A PHASE MATCHING, ADIABATIC ACCELERATOR

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Abstract

Tabletop accelerators are highly sought after. One way of reducing accelerator footprints is to down-scale electromagnetic wavelengths; however, without correspondingly high field gradients, particles will be more susceptible to phase-slippage – especially at low energy. We investigate how an adiabatically-tapered dielectric-lined waveguide could maintain phase-matching between the accelerating mode and electron bunch at moderate field strength. We describe our analytical model, compare it with CST and implement it into ASTRA and provide a glimpse into the beam dynamics.

INTRODUCTION

Particle accelerators have emerged as essential tools to explore fundamental science. Particle colliders for example, have extended our knowledge of the fundamental forces and essentially contributed to the development of the standard model. More recently, the advent of light sources based on synchrotron radiation from high-energy (GeV) electron bunches has extended our vision to Angstrom-scale interactions, allowing us to peer onto biological and chemical interactions at femtosecond timescales. Electron-diffraction imaging offers a significantly lower-energy route (~MeV) to observing Angstrom-scale phenomena with unprecedented resolution.

Exploring smaller and faster processes generally requires more energetic particle beams which has driven the development of large-scale accelerators such as the LHC, LCLS, and European XFEL. Modern accelerators are based on radio frequency (RF) technology with wavelengths on O(10 cm) and corresponding accelerating gradients O(100 MV/m). Scaling to smaller wavelengths has its own complications. A shorter wavelength requires a relatively short bunch (σt ≪ λ) to maintain reasonable energy spreads under acceleration. Moreover, from the normalized potential d0 ≪ E0λ, obtaining the same acceleration dynamics in a scaled accelerator requires E0 ∝ λ−1 [1]. Realizing GV/m+ acceleration gradients is difficult and also presents challenges such as tight tolerances and breakdowns [2] in metallic structures.

These limitations have motivated research into alternative acceleration techniques which fall into two general categories operating in sub-millimeter-scale regimes: beam-driven and laser-driven acceleration. Beam-driven acceleration uses a high-impedance medium (e.g. dielectric lined waveguide (DLW) or plasma) to transfer energy between two ultrarelativistic bunches [3]. In laser-driven acceleration, a high-power laser is used to accelerate an electron bunch either directly (i.e. direct laser acceleration (DLA) [4,5]) or via a medium e.g. plasmas [6,7]. A principle limitation to these techniques is phase-slippage where the accelerating electron bunch velocity is not matched to the phase velocity (vph) of the structure or plasma. In the ultrarelativistic limit this problem is mitigated; however, acceleration at low energies has been predominantly accomplished with an ultra-high power laser pulse in the multi-TW regime.

Here we describe a technique to abridge low-energy acceleration with small wavelengths and relatively small accelerating fields. We accomplish this with an adiabatically-tapered DLW to maintain phase matching between the longitudinally-dependent phase velocity of the structure and the accelerating electron bunch. We discuss the general theory of the structure, its limitations, we provide an example with λ = 1 mm and discuss the resulting beam dynamics.

THEORY

Dielectric-lined waveguides (DLWs) have attracted interest in the accelerator community for their versatility to accelerate, manipulate and characterize [8–13] electron beams. A DLW is generally composed of a hollow core surrounded by a dielectric lining with an outer metallic coating. Here we consider a cylindrical-DLW with inner radius a, outer radius b and relative dielectric permittivity εr. The accelerating (TM01) mode has the following general field solutions in cylindrical coordinates (r, φ, z) [14]

\[ E_r = \frac{E_0 k_r}{k_1} I_1(k_1r) \cos(\omega t - k_z z + \psi) \]
\[ H_\phi = -\frac{\omega E_0}{k_1} I_1(k_1r) \sin(\omega t - k_z z + \psi) \]

\[ E_z = E_0 i_0(r_1 k_1) \sin(\omega t - k_z z + \psi) \]
where $k_1 = \omega \sqrt{\frac{1}{v_p^2} - \frac{1}{c^2}}$, $k_2 = \omega \sqrt{\frac{\varepsilon_r}{c^2} - \frac{1}{v_p^2}}$, and $k_z = \frac{\omega}{v_p}$.

Here $I$ is the modified Bessel function of the first kind, $E_0$ is the longitudinal field amplitude, $k_1$ and $k_2$ are the radial components of the wavenumber $k$ in the vacuum core and dielectric structure respectively, $k_z$ is the longitudinal wavenumber, $v_p$ is the phase velocity, $\omega$ is the angular frequency and $\psi$ is a phase constant. Solutions to the characteristic equation depend on the DLW structure parameters ($a, b, \varepsilon_r$) and yield propagating modes for $(\omega, k_z)$.

We make the following ansatz for the fields in a tapered waveguide

$$E_z = E_0 I_0 (r k_1) \sin(\omega t - \int_0^z dz k_z + \psi)$$
$$E_r = \frac{E_0 k_z}{k_1} I_1 (r k_1) \cos(\omega t - \int_0^z dz k_z + \psi)$$
$$H_\phi = \frac{E_0 \omega k_0}{k_1} I_1 (r k_1) \cos(\omega t - \int_0^z dz k_z + \psi),$$

where $E_0, k_1$ and $k_z$ now depend on $z$.

Checking with Maxwell’s equations,

$$\frac{\partial}{\partial z} E_z = -\frac{1}{r} \frac{\partial}{\partial r} (r E_r),$$
$$\frac{\partial}{\partial t} E_z = -\frac{1}{r c^2} \frac{\partial}{\partial r} (r B_\phi),$$

the ansatz is satisfied and adiabatic for the following conditions

$$\left| \frac{r k_1 I_1 (k_1 r)}{k_z I_0 (k_1 r)} \right| = \frac{v_p}{\sqrt{1 - \frac{1}{v_p^2}}} \frac{I_1 (k_1 r)}{I_0 (k_1 r)} \ll 1,$$

$$\left| \frac{E_r'}{E_0 k_z} \right| = \left| \frac{E_0' v_p}{E_0 \omega} \right| \ll 1.$$

To include particle acceleration, we solve the coupled differential equations

$$\frac{\partial z}{\partial t} = \beta c,$$
$$\frac{\partial \beta}{\partial t} = \frac{e E_0}{\gamma^2 m c^2} \sin(\omega t - \int_0^z k_z dz + \phi).$$

We developed a C++ code to iteratively solve the dispersion relations and field equations; for a given inner radius $a$, we solve the system of equations to obtain $E_0$ and the appropriate taper thickness $\delta$ at a particular $v_p$; now however, $v_p$ corresponds to the normalized velocity of the accelerating particle $\beta$. We implemented our ansatz field equations into ASTRa [15]. In the following section we discuss simulation results for a 300 GHz, 100 MV/m structure.

\textbf{EXAMPLE}$\lambda=1 \text{ mm, } E_0=100 \text{ MV/m}$

We now consider an example based on a 300 GHz ($\lambda=1 \text{ mm}$) driving field in a structure with constant inner radius $a=0.5 \text{ mm}$ and varying $b$ made of Quartz ($\varepsilon_r=4.41$) – limiting $v_p > c/\sqrt{\varepsilon_r} \approx 0.48c$. We consider a bunch with initial energy of $E_i=205 \text{ keV}$, corresponding to $\beta = 0.7$.

![Figure 1: The tapered structure in our example for a fixed inner radius $a=0.5 \text{ mm}$ (blue) and a tapered Quartz ($\varepsilon_r=4.41$) dielectric thickness with outer radius (shown red). The taper is matched for an initial energy of $E_i=205 \text{ keV}$ with a maximum accelerating field of $E_0=100 \text{ MV/m}$ EXAMPLE](image)

The structure (shown in Fig. 1(top)) is matched to $E_0=100 \text{ MV/m}$ (note $E_0=100 \text{ MV/m}$ for $v_p = c$ (e.g. $k_1=0$, at $z=0$) but $v_p = 0.7c$, $E_0 \approx 20 \text{ MV/m}$). From Eq. 4 the condition for $r=100 \mu m$ at $z=0$ gives $0.0017$ and approaches $10^{-7}$ toward the end of the structure. For completeness we compared our analytical model with CST MWS [16] (see Fig. 1(bottom)) for the first 2 cm of the structure where the majority of the taper occurs; discrepancies arise at the entrance and exit of the structure which are excluded in our theoretical model.

We can gain some insights into the longitudinal dynamics with a single particle; illustrated in Fig. 2(top), the final energy is plotted as a function of the injection phase for various accelerating fields; larger gradients enable a larger acceptance while no efficient acceleration is possible for lower gradients. Fig. 2(bottom) shows the end phase as a function of starting phase, a plateau in the curve implies the end phase is within a certain interval independent of the initial phase, i.e. the bunch is compressed.

Finally we look at the acceleration dynamics of a 100 fC spherical electron bunch radius $\sigma_r=10 \mu m$ with zero emittance and energy spread. We used a 1.2 T magnetic field to counter the defocussing forces. We scan over the in-
We have described a new method of accelerating low-energy particles by maintaining phase-matching between an accelerating bunch and the phase of the longitudinal electric field with an adiabatically tapered dielectric lined waveguide. We developed an analytical model which was validated with Maxwell’s equations and CST MWS. The analytical model was implemented into Astra and we provide a beam dynamics example. Our future work will look at other e.g. slab structures; and optimizations. We also investigate ways to reduce the required magnetic field strengths.

**CONCLUSION**

We have described a new method of accelerating low-energy particles by maintaining phase-matching between an accelerating bunch and the phase of the longitudinal electric field with an adiabatically tapered dielectric lined waveguide. We developed an analytical model which was validated with Maxwell’s equations and CST MWS. The analytical model was implemented into Astra and we provide a beam dynamics example. Our future work will look at other e.g. slab structures; and optimizations. We also investigate ways to reduce the required magnetic field strengths.
REFERENCES


