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Longitudinal Effects in Space Charge Dominated Cooled Bunched Beams

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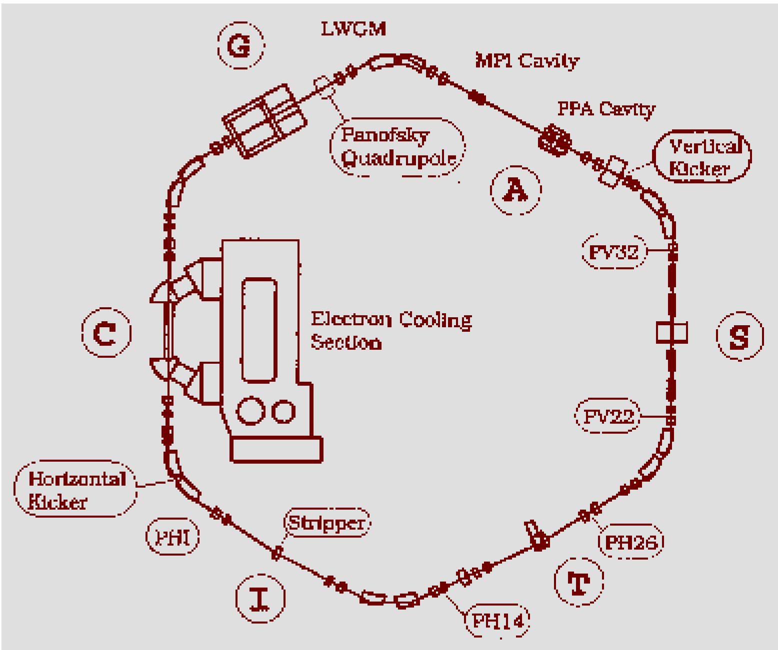
Summary (in advance)

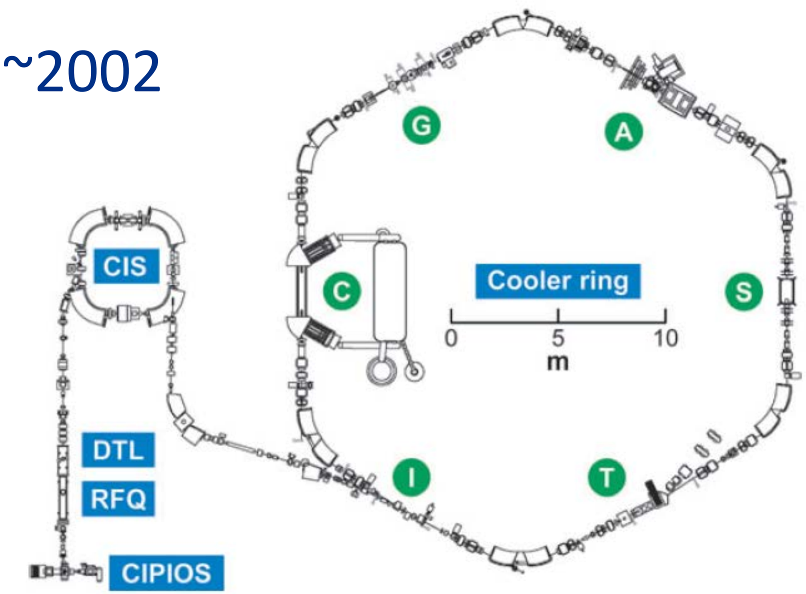
- Longitudinal space-charge effects manifest themselves in several fascinating ways:
  - Self-consistent stationary longitudinal distribution
  - Synchrotron modes shifts with beam current
- These effects are easily accessible in low-energy electron-cooled ion beams

- This talk is based on my research at the Indiana University Cyclotron Facility (IUCF) Cooler, but easily applicable to all other cooler rings

IUCF Cooler synchrotron (1988 – 2002)

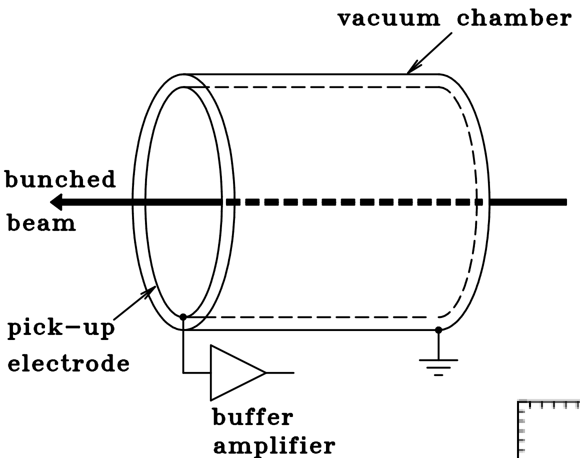


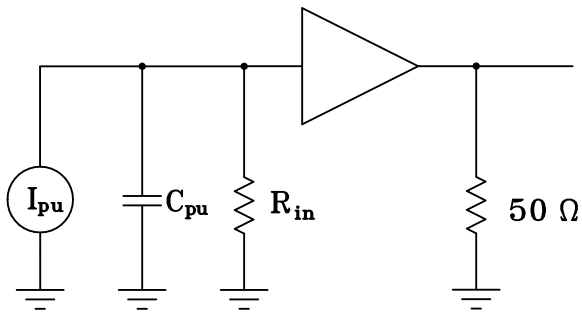
~2002



- These experiments were conducted with electron-cooled proton beams (45 – 150 MeV) below transition energy.
- Longitudinal and transverse profiles were measured for a range of beam currents. Also, the synchrotron dipole and quadrupole frequencies were measured.

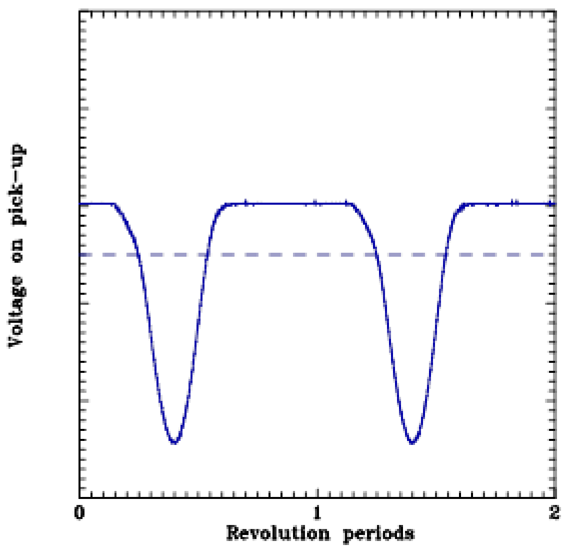
Measurements

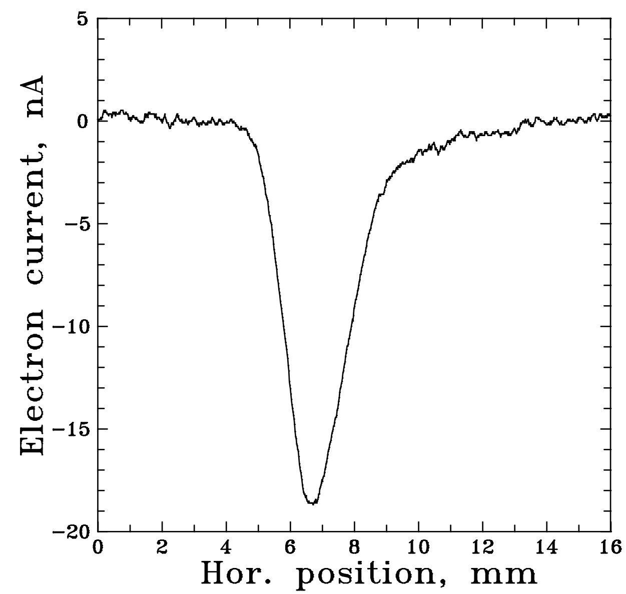




Flying wire signal -- Transverse profile

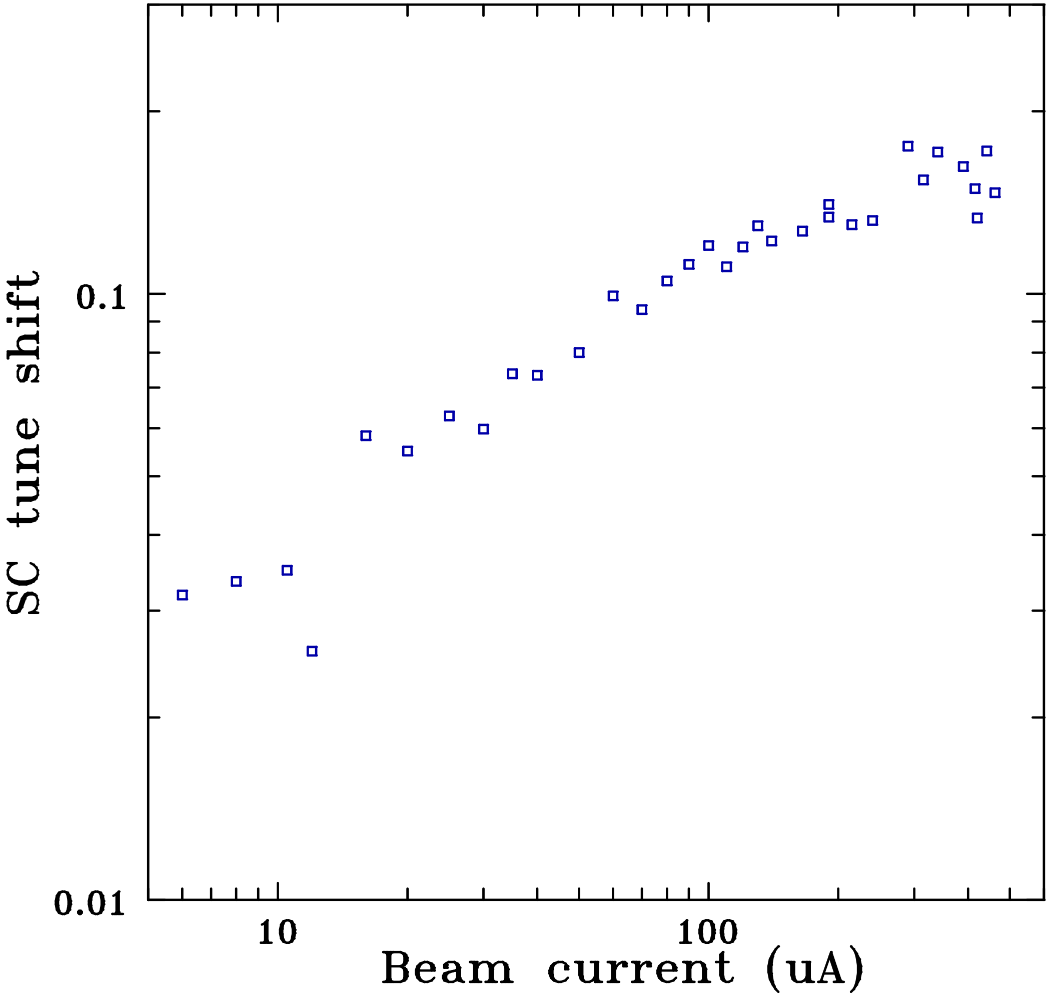
Voltage on the pick-up electrode for τ » T (Rin = 1 MΩ).





Longitudinal profile

From measured long and transv beam profiles we can calculate the SC tune shifts:



$$\Delta \nu = -\frac{BF \times N \ r_p}{4\pi \beta \gamma^2 \varepsilon_n},$$

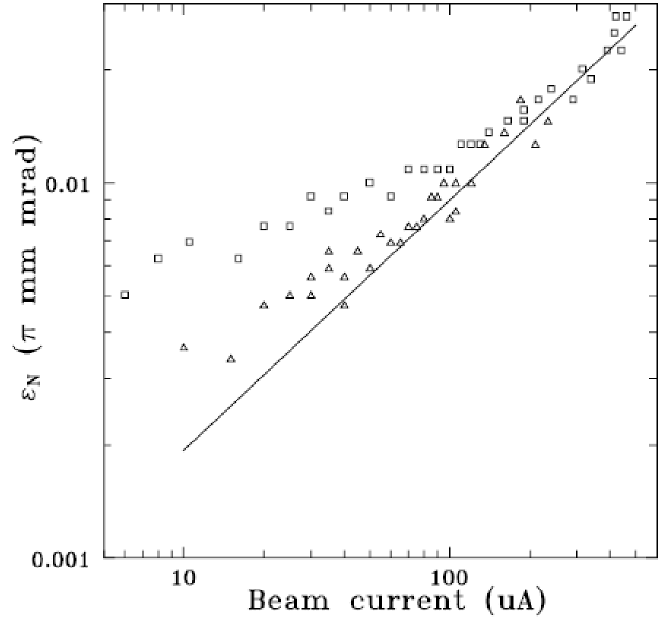
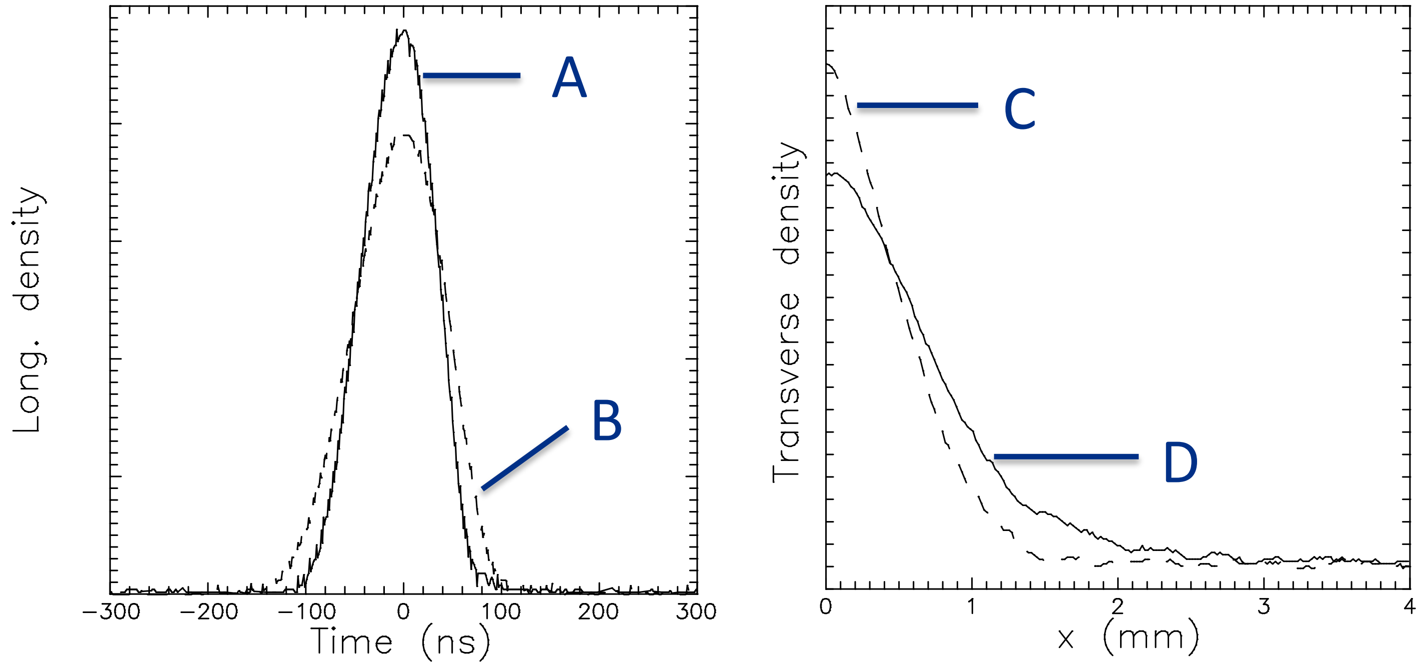


Figure 4. Normalized rms emittance as a function of the average bunched proton beam current before (□) and after (△) the alignment of electron and proton beams. Solid line is $I^{2/3}$.

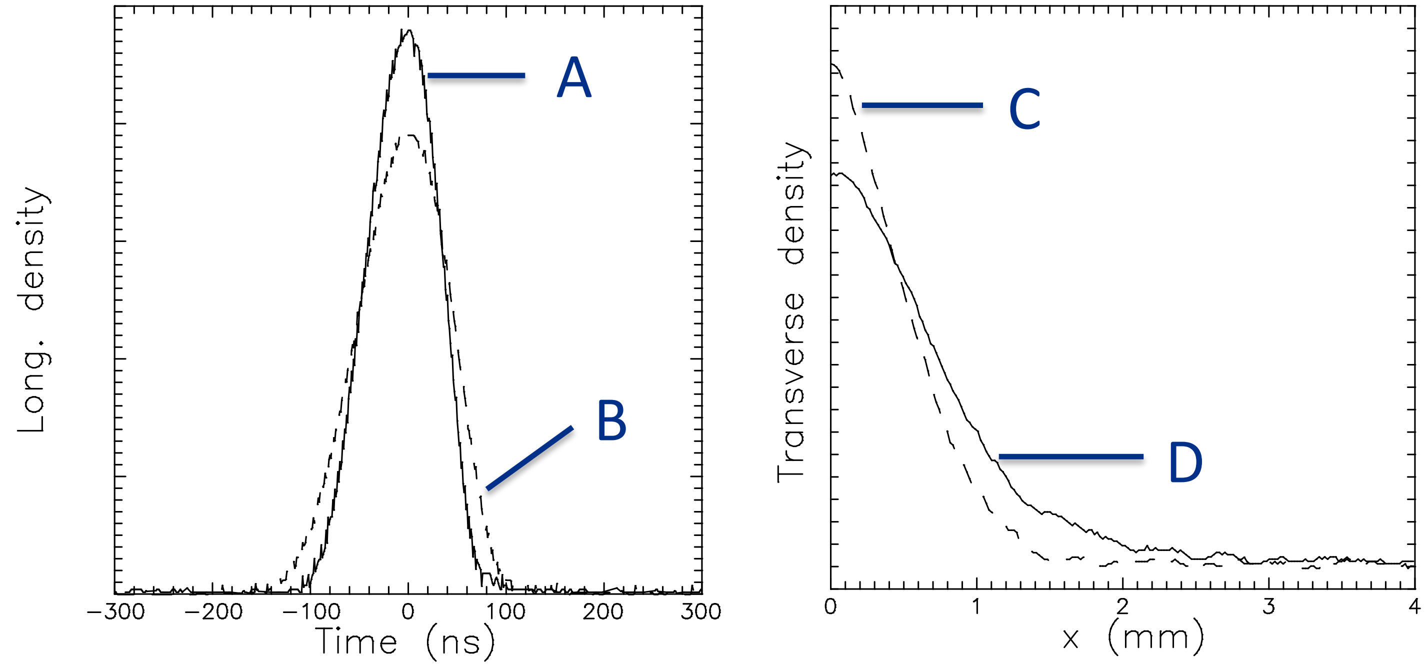
- This tune shift never exceeded -0.2

…First, a quiz…



- These are measured stationary beam distribution profiles for a 150 MeV proton beam with continuous electron cooling. One profile is taken with “good” vacuum, one with “bad” vacuum.
- Question: Which profiles correspond to “bad” vacuum?
- Bad vacuum: (1) A and C; (2) B and C; (3) A and D; (4) B and D

The correct answer is (3)



- (3) A and D;
- Transverse beam profile gets broader (D) from Coulomb scattering (because of “bad” vacuum) but the bunch length gets shorter (A) because the longitudinal space-charge defocusing is reduced.

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LONGITUDINAL MOTION IN HIGH CURRENT ION BEAMS - A SELF-CONSISTENT PHASE SPACE DISTRIBUTION WITH AN ENVELOPE EQUATION*

David Neuffer**

Presented at the Particle Accelerator Conference, San Francisco, CA March 12-14, 1979

The ions in the bunch experience a space charge force given by [4)]

$$F_z = \frac{-g}{\gamma^2} q^2 e^2 \frac{d\lambda}{dz}$$

SC defocusing

RF focusing

where q is the ion charge state, λ is the number of ions per unit length, and g is a geometrical factor of order unity.

$$\frac{\partial f}{\partial s} + z' \frac{\partial f}{\partial z} + \left(- A \frac{\partial \lambda}{\partial z} - Kz\right) \frac{\partial f}{\partial z'} = 0 \qquad (5)$$

If we choose f such that f = f(H) then the Vlasov equation requires that $\frac{\partial f}{\partial s} = 0$ and we have a stationary distribution. We must also choose an f that is self-consistent; that is

$$\int f(z', z, s)\, dz' = \lambda(z, s) \qquad (6)$$

As a simplest solution we desire λ(z) to be parabolic:

$$\lambda = \frac{3}{4} \frac{N}{z_m} \left(1 - \frac{z^2}{z_m^2}\right) \equiv \lambda_o \left(1 - \frac{z^2}{z_m^2}\right) \qquad |z| < z_m$$

$$\lambda = 0 \qquad |z| > z_m \qquad (7)$$

With electron cooling, the Vlasov equation becomes the Fokker-Plank equation

$$\frac{\partial \Psi}{\partial t} + \dot{\phi} \frac{\partial \Psi}{\partial \phi} + \dot{\delta} \frac{\partial \Psi}{\partial \delta} = \frac{\partial}{\partial \delta} \left(\lambda \delta \Psi + \frac{D}{2} \frac{\partial \Psi}{\partial \delta} \right),$$

$$\dot{\phi} = h \eta \omega_o \delta , \quad \dot{\delta} = f_o \frac{e V_{rf}}{\beta^2 E} \sin(\phi) + \frac{Z_o g e^2 N f_o}{\gamma^2 \beta^2 E} \frac{c h^2}{R} \frac{\partial \rho(\phi, t)}{\partial \phi},$$

$$\rho(\phi, t) = \int_{-\infty}^{+\infty} \Psi(\phi, \delta, t) d\delta .$$

For a uniform transverse distribution

g = log[chamber radius/beam radius] + ½ -- geometric factor

- This equation can be solved analytically for a stationary distribution function

The stationary distribution

$$\Psi_o(\phi,\delta) = \frac{1}{(2\pi)^{1/2}\sigma} e^{-\delta^2/2\sigma^2} \rho_o(\phi),$$

$$\rho_o(\phi)\, e^{-\alpha\,\rho_o(\phi)} = \rho_o(0)\, e^{-\alpha\,\rho_o(0)} \exp\left[\kappa\left(1-\cos(\phi)\right)\right].$$

$\rho_o(0)$ must be chosen such that $\rho_o(\phi)$ is normalized to unity

$$\alpha = \frac{Z_o\, g\, e^2 N}{2\pi\gamma^2\beta^2\sigma^2\eta E}\frac{c\,h}{R};\ \kappa = \frac{1}{2\pi\sigma^2\beta^2\eta\, h}\frac{e V_{rf}}{E}.$$

- There are two unknowns: σ and g, the rms momentum spread and the geometric factor.
- For vanishing space-charge ($\alpha = 0$) the linear density $\rho_o(\phi)$ becomes Gaussian.
- For vanishing momentum spread ($\sigma \to 0$) the linear density becomes

$$\rho_o(\phi) = \frac{\gamma^2}{g\,h^2}\frac{V_{rf}\,R}{Z_o\, e\, N\, c}\left(\cos(\phi) - \cos(\phi_o)\right),$$

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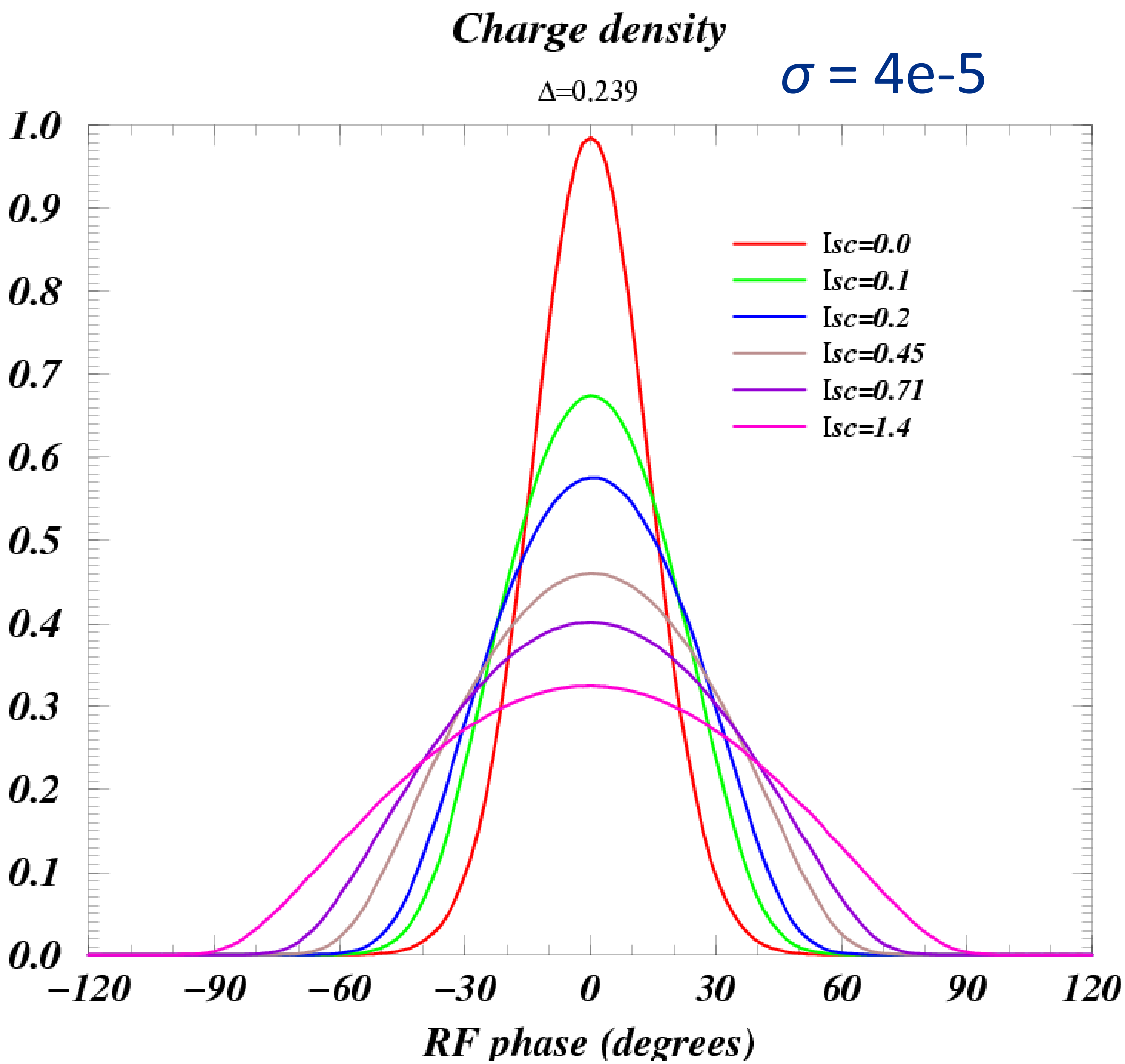
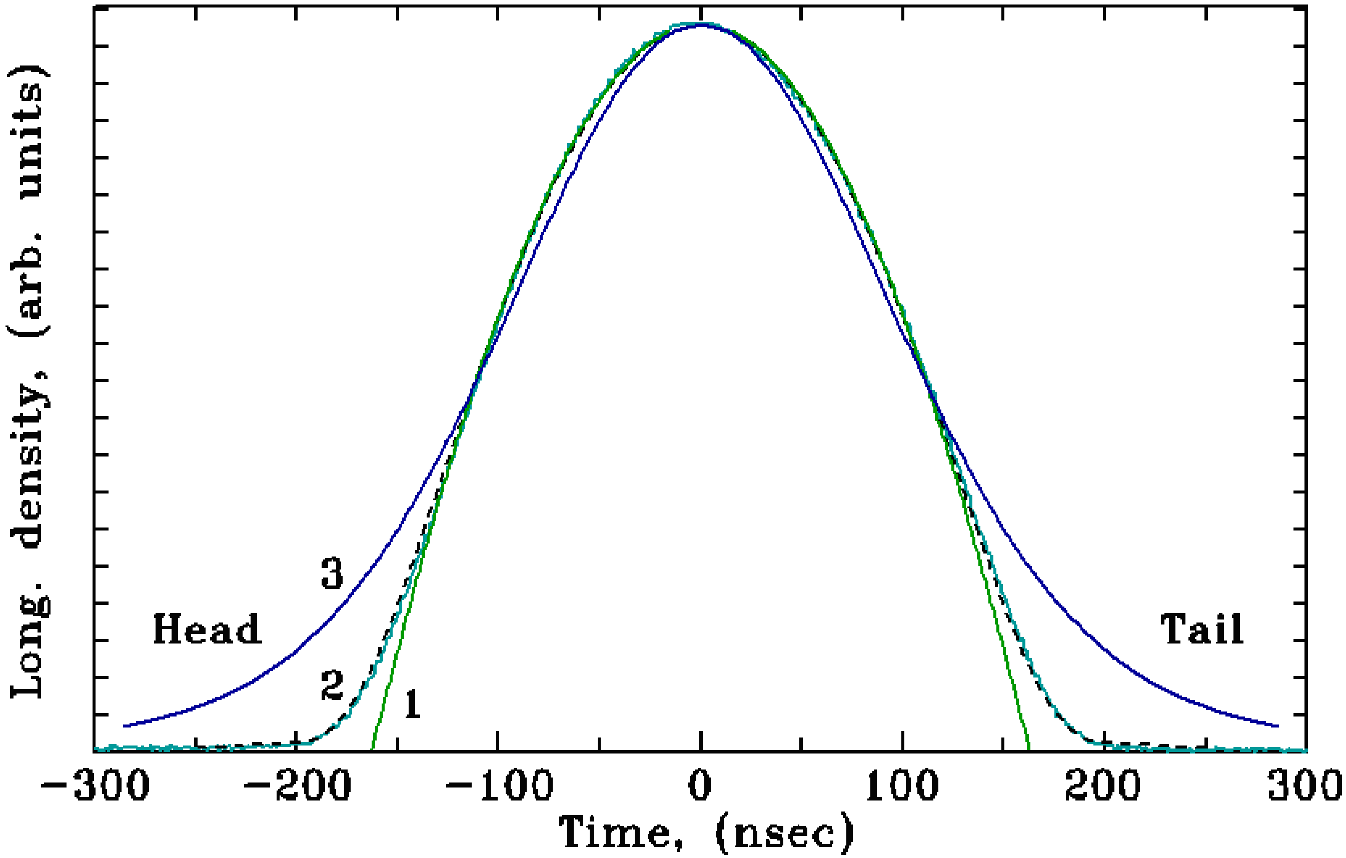
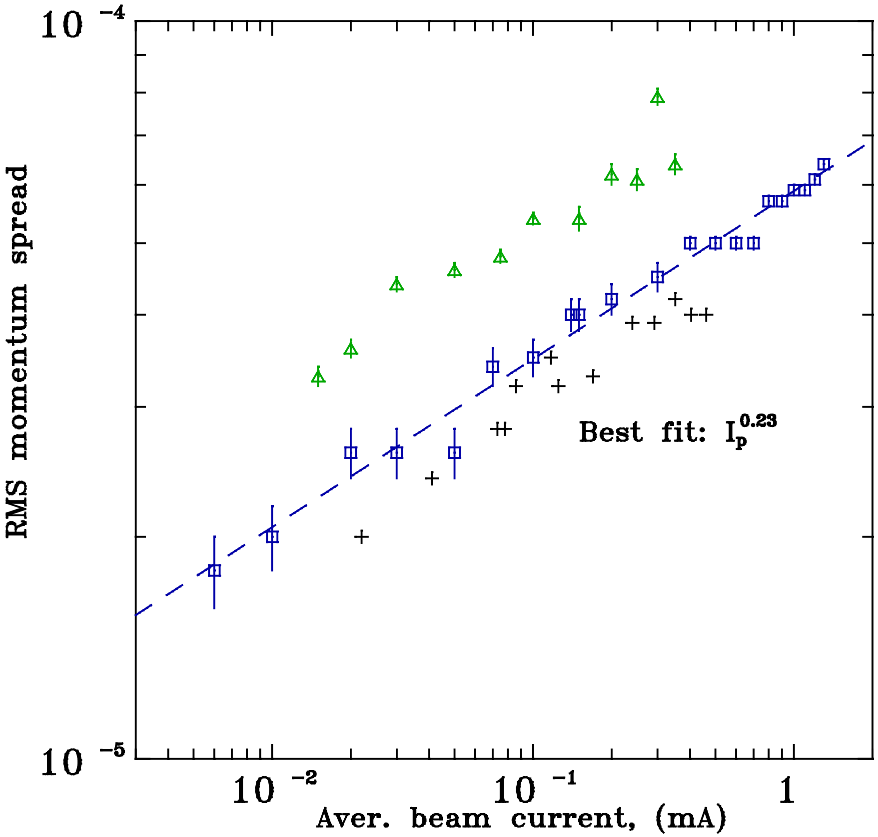
Stationary longitudinal bunch charge density distributions for a fixed momentum spread (σ) and various bunch charges





Measured (solid, 2) and theoretical (dashed, 2) linear density. Cosine (1) and Gaussian (3) fits are also presented. $I_o \approx 400\ \mu A$, $V_{rf} \approx 12$ V. IUCF Cooler, 1994

The fit with two parameters (σ and g) is unambiguous



Proton beam rms momentum spread vs. beam current. V_{rf}= 12 V (□), V_{rf}= 126.4 V (Δ), and V_{rf}= 13.8 V (+). Derived from the bunch shape fitting.

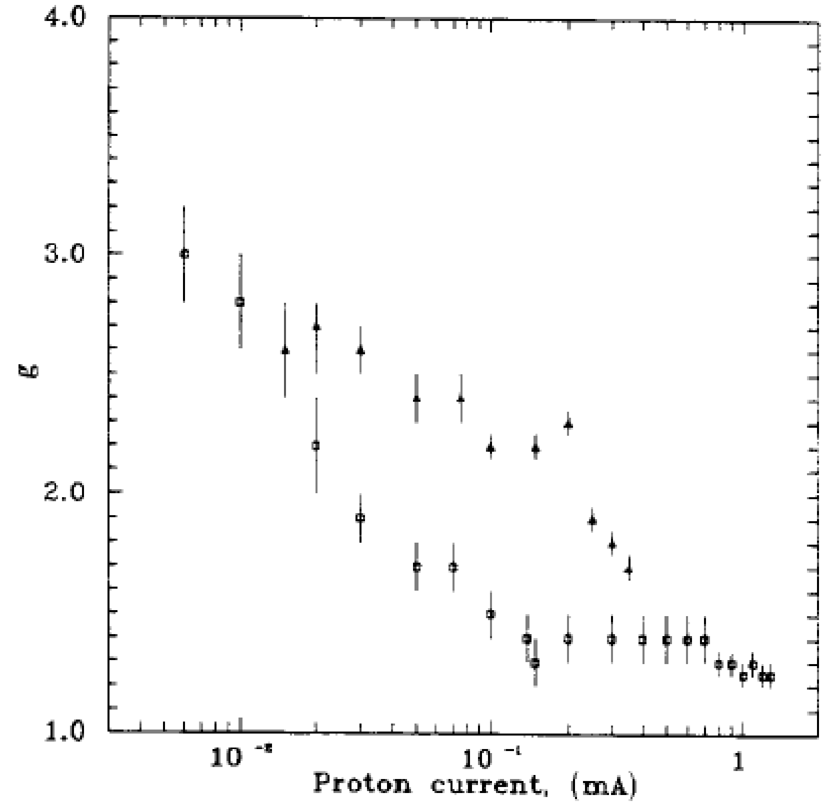


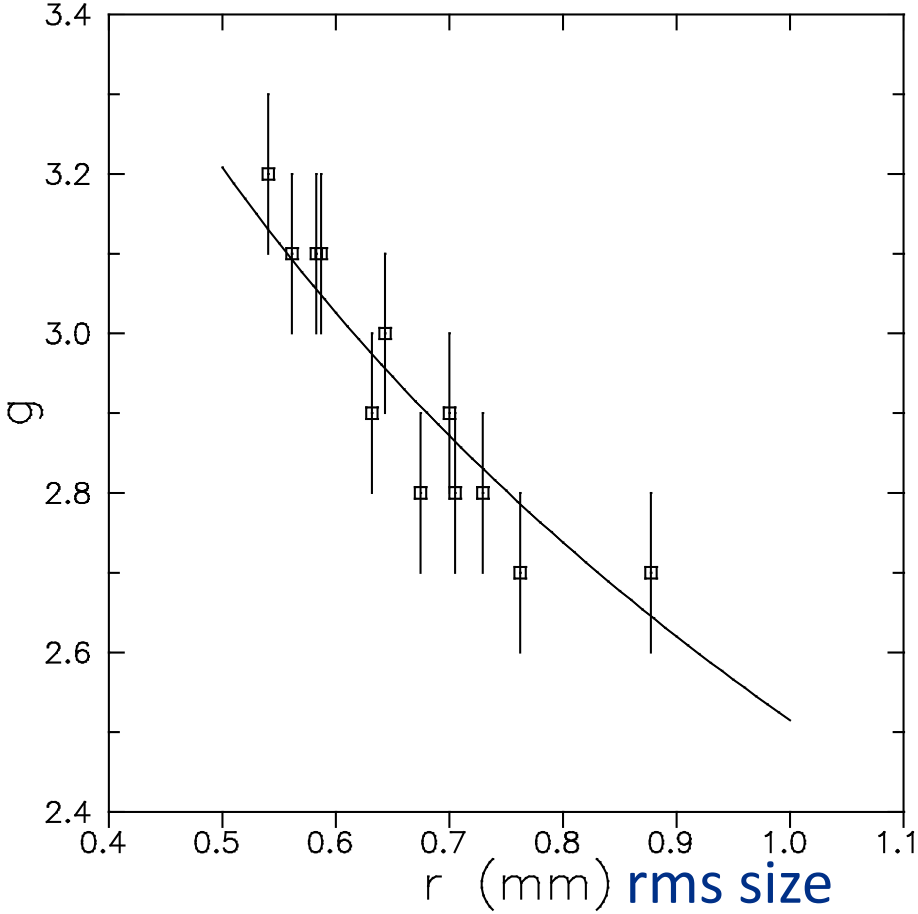
Figure 4. Geometric factor g vs. proton beam current. V_{rf} = 10 V (□), V_{rf} = 126.4 V (Δ).

For gaussian beam distributions:

(arXiv:1508.00153, R. Baartman)

$$g = \frac{1}{4} + \ln\left(\frac{b}{2 \times \text{rms size}}\right)$$

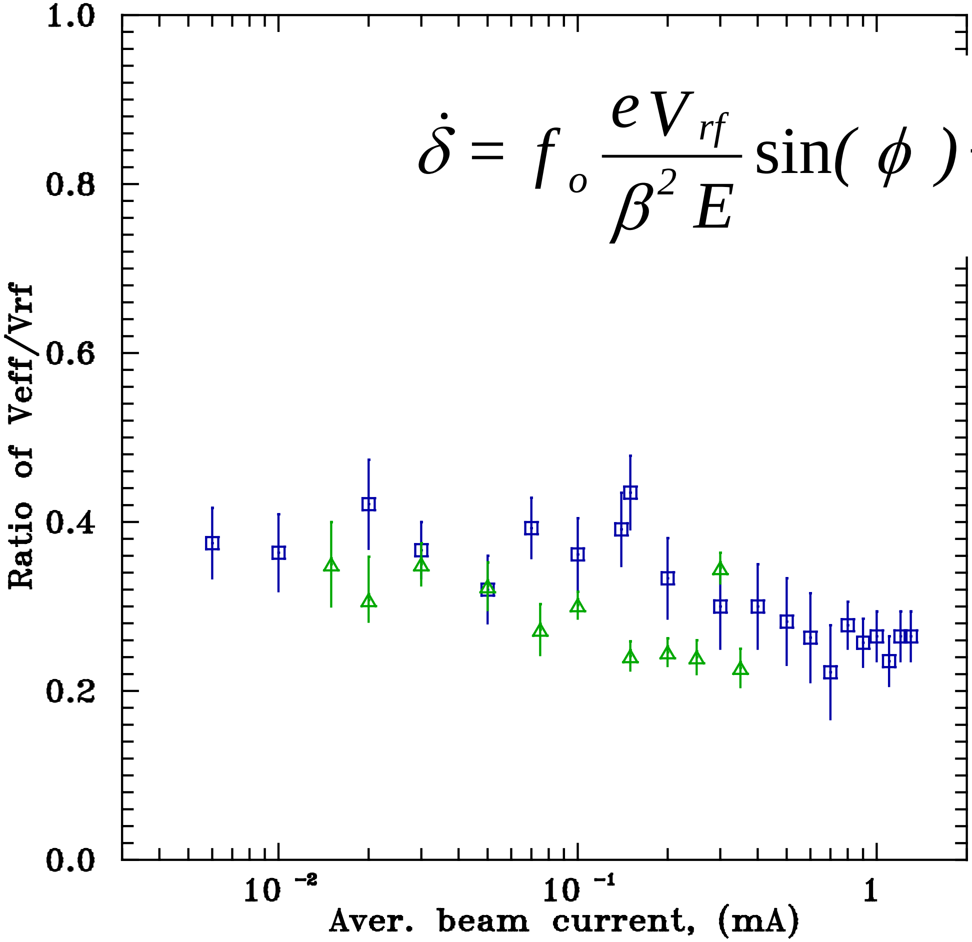
Cross-checking *g* vs transverse beam radius



$$g = \frac{1}{4} + \ln\left(\frac{b}{2\times \text{rms size}}\right)$$

- Best fit corresponds to $b \approx 20$ mm
- The IUCF cooler had the average beam pipe radius of 25 mm

$$\dot{\delta} = f_o \frac{e V_{rf}}{\beta^2 E} \sin(\phi) + \frac{Z_o\, g\, e^2 N f_o}{\gamma^2 \beta^2 E} \frac{c\, h^2}{R} \frac{\partial \rho(\phi, t)}{\partial \phi},$$



Ratio of the effective rf voltage to the applied rf voltage derived from the bunch shape fitting. V_{rf}= 12 V (□), V_{rf}= 126.4 V (Δ).

Longitudinal dynamics of space-charge dominated bunched beams

- Linear rf voltage model (from Neuffer's 1979 paper)
- Long envelope equation: $\ddot{\phi}_o = \left(2\pi h|\eta| f_o\right)^2 \left(\frac{\varepsilon_L^3}{\phi_o^3} + \frac{N}{N_o}\frac{1}{\phi_o^2} - A\phi_o \right),$
- The dipole synchrotron frequency is not affected by SC
- After linearization, one obtains for the small-amplitude quadrupole (bunch length) oscillation frequency:

$$\left(\frac{\omega_q}{\omega_{so}}\right)^2 = \frac{V_{eff}}{V_{rf}} + 3 .$$

- With no SC: $\omega_q = 2\omega_{so}$ -- it's a simple bunch rotation
- With zero momentum spread: $\omega_q = \sqrt{3}\omega_{so}$
- For the sin() rf voltage the situation is more complicated

Long. bunch modes with sin() rf voltage

- We found approx. analytic solutions for bunch modes with zero momentum spread. The modes are solutions to this equation

$$\frac{d}{d\phi}\left(\frac{d\rho_n}{d\phi}\left(\cos(\phi)-\cos(\phi_o)\right)\right)+\left(\frac{\omega_n}{\omega_{so}}\right)^2\rho_n=0\,.$$

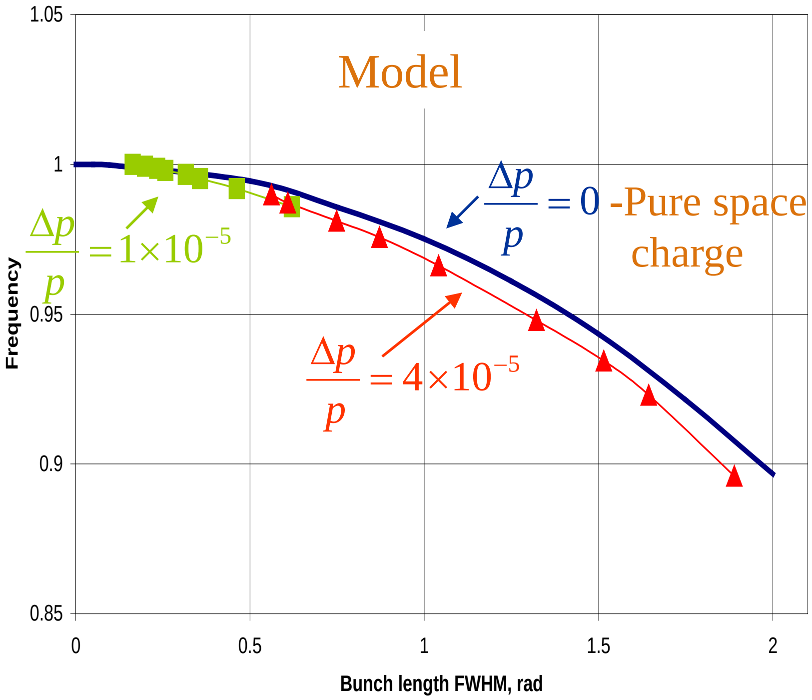
$$\left(\frac{\omega_1}{\omega_{so}}\right)^2\approx\frac{\int_{-\phi_o}^{+\phi_o}(\cos(\phi)-\cos(\phi_o))\cos(\phi)\,d\phi}{\int_{-\phi_o}^{+\phi_o}(\cos(\phi)-\cos(\phi_o))\,d\phi}.$$

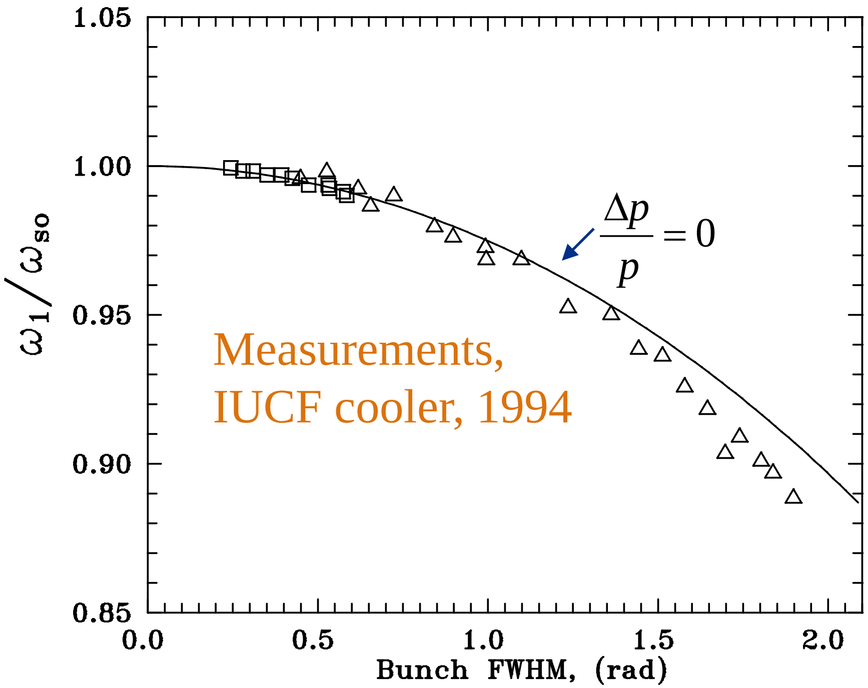
-- the dipole mode

The quadrupole mode was found numerically

- For non-zero momentum spread we solved the time-dependent F-P equation numerically
- Both modes were measured in the IUCF cooler

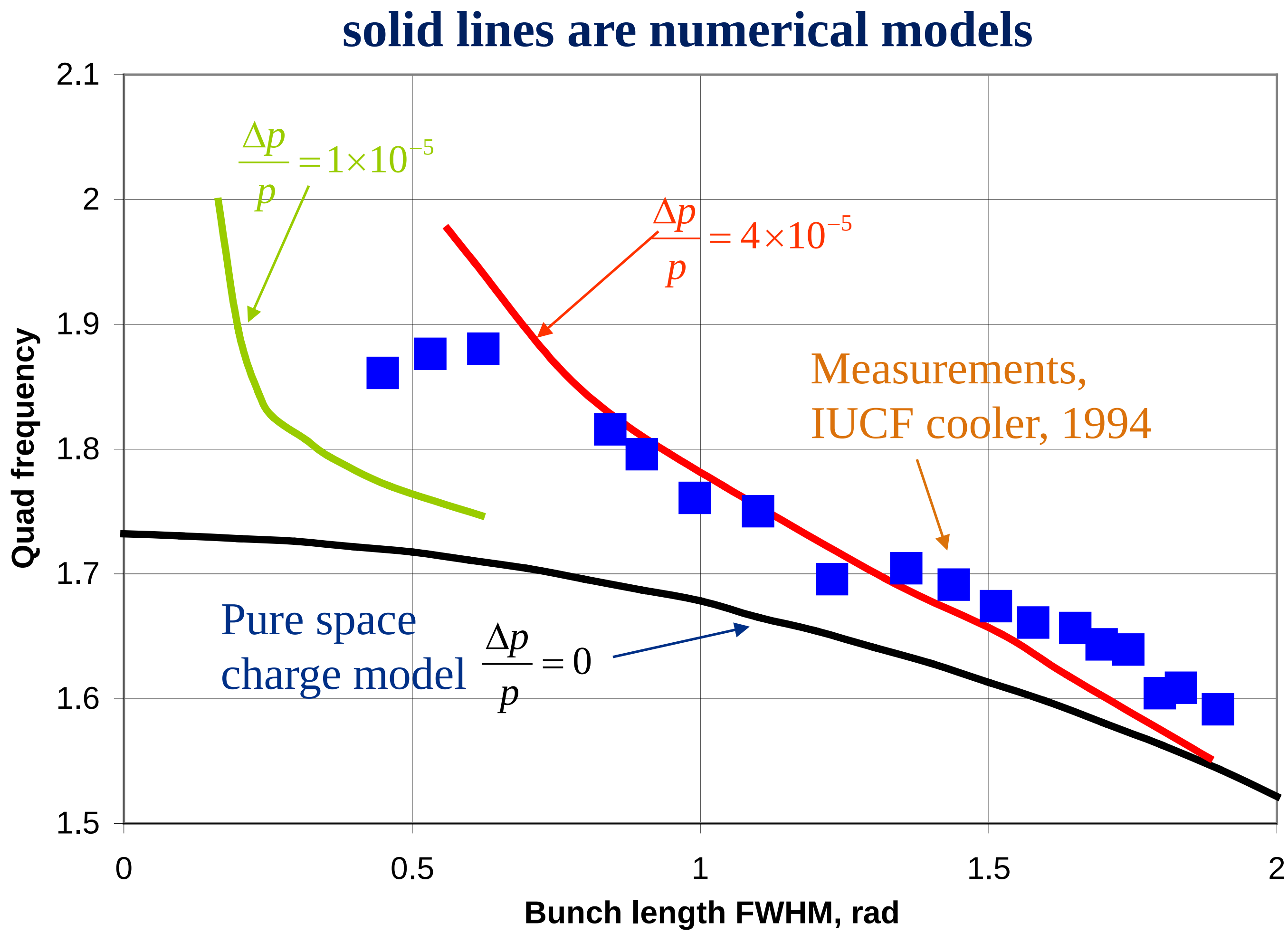
The dipole mode





For a space-charge dominated regime the bunch length is primarily a function of a beam current, thus, for a sinusoidal rf voltage, the synchrotron frequency depends on the beam current! (solid lines are numerical models)

The quadrupole mode



Summary

- The beam momentum spread can not be determined from the bunch length alone! Need to perform a fit with two parameters.
- Electron-cooled bunched beams are typically space-charge dominated longitudinally: the effective rf voltage amplitude in the bunch is small, ~20% of external rf voltage.
  - Need to take this into account in modeling of IBS, etc
- The coherent synchrotron (dipole) frequency depends on the beam current! Also, no decoherence in large amplitude synchrotron oscillations is observed.