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Heavy-Quark Symmetry Implies Stable Heavy Tetraquark Mesons $Q_iQ_j\bar{q}_k\bar{q}_l$

Estia J. Eichten* and Chris Quigg†
Fermi National Accelerator Laboratory
P.O. Box 500, Batavia, Illinois 60510 USA

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For very heavy quarks Q, relations derived from heavy-quark symmetry predict the existence of novel narrow doubly heavy tetraquark states of the form $Q_iQ_j\bar{q}_k\bar{q}_l$ (subscripts label flavors), where q designates a light quark. By evaluating finite-mass corrections, we predict that double-beauty states composed of $bb\bar{u}\bar{d}$, $bb\bar{u}\bar{s}$, and $bb\bar{d}\bar{s}$ will be stable against strong decays, whereas the double-charm states $cc\bar{q}_k\bar{q}_l$, mixed beauty+charm states $bc\bar{q}_k\bar{q}_l$, and heavier $bb\bar{q}_k\bar{q}_l$ states will dissociate into pairs of heavy-light mesons. Observation of a new double-beauty state through its weak decays would establish the existence of tetraquarks and illuminate the role of heavy color-antitriplet diquarks as hadron constituents.

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Following the discovery of the charmonium-associated state $X(3872)$ by the BELLE collaboration [1], experiments have led a renaissance in hadron spectroscopy [2].

Many of the newly observed states invite identification with compositions less spare than the traditional quark–antiquark meson and three-quark baryon schemes [3]. Tetraquark states composed of a heavy quark and antiquark plus a light quark and antiquark have attracted much attention. The observed candidates all fit the form $c\bar{c}q_k\bar{q}_l$, where the light quarks q may be $u, d,$ or s. No such states are observed significantly below threshold for strong decays into two heavy-light meson states $\bar{c}q_k + c\bar{q}_l$; all have strong decays to $c\bar{c}$ charmonium + light mesons.

In this Letter we examine the possibility of tetraquark configurations for which all strong decays are kinematically forbidden. We show that, in the heavy-quark limit, stable—hence exceedingly narrow—$Q_iQ_j\bar{q}_k\bar{q}_l$ mesons must exist. To apply this insight, we take into account corrections for finite heavy-quark masses to deduce which tetraquark states containing b or c quarks should be stable. The most promising example is a $J^P = 1^+$ isoscalar double-b meson, $\mathcal{T}^{\{bb\}-}_{[\bar{u}\bar{d}]}$.

In the heavy-quark limit, the lowest-lying tetraquark configurations resemble the helium atom, a factorized system with separate dynamics for the compact heavy color-$\bar{\mathbf{3}}$ Q_iQ_j "nucleus" and for the light quarks bound to the stationary color charge. (We recall that the one-gluon-exchange interaction is attractive for two quarks forming a color antitriplet, with half the strength of the attraction between a quark and antiquark bound in a color singlet.) At large $Q_i - Q_j$ separations, which become increasingly important as the heavy-quark masses decrease, the light $\bar{q}_k\bar{q}_l$ cloud screens the Q_iQ_j interaction, so that the $Q_iQ_j\bar{q}_k\bar{q}_l$ complex may rearrange into a pair of heavy-light mesons [4]. For heavy quarks Q_iQ_j bound in a color $\bar{\mathbf{3}}$ by an effective potential of the "Cornell" Coulomb + linear form at half strength for both components [5], the rms core radii are $\langle r^2\rangle^{1/2} = 0.28\ \text{fm}\,(cc); 0.24\ \text{fm}\,(bc); 0.19\ \text{fm}\,(bb)$, all considerably smaller than the size of the associated tetraquark states. Hence the core-plus-light (anti)quarks idealization should be a reliable guide to the masses of ground-state tetraquarks containing charms and bottoms.

The ground state of the attractive $\bar{\mathbf{3}}$ Q_iQ_j configuration may have total spin $S_{Q_iQ_j} = 1$ for identical quarks $(i = j)$ or for quarks of different flavors $(i \neq j)$ in a symmetric flavor configuration $\{Q_iQ_j\}$ or total spin $S_{Q_iQ_j} = 0$ for quarks of different flavors $(i \neq j)$ in an antisymmetric flavor configuration $[Q_iQ_j]$. To construct a color-singlet $Q_iQ_j\bar{q}_k\bar{q}_l$ state, the light $\bar{q}_k\bar{q}_l$ must be in a color-$\mathbf{3}$. For the tetraquark ground state, both the heavy Q_iQ_j and light $\bar{q}_k\bar{q}_l$ pairs must be in $(\ell = 0)$ s-waves. To satisfy the Pauli principle, the flavor-symmetric $\{\bar{q}_k\bar{q}_l\}$ state must have total (light-quark) spin $j_\ell = 1$, whereas the flavor-antisymmetric $[\bar{q}_k\bar{q}_l]$ must have $j_\ell = 0$.

Stability in the heavy-quark limit. For very heavy quarks, a hadron mass receives negligible contributions from the motion of the heavy quarks and spin interactions. Accordingly, the following relations hold among the masses of heavy-light and doubly-heavy-light mesons and baryons [6]:

$$\begin{aligned}
m(\{Q_iQ_j\}\{\bar{q}_k\bar{q}_l\}) - m(\{Q_iQ_j\}q_y) &= m(Q_x\{q_kq_l\}) - m(Q_x\bar{q}_y)\\
m(\{Q_iQ_j\}[\bar{q}_k\bar{q}_l]) - m(\{Q_iQ_j\}q_y) &= m(Q_x[q_kq_l]) - m(Q_x\bar{q}_y)\\
m([Q_iQ_j]\{\bar{q}_k\bar{q}_l\}) - m([Q_iQ_j]q_y) &= m(Q_x\{q_kq_l\}) - m(Q_x\bar{q}_y)\\
m([Q_iQ_j][\bar{q}_k\bar{q}_l]) - m([Q_iQ_j]q_y) &= m(Q_x[q_kq_l]) - m(Q_x\bar{q}_y)\,.
\end{aligned} \tag{1}$$

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(In the limit, a heavy core is a heavy core.)

It is easy to see that the dissociation of $Q_iQ_j\bar{q}_k\bar{q}_l$ into two heavy-light mesons is kinematically forbidden, for sufficiently heavy quarks. The $\mathcal{Q}$ value for the decay is

$$\begin{aligned}\mathcal{Q} \equiv m(Q_iQ_j\bar{q}_k\bar{q}_l) - [m(Q_i\bar{q}_k) + m(Q_j\bar{q}_l)] = \\ \Delta(q_k, q_l) - \tfrac{1}{2}\left(\tfrac{2}{3}\alpha_{\mathrm{s}}\right)^2[1 + O(v^2)]\overline{M} + O(1/\overline{M}) \ ,\end{aligned} \tag{2}$$

where $\Delta(q_k, q_l)$, the contribution due to light dynamics, becomes independent of the heavy-quark masses, $\overline{M} \equiv (1/m_{Q_i} + 1/m_{Q_j})^{-1}$ is the reduced mass of Q_i and Q_j, and α_{s} is the strong coupling. The velocity-dependent hyperfine corrections, here negligible, are calculable in the nonrelativistic QCD formalism [7]. For large enough values of $\overline{M}$, the middle term dominates, so the tetraquark is stable against decay into two heavy-light mesons.

The other possible decay channel is to a doubly heavy baryon and a light antibaryon,

$$(Q_iQ_j\bar{q}_k\bar{q}_l) \to (Q_iQ_jq_m) + (\bar{q}_k\bar{q}_l\bar{q}_m) \ . \tag{3}$$

By Eq. 1, we have

$$m(Q_iQ_j\bar{q}_k\bar{q}_l) - m(Q_iQ_jq_m) = m(Q_xq_kq_l) - m(Q_x\bar{q}_m) \ . \tag{4}$$

In the heavy-quark regime, the flavored-baryon–flavored-meson mass difference on the right-hand side of Eq. 4 has the generic form $\Delta_0 + \Delta_1/M_{Q_x}$. Using the observed mass differences, $m(\Lambda_c) - m(D) = 416.87$ MeV and $m(\Lambda_b) - m(B) = 340.26$ MeV, and choosing effective quark masses $m_c \equiv m(J/\psi)/2 = 1.55$ GeV, $m_b \equiv m(\Upsilon)/2 = 4.73$ GeV, we find $\Delta_1 = 176.6\ \mathrm{MeV}^2$ and $\Delta_0 = 303$ MeV, hence the mass difference in the heavy-quark limit is 303 MeV. All of these mass differences are smaller than the mass of the lightest antibaryon, $m(\bar{p}) = 938.27$ MeV, so we conclude that no decay to a doubly heavy baryon and a light antibaryon is kinematically allowed. *This completes the demonstration that, in the heavy-quark limit, stable $Q_iQ_j\bar{q}_k\bar{q}_l$ mesons must exist.*

Beyond the heavy-quark limit. To ascertain whether stable tetraquark mesons might be observed, we must estimate masses of the candidate configurations. Numerous model calculations exist in the literature [8], but it is informative to make estimates in the spirit of heavy-quark symmetry.

The leading-order corrections for finite heavy-quark mass correspond to hyperfine spin-dependent terms and a kinetic energy shift that depends only on the light degrees of freedom,

$$\delta m = \mathcal{S}\frac{\vec{S}\cdot\vec{j}_\ell}{2\mathcal{M}} + \frac{\mathcal{K}}{2\mathcal{M}} \ , \tag{5}$$

where $\mathcal{M} = m_{Q_i}$ or $m_{Q_i} + m_{Q_j}$ denotes the mass of the heavy-quark core for hadrons containing one or two heavy quarks and the coefficients $\mathcal{S}$ and $\mathcal{K}$ are to be determined from experimental data summarized in Table I. The spin splittings lead directly to the coefficients $\mathcal{S}$ tabulated in the last column. The pattern of the spin coefficients is entirely consistent with the expectations of heavy-quark symmetry.

The kinetic energy shift due to light quarks will be different in $Q\bar{q}$ mesons and Qqq baryons. By comparing the centroid (or center-of-gravity, c.g.) masses for the charm and bottom systems we can extract the difference of the kinetic-energy coefficients $\mathcal{K}$ for states that contain one or two light quarks, viz. $\delta\mathcal{K} \equiv \mathcal{K}_{(ud)} - \mathcal{K}_d$. For example,

$$\begin{aligned}\{m[(cud)_{\mathbf{\bar{3}}}] - m(c\bar{d})\} - \{m[(bud)_{\mathbf{\bar{3}}}] - m(b\bar{d})\} \\ = \delta\mathcal{K}\left(\frac{1}{2m_c} - \frac{1}{2m_b}\right) = 5.11\ \mathrm{MeV} \ ,\end{aligned} \tag{6}$$

from which we extract $\delta\mathcal{K} = 0.0235\ \mathrm{GeV}^2$. The resulting mass shifts are

$$\begin{aligned} m[\{cc\}(\bar{u}\bar{d})] - m(\{cc\}d) &: \ \frac{\delta\mathcal{K}}{4m_c} = 2.80\ \mathrm{MeV} \\ m[(bc)(\bar{u}\bar{d})] - m(\{bc\}d) &: \ \frac{\delta\mathcal{K}}{2(m_c + m_b)} = 1.87\ \mathrm{MeV} \\ m[\{bb\}(\bar{u}\bar{d})] - m(\{bb\}d) &: \ \frac{\delta\mathcal{K}}{4m_b} = 1.24\ \mathrm{MeV} \end{aligned} \tag{7}$$

These values are small—only slightly larger than the isospin breaking effects that we neglect as too small to affect the question of stability [12].

Combining the heavy-quark-symmetry relations of Eq. 1 with the leading-order corrections we obtain the masses of ground-state $Q_iQ_j\bar{q}_k\bar{q}_l$ tetraquarks summarized in Table II [13]. As inputs for the doubly heavy baryons not yet experimentally measured, we use the model calculations of Karliner and Rosner [14].

Narrow Tetraquark States. As we explained in the discussion surrounding Eq. 4, strong decays of $Q_iQ_j\bar{q}_k\bar{q}_l$ tetraquarks to a doubly heavy baryon and a light antibaryon are kinematically forbidden for all the ground states. Strong decay to a pair of heavy-light mesons will occur if the tetraquark state lies above threshold. For $J^P = 0^+$ or 2^+, a $Q_iQ_j\bar{q}_k\bar{q}_l$ meson might decay to a pair of heavy-light pseudoscalar mesons while for $J^P = 1^+$ the allowed decay channel would be a pseudoscalar plus a vector meson. According to our mass estimates, the only tetraquark mesons below threshold are the axial vector $\{bb\}[\bar{u}\bar{d}]$ meson, $\mathcal{T}^{\{bb\}-}_{[\bar{u}\bar{d}]}$, that is bound by 121 MeV and the axial vector $\{bb\}[\bar{u}\bar{s}]$ and $\{bb\}[\bar{d}\bar{s}]$ mesons bound by 48 MeV. We expect all the other $Q_iQ_j\bar{q}_k\bar{q}_l$ tetraquarks to lie at least 78 MeV above the corresponding thresholds for strong decay [16]. Promising final states include $\mathcal{T}^{\{bb\}-}_{[\bar{u}\bar{d}]} \to \Xi^0_{bc}\bar{p}$, $B^-D^+\pi^-$, and $B^-D^+\ell^-\bar{\nu}$ (which establishes a weak decay), $\mathcal{T}^{\{bb\}-}_{[\bar{u}\bar{s}]} \to \Xi^0_{bc}\bar{\Sigma}^-$, $\mathcal{T}^{\{bb\}0}_{[\bar{d}\bar{s}]} \to \Xi^0_{bc}(\bar{\Lambda}, \bar{\Sigma}^0)$, and so on.

TABLE I. Representative masses [9], in MeV, and derived quantities for ground-state hadrons containing heavy quarks.

State[a]	j_ℓ	Mass $(j_\ell + \frac{1}{2})$	Mass $(j_\ell - \frac{1}{2})$	Centroid	Spin Splitting	$\mathcal{S}$ [GeV^2]
$D^{(*)}$ $(c\bar{d})$	$\frac{1}{2}$	2010.26	1869.59	1975.09	140.7	0.436
$D_s^{(*)}$ $(c\bar{s})$	$\frac{1}{2}$	2112.1	1968.28	2076.15	143.8	0.446
Λ_c $(cud)_{\bar{\mathbf{3}}}$	0	2286.46	$\cdots$	$\cdots$		$\cdots$
Σ_c $(cud)_{\mathbf{6}}$	1	2518.41	2453.97	2496.93	64.44	0.132
Ξ_c $(cus)_{\bar{\mathbf{3}}}$	0	2467.87	$\cdots$	$\cdots$		$\cdots$
Ξ_c' $(cus)_{\mathbf{6}}$	1	2645.53	2577.4	2622.82	68.13	0.141
Ω_c $(css)_{\mathbf{6}}$	1	2765.9	2695.2	2742.33	70.7	0.146
Ξ_{cc} $(ccu)_{\bar{\mathbf{3}}}$	0	3621.40[b]	$\cdots$		$\cdots$	
$B^{(*)}$ $(b\bar{d})$	$\frac{1}{2}$	5324.65	5279.32	5313.32	45.33	0.427
$B_s^{(*)}$ $(b\bar{s})$	$\frac{1}{2}$	5415.4	5366.89	5403.3	48.5	0.459
Λ_b $(bud)_{\bar{\mathbf{3}}}$	0	5619.58	$\cdots$		$\cdots$	
Σ_b $(bud)_{\mathbf{6}}$	1	5832.1	5811.3	5825.2	20.8	0.131
Ξ_b $(bds)_{\bar{\mathbf{3}}}$	0	5794.5	$\cdots$		$\cdots$	
Ξ_b' $(bds)_{\mathbf{6}}$	1	5955.33	5935.02	5948.56	20.31	0.128
Ω_b $(bss)_{\mathbf{6}}$	1		6046.1			
B_c $(b\bar{c})$	$\frac{1}{2}$	6329[c]	6274.9	6315.4[c]	54[c]	0.340[c]

[a] Subscripts denote flavor-SU(3) representations for heavy baryons.
[b] From the LHCb observation, Ref. [10].
[c] Inferred from the lattice QCD calculation of Ref. [11].

TABLE II. Expectations for ground-state tetraquark masses, in MeV.[a] The column labeled "HQS Relation" is the result of our heavy-quark symmetry relations and is explicitly given by the sum of the right-hand-side of Eq. 1 and the kinetic-energy mass shifts of Eq. 7. Here q denotes an up or down quark. For stable tetraquark states the $\mathcal{Q}$ value is highlighted in a box.

State	J^P	j_ℓ	$m(Q_iQ_jq_m)$ (c.g.)	HQS relation	$m(Q_iQ_j\bar{q}_k\bar{q}_l)$	Decay Channel	$\mathcal{Q}$ [MeV]
$\{cc\}[\bar{u}\bar{d}]$	1^+	0	3663[b]	$m(\{cc\}u) + 315$	3978	D^+D^{*0} 3876	102
$\{cc\}[\bar{q}_k\bar{s}]$	1^+	0	3764[c]	$m(\{cc\}s) + 392$	4156	$D^+D_s^{*-}$ 3977	179
$\{cc\}\{\bar{q}_k\bar{q}_l\}$	$0^+, 1^+, 2^+$	1	3663	$m(\{cc\}u) + 526$	4146, 4167, 4210	D^+D^0, D^+D^{*0} 3734, 3876	412, 292, 476
$[bc][\bar{u}\bar{d}]$	0^+	0	6914	$m([bc]u) + 315$	7229	B^-D^+/B^0D^0 7146	83
$[bc][\bar{q}_k\bar{s}]$	0^+	0	7010[d]	$m([bc]s) + 392$	7406	B_sD 7236	170
$[bc]\{\bar{q}_k\bar{q}_l\}$	1^+	1	6914	$m([bc]u) + 526$	7439	B^*D/BD^* 7190/7290	249
$\{bc\}[\bar{u}\bar{d}]$	1^+	0	6957	$m(\{bc\}u) + 315$	7272	B^*D/BD^* 7190/7290	82
$\{bc\}[\bar{q}_k\bar{s}]$	1^+	0	7053[d]	$m(\{bc\}s) + 392$	7445	DB_s^* 7282	163
$\{bc\}\{\bar{q}_k\bar{q}_l\}$	$0^+, 1^+, 2^+$	1	6957	$m(\{bc\}u) + 526$	7461, 7472, 7493	BD/B^*D 7146/7190	317, 282, 349
$\{bb\}[\bar{u}\bar{d}]$	1^+	0	10 176	$m(\{bb\}u) + 306$	10 482	$B^-\bar{B}^{*0}$ 10 603	$\boxed{-121}$
$\{bb\}[\bar{q}_k\bar{s}]$	1^+	0	10 252[c]	$m(\{bb\}s) + 391$	10 643	$\bar{B}\bar{B}_s^*/\bar{B}_s\bar{B}^*$ 10 695/10 691	$\boxed{-48}$
$\{bb\}\{\bar{q}_k\bar{q}_l\}$	$0^+, 1^+, 2^+$	1	10 176	$m(\{bb\}u) + 512$	10 674, 10 681, 10 695	B^-B^0, B^-B^{*0} 10 559, 10 603	115, 78, 136

[a] Masses of the unobserved doubly heavy baryons are taken from Ref. [14]; for lattice evaluations of b-baryon masses, see Ref. [15]
[b] Based on the mass of the LHCb Ξ_{cc}^{++} candidate, 3621.40 MeV, Ref. [10].
[c] Using the s/d mass differences of the corresponding heavy-light mesons.
[d] Evaluated as $\frac{1}{2}[m(c\bar{s}) - m(c\bar{d}) + m(b\bar{s}) - m(b\bar{d})] + m(bcd)$.

As others have noted [8, 17], unstable doubly heavy tetraquarks might be reconstructed as resonances in the "wrong-sign" combinations of DD, DB, and BB. The doubly charged $\mathcal{T}_{[\bar{d}\bar{s}]}^{\{cc\}++} \to D^+D_s^+$, etc. would stand out as *prima facie* evidence for a non-$q\bar{q}$ level.

While the production of $Q_iQ_j\bar{q}_k\bar{q}_l$ mesons is undoubtedly a rare event, we draw some encouragement for near-term searches from the large yield of B_c mesons recorded in the LHCb experiment [18] and the not inconsiderable rate of Double-Υ production observed in 8-TeV pp collisions by the CMS experiment, $\sigma(pp \to \Upsilon\Upsilon +$ anything$) = 68 \pm 15$ pb [19]. The ultimate search instrument might be a future electron–positron Tera-Z factory, for which the branching fractions [9] $Z \to b\bar{b} = 15.12 \pm 0.05\%$ and

$Z \to b\bar{b}b\bar{b} = (3.6 \pm 1.3) \times 10^{-4}$ offer hope of many events containing multiple heavy quarks.

Concluding remarks. We have shown that, in the heavy-quark limit, stable $Q_iQ_j\bar{q}_k\bar{q}_l$ tetraquarks must exist. Our estimates of tetraquark masses lead us to expect that strong decays of the $J^P = 1^+$ $\{bb\}[\bar{u}\bar{d}]$, $\{bb\}[\bar{u}\bar{s}]$, and $\{bb\}[\bar{d}\bar{s}]$ states are kinematically forbidden, so that these states should be exceedingly narrow, decaying only through the charged-current weak interaction. Observation of any of these states would signal the existence of a new form of stable matter, in which the doubly heavy color-$\mathbf{\bar{3}}$ Q_iQ_j diquark is a basic building block. The unstable $Q_iQ_j\bar{q}_k\bar{q}_l$ tetraquarks—particularly those with small $\mathcal{Q}$ values—may be observable as resonances decaying into pairs of heavy-light mesons, if they are not too broad to stand out above backgrounds.

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Note added.—We recently learned of interesting calculations of tetraquark masses that also highlight the likelihood of a stable doubly heavy tetraquark [20].

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[*] Email: eichten@fnal.gov; ORCID: 0000-0003-0532-2300
[†] Email: quigg@fnal.gov; ORCID: 0000-0002-2728-2445

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