NONLINEAR PHASE DISTORTION IN A Ti:SAPPHIRE OPTICAL AMPLIFIER FOR OPTICAL STOCHASTIC COOLING

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Abstract

Optical Stochastic Cooling (OSC) has been considered for future high-luminosity colliders as it offers much faster cooling time in comparison to the micro-wave stochastic cooling. The OSC technique relies on collecting and amplifying a broadband optical signal from a pickup undulator and feeding the amplified signal back to the beam. It creates a corrective kick in a kicker undulator. Owing to its superb gain qualities and broadband amplification features, Titanium:Sapphire medium has been considered as a gain medium for the optical amplifier (OA) needed in the OSC [1]. One important characteristic of any OA used in OSC is the value of nonlinear phase distortions. In this paper we experimentally measure phase distortions by inserting a single-pass OA into one leg of a Mach-Zehnder interferometer. The data are used to infer the reduction of the corrective kick applied to the particle in the kicker undulator due to these phase distortions, coupled with host dispersion and gain narrowing.

INTRODUCTION

Optical Stochastic Cooling (OSC) is a method of beam cooling [2] which, if brought to fruition, could dramatically reduce the damping times in future hadron machines and thus could lead to an increase in achievable luminosity. OSC is similar to the stochastic cooling typically operating in the microwave frequencies but OSC’s potential is determined by its much larger amplifier bandwidths due to transitioning to much higher frequencies in the optical range. In short OSC makes corrective kicks to a particle in a kicker undulator (the kicker) using the same particle radiation generated in the pickup undulator located upstream. A bypass chicane is used to make room for an optical system (lens and amplifier) and to merge the particle and its focused and amplified radiation in space and time in the kicker undulator.

The goal of the optical system is to preserve the time structure of the radiation generated in the pickup, to increase its amplitude, and superimpose its image in the kicker undulator with radiation generated in the kicker undulator having an identical radiation field. These two fields can then add constructively or destructively depending on their phase difference, which generates a longitudinal kick. With appropriately designed beam optics the particles momentum and betatron deviations are damped. Details of OSC can be found in [3, 4], here we focus amplification in Ti:Sapphire (Ti:Saph) based optical amplifier.

Below we consider how the light generated in the pickup into a cone subtended by an angle θ ≈ 1/γ (where γ is the relativistic Lorentz factor of the radiating particle) is amplified and focused into the kicker. The radiation bandwidth is determined by the number of undulator periods and the angle θ and is quite large, ≈ 10 – 20%. That makes it nontrivial to keep phase and amplitude distortions at acceptable levels across the entire band. Specifically we consider nonlinear phase distortions related to amplification and additionally account for distortion from host dispersion and the gain narrowing.

AMPLIFIER PULSE DISTORTION

The modification of the electric field harmonic of a broadband light pulse traveling through an amplifier is given by [5]

$$E_2(\omega, z) = E_1(\omega) \exp \left[ iz \beta (1 + \frac{\chi'}{2}) + \frac{z \beta \chi''}{2} \right], \quad (1)$$

where $E_1(\omega)$ is the original radiated field from the pickup undulator, $\beta \equiv \frac{\omega - \omega_0}{c}$, with $n$ is index of refraction and $c$ is the speed of light in vacuum. $\chi_{at} = \chi'(\omega, z) + i \chi''(\omega, z)$ is the complex Lorentzian atomic line shape given as

$$\chi_{at} = -\chi'' \left[ \frac{\Delta \chi}{1 + \Delta \chi^2} + i \frac{1}{1 + \Delta \chi^2} \right], \quad (2)$$

where $\Delta \chi \equiv \frac{2 \omega - \omega_0}{\Delta \omega}$ with $\omega_0$ being the midband angular frequency and $\Delta \omega$ the FWHM amplifier bandwidth. The amplitude of line shape is

$$\chi'' = \frac{3}{4 n^2} \frac{\Delta N(\tau) \lambda^3}{\Delta \omega \tau n^2}, \quad (3)$$

where $\tau$ is the upper state lifetime of the gain medium. The $z$-dependence of population inversion, $\Delta N$, stems from absorption of the pump laser as it propagates through the amplifier. Note that we can safely neglect saturation effects of the amplified light as the undulator radiation is too weak to give rise to them. All quantities in the above formula use midband values.

The first term in the exponent of Eq. 1 is associated with dispersion of the host medium (sapphire) of the amplifier. In the absence of dispersion compensation it gives the largest contribution to pulse distortion. Since complete dispersion compensation does not seem possible for OSC amplification,
a highly doped gain medium must be used to minimize the length of amplifier and, consequently, its phase distortions. The need for a highly doped crystal is also consistent with the requirement imposed by the chicane. The coefficient $\beta$ can be calculated using the Sellmeier’s equation with the coefficients found in [9]. The real part of the lineshape gives rise to a frequency-dependent phase shift. The total shift is found by integrating $\chi'(\omega, z)$ over the length of the amplifier which is essentially an integration over the population inversion and can be related to the signal growth as $\frac{dI}{dz} = \sigma \Delta N I$.

The phase shift becomes

$$\phi_{\text{amp}} = \frac{\beta}{2} \int \chi'(\omega, z) dz = -\frac{1}{2} \frac{\Delta x}{(1 + \Delta x^2)} \ln(G),$$

(4)

with $G$ the gain in intensity in the absence of detuning. In simplifying the above we make the approximation $\sigma = \frac{1}{2} \frac{\chi'}{\Delta \omega_a \sigma_n^2}$. Formulas for calculating the gain of a Ti:Sapph amplifier for OSC are given in [11]. Measurements of this shift are discussed in the next section.

Identical integration of the imaginary part of the lineshape gives an amplitude growth. Finally Eq. 1 is rewritten as

$$E_2(\omega, z) = E_1(\omega) \exp[i(\omega \beta + \phi_{\text{amp}})]G^{\frac{1}{2(1+\Delta x^2)}}.$$  

(5)

**AMPLIFIER PHASE SHIFT**

**Measurements**

To measure $\phi_{\text{amp}}$ introduced in the previous section we inserted a 2-mm thick Ti:sapph crystal into one leg of a Mach-Zhender interferometer. The crystal had an absorption constant $\alpha \approx 10 \text{ cm}^{-1}$ at 514 nm.

For the seed a Spectra-Physics Tsunami laser, capable of CW or mode-locked operation, was used. The short pulses generated while mode-locked were used to equalize the optical path lengths of the two interferometer legs and detect amplification, but phase measurements were done in CW operation to obtain a well defined laser frequency. Measurements were taken from 740 to 810 nm with 10 nm spacing. The crystal was pumped with a Q-switched Spectra-Physics Quanta Ray Nd:Yag laser with a frequency doubling crystal. The pump laser produced 10 ns pulses with 65 mJ per pulse at 10 Hz and a wavelength of 532 nm. A lens focused the pump laser pulse to a spot size of $\approx 1.5 \text{ mm}$ on the crystal.

In order to avoid damage of the interferometer optics we had a small ($\approx 10^\circ$) angle between the pump and seed beams. Under these conditions we typically observe $G \approx 3$ with a seed pulses centered at 780 nm with 20 nm (FWHM) bandwidth. The spectra and central wavelength were measured with an Ocean Optics spectrometer. A Dicam Pro intensifier camera recorded the interference pattern. The double-shutter mode of the camera allows for two images to be taken in rapid succession, in our case we chose 2 $\mu$s time lapse between shots with 40 ns exposure time. A beam splitter between the interferometer output and the camera was used to simultaneously observe the signal with the camera and a photo-diode connected to a fast oscilloscope.

The camera was triggered off the Q-switch of the pump laser and a delay unit was used to adjust the timing so that the first shot is taken before amplification ($\approx 1.5 \mu$s) and the second after ($\approx 0.5 \mu$s).

At each wavelength 20 shots were taken and the interference pattern was projected along the horizontal axis. We then found the change in the peak position of the interference pattern yielding the total phase shift.

In addition to the phase shift derived in the previous Section a non-resonant change in the refraction index of Ti:Sapph has been reported [12]. This results in a constant phase shift for all frequencies. The shift from this has no consequence for OSC but we compensate for it in our data by adding a constant term to set $\phi_{\text{amp}} = 0$ for the midband frequency in agreement with Eq. 4.
Discussion

\(\phi_{amp}\) is essentially linear near its midband value. From Eq. 4 the slope (with respect to \(\omega\)) at the midband is 
\[
\frac{\ln(G)}{180 \pi \Delta \omega_a} = 105 \text{ deg-fs}
\]
and a linear fit of the data shown on the right side of Fig. 2 gives a slope of 81 ± 5 deg-fs, in rough agreement. A linear shift in phase would correspond to a net delay of the pulse but no distortions, and hence has no effect on OSC. The data presented here is not accurate enough to measure the nonlinear contribution directly.

We finally explore the possible use of the characterized crystal for a hypothetical OSC demonstration experiment with an electron beam. We consider an amplifier with 20 dB of gain. For this gain we require 2.09 MW/cm² of CW pumping intensity. Cryogenic cooling is required to reduce thermal effects to manageable levels.

Table 2: Beam and Undulator Parameters assumed for a possible OSC proof-of-principle experiment using a Ti:Sapph lasing medium.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Electron Energy</td>
<td>150 MeV</td>
</tr>
<tr>
<td>Peak magnetic field (B)</td>
<td>1434 G</td>
</tr>
<tr>
<td>Number of periods</td>
<td>10</td>
</tr>
<tr>
<td>Period</td>
<td>8 cm</td>
</tr>
<tr>
<td>Undulator parameter (K)</td>
<td>1.1</td>
</tr>
<tr>
<td>On-axis radiation wavelength</td>
<td>730 nm</td>
</tr>
</tbody>
</table>

To estimate the effects of pulse distortions for OSC we use Synchrotron Radiation Workshop (SRW) [10] to compute the on-axis radiation from an undulator after being focused by a lens. The parameters for the undulator are given in table 2, they represent reasonable values for a potential demonstration of OSC with electrons.

Next the spectrum is modified using Eq. 5 and both the original and amplified spectra are converted into the time domain and a correlation function

\[
\gamma_{12}(\tau) = \frac{\langle E_1(t)E_2^*(t+\tau)\rangle}{\left[\langle |E_1|^2\rangle\langle |E_2|^2\rangle\right]^{1/2}}
\]

is computed between them. Without distortion this function has a maximum value of 1, the reduction of this maximum value estimates a decrease in the corrective kick for OSC from pulse distortions. If we only consider distortions from \(\phi_{amp}\) and gain narrowing the maximum \(\gamma_{12} = 0.87\), including host dispersion lowers this to 0.68. This reduction in the kick is compensated by the increase in field amplitude which for a 20 dB amplifier is a factor of 10. Therefore a 20 dB Ti:Sapph amplifier yields about a factor of 7 increase in the corrective kick.

CONCLUSION

Ti:Sapph is one of the most promising materials for a single pass amplification scheme for OSC. Amplifying with a CW pump has a practical advantage in that, owing to the relatively weak signal coming from the pickup, amplification gain and pump power requirements are independent of the particle bunch length and repetition rate of the accelerator. However pulse lengthening from gain narrowing and host dispersion limit the amount of amplitude growth the amplifier can give to the signal. The nonlinear phase changes from amplification also play a, albeit much smaller, role in this scheme. When all distortion effects are accounted for the corrective kick is reduced by a factor of 0.68 compared to perfect amplification, with the largest contribution coming from host dispersion.

REFERENCES


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1 The experimental OSC test planed at the IOTA ring is foreseen to be based on an alternative amplifying medium; see Ref. [11] for details.