

Decay constants f_B and f_{B_s} and quark masses m_b and m_c from HISQ simulations

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We present a progress report on calculations of the decay constants f_B and f_{B_s} from lattice-QCD simulations with highly-improved staggered quarks. Calculations are carried out with several heavy valence-quark masses on $(2+1+1)$ -flavor ensembles that include charm sea quarks. We generate data at several lattice spacings and values of light sea-quark masses, including an approximately physical-mass ensemble at all but the smallest lattice spacing, 0.03 fm. This range of parameters provides excellent control of the continuum extrapolation to zero lattice spacing and of heavy-quark discretization errors. We also obtain charm- and bottom-quark masses from the heavy-light meson masses, obtained from the decay-constant correlation functions, using an approach based on heavy-quark effective theory.

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1. Introduction

One way of searching for new physics is by looking for discrepancies between precise experimental measurements and equally precise theoretical calculations within the Standard Model. To this end, the study of heavy-light mesons provides a rich area for investigation. On the one hand, the leptonic decays of pseudoscalar mesons enable determinations of Cabibbo-Kobashi-Maskawa (CKM) quark-mixing matrix elements within the Standard Model. On the other hand, the study of heavy-light meson masses within the framework of heavy quark effective theory (HQET) enables determinations of charm- and bottom-quark masses and also some low-energy constants (LECs) appearing in HQET, which in turn can be used for inclusive determinations of $|V_{ub}|$ and $|V_{cb}|$.

Here we provide a progress report on our effort to calculate the leptonic decay constants f_B and f_{B_s} in four-flavor lattice QCD [1]. The calculations are done using highly improved staggered quarks (HISQ) [2–5] with masses heavier than the charm-quark mass. For details about the method for extracting the decay constants from two-point correlation functions, see Ref. [6]. We also extend this work to study the masses of heavy-light mesons. Within the framework of HQET, we present a method to organize the heavy-quark mass dependence of heavy-light mesons. This method leads to lattice-QCD determinations of quantities such as $\bar{\Lambda}$ and μ_π^2 that appear in HQET. Equivalently, this method provides a way to calculate the charm- and bottom-quark masses.

The range of valence heavy-quark masses and lattice-spacings for the QCD gauge-field ensembles in this study is shown in the left panel of Fig. 1. Compared with Ref. [1], our analysis now includes an $a \approx 0.03$ fm, $m'_l/m'_s = 0.2$, ensemble for which $am_b \approx 0.6$, and thus no extrapolation from lighter heavy-quark masses is needed. In order to avoid large lattice artifacts we drop data with $am_h > 0.9$ and parameterize the heavy-quark mass dependence in our fits at smaller values guided by HQET.

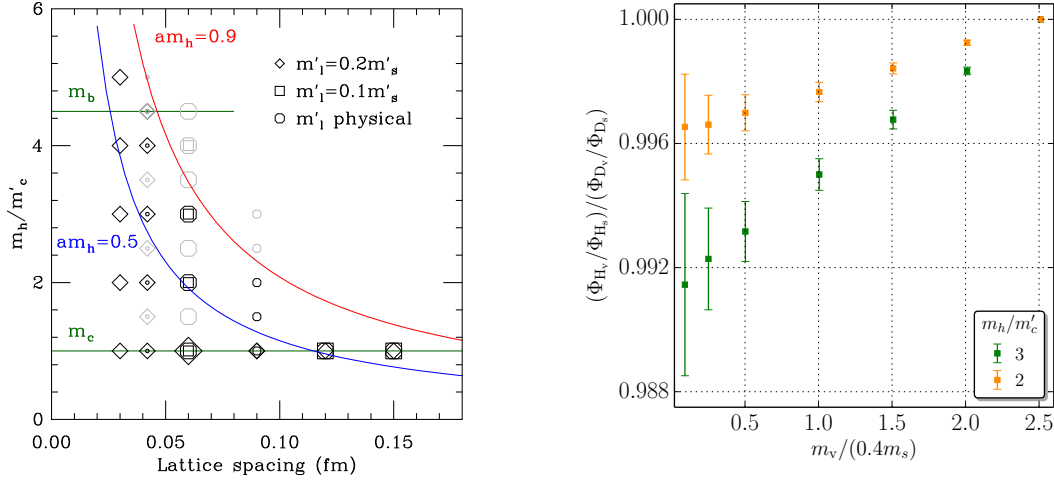


Figure 1: Left: valence heavy-quark masses and lattice-spacings of ensembles in this study for different light-to-strange sea-quark-mass ratios m'_l/m'_s . (Primes on the masses indicate the simulation mass values.) The symbol radius is proportional to the data sample size. The red line indicates the cut $am_h = 0.9$; the blue, $am_h = 0.5$. Right: double ratio of decay constants for the physical-mass ensemble at 0.06 fm as a function of valence light-quark mass m_v . In this double ratio the leading-order terms in HQET and $S\bar{\chi}PT$ cancel, revealing the higher-order terms that depend upon both light- and heavy-quark masses.

2. Chiral-continuum-HQET fit of decay constants

We use HQET to model the heavy-quark mass dependence of the decay constants. In heavy-quark physics, the conventional pseudoscalar-meson decay constant is $\Phi_H = f_H \sqrt{M_H}$. Let us start with massless light quarks. The decay constant in this limit, denoted by Φ_0 , can be expanded as

$$\Phi_0 = C \left(1 + k_1 \frac{\Lambda_{\text{HQET}}}{M} + k_2 \left(\frac{\Lambda_{\text{HQET}}}{M} \right)^2 + \dots \right) \tilde{\Phi}_0, \quad (2.1)$$

where $\tilde{\Phi}_0$ is the matrix element of the HQET current in the infinite-mass limit and the Wilson coefficient C arises from matching of the QCD current and the HQET current

$$C = \left[\alpha_s(m_Q) \right]^{-2/\beta_0} \left(1 + \mathcal{O}(\alpha_s) \right), \quad (2.2)$$

where m_Q is the heavy-quark mass and $\beta_0 = 11 - 2n_f/3 = 25/3$ in our simulations.¹ Within the framework of heavy-meson, rooted, all-staggered chiral perturbation theory (HMrAS χ PT) [7], this relation can be extended to include the light-quark mass dependence and taste-breaking discretization errors. This provides a suitable fit function to perform a combined fit to lattice data at multiple lattice spacings and various valence- and sea-quark masses.

The fit function that we use in this analysis has the following schematic form

$$\Phi_{H_v} = C \left(1 + k_1 \frac{\Lambda_{\text{HQET}}}{M_{H_s}} + k_2 \left(\frac{\Lambda_{\text{HQET}}}{M_{H_s}} \right)^2 + k'_1 \frac{m_c}{m'_c} \right) \left(1 + \text{log/analytic terms} \right) \left(\frac{m'_c}{m_c} \right)^{3/27} \tilde{\Phi}_0, \quad (2.3)$$

where the ‘‘log/analytic’’ terms include the next-to-leading order (NLO) staggered chiral logarithms, and the NLO, NNLO, and NNNLO analytic terms in the valence and sea-quark masses. The NLO staggered chiral logarithms are given in equation 177 of Ref. [7], and the NLO analytic terms (at a fixed heavy-quark mass) are

$$L_s (2m_l + m_s) + L_v m_v + L_a a^2. \quad (2.4)$$

Because we have a wide range of heavy-quark masses from near charm to bottom (at the finest lattice spacings) it is important to take the heavy-quark mass dependence of L_s and L_v into account. The importance of this dependence is shown in the right panel of Fig. 1, where a double ratio of decay constants $(\Phi_H/\Phi_{H_s})/(\Phi_D/\Phi_{D_s})$ is constructed to be sensitive to higher-order terms that depend upon both the light- and heavy-quark masses. Based on the observed quark-mass dependence of the double ratio, we allow the NLO analytic-term LECs to have M_{H_s} dependence, replacing

$$L_s \rightarrow L_s + L'_s \frac{\Lambda_{\text{HQET}}}{M_{H_s}} + L''_s \left(\frac{\Lambda_{\text{HQET}}}{M_{H_s}} \right)^2, \quad (2.5)$$

$$L_v \rightarrow L_v + L'_v \frac{\Lambda_{\text{HQET}}}{M_{H_s}} + L''_v \left(\frac{\Lambda_{\text{HQET}}}{M_{H_s}} \right)^2. \quad (2.6)$$

Because we have very precise data, the NLO terms in HMrAS χ PT are insufficient to describe the quark-mass dependence; we therefore include all NNLO and NNNLO mass-dependent analytic

¹Here, $\tilde{\Phi}_0$ is a renormalization-group invariant quantity, where the renormalization scale and scheme dependence of the HQET current and the Wilson coefficient C cancel. Because of this, C in Eq. (2.2) does not depend on the scale of the effective theory.

terms in our preferred fit. We do not include the NNLO and NNNLO analytic terms that depend on the lattice spacing. Similar to L_v and L_s , the LECs appearing at higher orders in principle have heavy-quark mass corrections, although in practice most of these corrections are not needed to obtain an acceptable fit. A heavy-quark mass dependence also appears implicitly in the NLO chiral logarithms through the $M_{H_s^*} - M_{H_s}$ hyperfine splitting and heavy-light flavor splittings. The factor $(m'_c/m_c)^{3/27}$ in Eq. (2.3) incorporates the leading effect of mistunings in the simulated sea charm-quark mass m'_c compared to the physical charm mass m_c . The coefficient $\tilde{\Phi}_0$ in Eq. (2.3), which is a constant in the continuum limit, has a generic lattice-spacing dependence, which we parameterize as

$$\tilde{\Phi}_0 \rightarrow (1 + c_1 \alpha_s (a\Lambda)^2 + c_2 (a\Lambda)^4 + c_3 \alpha_s (am_h)^2 + c_4 (am_h)^4 + c_5 \alpha_s (am_h)^4) \tilde{\Phi}_0. \quad (2.7)$$

The LECs appearing at NLO and higher orders also have lattice artifacts, but we do not incorporate them into our fit here.

In total, our base fit function in this analysis has 23 parameters. Figure 2 shows two projections of the resulting fit. From the fit function evaluated at zero lattice spacing and physical sea-quark masses, we obtain the decay constants as a function of M_{H_s} and the valence light-quark mass.

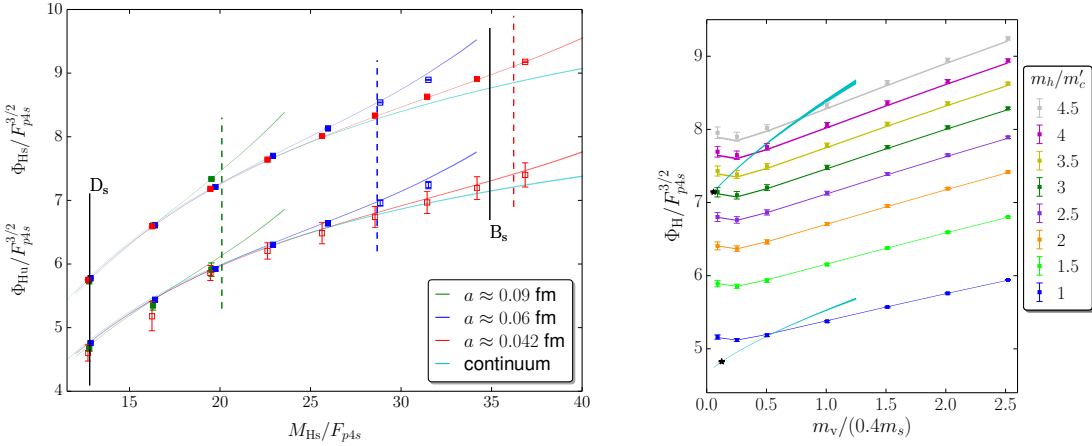


Figure 2: Chiral-continuum-HQET fit of decay-constant data at 6 lattice spacings with valence heavy-quark masses in the range $am'_c \leq am_h < 0.9$. This fit has a correlated $\chi^2/\text{dof} = 265/305$, giving $p = 0.95$. Left: decay constants plotted in units of F_{p4s} vs. the heavy-strange meson mass for three lattice spacings and the continuum extrapolation (see Ref. [6] for the definition and determination of the scale F_{p4s} on the HISQ ensembles). Data points to the right of the dashed vertical line of the corresponding color are excluded from the fit. The open symbols indicate the data points that are omitted. Right: decay constants for the $a \approx 0.042$ fm, $m'_l/m'_s = 0.2$ data vs. the valence light-quark mass. The full-QCD, continuum-limit results at physical c - and b -quark masses are shown in cyan. The stars indicate the physical light-quark-mass results for the B^+ and D^+ mesons.

2.1 Error budget

To estimate the systematic errors on the decay constants, we rerun the analysis with alternative fit functions, including or dropping the coarsest ensembles, and with various choices for scale-setting quantities and tuned quark masses. (For details see Refs. [1, 6].) After rejecting the fits with $p < 0.05$, we take the extremes of the histograms as our estimate of the systematic error from the chiral-continuum-HQET fit. Preliminary error budgets for f_B and f_{B_s} are presented in Table 1.

Table 1: Preliminary error budgets. The first error is the statistical error from our base fit. The second error, determined from histograms, is our estimate of the fit systematic error. The third error comes from propagating the error of the experimental value of f_π , which is the physical quantity used to set the lattice scale.

	f_{B^+} MeV	f_{B_s} MeV
Statistics	0.4	0.3
Alternative fit choices	1.1	0.8
Exp. f_π	0.3	0.4
Total	1.2	1.0

Compared with our previous report [1], the uncertainty has been decreased, primarily because our analysis now includes more data at 0.042 fm and an even finer ensemble at 0.03 fm.

3. HQET and heavy-light meson masses

Similar to the case of decay constants of heavy-light mesons, we employ effective field theories to construct a fit function for meson masses. For the mass of a heavy-light pseudoscalar meson, containing a heavy-quark with mass m_Q , HQET gives

$$M_H = m_Q + \bar{\Lambda} + \frac{\mu_\pi^2 - \mu_G^2(m_Q)}{2m_Q} + \mathcal{O}(1/m_Q^2). \quad (3.1)$$

The LECs appearing in this relation have a simple physical interpretation [8]: $\bar{\Lambda}$ is the energy of the light quark and gluons; $\mu_\pi^2/2m_Q$ is the kinetic energy of the heavy quark; and $\mu_G^2/2m_Q$ corresponds to the hyperfine energy of the interaction between the heavy quark's spin and the chromomagnetic field inside the meson. These LECs also appear in the heavy-quark expansions of inclusive semileptonic decay rates. Because of the nonperturbative nature of these LECs, lattice gauge theory can play an important role in their determination.

The detailed definition of the quantities on the right-hand side of Eq. (3.1) depend on how the HQET is renormalized. In the $\overline{\text{MS}}$ scheme, m_Q would be the pole mass. The relation between the pole mass and the $\overline{\text{MS}}$ mass is known through order α_s^4 for n_l massless quarks and n_h massive (with mass m_Q) quarks [9]. (Assuming that the sea charm quark is decoupled, the values of $n_l = 3$ and $n_h = 0$ are relevant to our ensembles with $n_f = 2 + 1 + 1$ flavors.) Due to the renormalon problem in the pole mass, we replace the pole mass with the so-called renormalon-subtracted (RS) mass [10], which is defined by subtracting the leading infrared renormalon from the pole mass. We also follow Ref. [11] to decouple the sea charm quark from the theory, and, in turn, express the RS mass in terms of the $\overline{\text{MS}}$ mass in a theory with three active sea quarks. We then introduce a reference mass denoted by m_{c^*} , which can be the charm-quark mass, and we formulate the relations in terms of the ratio of quark masses and the reference mass.

We intend to exploit the expansion in Eq. (3.1) to construct a function to fit to the heavy-light meson masses calculated on the lattice. To this end, we need to relate the bare mass of a quark on the lattice and its $\overline{\text{MS}}$ mass. This relation can be calculated through their connections to the pole

mass. The one-loop calculation for staggered quarks gives [12]

$$m^{\text{pole}} = \frac{am}{a} \left[1 + \alpha_{\text{lat}} \left(-\frac{2}{\pi} \log(am) + A_{10} \right) + \mathcal{O}(\alpha_{\text{lat}}^2) \right], \quad (3.2)$$

$$A_{10} = K_0 + K_1(am)^2 + K_2(am)^4 + \dots. \quad (3.3)$$

Now, considering the relation between the pole mass and the $\overline{\text{MS}}$ mass, one can show that

$$\begin{aligned} \frac{m_h^{\overline{\text{MS}}}(\mu)}{m_{c^*}^{\overline{\text{MS}}}(\mu)} &= \frac{am_h}{am_{c^*}} \left[1 + \alpha_{\overline{\text{MS}}}(\mu) \left(K_1((am_h)^2 - (am_{c^*})^2) + \dots \right) + \dots \right], \\ &= \frac{am_h}{am_{c^*}} + \mathcal{O}((am)^2\alpha), \end{aligned} \quad (3.4)$$

where m_h and m_{c^*} denote the masses of a generic quark and the reference quark, respectively. (In this analysis am_{c^*} is the tuned charm-quark mass.) The lattice artifacts appearing in the right hand side of Eq. (3.4) vanish in the continuum limit for tuned quark masses. This relation can be exploited to connect the continuum expressions to lattice simulations.

We now organize a fit function as

$$M_H = m_h^{\text{RS}} + \delta_a + \bar{\Lambda}^{\text{RS}} + \frac{\mu_\pi^2 - \mu_G^2(m_h^{\text{RS}})}{2m_h^{\text{RS}}} + \frac{\rho}{(2m_h^{\text{RS}})^2}, \quad (3.5)$$

with $\bar{\Lambda}^{\text{RS}}$, μ_π^2 , $\mu_G^2(m_b)$ and ρ as fit parameters. We set the prior value of $\mu_G^2(m_{c^*})$ based on the hyperfine splitting of $M_{B^*} - M_B$. The RS mass in Eq. (3.5) is constructed from the $\overline{\text{MS}}$ mass, which in turn is obtained from

$$m_h^{\overline{\text{MS}}}(\mu) \Big|_{\mu=\bar{m}_{c^*}} = \bar{m}_{c^*} \frac{am_h}{am_{c^*}}. \quad (3.6)$$

The lattice artifacts in relating the ratio of bare masses to the ratio of $\overline{\text{MS}}$ masses are absorbed in δ_a :

$$\delta_a = K_1 \bar{m}_{c^*} ((am_h)^2 - (am_{c^*})^2) \alpha. \quad (3.7)$$

We are currently considering \bar{m}_{c^*} , which is the mass of the reference quark, as a free parameter in this analysis, allowing the fit to absorb perturbative and scale errors coming from the relation between \bar{m}_{c^*} and am_{c^*} . With this parameterization, our fit function has 6 fit parameters that are to be determined by lattice data. If required one can improve the fit function by adding more terms.

Figure 3 illustrates a sample fit based on the method presented here. This shows qualitatively that the lattice artifacts are well modeled by our parameterization of the discretization errors. Further, the observed agreement of the simulation M_H for different lattices spacings indicates that heavy-quark discretization errors are small. From the continuum extrapolation, we obtain values of meson masses as a function of the heavy-quark mass ratios and the mass of the reference quark \bar{m}_{c^*} , which is determined by the fit. By fixing the meson mass to the mass of D and B mesons, we can then determine the charm- and bottom-quark masses. The quantitative results will be presented in a future paper.

4. Conclusion

We have presented the status of our analysis of f_B and f_{B_s} from a lattice-QCD calculation with the HISQ action for all quarks. We anticipate that our calculations when completed will be the most precise to date. We also presented our method of extracting charm- and bottom-quark masses from heavy-light meson masses. This method also yields the quantities $\bar{\Lambda}$ and μ_π^2 of HQET.

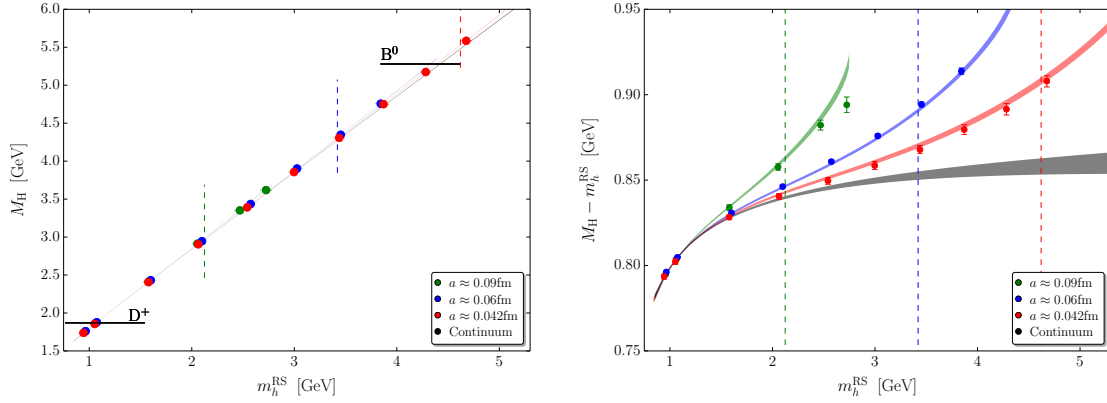


Figure 3: HQET fit of heavy-light-meson-mass data with valence heavy-quark masses $am_h^l \leq am_h < 0.9$. The dashed vertical lines indicate the cut $am_h = 0.9$ for each lattice spacing.

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