Abstract — Multiphysics analyses for superconducting cavities are essential in the course of cavity design to meet stringent requirements on cavity frequency detuning. Superconducting RF cavities are the core accelerating elements in modern particle accelerators whether it is proton or electron machine, as they offer extremely high quality factors thus reducing the RF losses per cavity. However, the superior quality factor comes with the challenge of controlling the resonance frequency of the cavity within few tens of hertz bandwidth. In this paper, we investigate how the multiphysics analysis plays a major role in proactively minimizing sources of frequency detuning, specifically; microphonics and Lorentz Force Detuning (LFD) in the stage of RF design of the cavity and mechanical design of the niobium shell and the helium vessel.

Index Terms — Superconducting RF cavities, Microphonics, Lorentz force detuning, multiphysics analysis.

I. INTRODUCTION

Particle accelerator technology evolves gradually towards improving reliability and efficiency of the accelerator machines, which would reduce their cost for current applications and even make them more accessible to new industrial applications. Superconducting cavities are the cornerstones of modern particle accelerators with their ultra-low electrical resistivity and extremely high quality factor. However, one of the technical challenges facing the superconducting technology for RF cavities is the fact that superconducting cavities are hypersensitive to frequency fluctuations as any minor drift (few tens of Hz) in the operating frequency would get the cavity seriously detuned.

Frequency detuning is mainly caused by either the fluctuations in the applied Helium path pressure; microphonics or by the radiation pressure of the electromagnetic field inside the cavity; Lorentz Force Detuning (LFD) [1].

Compensation of frequency detuning is conventionally realized by employing robust slow and fast tuning mechanisms and meanwhile, devising enough reserve in the power of the RF source to get the cavity over-coupled if necessary. Over-coupling the cavity, which in effect widens the cavity’s bandwidth, would help to mitigate the frequency detuning, but unfortunately would increase both the capital and operation cost of the particle accelerator machine, as more RF energy is basically needed and is partially wasted in this case.

On the other hand, increasing the mechanical stiffness of the cavity could potentially reduce the undesired detuning effects, whether it is microphonics or LFD; however it will essentially also limit the desired capability of tuning the cavity frequency and will complicate the tuner design.

II. SOURCES OF DETUNING IN SRF CAVITIES

The geometry of superconducting RF cavity inherently couples the electromagnetic and mechanical aspects of the structure. An SRF cavity would consist of a relatively thin shell made typically from niobium that encloses the electromagnetic resonating domain and is jacketed by typically a stainless steel or titanium vessel that carries the refrigeration liquid (superfluid helium) to the cavity surface. Figure 1 shows the geometry of 9-cell elliptical cavity.

Perturbing the volume of a cavity would directly reflect on its resonance frequency. The frequency shift due to this shape perturbation is proportion to the change of the magnetic and electric stored energies. However, the sign of the frequency shift being positive or negative is dependent on where the perturbation is located being in a high magnetic field or electric field dominated areas.

Slater states that the fractional change in a cavity’s resonance frequency is proportional to the fractional change in its stored energy [2], such that

$$\frac{\omega - \omega_0}{\omega_0} = \frac{\int_V \mu |H_0|^2 - \varepsilon |E_0|^2 \, dv}{\int_V \mu |H_0|^2 + \varepsilon |E_0|^2 \, dv}$$

Where \(\omega\), and \(\omega_0\), are the perturbed and unperturbed angular resonance frequencies, respectively. \(\mu\), and \(\varepsilon\) are the permeability and permittivity of the medium inside the cavity. \(H_0\) is the magnetic field, \(E_0\) is the electric field, and \(V\) is the volume of the cavity.

Conventionally, the different frequency detuning mechanisms are classified according to their sources.

Fig. 1. Sources of frequency detuning in superconducting RF cavities.

The most common frequency detuning sources in the context of SRF cavities [1], as illustrated in Fig. 1, are the following:

- **Microphonics**
- **Lorentz Force Detuning (LFD)**

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**Index Terms** — Superconducting RF cavities, Microphonics, Lorentz force detuning, multiphysics analysis.
a) Helium bath pressure fluctuations

Helium pressure in the cryogenic system inherently varies causing continuous pressure fluctuations on the cavity walls. Pressure fluctuations then translate to continuous perturbations to the cavity’s volume that would induce detuning to the resonant frequency.

Conventionally, the cavity sensitivity to pressure fluctuation is expressed by the term $df/dp$, which basically quantifies the expected frequency change due to certain change in the helium bath pressure. The sensitivity coefficient $df/dp$ actually could have positive or negative sign depending on where most of the deformation is happening, whether it is in the high magnetic field or high electric field areas, respectively. In fact, the value of $df/dp$ is especially critical for continuous wave particle accelerator machines, as it is the dominant detuning factor in such machines.

Figure 2 demonstrates the displacement under 1 bar pressure load calculated using Ansys workbench [3] for a single spoke superconducting cavity with resonance frequency 325 MHz.

b) Electromagnetic radiation pressure (Lorentz Force Detuning)

Electromagnetic fields inside the cavity exert radiation pressure on inside walls of the cavity. This pressure is defined as

$$P_{rad} = \frac{1}{4} \left( \mu |H|^2 - \varepsilon |E|^2 \right)$$

It is worth noting here that the radiation pressure exercised by the magnetic field is positive pushing pressure, while it is negative pulling for the electric field.

Lorentz force detuning coefficient is commonly used to express the cavity frequency sensitivity to radiation pressure. It is basically defined as

$$LFD = \frac{\Delta f}{E_{acc}^2}$$

Where $\Delta f$ is the cavity’s resonance frequency shift observed while the cavity maintained $E_{acc}$ voltage gradient. $LFD$ is conventionally expressed in the units of Hz/(MV/m$^2$).

The $LFD$ coefficient typically has a negative sign as both the repulsive magnetic field forces and the attractive electric field forces work together to decrease the resonance frequency of the deformed cavity. Figure 3 shows, for illustration, the Lorentz forces exerted on the 650 MHz $\beta$=0.9 single cell cavity ahead with the radiation pressure values in mbar at the 3.5 MV single cell cavity voltage.

c) Mechanical vibrations

Mechanical vibrations that already exist in the cavity system due to pumps and motors travel through the pipes and reach the cavity [1]. If for any reason, it coincides with the natural mechanical modal frequencies of the cavity structure, the cavity will start to mechanically vibrate which will then modulate the resonance frequency causing detuning. For instance, Figure 4 shows the modal frequencies of a bare 650 MHz $\beta$=0.9 5-cell cavity. The frequency detuning due to these mechanical vibrations is random.

Special attention during design has to be made to make sure that the modal frequencies are far enough from the 50 Hz electricity oscillation.

III. MULTIPHYSICS MODELING OF FREQUENCY DETUNING

The modeling of the detuning effects in the course of cavity design requires multiphysics analyses, where the electromagnetic and mechanical aspects of the problem are coupled together. Modeling the frequency detuning regardless of its source follows the scheme shown in Fig. 5, summarized in the following steps.
1. Utilizing a full 3D electromagnetic solver, find the Eigen-frequencies of the cavity.

2. By means of a solid mechanics solver, find the displacement that occurs to the cavity structure under certain boundary loads. In case of computing frequency sensitivity to pressure, the boundary load is the helium bath pressure, while it is the radiation pressure in the case of Lorentz force detuning.

3. Deform the geometrical mesh of the structure according to the computed displacement.

4. Recalculate the Eigen-frequencies of the deformed cavity.

Single spoke resonators (SSR1) are currently under production for Project X injection experiment (PXIE), which is planned to test integrated systems for future proton accelerators [4]. The gap distance of SSR1 cavity was designed to accelerate particle at relative velocity $\beta = 0.22$. Cavity operates at 325 MHz with bandwidth of 90 Hz. The nominal gain per cavity is 2 MeV with projected maximum magnetic field of 60 mT and max surface electric field of 39 MV/m. The SSR1 cavities have been optimized to minimize the $df/dp$ coefficient to less than 10 Hz/mbar utilizing a series of multiphysics analyses using Comsol [5] and following the minimization approach in [6].

A total of 10 single spoke resonators of SSR1 have been fabricated. Figure 6(a) shows SSR1-107 cavity prototype. The prototype cavity have been extensively tested to measure the frequency detuning coefficients. Figure 6(b) and (c) compares simulated versus measured values for $df/dp$ and $LFD$, respectively. Simulated values are in good agreement with measured ones.

IV. CONCLUSION

Modeling of frequency detuning in superconducting cavities is essential to minimize the detuning coefficients and plan ahead for the active frequency control needed during actual operation of the particle accelerator.

REFERENCES


