Abstract

Emittance exchange mediated by wedge absorbers is required for longitudinal ionization cooling and for final transverse emittance minimization for a muon collider. A wedge absorber within the MICE beam line could serve as a demonstration of the type of emittance exchange needed for 6-D cooling, including the configurations needed for muon colliders, as well as configurations for low-energy muon sources. Parameters for this test are explored in simulation and possible experimental configurations with simulated results are presented.

INTRODUCTION

Ionization cooling has been proposed as a potentially useful method for cooling particle beams.[1, 2] It could cool muons to a level suitable for a high-energy lepton collider or increase the intensity of muon beams for neutrinos and lower-energy experiments. It could also be used in some proton and ion beam scenarios. In ionization cooling the beam passes through a material, where energy loss is opposite the direction of momentum, followed by longitudinal acceleration restoring the average momentum. This reduces transverse emittances (ε∥,N), following the cooling equation:

$$\frac{d\varepsilon_N}{ds} = -\frac{\rho_i}{\beta_N^2} e_N + \frac{\beta_i E_i^2}{2\rho_i m_e c^2 L_N E}$$

where the first term is the frictional cooling effect and the second is the multiple scattering heating term. Here \(L_N\) is the material radiation length, \(\beta_i\) is the betatron focusing function, and \(E_i\) is the characteristic scattering energy (~14 MeV), and \(\rho_i\) is the transverse partition number.

Longitudinal cooling depends on having the energy loss mechanism such that higher-energy muons lose more energy. The natural dependence of dE/ds on E is weak and is antidamping for \(E < \sim 300\) MeV. The cooling rate is enhanced when absorbers are placed at non-zero dispersion and the absorber thickness depends upon position, such as in a wedge absorber. With wedge cooling, the longitudinal and transverse partition numbers are coupled, exchanging transverse and longitudinal cooling rates:

$$g_L \Rightarrow g_{L,0} + \frac{\eta_0}{\rho_0}; g_\perp \Rightarrow 1 + \frac{\eta_0}{\rho_0}$$

where \(\rho_0\) is the change in density with transverse position, \(\rho_0\) is the density associated with dE/ds, and \(\eta_0\) is the dispersion (\(\eta = dx/d(\Delta p/\rho_0)\)). This coupling is essential for effective longitudinal cooling and is needed in any multistage system requiring large cooling factors.

High-luminosity \(\mu^+\mu^-\) colliders also require a “final cooling” stage in which transverse emittance is reduced, while longitudinal emittance may increase. This may be done by explicit emittance exchange techniques, such as energy loss in a wedge absorber.[4,5]

The goal of the MICE experiment [6] is to explore the conditions for ionization cooling and to demonstrate the components of a \(\mu\) cooling system. Wedge absorber emittance exchange is an essential component in cooling systems and MICE would be greatly enhanced by a wedge demonstration. Large exchanges can be obtain within a single wedge and readily measured within MICE. In this note we explore use of single polyethylene wedges to demonstrate the basic principles of emittance exchange within ionization cooling.

WEDGE EXCHANGE EFFECTS

A transport matrix based formalism can be used to estimate the exchange effects of a single wedge.[6] Figure 1 shows a stylized view of the passage of a beam with dispersion \(\eta_0\) through a wedge absorber. The wedge is approximated as an object that changes particle momentum offset \(\delta = \Delta p/P_0\) as a function of \(x\), and the wedge is shaped such that that change is linear in \(x\). (The change in average momentum \(P_0\) is ignored in this approximation, as well as energy straggling and multiple scattering.) The rms beam properties entering the wedge are given by the transverse emittance \(\varepsilon_0\), betatron amplitude \(\beta_0\), dispersion \(\eta_0\) and relative momentum width \(\delta_0\). The wedge transforms the \(\delta\) of particles depending on position \(x\):

$$\frac{\Delta p}{p} = \delta \rightarrow \delta - \frac{2(\Delta p/\rho) \tan \frac{\delta L}{P_0}}{\delta x = \delta - \delta x}$$

\(d\rho/\rho\) is the momentum loss rate in the material. The dispersion + wedge can be represented by a matrix:

$$M_{\eta_0} = \begin{bmatrix} 1 & \eta_0 \\ -\delta' & 1 - \delta'\eta_0 \end{bmatrix}$$

Figure 1. Schematic view of a muon beam passing through a wedge (left). Wedge as will be constructed for MICE (right), designed to fit in the same absorber holder as the LiH absorber.

Writing the \(x,\delta\) beam distribution as a phase-space ellipse:

$$g_0x^2 + b_0\delta^2 = \sigma_0\delta_0$$

and transforming the ellipse by standard
techniques obtains new coefficients $b_i, g_i, \eta_i$. [6] The moment-width is changed to:

$$\delta_1 = \sqrt{\delta_0^2 + \sigma_0^2} = \delta_0 \sqrt{1 - (\eta_0 \delta_0)^2 + \frac{\sigma_0^2}{\delta_0^2}}^{1/2}.$$  

The bunch length is unchanged. The longitudinal emittance is therefore changed by the factor $\delta_1/\delta_0$. The transverse emittance is changed by the inverse of this factor. The betatron functions ($\beta_i$, $\eta_i$) are also changed. Wedge parameters can be arranged to obtain large exchange factors in a single wedge.

**WEDGES FOR FINAL COOLING**

In final cooling for a $\mu$ collider, we wish to reduce transverse emittance at the cost of increased longitudinal emittance. [3, 4, 5] To obtain a large exchange in a single wedge, the $\delta p$ spread induced by the wedge should be much greater than the initial value: $\delta_0 << \sigma_0$. Thus the incident beam should have a small momentum spread and small momentum $P_0$ and the wedge should have a large tan($\theta/2$), large $\delta p/ds$ and a large $\sigma_0 = (2\sigma_p P_0)$). Beam from a final cooling segment is likely to have $P_0 \approx 100-150$ MeV/c, and $\delta p \approx 3$ MeV/c. $\delta p$ should be reduced to ~0.5 MeV/c, and this can be done by rf debunching of the beam to a longer bunch length. The best wedge material is a high-density low-Z material (Be or C (diamond density) or Be:C (almost as good)). At these parameters emittance exchange by a factor of 5 to $\epsilon_0 \approx 25 \mu$ can be obtained from a single C wedge. This matches the required emittance of a high energy collider, indicating that thick wedge cooling can be an important part of a collider cooling system. Table 1 and figs. 2 show parameters and ICOOL [9] simulation results of this wedge exchange.

<table>
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<th>Case</th>
<th>$P_0$ (MeV/c)</th>
<th>$\epsilon_x$ (mm)</th>
<th>$\epsilon_y$ (mm)</th>
<th>$\epsilon_z$ (mm)</th>
<th>$\delta E_{rms}$ MeV</th>
<th>$\eta$ (m)</th>
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<td>2.93</td>
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<td>4.3</td>
<td>32.9</td>
<td>7.38</td>
<td>-0.1</td>
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</table>

Table 2: Beam parameters at entrance and exit of a $w=6.5$ cm, $\theta=45\degree$ $C_2H_4$ wedge. G4beamline simulations results:

Case I: Reverse emittance exchange $\beta^*=0.77m$.

Case II: Direct emittance exchange $\beta^*=0.45m$.

**EXPERIMENT AT MICE PARAMETERS**

The MICE experiment has considered inserting a wedge absorber into the beam line [7]. The layout would be a scale model of final cooling wedge examples (~10× larger). For an exploration of wedge emittance exchange at present parameters, we would like use Polyethylene wedges, such as a $w=6.5$cm, $\theta=45\degree$ case. (Maximal exchange occurs with a $\sigma = -w/2$ beam.) Poly ($C_2H_4$) is inexpensive, readily available and can be easily machined into the desired shapes. It is a relatively low-Z material (C and H) with relatively little multiple scattering. (Lower Z materials (Li, H, LiH, Be) would be better for cooling, but $C_2H_4$ is adequate for this initial demonstration.)

Fig. 3 shows an overview of the MICE step 4 cooling channel, as it is currently installed for the 2016-17 run. Muon beam particles from the MICE beam line enter the upstream spectrometer (US) where they are measured, then through the matching coils M2 and M1 to the Focus coil FC where the energy absorber or wedge would be. Matching coils transport the beam through the downstream solenoid DS where the beam is measured. Comparison of US and DS measurements establish the cooling/emittance exchange performance.

For the $w=6.5$ cm, $\theta=45\degree$ wedge, the incident beam could be matched to $\sigma_x = 3.3$ cm, ($c_m=3$ mm, $\beta^*=77$cm) $P_0=200$ MeV/c, corresponding to a baseline MICE beam setting [8], but with $\delta p = 2$ MeV/c. (This is setting 2016-05-3 which has M2 at 250 A and M1 at 186 A, FC in 129A flip mode, downstream M1, M2 off and 3T-3T in US/DS.) The small $\delta p$ is obtained by software selection of the incident beam. This example obtains an increase in $\delta p$

Figure 3: CAD drawing of the step 4 MICE cooling channel, showing the cryomodules and the magnetic coils, with the upstream and downstream spectrometers (US and DS) for measuring the beam, and matching and focus coils (M1, M2, and FC). The wedge would sit within the focus coils. The downstream M1 coil is non-operational, restricting the current optics choices.

Figure 3: Momentum spread distributions before and after a final cooling wedge. (Initial transverse distribution is non-gaussian.)

Figure 3: CAD drawing of the step 4 MICE cooling channel.
by a factor of ~4 accompanied by a reduction in $\varepsilon_t$ by a factor of ~4. This example was simulated in G4beamline [10], with results presented in table 2 (case I) and displayed in Fig. 4. The resulting scenario would be an interesting scaled model of a final cooling scenario and would test the basic physics and optics of the exchange configuration. Note that verification of fig. 4 in MICE should be relatively straightforward because it is a large effect, and it would be measuring a critical ingredient in a collider scenario.

Figure 4: Momentum distribution before and after absorber, Case I. (Gaussian initial beams.)

The same experimental setup can measure direct emittance exchange, by introducing a dispersion into the beam and using a larger initial $\delta p$, by software selection from a large initial beam sample. A different focusing solution is necessary to match the beam size to the wedge. Changing M2 and M1 to 180 and 130 A, and FC to 165 A, changes $\beta_t$ to ~0.45m at the absorber. An initial beam with $\sigma_{LN} = 3$mm, $\delta p = 10$ MeV/c and dispersion $\eta = 0.5$m, has $\sigma = 3.3$cm at the wedge, and is matched to the wedge size. Results are shown in Table 2 case II and figure 5. The dispersion is reduced to ~0.1m. (see fig. 5) The eigenemittance evaluation changes modes of 3.1 mm and 2.9 mm (transverse) to 4.3 and 2.8 mm while the longitudinal decreases from 38 to 32 mm ($\sigma E$ decreases from 8.85 to 7.38 MeV). The example demonstrates longitudinal cooling by the wedge, complementary to the transverse emittance reduction of case I.

P. Snopok et al. have considered a more precisely matched preselected 140 MeV/c beam, with larger initial transverse emittance, to obtain cases with simultaneous transverse and longitudinal cooling.[11] The MICE experiment should include 140 MeV/c and 200MeV/c beam examples, with appropriate optics.[12] Selecting initial beam profiles for both cases will enable the exploration of a variety of beam optics cases, and map out many of the beam manipulation options possible with wedge absorbers.

Figure 5. Y-P phase space before and after direct emittance exchange (Case II). The dispersion is greatly reduced, while the momentum spread decreases.

CONCLUSION

Beam transport through wedge absorbers and the resulting changes in beam phase space are a critical ingredient in complete ionization cooling systems; they are essential for longitudinal cooling and important in phase-space manipulations. A demonstration of these effects in MICE would enhance its performance as a first demonstration of the components of ionization cooling systems.

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REFERENCES