Wave-Optics Modeling of the Optical-Transport Line for Passive Optical Stochastic Cooling

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Abstract

Optical stochastic cooling (OSC) is expected to enable the fast cooling of dense particle beams owing to the large bandwidth supported by optical systems in the visible and infrared regions. A subsystem critical to the OSC scheme is the focusing optics used to image radiation from the upstream “pickup” undulator to the downstream “kicker” undulator. In this paper, we present simulation results using wave-optics calculation carried out with the Synchrotron Radiation Workshop (SRW). Our simulations are performed in support to a proof-of-principle experiment planned at the Integrable Optics Test Accelerator (IOTA) at Fermilab. The calculations provide an estimate of the energy kick received by a 100-MeV electron as it propagates in the kicker undulator and interacts with the electromagnetic pulse it produced at an earlier time while traveling through the pickup undulator.

Keywords: beam-cooling technique, electron-laser interaction, undulator radiation, beam dynamics

1. Introduction

The optical stochastic cooling (OSC) scheme is a variant of the microwave-stochastic cooling which relies on an optical signal to carry information on the beam distribution and apply the corresponding cooling force \cite{1, 2}; see Figure 1. In OSC a particle radiates an electromagnetic wave while passing through an undulator magnet [henceforth referred to as the pickup undulator (PU)]. The radiation pulse passes through a series of lenses and an optical amplifier and is imaged at the location of a downstream undulator magnet dubbed as kicker undulator (KU). The particle propagates through a bypass chicane (B\textsubscript{1}, B\textsubscript{2}, B\textsubscript{3}, B\textsubscript{4}) which provides an energy-dependent path length (i.e., time of flight) in addition to the path length variation due to the betatron coordinate. The chicane also provides the space to house the optical components necessary to the optical-pulse manipulation and amplification. The imaged PU-radiation field and particle copropagate in the KU and a net energy exchange between the two occurs. When the particle time of arrival is properly selected a corrective energy kick is applied to the particle and so damping of the particles synchrotron oscillations occurs as the process is repeated over many turns in the circular accelerator. If the KU is located in a dispersive section the corrective kick can also yield to cooling in the dispersive plane. Furthermore if the horizontal and vertical degrees of freedom are coupled outside of the cooling insertion the OSC can provide 6D phase-space particle cooling.

Although the nominal OSC scheme discussed in most of the literature involves an optical amplifier, the experiment planned in the 100-MeV IOTA electron ring at Fermilab \cite{3} will not incorporate an optical amplifier in its first phase. This latter version of OSC is referred to as passive OSC (POSC) and it is the variant under consideration throughout this paper.

A comprehensive treatment of the OSC can be found in Ref. \cite{4} where the kick amplitude is computed semi-analytically by considering a single focusing lens placed between the two undulators separated by a distance much larger than their length. In doing so the depth of field associated with the finite length of the undulators is suppressed. Although theoretically convenient, this focusing scheme is not practical and a three-lens configuration is instead adopted with focal lengths \(f_i\) and distances \(L_i\) fulfilling \cite{4}

\[ f_1 = L_2 \quad \text{and} \quad f_2 = -\frac{L_4^2}{2(L_1 - L_2)}, \quad (1) \]
where the parameters are defined in Fig. 1. The resulting transfer matrix between the KU and PU defined in the position-divergence coordinate system $\mathbf{X} = (x, x')$ is $M_{KU \to PU} = -I$, where $I$ is the $2 \times 2$ identity matrix. The three-lens telescope configuration supports a longitudinal point-to-point imaging between the PU and KU while also flipping the focal spot across the horizontal kicker axis. Correspondingly the telescope addresses the depth-of-field issue and the results derived for a single lens are directly applicable. The parameters of the optical telescope and undulators (PU and KU are identical) are listed in Tab. 1. Note that both undulators are providing a vertical magnetic field $\mathbf{B} = By$ so that the oscillatory trajectory lies in the $(x, z)$ plane. Finally we defined the resonant wavelength as $\lambda_r = \frac{\lambda_0}{2 \gamma} \left[ 1 + \frac{K^2}{2} + (\gamma \theta)^2 \right]$ where the parameters are defined in Tab. 1 and $\theta$ is the observation angle w.r.t the electron direction. Specifically, we defined the on-axis resonant wavelength as $\lambda_0 \equiv \lambda_r(\theta = 0)$.

### Table 1: Parameters for the optical telescope and undulators for the proposed POSC experiment at IOTA.

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<th>parameter, symbol</th>
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<tr>
<td>electron Lorentz factor, $\gamma$</td>
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</table>

2. Single-lens focusing

A wave-optics model of single-lens focusing was implemented in the SYNCHROTRON RADIATION WORKSHOP (SRW) program [5] to benchmark our numerical implementation with the analytical model obtained for a single lens configuration [4].

Considering the case of POSC, taking $K_u \ll 1$, and assuming an infinite numerical aperture of the focusing lens, the on axis electric field imaged in the KU is given by

$$E_x(x = y = 0) = \frac{4}{3} eK^2 \kappa^3,$$

where $\kappa_u \equiv 2\pi/\lambda_u$ ($\lambda_u$ is the undulator period) and $\gamma$ the Lorentz factor. The transverse velocity of the particle is $v_x = Kc/\gamma \sin(k_u z)$ and the kick amplitude is approximately

$$\Delta E = e \int_0^{L_u} \frac{E_x \kappa_u}{\gamma} \sin^2(k_u z) dz = \frac{eE_xK_uL_u}{2\gamma},$$

where $L_u$ is the undulator length. Combining the latter equation with Eq. 2 yields

$$\Delta \varepsilon = \frac{2\pi}{3} (eK\gamma)^2 k_u N_u,$$

where $N_u$ is the number of undulator periods. Intuitively Eq. 4 is just equal to the total energy loss as the electron travels through one undulator. When $K_u$ is increased (thereby resulting in an increased angular deflection) and the finite angular acceptance of the lens, $\theta_m$, taken into account, the on-axis electric field $E_x(x = y = 0)$ in the KU is reduced by a factor $E_h(K_u, \gamma \theta_m) \leq 1$. The expression of $E_h(K_u, \gamma \theta_m)$ is derived in [4] and its dependence on $K_u$ appears in Fig. 2 for three cases of $\gamma \theta_m$. There is an additional efficiency factor, $F_u(\kappa_u) = J_0(\kappa_u) - J_1(\kappa_u)$, when computing the kick value arising from the longitudinal oscillation [given by $\frac{K^2}{8\pi^2} \sin(2k_u z)$] of the particle in the KU where $\kappa_u \equiv K^2/4(1 + K^2/2)$. The kick amplitude from Eq. 4 is thus reduced by the factor of $F_h(K_u, \theta_m \gamma) \times F_u(\kappa_u)$.

By propagating each frequency component of the PU field amplitude in the time domain inside the KU can be computed [6]. This is first done for the case of a single focusing lens using $L_u$ and $\lambda_u$ from Tab. 1, but varying $N_u$ and other parameters appropriately. For this benchmarking simulation, the distance between the PU and KU centers is taken to $L_d = 19.5$ m (i.e. $L_d \gg L_u$) in order to suppress the depth-of-field effect and the focal length of the lens is $f = L_d/2$. The simulated value for $E_x(K_u, \gamma \theta_m)$ are found to be in excellent agreement (relative difference below 5%) as shown in Fig. 2.

![Figure 2](image-url)
3. Imaging with a three-lens telescope

We now focus on the imaging scheme proposed for the POSC experiment at IOTA with parameters summarized in Tables 1. The point-to-point imaging of the KU radiation in the PU is accomplished with a three-lens telescope. First, the field amplitude at the KU longitudinal center is compared with the expected value from theory: using Eq. 2 and $F_0(1.038, 0.8) = 0.25$ yields $E_x = 11.8$ V/m while SRW gives 10.9 V/m corresponding to a relative discrepancy < 7%. The kick amplitude using Eq. 4 and $F_0(0.18) = 0.91$ yields $\Delta E = 22$ meV while directing computing the kick in the same way with the SRW result gives a value of 20.1 meV. Therefore the agreement between theoretical predictions and numerical simulations is reasonable as already observed in the previous Section.

It should be noted that with SRW the longitudinal and transverse dependence of the electric field neglected in theory can also be accounted. The latter of which is from the effective numerical aperture being less at the edges of the undulator than it is at the center. To find the kick value from SRW, the time-domain field was computed along the kicker every 3.2 mm. The average forward velocity of the particle is $\langle v_x \rangle \equiv \beta c = c \beta (1 - K_u^2/4\gamma^2)$ where $c$ denotes the velocity of light. Therefore as the particle advances through the kicker it falls back relative to the radiation packet by an amount

$$\delta z = z_l \frac{1 - \beta}{c} + \frac{K_u^2 \sin(2k_u z)}{2\gamma^2 k_u},$$

with $z_l$ the location of radiation packet in the KU referenced to its entrance. The latter equation, which also accounts the electron’s longitudinal oscillatory motion, is used to compute the electric field $E_x(x, z)$ experienced by the electron as it propagates through the KU. The change in energy is then obtained via the numerical integration of

$$\Delta E = \int_{z=0}^{z=L_u} v_x E_x(x, z) dz.$$  \hspace{1cm} (6)

It is also being tacitly assumed that the arrival time of the particle is such that $E_x(x, z)$ maximizes the kick. A plot of the electric field in the undulator mid plane $E_x(x, y = 0, z)$ appears in Fig. 3(a) with the trajectory of the electron overlapped. The corresponding evolution of the electron energy along the KU is displayed in Fig. 3(b).

The kick amplitude is found to be 18 meV. A reduction of 10.4 % comes from the longitudinal dependence of the field amplitude along the KU. The maximum transverse displacement of the particle in the KU is 93 µm allowing the particle to experience electric fields values reduced by ∼ 5 % w.r.t. to the maximum on-axis value. Such an effect reduces the kick by only 1.1 %. This is expected since the instantaneous energy transfer to the particle is proportional to $v_x$ which attains its maximum value on axis. As the electron’s transverse offset increases the velocity decreases to eventually vanishes when the electron reaches its maximum offset. Such a dependence of the velocity $v_x(x)$ mitigates the impact of the off-axis field reduction. Furthermore for the particle receiving the largest energy kick the phase of the wave (as seen in the co-moving frame of the particle) is such that the field is zero when the particle is the farthest off axis.

Our simulations also allow for the kick to be computed as a function of $\tau$ the delay relative to a reference particle as is shown in Figure 4. The envelope, $w(\tau)$, of the kick is approximately Gaussian with a value $\sigma_\tau = 13.5$ fs. A common approximation to the pulse length is $t_1 = n_w \lambda/c$ which for the undulator parameters for IOTA is 51.3 fs. Since the telescope focuses light from one location in the
PU to the corresponding location in the KU, the shape of the wave packet modulates while propagating through the KU. This modulation reduces the effective length of the wave packet at any particular location in the KU. Since the transverse dimensions of the wave packet (∼ 520 µm for the half-waist) are larger than the transverse beam size, the wave packet can be thought of as slicing the beam only along the longitudinal direction. Assuming that the bunch density is constant over the length of the wave packet the number of particles in a slice can be approximated as:

\[ N_s \approx \frac{N}{l_b} \int w(\tau)d\tau = \frac{N\sigma \sqrt{2\pi}}{c l_b} \]  

(7)

where \( l_b \) is the bunch length. The expected bunch length in IOTA during the OSC experiment, prior to cooling, is 14.2 cm giving \( N_s/N = 7.1 \times 10^{-5} \).

4. Conclusion

We used SRW to perform a simulation of the kick amplitude for OSC. We compared our results to the semi-analytic theory developed in [4] and found agreement better than 5% for a range of K values and angular acceptances of the focusing lens. Going further we computed the kick expected for the test of the OSC to be done in the IOTA ring at Fermilab. It was found the decreasing of the effective numerical aperture for points away from the kicker center reduced the kick by approximately 10% while the particles transverse dependence of the field amplitude had a much smaller effect on the kick amplitude.

5. Acknowledgments

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References