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Suppressing Transverse Beam Halo with Nonlinear Magnetic Fields

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High intensity proton storage rings are central for the development of advanced neutron sources, drivers for the production of pions in neutrino factories or muon colliders, and transmutation of radioactive waste. Fractional proton loss from the beam must be very small to prevent radioactivation of nearby structures, but many sources of beam loss are driven by collective effects that increase with intensity. Recent theoretical work on the use of nonlinear magnetic fields to design storage rings with integrable transverse dynamics is extended here to include collective effects, with numerical results showing validity in the presence of very high beam current. Among these effects is the formation of beam halo, where particles are driven to large amplitude oscillations by coherent space charge forces. The strong variation of particle oscillation frequency with amplitude results in nonlinear decoherence that is observed to suppress transverse halo development in the case studied. We also present a necessary generalization of the Kapchinskij-Vladimirskij equilibrium distribution, which was introduced over 50 years ago for modeling linear dynamics in particle accelerators.

When a high-intensity storage ring, such as the Spallation Neutron Source (SNS), is designed, heavy emphasis is placed upon avoiding beam loss due to space charge driven instabilities [1–4]. GeV scale proton beams must be stored with no more than 1 W/m loss of particles to the beam pipe to avoid unsafe levels of activation. Thus, finding novel ways to mitigate beam loss to space charge driven effects is crucial for pushing the intensity frontier.

Because the forces that steer and store a beam in an accelerator are mostly linear with some nonlinearities, a dense set of resonances exist that must be avoided during operation. It is well-understood that intense beams can shift the designed betatron frequency of a beam over one of these resonances [5–7]. Furthermore, in a linear lattice of steering and focusing magnets, a beam that is mismatched will undergo breathing oscillations, which can resonantly drive particles in the beam core to a halo surrounding the beam [8–10]. This halo can strike the beam pipe, causing activation.

For linear dynamics there is a single frequency for the single particle trajectories, which can lead to resonances. A periodic forcing remains in phase with the oscillations, thereby depositing energy into the particles and driving them to larger amplitudes. Nonlinear systems generally have amplitude-dependent frequencies, which can mitigate resonances by shifting the frequency as amplitude increases, so particles do not stay in resonance very long. However, nonlinear potentials can cause poorly confined trajectories and undesirable increases in the beam size or decrease in beam lifetime.

Despite these shortcomings, the "nonlinear decoherence" that arises when amplitude-dependent frequency shifts become large enough could mitigate many of the instabilities and resonances that prevent pushes into increasing beam intensity. A recent innovation to achieve this strong nonlinear decoherence [11–13] shows how to use highly nonlinear elements which lead to integrable or chaotic bounded single particle trajectories in zero-current. In this letter, we consider the integrable elliptic potential described in Sec. V.A.[11], although we observed similar results for the octupole potential in Sec. IV. The use of nonlinear decoherence in a nearly integrable configuration has been proposed [14] and shown to reduce beam halo [15]. We consider a highly nonlinear integrable lattice design in the presence of space charge for the first time, and observe beam halo suppression. We also show how to generalize the Kapchinskij-Vladimirskij (KV) distribution, first described in [5] for linear lattices, for these nonlinear lattices.

For most high intensity applications, the stored beam will be substantially longer than it is wide. Furthermore, the longitudinal synchrotron oscillations are much slower than the transverse betatron oscillations. We therefore test these nonlinear lattice concepts with a two dimensional model which neglects the longitudinal motion of the particles in the beam, as well as any longitudinal space charge forces. These effects will be the subject of future study.

Previous work [16] shows that a KV distribution that is not matched to the lattice properly combined with another low-density will drive particles in a properly matched KV distribution (pre-halo) into the halo rapidly. The origin of this behavior can be explained as follows – in a linear lattice, a mismatched beam will rotate rigidly in the $x - p_x$ and $y - p_y$ plane at the frequency of betatron oscillations (the *betatron tune*). This leads to the projection in the $x - y$ plane expanding and contracting at twice the betatron tune. In a KV distribution,

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the space charge forces are linear, and the pre-halo particles which are not undergoing these rotations see a resonantly oscillating linear force that pushes them to the outside of the beam. Once they are outside the beam, the space charge forces are nonlinear, and they settle into large radius nonlinear oscillations. This is a direct result of the core undergoing simple harmonic motion in the transverse phase space coordinates, which manifests as breathing modes in the $x - y$ plane. This result for the KV distribution is well-known [9]. To compare this result to the nonlinear lattices properly, we develop a generalization of the classic KV distribution to deal with these new lattices.

Any function of the invariants of motion form a stationary solution to the Vlasov equation [17, 18]. In 1958, Courant and Snyder derived their eponymous invariant [19] for the piecewise linear magnetic fields that are used in every large accelerator today. The KV distribution is taken to be a delta function

$$f_{KV}(\vec{p}_N, \vec{q}_N) = \delta\left[I(\vec{p}_N, \vec{q}_N) - \varepsilon_0\right] \tag{1}$$

where I is the Courant-Snyder invariant,

$$I = \gamma \vec{q}_N^2 + 2\alpha \vec{q}_N \cdot \vec{p}_N + \beta \vec{p}_N^2 \tag{2}$$

which has the form of an harmonic oscillator Hamiltonian. Here α, β, and γ are the *Twiss parameters* (see, e.g., Chapter 2 of [20]) that vary at each point around the accelerator ring, and q_N and p_N are the normalized transverse coordinate and momentum given by

$$\vec{q}_N = \vec{q}/\sqrt{\beta} \qquad \vec{p}_N = \vec{p}\sqrt{\beta} - \beta' \vec{q}/(2\sqrt{\beta}) \tag{3}$$

where here q is either x or y, and p is either p_x or p_y. The avenue to generalizing the above KV distribution would be to find another, similar function that reduces to the Courant-Snyder invariant for a purely linear lattice when the nonlinear potential strengths are zero.

It was shown in [11] that for a careful choice of nonlinear potential, the Hamiltonian

$$H = \frac{1}{2}(p_{xN}^2 + p_{yN}^2) + \frac{1}{2}(x_N^2 + y_N^2) + U(x_N, y_N) \tag{4}$$

is independent of time, and is therefore a conserved quantity like the Courant-Snyder invariant. The above H is an invariant of the motion and allows for a treatment similar to the KV distribution. The generalized KV distribution given by

$$f = \delta\big[H(\vec{p}_N, \vec{q}_N) - \varepsilon_0\big] \tag{5}$$

is a stationary solution for the Vlasov equation. Here, ε_0 reduces to the geometric transverse emittance in the linear limit. Like the original KV distribution, the generalized KV distribution fills a 4D shell of phase space uniformly, defined by ε_0, and because of its functional form is isotropic in $\vec{p}_N$. However, while the original KV distribution uniformly fills every 2D projection of phase space as uniform ellipses, the generalized KV distribution fills more exotic shapes due to the additional $U(x_N, y_N)$ potential.

To test these ideas numerically, we use the PyORBIT software [21] and consider a long, high-current proton beam in a model periodic focusing lattice. This lattice consists of a 2 meter drift capped by a thin lens effective element that is focusing in both planes (see Fig.'s 1 and 2 of [12] for a discussion of this element). It is necessary to have this to have equal beta functions in the horizontal and vertical plane to satisfy one of the conditions in [11]. In the absence of the elliptic potential, we consider this linear lattice with a fractional betatron tune of 0.3. In the zero-current limit, the beta function is known. For finite current, the focusing strength is increased, on the order of 1%, to keep the beta function periodic and with a constant minimum value to restrict the maximum radius of the beam. As a consequence, this tends to keep the betatron tune fixed.

For the integrable elliptic lattice (IEL), we fill the drift with the properly scaled elliptic magnet element described in [11] with a normalized strength 60% of the strength at which linear confinement in the vertical direction is lost. All other parameters are kept the same, although the quadrupole term in the elliptic element causes the betatron tune to increase in the horizontal plane and decrease in the vertical plane.

For the linear lattice with pre-halo, we observed the onset of halo formation within 500 passes (see Fig. 1). Here, since all the particles have almost the same frequency of oscillation regardless of amplitude, the pre-halo particles remain in resonant interaction with the periodic space charge forces created by the breathing mode in the transverse $x - y$ plane, as described in [9]. The criterion to be counted in the halo was that a particle be two RMS outside the average momentum or position, and these particles are depicted with blue dots.

Using the same parameters but including the elliptic element, the IEL with matched beam saw some transient motion as the beam equilibrated with space charge forces, but no halo developed (see Fig. 2). The reason for this is two-fold. First, because the initially mismatched core beam experiences nonlinear decoherence, its initial mismatch does not lead to a coherent breathing mode as with the linear case. This was observed numerically. The second reason is that, whatever breathing mode periodicity might occur, the pre-halo particles are only in resonance over a small range of amplitudes, after which they fall out of resonance.

This interpretation is supported by the tune diagram for the individual particle Poincaré surfaces of section. By taking the Fourier transform of the x and y positions as a function of time, we are able to extract the tune information for given particles in the pre-halo. A sam-

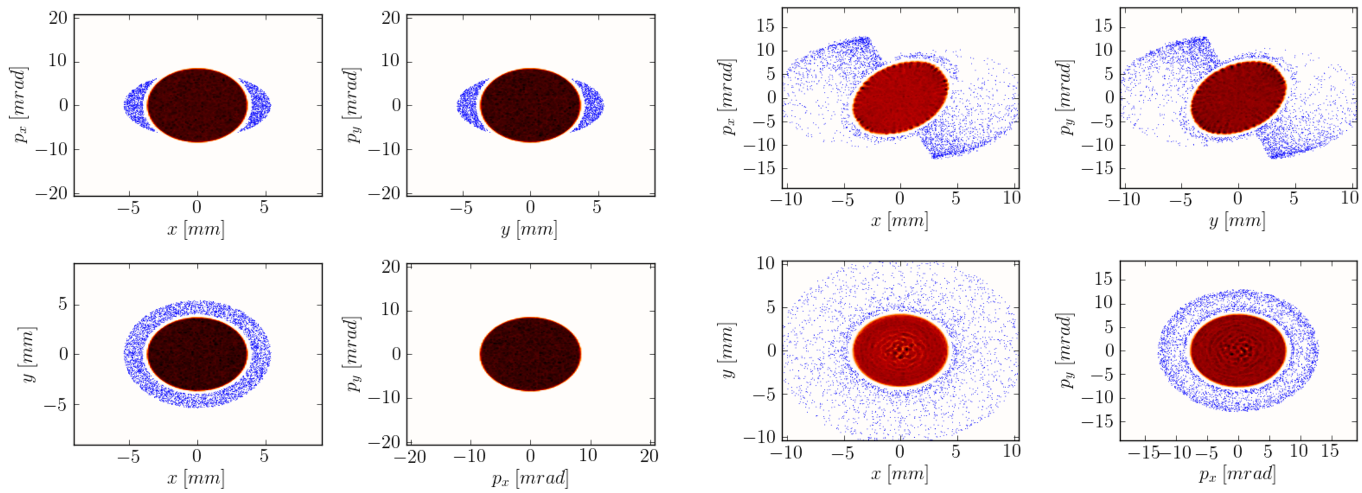

FIG. 1: Histogram plots of the 2D phase space projections initially (left) and after 500 passes (right) for the linear lattice. Blue dots indicate particles outside of 2 RMS beam radius. The pre-halo indicated by the blue dots uniformly fills the projections and accounts for 1% of the total beam current.

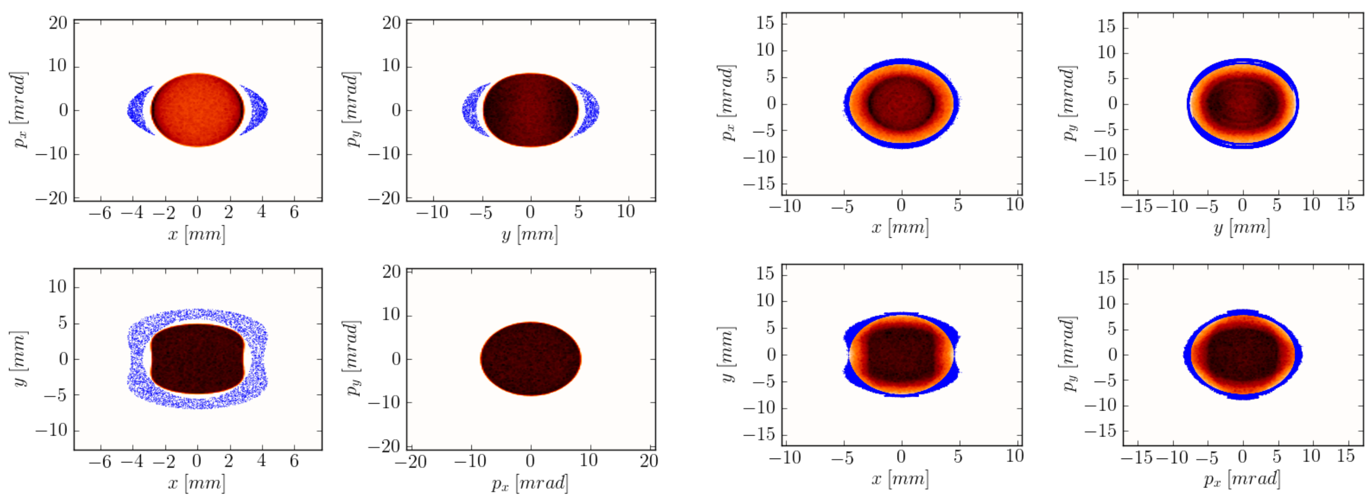

FIG. 2: Histogram plots initially (left) and after 500 passes (right) for the IEL. Note the hourglass shape of the properly matched IEL beam.

ple set of Poincaré surfaces of section for five particles is shown in figure 3.

By appearance this would seem to indicate that space charge has broken the integrability of the trajectories, but they remain bounded. The exact details of this plot are difficult to divine in real space, but in Fourier space it is transparent. The tune diagram in figure 4 shows that particles in the IEL, even with similar amplitudes for their nonlinear oscillations, have different frequencies of motion, and in many cases have relatively strong subharmonics. Therefore, if space charge drives a particle from one amplitude to another in the IEL, its oscillations will have a different tune. We observed similar effects in the chaotic bounded octupole lattice and nearly-integrable FODO lattice cases, and will elaborate on this in future publications.

The recent work in [11] has developed a new paradigm for designing highly-nonlinear particle accelerator lattices that simultaneously demonstrate strong frequency shift with amplitude and integrable two-dimensional single-

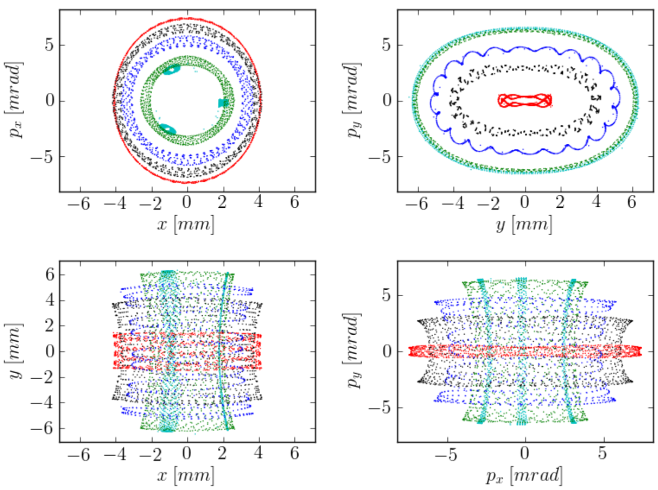

FIG. 3: Poincaré surfaces of section for five test particles after 1,000 passes.

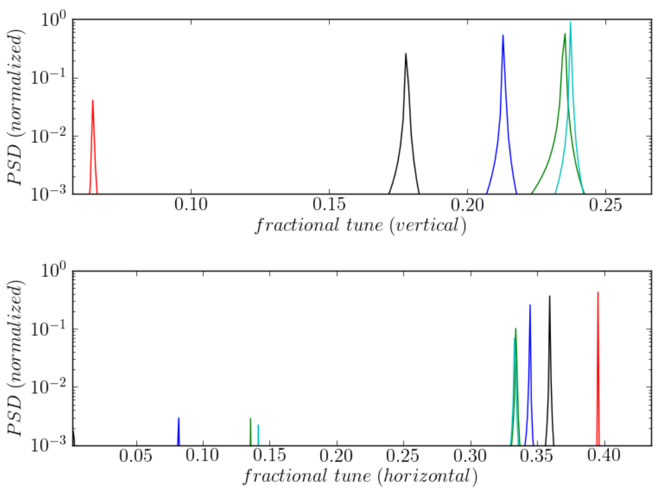

FIG. 4: Power spectral density for the nonlinear oscillations of the Poincaré particles in figure 3.

particle dynamics. For this integrable lattice, we have shown that this paradigm is applicable for long bunches with currents a thousand times greater than the current state-of-the-art in proton storage rings. The combination of strong amplitude-dependent frequency shift and stable long-time dynamics provides the nonlinear decoherence that suppresses potentially harmful resonances. In particular, we showed above that for a mismatched beam that drives nearby particles out into a halo via the particle-core model [9, 16] that the integrable lattice suppresses the core oscillations and prevents the halo from forming. This work resulted in a nonlinear generalization of the KV particle distribution that enables the creation of uniformly-filled beams that are well-matched to this new type of nonlinear lattice. This work shows a path towards future high-intensity hadron accelerators with order-of-magnitude lower beam loss.

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[1] J. A. Holmes, V. Danilov, J. Galambos, D. Jeon, and D. K. Olsen, Phys. Rev. ST-AB **2** (1999).
[2] J. Wei *et al.*, Phys. Rev. ST-AB **3** (2000).
[3] J. Wei, Rev. Mod. Phys. **75** (2003).
[4] S. Henderson *et al.*, "The Spallation Neutron Source Accelerator System Design," (2012), Submitted for publication (Nucl. Instrum. Methods).
[5] I. M. Kapchinskij and V. V. Vladimirskij, in *Proc. of Int'l. Conf. on High Energy Acc.* (CERN, 1959) pp. 274–288.
[6] F. Sacherer, *Transverse space-charge effects in circular accelerators*, Ph.D. thesis, University of California, Berkeley (1968).
[7] L. J. Laslett, in *Summer Study on Storage Rings, BNL Report* (1963) pp. 324–367.
[8] J. O'Connell, T. Wangler, R. Mills, and K. Crandall, in *Proc. of Part. Accel. Conf.* (1993) p. 3657.
[9] R. Gluckstern, Phys. Rev. Lett. **73**, 1247 (1994).
[10] R. Jameson, *Self-Consistent Beam Halo Studies & Halo Diagnostic Development in a Continuous Linear Focusing Channel*, Tech. Rep. LA-UR-94-3753 (Los Alamos National Laboratory, 1994).
[11] V. Danilov and S. Nagaitsev, Phys. Rev. ST-AB **13** (2010).
[12] S. Nagaitsev, A. Valishev, and V. Danilov, in *Proceedings of HB2010*, THO1D01 (2010).
[13] S. Nagaitsev and V. Danilov, in *Proceedings of IPAC '10*, THPE094 (2010).
[14] K. G. Sonnad and J. R. Cary, Phys. Rev. E **69** (2004).
[15] K. G. Sonnad and J. R. Cary, Phys. Rev. ST-AB **8** (2005).
[16] D. L. Bruhwiler, in *AIP Conf. Proc.*, Vol. 377 (1995) pp. 219–233.
[17] D. Nicholson, *Introduction to Plasma Theory* (J. Wiley & Sons, 1983).
[18] N. Krall and A. Trivelpiece, *Principles of Plasma Physics* (San Francisco Press, 1986).
[19] E. D. Courant and H. S. Snyder, Ann. Phys. **3**, 1 (1958).
[20] S. Y. Lee, *Accelerator Physics* (World Scientific, 2004).
[21] A. Shishlo, J. A. Holmes, and T. Gorlov, in *Proc. of ICAP 2009*, THPSC052 (2009).