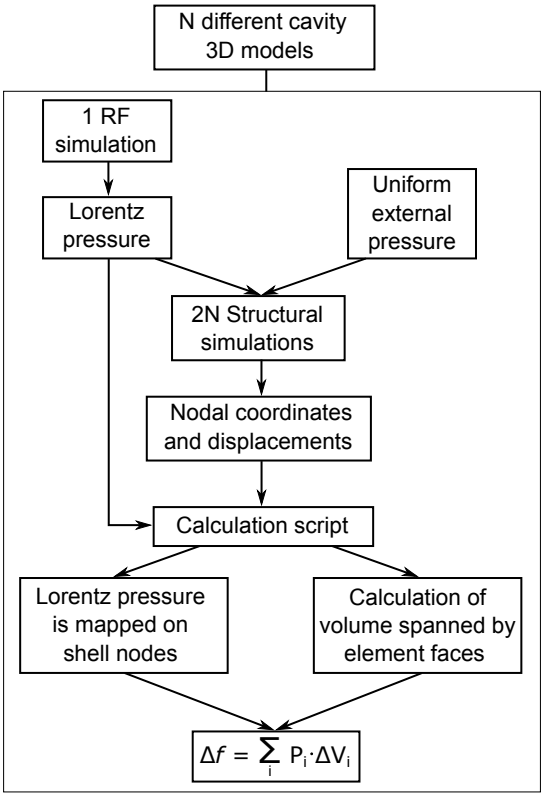


Figure 5. Implementation of the numerical model: for the evaluation of the LFD and pressure sensitivity characterization of N different cavity models, one RF and 2N structural simulations are necessary. The current approach would require 2N RF and 2N multiphysics simulations.

of the load are calculated and so are the normal unit vectors and the spanned volume height vector. Fig. 6 shows a portion of the model representing the element faces; a coarse mesh is used here for clarity.

ΔV is then calculated from the volume of the convex hull defined by six points: three are the nodes belonging to the element face before the application of the load and three are the corresponding nodes after the load is applied. The sign of the projection of the height vector on the normal unit vector is then multiplied to ΔV and the frequency shift can be calculated according to Eq. 2.1.

Tables 3 and 4 summarize the results obtained regarding the pressure sensitivity and the LFD respectively. The maximum deviation is 2.2% for the estimation of $\frac{df}{dp}$ and 1% for the estimation of the LFD. The computational time is another important aspect to be considered. Especially because cavity shells usually contain small feature and curvatures that requires finer meshes. Furthermore a realistic cavity model could have no applicable symmetry plane, which which increases the computation time accordingly. For the case of the Pillbox cavity for instance, the number of mesh cells would be eight times larger if no symmetry plane would have been applied.

To give an estimate of the time that can be saved using the proposed methodology, all Comsol and Ansys simulations ran on the same machine and so is the Matlab script.

The computational time required by each simulation is:

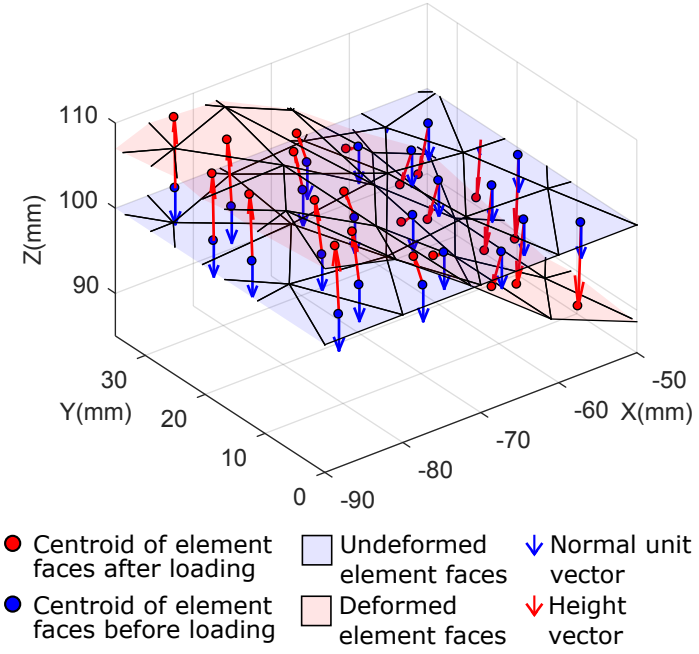


Figure 6. Element faces of the cavity shell inner surfaces before and after the application of the load are represented in light blue and light red respectively. The normal unit vectors, with origin on the undeformed element face centroids, point always inwards. The height vectors point from the centroid of the undeformed to the centroid of the deformed element face. The projection of the height vector can therefore be used to establish the sign of each ΔV for each element face.

Table 3. Frequency shifts comparison for $\frac{df}{dp}$ (Hz)

Frequency shifts	Model *a*	Model *b*	Model *c*	Model *d*
$\frac{df}{dp}_{Comsol}$	17.774	16.641	15.531	12.926
$\frac{df}{dp}_{Matlab}$	18.075	16.896	15.875	13.118
Error	1.7%	1.5%	2.2%	1.5%

Table 4. Frequency shifts comparison for LFD (kHz)

Frequency shift	Model *a*	Model *b*	Model *c*	Model *d*
LFD_{Comsol}	-0.456	-0.258	-0.415	-0.210
LFD_{Matlab}	-0.458	-0.261	-0.417	-0.212
Error	0.5%	1%	0.5%	0.9%

- One multiphysic simulation with Comsol required between 6.5 and 7.5 minutes each;
- One structural simulation with Ansys required 1 minute;
- The Matlab script requires 15 seconds to compute all the results shown in Tables 3 and 4.

For the four different cavity models analyzed in this paper, using the standard approach led to $7' \cdot 4 \cdot 2 = 56$ minutes in total of computational time. The use of the proposed method cut the total effort to $(1' \cdot 4 \cdot 2) + 15" + 2' \simeq 10$ minutes, where the 2 minutes represent the required RF simulation computational time. A similar approach was also for the design of single and triple spoke cavities at Fermilab [9].

4 Conclusion

The proposed method and numerical model represent a tool that allow to fully characterize the detuning behavior of a SRF cavity using only one simple structural analysis and provide an estimation of $\frac{df}{dp}$ and LFD with only a small deviation compared to the classical approach, which involves a multiphysic simulation. Though the proposed method was studied for a simple pillbox cavity it can be generally applied to any cavity shapes and dimensions. Moreover the required computational time has been reduced considerably.

Acknowledgments

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