

# LUMINOSITY GOALS FOR A 100-TeV PP COLLIDER

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## Abstract

We consider diverse examples of science goals that provide a framework to assess luminosity goals for a future 100-TeV proton-proton collider.

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# 1 Introduction

All experimental measurements benefit from larger data sets, since statistical uncertainties diminish. Some measurements are ultimately limited by backgrounds or by systematic uncertainties, but additional data can help to reduce these, or provide alternative and independent measurements. For example, multipurpose experiments such as those carried out at particle colliders (from the  $B$  factories up to the highest energy hadron colliders) explore a broad spectrum of available observables, including those that have very small rates that will continue to benefit as data sets increase.

Nevertheless, practical, technical, and financial considerations limit the integrated luminosity that an accelerator will ultimately be able to deliver, so it is important both to aim high and to anticipate what the minimum luminosity must be to guarantee significant new results. The measurement of specific processes can be used to define such minimal goals. This is a well-posed problem in the case of measurements of known processes, where the goal is, for example, a given precision. In the case of searches for new phenomena, things are less clear. The searches for the top quark and the Higgs boson, whose mass ranges and properties were well defined, set reliable luminosity requirements that were used in setting the accelerator specifications of the Tevatron, of its Run 2 upgrade, and of the Large Hadron Collider. But after the Higgs discovery, we lack a well-defined direction for the appearance of new physics phenomena that can be guaranteed (or at least anticipated with a high degree of confidence). Discoveries in Run 2 of the LHC and beyond could change this situation.

The absence of a clear target leads, for now, to large uncertainties in the definition of discovery-driven parameters of future colliders. This is true both of possible discoveries at the highest mass reach and of discoveries that might result if deviations from the standard model were seen in precision studies of electroweak observables, or of Higgs decays. In both cases one should simply aim at the most aggressive possible performance (in energy and luminosity) allowed by the balance of technological challenge and costs and then assess the impact of such measurements. The impact must be large enough to both motivate the experimental community to participate and justify the cost of undertaking a major new project.

As the high energy physics community starts discussing scenarios for future hadron colliders in the energy range of 100 TeV [1,2], it is natural to ask what the appropriate luminosity goals should be. A generic argument, based on the scaling properties of cross sections as a function of the partonic center-of-mass energy suggests that in order for the increase in discovery reach to match the increase in collider energy,  $\sqrt{s}$ , the luminosity should scale as  $s$ , the square of the center of mass energy [3,4]. Scaling violations in the partonic densities can be used to support an argument for even faster luminosity growth [5,6]. This scaling argument has the virtue of simplicity, but the conclusions are sensitive to the choice of starting parameters. It is worth recalling that, because of the fixed size of the LEP tunnel, the LHC compensated for constrained energy by setting aggressive luminosity goals. In different circumstances, the energy–luminosity optimization might take a different path.

In this note, we consider from a broader perspective the physics opportunities that a 100-TeV hadron collider should address, among them, extending the mass reach for discovery. Specifically, we examine several physics cases that drive the luminosity goals. In the context set by those goals, we ask how high a luminosity is desirable and whether we can reasonably set a minimum acceptable luminosity [7].

We set as a first requirement that *the initial luminosity of a new hadron collider should be sufficiently high to surpass the exploration potential of the LHC very quickly*, certainly within the first year of operation. We consider the luminosity demands of four areas of investigation.

1. The search for new phenomena, inaccessible to the LHC, at high mass scales;
2. Increased sensitivity to rare or high-background processes at mass scales well below the kinematical limit of the 100 TeV collider;
3. Increased precision for studies of new particles within the ultimate discovery reach of the LHC;

4. Incisive studies of the Higgs boson, both in the domain of precision, and in the exploration of new phenomena.

## 2 Luminosity Needs of the Physics Criteria

### 2.1 Extending the discovery reach at high mass scales

We consider, as a first example, the case of a possible sequential  $W'$  boson, a massive electroweak gauge boson with couplings identical to those of the standard-model  $W^\pm$  boson. The production proceeds via quark anti-quark annihilation ( $q\bar{q}$ ). Setting the discovery threshold at 100 total produced  $W'$  bosons (leading to  $\sim 20$  events in the clean and background-free leptonic final states with electrons and muons) gives the luminosity requirements displayed in the left plot of Fig. 1, as a function of the  $W'$  mass  $M(W')$ .<sup>1</sup> In the luminosity range of  $0.1\text{--}10^3\text{ ab}^{-1}$ , the increase in mass reach is well approximated by a logarithmic behaviour, with a  $\sim 7\text{ TeV}$  increase in mass for a tenfold luminosity increase:  $M(L) - M(L_0) \sim 7\text{ TeV} \log_{10}(L/L_0)$  (a simple argument for this scaling relation is given in Appendix A). The relative gain in mass reach therefore diminishes as the total luminosity is increased, as shown in the right plot of Fig. 1. This displays the relative extension in mass reach achieved with a factor of 10 increase in luminosity. For example, if for a given integrated luminosity  $L_0$  we are sensitive to a mass  $M_{W'} = 20\text{ TeV}$ ,  $10 \times L_0$  will give sensitivity to a mass a factor of  $\sim 1.4$  times larger, namely  $28\text{ TeV}$ . The additional sensitivity gain given by a factor of 10 increase in luminosity drops below 20% at around  $40\text{ TeV}$ , the discovery reach corresponding to about  $10\text{ ab}^{-1}$  (see the left plot of Fig. 1). The conclusion is that higher luminosity is of greater benefit in the exploration of lower, rather than higher, masses. To illustrate the interplay between collider energy and luminosity, we show in Fig. 2 how cross sections increase as the c.m. energy is raised above  $\sqrt{s} = 100\text{ TeV}$ . For a mass of  $40\text{ TeV}$ , an increase in energy from  $100\text{ TeV}$  to  $130\text{ TeV}$  would be equivalent to a factor of 10 increase in luminosity at  $\sqrt{s} = 100\text{ TeV}$ .

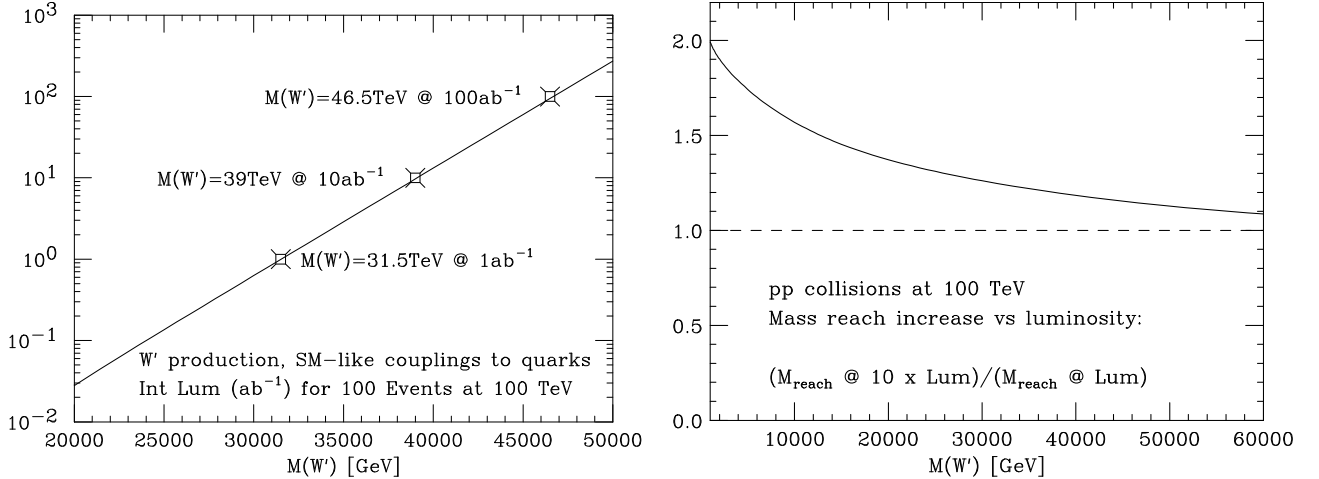


Figure 1: Left plot: integrated luminosity ( $\text{ab}^{-1}$ ) required to produce 100 events of a sequential standard-model  $W'$  boson at  $100\text{ TeV}$ , as a function of the  $W'$  mass. Right plot: mass reach increase for a sequential  $W'$  from a factor of 10 increase in luminosity.

Qualitatively similar conclusions can be reached considering processes dominated by a  $gg$  initial state, rather than  $q\bar{q}$ . The pair-production of massive color-triplet quarks and squarks, and of gluino-like states, is shown in Fig. 3. As exhaustive list of additional examples is given in Ref. [6].

The above qualitative analysis can be illustrated using more complete studies done for the LHC luminosity upgrade, as shown for example in Table 1, which gives ATLAS and CMS's estimates for

<sup>1</sup>The  $W'$  cross sections are calculated at LO, using the PDF sets CTEQ6.6 [14] and scale  $Q = M_{W'}$ .

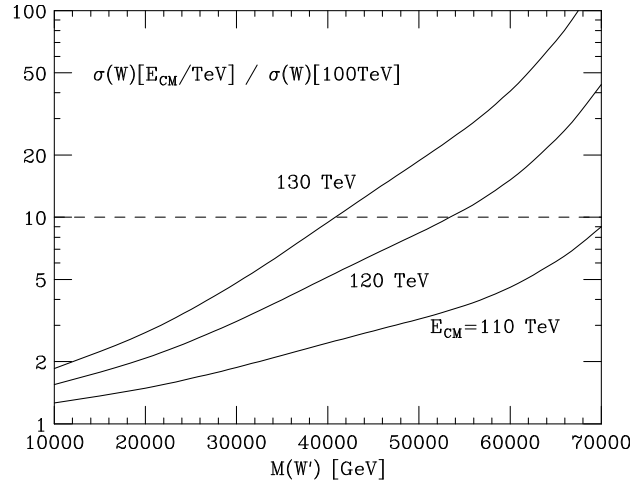


Figure 2: Ratio of  $W'$  production cross sections at different values of  $\sqrt{s}$  to those at  $\sqrt{s} = 100$  TeV, as a function of the  $W'$  mass.

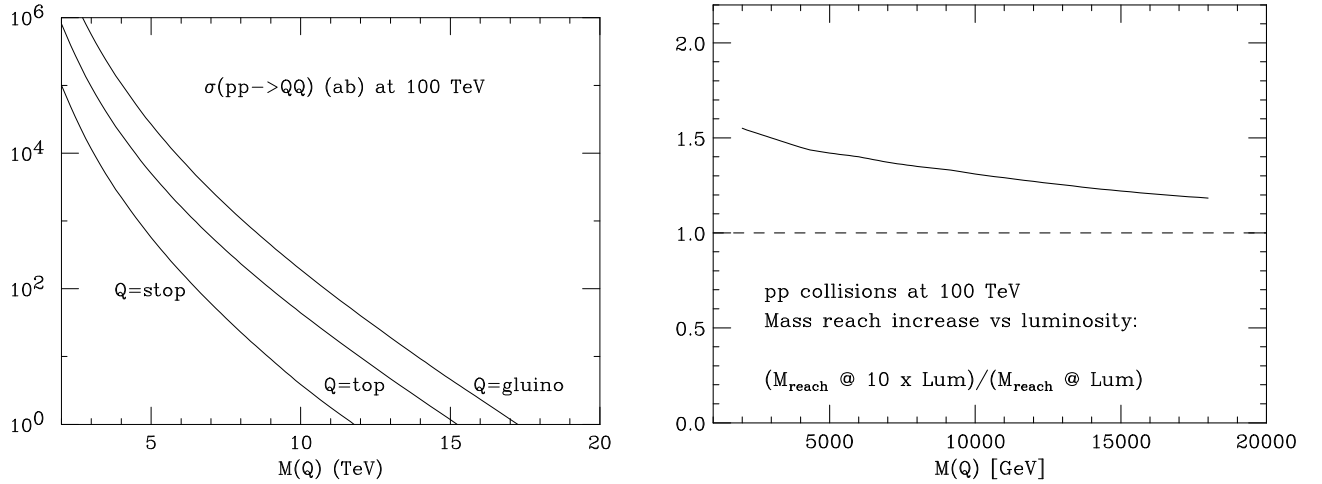


Figure 3: Left plot: cross sections for pair-production of colour-triplet scalars (“stop”), fermions (“top”) and gluinos, as a function of their mass. Right plot: mass reach increase for heavy quark pair production from a factor of 10 increase in luminosity.

the exclusion and discovery reach of a sequential standard-model  $Z'$  gauge boson decaying to leptons. The mass reach increases by 20% as the integrated luminosity increases from 300 to 3000  $\text{fb}^{-1}$ . One could therefore argue that, from the perspective of simply increasing the mass reach at the high end, the LHC will already have almost saturated its discovery potential after 300  $\text{fb}^{-1}$ . Indeed the main motivations for its upgrade to 3000  $\text{fb}^{-1}$  come from the need to study with greater statistics the Higgs boson, or to search in greater detail for elusive signatures of beyond-the-standard-model phenomena in the TeV mass region (see, e.g., the studies performed in the context of the ECFA Workshop on HL-LHC [12]). Assuming 300  $\text{fb}^{-1}$  as a reference to scale the luminosity by the factor of  $s$ , we obtain a target integrated luminosity of  $300 \times (100/14)^2 \text{ fb}^{-1} \sim 15 \text{ ab}^{-1}$ , a figure consistent with the current parameters of the FCC-hh machine design [1].

## 2.2 Enhancing the discovery reach at low mass

By low mass we mean masses, or parton subenergies  $\sqrt{\hat{s}}$ , small relative to the kinematical limit of the collider,  $\sqrt{s}$ : this category would include the top quark and the Higgs boson, as well as new particles such as sleptons or charginos. For these particles the discovery can be limited by the smallness of the

Integrated Luminosity	300 fb <sup>-1</sup>	3000 fb <sup>-1</sup>
95% CL exclusion limit (ATLAS [8])	6.5 TeV	7.8 TeV
5 $\sigma$ discovery limit (CMS [9])	5.1 TeV	6.2 TeV

Table 1: Projected sensitivity, at  $\sqrt{s} = 14$  TeV, for the exclusion and discovery of a  $Z'$  gauge boson with standard-model couplings.

cross sections, by the rarity of, or low efficiency for the signal, by large backgrounds, or by important systematic uncertainties. The discussion of the optimal luminosity is therefore very much dependent on the process and on what the limiting factors are in its case.

If backgrounds are negligible, the searches for rare or forbidden decays of a given particle, or for new particles with low-rate but clean signatures, will benefit linearly from an increase in luminosity.<sup>2</sup> The required amount of luminosity depends on the specific rate targets that make these specific processes interesting. No general statement can be made, and arguments such as scaling the luminosity proportionally to  $s$  do not necessarily apply.

How the discovery reach improves for low-efficiency and large-background final states, *e.g.*, searches that rely on small missing- $E_T$  signatures, is strongly affected by the detector performance. Improvements in sensitivity from increasing statistics through higher instantaneous luminosity will be limited when systematics uncertainties dominate. Clear examples appear in the projections being made for the HL-LHC. For example, Fig. 4 shows the discovery and exclusion reach for bottom squarks at the LHC, at 300 and 3000 fb<sup>-1</sup>, using  $\tilde{b} \rightarrow b\chi_0$  decays. The reduced sensitivity to final states with small missing  $E_T$  strongly limits the possible progress in the regions of parameter space corresponding to compressed mass spectra, which are shown to the right of the "forbidden" line on this plot.

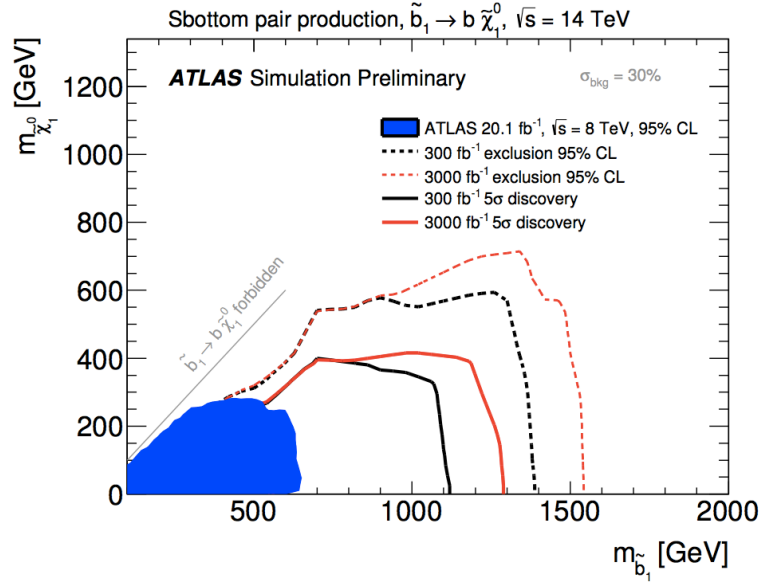


Figure 4: Projected evolution with luminosity of the exclusion and discovery reach for bottom squarks at the LHC [10].

Another example is given in Fig. 5, showing the luminosity evolution of the discovery reach at 100 TeV for top squarks. The upper mass reach goes from 6 to 8 TeV for  $L = 3 \rightarrow 30$  ab<sup>-1</sup>, consistent with the statistical scaling shown in Fig. 3. The coverage in the rest of the  $(m_{\tilde{t}}, m_{\tilde{\chi}^0})$  plane does not grow as rapidly. It might be improved by further optimization of the analyses, and improvements in

<sup>2</sup>Examples could include pair production of doubly-charged Higgses, decaying to final states like  $e^+e^+\mu^-\mu^- + X$ , or FCNC top decays such as  $t \rightarrow cH$ , with  $H \rightarrow \gamma\gamma$  or  $\mu^+\mu^-$ .

detector-performance.

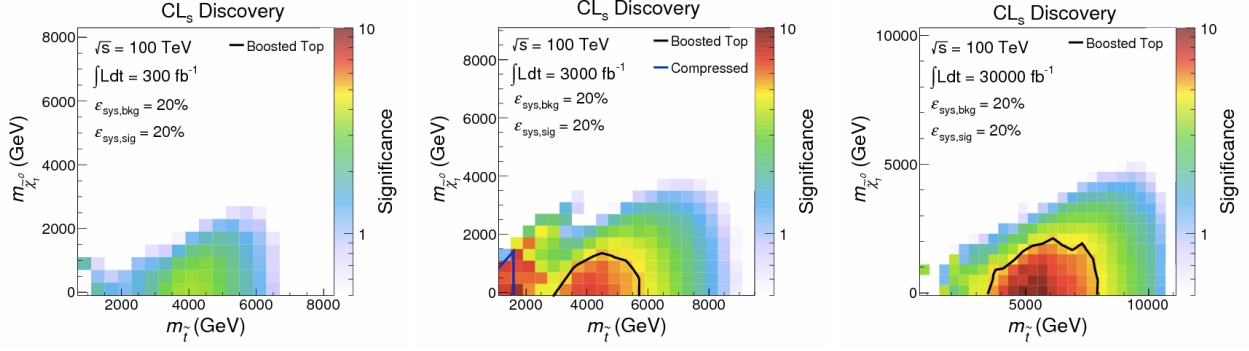


Figure 5: Top squark signal efficiency at 100 TeV, with 0.3, 3 and 30  $\text{ab}^{-1}$  (left to right, respectively), from Ref. [13]

These examples show that, for the exploration of physics at mass scales well below the kinematic limit, no generic scaling argument for luminosity can be given. In particular, for mass scales that are accessible to the LHC, one should recall that the increase in energy to 100 TeV will by itself lead to a substantial increase in production rates.

### 2.3 Precision studies of particles accessible to the LHC

If the LHC discovers new particles during its future runs, the production rates may not be sufficient to provide adequate precision in the determination of their properties. The 100-TeV collider should then aim to become a “factory” environment for these studies.

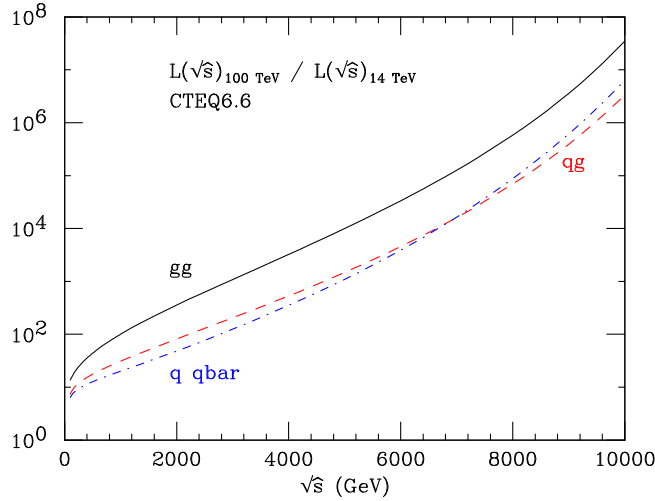


Figure 6: Ratio of partonic luminosities at 100 and 14 TeV, as a function of partonic center-of-mass energy  $\sqrt{\hat{s}}$ , for different partonic initial states. PDF set CTEQ6.6 [14],  $Q^2 = \hat{s}$ .

Consider, for example, particles at the upper limit of the HL-LHC discovery range, for example a gauge boson of mass around parton subenergy  $\sqrt{\hat{s}} = 6$  TeV produced singly in the  $q\bar{q}$  channel, or pair production of  $\sim 3$  TeV particles in the  $gg$  channel. Figure 6 shows the partonic luminosity ratios for various initial-state production channels ( $gg$ ,  $q\bar{q}$ ,  $q q$ ). In particular, in the cases at hand of  $q\bar{q}$  and  $gg$  we obtain a cross-section increase of  $10^4$  and  $10^5$ , respectively. When accompanied by an increase in integrated luminosity by a factor of  $\sim 10$ , this implies event samples up to a million times larger.

Process	$gg \rightarrow H$	$q\bar{q} \rightarrow WH$	$q\bar{q} \rightarrow WH$	$qq \rightarrow qqH$	$gg/q\bar{q} \rightarrow t\bar{t}H$	$gg \rightarrow HH$
$\sigma(100 \text{ TeV})/\sigma(14 \text{ TeV})$	14.7	9.7	12.5	18.6	61	42

Table 2: Ratio of cross sections at  $\sqrt{s} = 100 \text{ TeV}$  relative to  $\sqrt{s} = 14 \text{ TeV}$  for various Higgs production processes [15].

In the case of lighter particles, e.g., 1 TeV for a resonance in the  $q\bar{q}$  channel or 500 GeV for pair production in the  $gg$  channel, the rate increase due to the partonic luminosities is a factor of approximately 100. Once again, at low values of  $\sqrt{\hat{s}}/s$ , an increase in luminosity by an order of magnitude may be as advantageous as an increase in energy by a factor of seven. At high values of  $\sqrt{\hat{s}}$  there is a decisive advantage to increasing  $\sqrt{s}$ .

## 2.4 Study of Higgs-boson properties

The Higgs-boson inclusive production rate increases from 14 to 100 TeV by a factor in the range of 10–60, depending on the specific production process (see Table 2). These factors, together with the improvements in the theoretical systematics and the detector performance that one can confidently anticipate over the next 30 years, are large enough to promise an important improvement in the precision with which the Higgs properties can be studied at 100 TeV, even with a luminosity comparable to that of the LHC. It will be particularly true of channels such as associated production with top quarks,  $gg \rightarrow t\bar{t}H$ , and Higgs pair production in gluon fusion,  $gg \rightarrow HH$ , where the rate increases are the largest (60 and 40, respectively).

In the case of single Higgs production, detailed studies of the actual precision reach are lacking, and it is not possible at this time to anticipate the luminosity values at which systematic uncertainties will start to dominate. Preliminary studies [16–18] are however available for  $HH$  pair production, which will still be very poorly probed after completion of the HL-LHC program. A prime goal of  $HH$  studies is to extract the Higgs-boson self-coupling with a precision of 5% of the standard-model expectation. The preliminary studies suggest that this goal can be reached with  $30 \text{ ab}^{-1}$ , through the measurement of the cross section for Higgs pairs in the channel  $HH \rightarrow b\bar{b}\gamma\gamma$ .

## 3 Minimum goals for luminosity

Experience shows that no collider ever starts at the ultimate luminosity. It is interesting, therefore, to evaluate what minimum luminosity threshold opens the door on possible discoveries at 100 TeV.

If we consider dijet production as a probe of the shortest distances, we can extract a reference luminosity target from Fig. 7, which shows the leading-order cross section to produce central dijet pairs as a function of their invariant mass. The LHC has a sensitivity at the level of 1 event per  $\text{ab}^{-1}$  for dijet masses above  $\sim 9.5 \text{ TeV}$ . At this mass, the 100 TeV cross section is 6 orders of magnitude larger, which means that the HL-LHC sensitivity can be recovered within  $1 \text{ pb}^{-1}$ , i.e., in less than a day of running at a luminosity of  $10^{32} \text{ cm}^{-2}\text{s}^{-1}$ . The sensitivity to a mass range twice as large, 19 TeV, would require  $50 \text{ pb}^{-1}$ , namely of the order of one month at  $10^{32} \text{ cm}^{-2}\text{s}^{-1}$ , and one year of running at this luminosity would give us events with dijet mass well above 25 TeV.

If we consider particles just outside the possible discovery reach of the HL-LHC, which therefore the LHC could not have discovered, we find rate increases in the range of  $10^4$ – $10^5$  that we discussed earlier, for  $q\bar{q}$  and  $gg$  production channels, respectively. This means that integrated luminosities in the range of  $0.1$ – $1 \text{ fb}^{-1}$  are sufficient to push the discovery reach beyond what the HL-LHC has already explored. This can be obtained with initial luminosities as small as  $2 \times 10^{32} \text{ cm}^{-2}\text{s}^{-1}$ .

Finally, we project in Fig. 8 the temporal evolution of the expansion of discovery reach for various luminosity scenarios, relative to the reach of  $3 \text{ ab}^{-1}$  at 14 TeV. The left (right) plot shows results for a resonance whose couplings allow discovery at HL-LHC up to 6 TeV (1 TeV). Once again, we

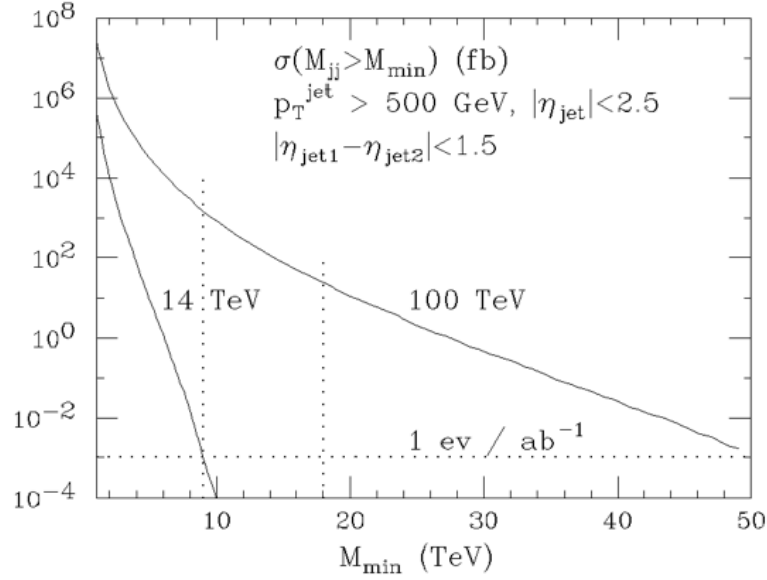


Figure 7: Cross sections for the production of dijet pairs with invariant mass  $M_{jj} > M_{\min}$ , at c.m. energies  $\sqrt{s} = 14$  and 100 TeV. The jets are subject to the  $p_T$  and  $\eta$  cuts shown in the legend.

notice that the benefit of luminosity is more prominent at low mass than at high mass. We also notice that, considering the multi-year span of the programme, and assuming a progressive increase of the luminosity integrated in a year, an early start at low luminosity does not impact significantly the ultimate reach after a fixed number of years.

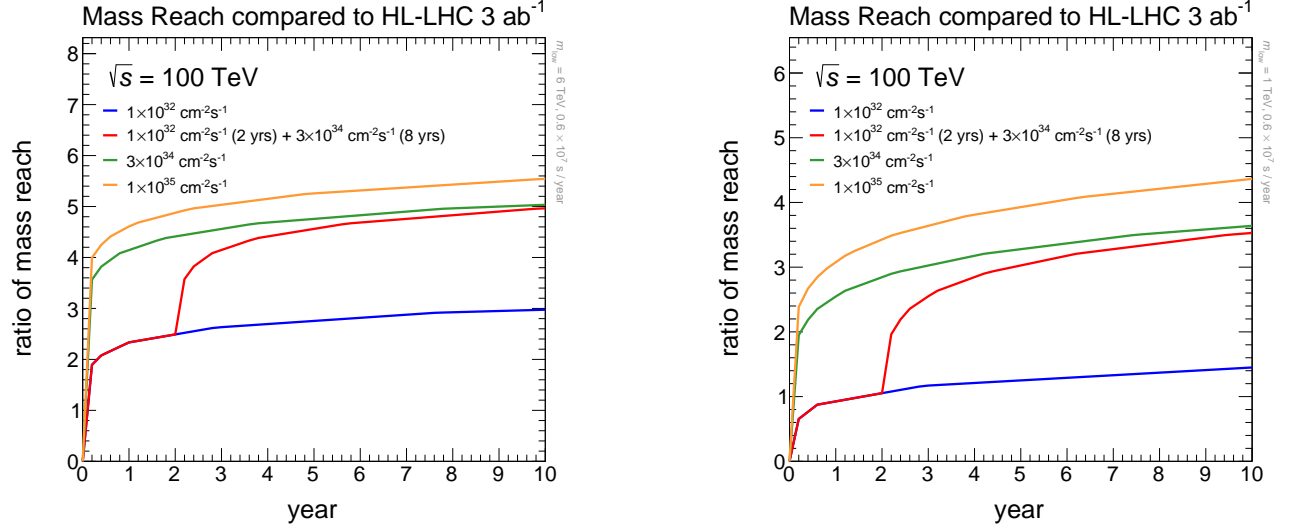


Figure 8: Evolution with time of the mass reach at  $\sqrt{s} = 100$  TeV, relative to HL-LHC, under different luminosity scenarios (1 year =  $6 \times 10^6$  sec). The left (right) plot shows the mass increase for a  $(q\bar{q})$  resonance with couplings enabling HL-LHC discovery at 6 TeV (1 TeV).

These results are not an argument for modest luminosity as an ultimate goal, but a reminder of the advantages of high collider energy. Should specific very-high-mass targets arise, the overall optimization of energy and luminosity need not be restricted to a single parameter.



## 4 Recommendations

The goal of an integrated luminosity in the range of 10-20  $\text{ab}^{-1}$  per experiment, corresponding to an ultimate instantaneous luminosity approaching  $2 \times 10^{35} \text{ cm}^{-2}\text{s}^{-1}$  [1], seems well-matched to our current perspective on extending the discovery reach for new phenomena at high mass scales, high-statistics studies of possible new physics to be discovered at (HL)-LHC, and incisive studies of the Higgs boson's properties. Specific measurements may set more aggressive luminosity goals, but we have not found generic arguments to justify them. The needs of precision physics arising from new physics scenarios to be discovered at the HL-LHC, to be suggested by anomalies observed during the  $e^+e^-$  phase of a future circular collider, or to be discovered at 100 TeV, may well drive the need for even higher statistics. Such requirements will need to be established on a case-by-case basis, and no general scaling law gives a robust extrapolation from 14 TeV. Further work on *ad hoc* scenarios, particularly for low-mass phenomena and elusive signatures, is therefore desirable.

For a large class of new-physics scenarios that may arise from the LHC, less aggressive luminosity goals are acceptable as a compromise between physics return and technical or experimental challenges. In particular, even luminosities in the range of  $10^{32} \text{ cm}^{-2}\text{s}^{-1}$  are enough to greatly extend the discovery reach of the 100 TeV collider over that of the HL-LHC, or to enhance the precision in the measurement of discoveries made at the HL-LHC.

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## A Scaling relations

The cross section  $\sigma$  is

$$\begin{aligned}\sigma &\sim L_p \cdot \hat{\sigma} \\ &\sim \frac{1}{\tau^a} \hat{\sigma},\end{aligned}\tag{1}$$

where  $\hat{\sigma}$  is the partonic cross section, and we have assumed that the parton luminosity  $L_p$  falls as a power law with increasing  $\tau = \hat{s}/s$ . In the signal process where the new physics particle mass scale is  $M$ , we will further assume that

$$\hat{\sigma} \propto \frac{1}{M^2}.\tag{2}$$

Next, we consider two different colliders with center of mass energies  $\sqrt{s_1}$  and  $\sqrt{s_2}$ , with integrated  $pp$  luminosity  $\mathcal{L}_1$  and  $\mathcal{L}_2$ , respectively. We assume the mass reaches of new physics at those two colliders are  $M_1$  and  $M_2$ , respectively. The corresponding parton fractions are  $\tau_i = M_i^2/s_i$ , ( $i = 1, 2$ ). Assuming

that the reach is obtained by the same number of signal events, we have

$$\frac{1}{\tau_1^a} \frac{1}{M_1^2} \mathcal{L}_1 = \frac{1}{\tau_2^a} \frac{1}{M_2^2} \mathcal{L}_2, \quad (3)$$

which means

$$\frac{M_2}{M_1} = \left( \frac{s_2}{s_1} \right)^{\frac{a}{2a+2}} \left( \frac{\mathcal{L}_2}{\mathcal{L}_1} \right)^{\frac{1}{2a+2}}. \quad (4)$$

For large  $a$ , this means energy is more important, and the gain with luminosity can be quite slow. In particular, if we require  $M_2/M_1 = E_2/E_1$ , we need  $\mathcal{L}_2 = (E_2/E_1)^2 \mathcal{L}_1$ , as emphasized in Refs. [3, 4]. However, this slow gain with luminosity also means that one would not lose too much mass reach by going to a much lower luminosity. As demonstrated here, this is ultimately due to the fact that the parton luminosity is steeply falling, in particular near the edge of the kinematical reach of a collider. The gain with luminosity is more important for smaller  $\alpha$  or lower  $\tau$  (lower mass).

Figure 9: The dependence of power  $a$  on mass scale  $M = \sqrt{\hat{s}} = \sqrt{s\tau}$

Some obvious approximations are made here. First of all, we ignored anomalous scaling. We also assumed that for the relevant range of  $\tau$ ,  $a$  remains approximately constant. This is certainly not true for full range of  $\tau$ . However,  $a$  does not vary too steeply with  $\tau$ , see Fig. 9. For comparing reaches, we often consider similar values of  $\tau$ .

Next we consider the gain luminosity with the same collider, i.e.,  $E_1 = E_2$ . We have

$$\frac{M_2}{M_1} = \exp \left( \frac{1}{2a+2} \log(\mathcal{L}_2/\mathcal{L}_1) \right) \simeq 1 + \frac{1}{2a+2} \log(\mathcal{L}_2/\mathcal{L}_1), \quad (5)$$

or

$$M_2 - M_1 \simeq \frac{M_1}{2a+2} \log(\mathcal{L}_2/\mathcal{L}_1) \quad (6)$$

For example, considering  $q\bar{q}$  initial state, around  $M_1 \simeq 40$  TeV,  $a \simeq 5.5$  (from Fig. 9), we have approximately

$$M_2 - M_1 \sim (7 \text{ TeV}) \times \log_{10}(\mathcal{L}_2/\mathcal{L}_1) \quad (7)$$

At the same time, for lower mass  $M_1 \simeq 20$  TeV,  $a \simeq 3$ , we have instead

$$M_2 - M_1 \sim (5.5 \text{ TeV}) \times \log_{10}(\mathcal{L}_2/\mathcal{L}_1) \quad (8)$$

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