

# The effect of the geomagnetic field on cosmic ray energy estimates and large scale anisotropy searches on data from the Pierre Auger Observatory

## The Pierre Auger Collaboration

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## Abstract

We present a comprehensive study of the influence of the geomagnetic field on the energy estimation of extensive air showers with a zenith angle smaller than  $60^\circ$ , detected at the Pierre Auger Observatory. The geomagnetic field induces an azimuthal modulation of the estimated energy of cosmic rays up to the  $\sim 2\%$  level at large zenith angles. We present a method to account for this modulation of the reconstructed energy. We analyse the effect of the modulation on large scale anisotropy searches in the arrival direction distributions of cosmic rays. At a given energy, the geomagnetic effect is shown to induce a pseudo-dipolar

pattern at the percent level in the declination distribution that needs to be accounted for.

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## 1. Introduction

High energy cosmic rays generate extensive air showers in the atmosphere. The trajectories of the charged particles of the showers are curved in the Earth's magnetic field, resulting in a broadening of the spatial distribution of particles in the direction of the Lorentz force. While such effects are known to distort the particle densities in a dramatic way at zenith angles larger than  $\sim 60^\circ$  [1–4], they are commonly ignored at smaller zenith angles where the lateral distribution function is well described by empirical models of the NKG-type [5, 6] based on a radial symmetry of the distribution of particles in the plane perpendicular to the shower axis.

In this article, we aim to quantify the small changes of the particle densities at ground induced by the geomagnetic field for showers with zenith angle smaller than  $\sim 60^\circ$ , focusing on the impacts on the energy estimator used at the Pierre Auger Observatory. As long as the magnitude of these effects lies well below the statistical uncertainty of the energy reconstruction, it is reasonable to neglect them in the framework of the energy spectrum reconstruction. As the strength of the geomagnetic field component perpendicular to the arrival direction of the cosmic ray,  $B_T$ , depends on both the zenith and the azimuthal angles  $(\theta, \varphi)$  of any incoming shower, these effects are expected to break the symmetry of the energy estimator in terms of the azimuthal angle  $\varphi$ . Such an azimuthal dependence translates into azimuthal modulations of the estimated cosmic ray event rate at a given energy. For any observatory located far from the Earth's poles, any genuine large scale pattern which depends on the declination translates also into azimuthal modulations of the cosmic ray event rate. Thus to perform a large scale anisotropy measurement it is critical to account for azimuthal modulations of experimental origin and for those induced by the geomagnetic field, as already pointed out in the analysis of the Yakutsk data [7]. Hence, this work constitutes an accompanying paper of a search for large scale anisotropies, both in right ascension and declination of cosmic rays detected at the Pierre Auger Observatory, the results of which will be reported in a forthcoming publication.

To study the influence of the geomagnetic field on the cosmic ray energy estimator, we make use of shower simulations and of the measurements performed with the surface detector array of the Pierre Auger Observatory, located in Malargüe, Argentina ( $35.2^\circ\text{S}$ ,  $69.5^\circ\text{W}$ ) at 1400 m a.s.l. [8]. The Pierre Auger Observatory is designed to study cosmic rays (CRs) with energies above  $\sim 10^{18}$  eV. The surface detector array consists of 1660 water Cherenkov detectors sensitive to the photons and the charged particles of the showers. It is laid out over an area of  $3000\text{ km}^2$  on a triangular grid and is overlooked by four fluorescence detectors. The energy at which the detection efficiency of the surface detector array saturates is  $\sim 3\text{ EeV}$  [9]. For each event, the signals recorded in the stations are fitted to find the signal at 1000 m from the shower core,  $S(1000)$ , used as a measure of the shower size. The shower size  $S(1000)$  is converted to the value  $S_{38}$  that would have been expected had the shower arrived at a zenith angle of  $38^\circ$ .  $S_{38}$  is then converted into energy using a calibration curve based on the fluorescence telescope measurements [10].

The influence of the geomagnetic field on the spatial distribution of particles for showers with zenith angle less than  $60^\circ$  is presented in Section 2, through a toy model aimed

43 at explaining the directional dependence of the shower size  $S(1000)$  induced by the geo-  
 44 magnetic field. The observation of this effect in the data of the Pierre Auger Observatory  
 45 is reported in Section 3. In Section 4, we quantify the size of the  $S(1000)$  distortions  
 46 with zenith and azimuthal angles by means of end-to-end shower simulations, and then  
 47 present the procedure to convert the shower size corrected for the geomagnetic effects into  
 48 energy using the Constant Intensity Cut method. In Section 5, the consequences on large  
 49 scale anisotropies are discussed, while systematic uncertainties associated with the primary  
 50 mass, the primary energy and the number of muons in showers are presented in Section 6.

## 51 **2. Influence of the geomagnetic field on extensive air showers**

52 The interaction of a primary cosmic ray in the atmosphere produces mostly charged  
 53 and neutral pions, initiating a hadronic cascade. The decay of neutral pions generates the  
 54 electromagnetic component of the shower, while the decay of the charged pions generates  
 55 the muonic one. Electrons undergo stronger scattering, so that the electron distribution is  
 56 only weakly affected by the geomagnetic deflections. Muons are produced with a typical  
 57 energy  $E_\mu$  of a few GeV (increasing with the altitude of production). The decay angle  
 58 between pions and muons is causing only a small additional random deflection, as they  
 59 almost inherit the transverse momentum  $p_T$  of their parents (a few hundred MeV/ $c$ ) so that  
 60 the distance of the muons from the shower core scales as the inverse of their energy. While  
 61 the radial offset of the pions from the shower axis is of the order of a few 10 m, it does  
 62 not contribute significantly to the lateral distribution of the muons observed on the ground  
 63 at distances  $r \geq 100$  m. Hence, at ground level, the angular spread of the muons around  
 64 the shower axis can be considered as mainly caused by the transverse momentum inherited  
 65 from the parental pions.

66 After their production, muons are affected by ionisation and radiative energy losses,  
 67 decay, multiple scattering and geomagnetic deflections. Below 100 GeV, the muon energy  
 68 loss is mainly due to ionisation and is relatively small (amounting to about  $2 \text{ MeV g}^{-1} \text{ cm}^2$ ),  
 69 allowing a large fraction of muons to reach the ground before decaying. Multiple scattering  
 70 in the electric field of air nuclei randomises the directions of muons to some degree, but  
 71 the contribution to the total angular divergence of the muons from the shower axis remains  
 72 small up to zenith angles of the shower-axis of about  $80^\circ$ .

73 Based on these general considerations, we now introduce a simple toy model aimed at  
 74 understanding the main features of the muon density distortions induced by the geomag-  
 75 netic field. We adopt the shower front plane coordinate system depicted in Fig. 1 [2]. In  
 76 the absence of the magnetic field, and neglecting multiple scattering, a relativistic muon  
 77 of energy  $E_\mu \simeq cp_\mu$  and transverse momentum  $p_T$  will reach the shower front plane after  
 78 traveling a distance  $d$  at a position  $r$  from the shower axis given by

$$r \simeq \frac{p_T}{p_\mu} d \simeq \frac{cp_T}{E_\mu} d. \quad (1)$$

79 On the other hand, in the presence of the magnetic field, muons suffer additional geomag-

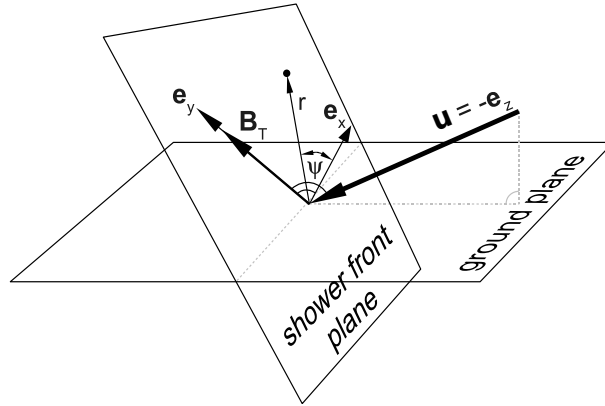


Figure 1: The shower front plane coordinate system [2, 4]:  $\mathbf{e}_z$  is anti-parallel to the shower direction  $\mathbf{u}$ , while  $\mathbf{e}_y$  is parallel to  $\mathbf{B}_T$ , the projection of the magnetic field  $\mathbf{B}$  onto the shower plane  $x$ - $y$ .  $(\psi, r)$  are the polar coordinates in the shower plane.

netic deflections. We treat the geomagnetic field  $\mathbf{B}$  in Malargüe as a constant field<sup>1</sup>,

$$B = 24.6 \mu\text{T}, \quad D_B = 2.6^\circ, \quad I_B = -35.2^\circ, \quad (2)$$

$D_B$  and  $I_B$  being the geomagnetic declination and inclination. The deflection of a relativistic muon in the presence of a magnetic field with transverse component  $B_T$  can be approximated with

$$\delta x_{\pm} \simeq \pm \frac{ecB_T d^2}{2E_{\mu}}, \quad (3)$$

where  $e$  is the elementary electric charge and the sign corresponds to positive/negative charged muons. The dependence of the geomagnetic deflections  $\delta x \equiv \delta x_+ = -\delta x_-$  on the distance to the shower axis  $r = \sqrt{x^2 + y^2}$  is illustrated in Fig. 2 obtained by comparing the position of the same muons in the presence or in the absence of the geomagnetic field in a simulated vertical shower of a proton at 5 EeV. The deviations expected from the expression for  $\delta x_{\pm}$  are also shown in the same graph (solid line). It was obtained by inserting muon energy and distance at the production point of the simulated muons into Eq. (3). It turns out that Eq. (3) estimates rather well the actual deviations, though the distance between the actual and the predicted deviations increases at large  $r$ . This is mainly because on the one hand  $d$  underestimates the actual travel length to a larger extent at larger  $r$ , while on the other hand the magnetic deviation actually increases while muons gradually lose energy during travel. Hence, from the muon density  $\rho_{\mu}(x, y)$  in the transverse plane in the absence of the geomagnetic field, the corresponding density  $\bar{\rho}_{\mu}(\bar{x}, \bar{y})$  in the presence of such a field can be obtained by making the following Jacobian transformation, in the same way as in the framework of very inclined showers [2],

$$\bar{\rho}_{\mu}(\bar{x}, \bar{y}) = \left| \frac{\partial(x, y)}{\partial(\bar{x}, \bar{y})} \right| \rho_{\mu}(x(\bar{x}, \bar{y}), y(\bar{x}, \bar{y})). \quad (4)$$

<sup>1</sup>In Malargüe the geomagnetic field has varied by about  $1^\circ$  in direction and 2% in magnitude over 10 years [11].



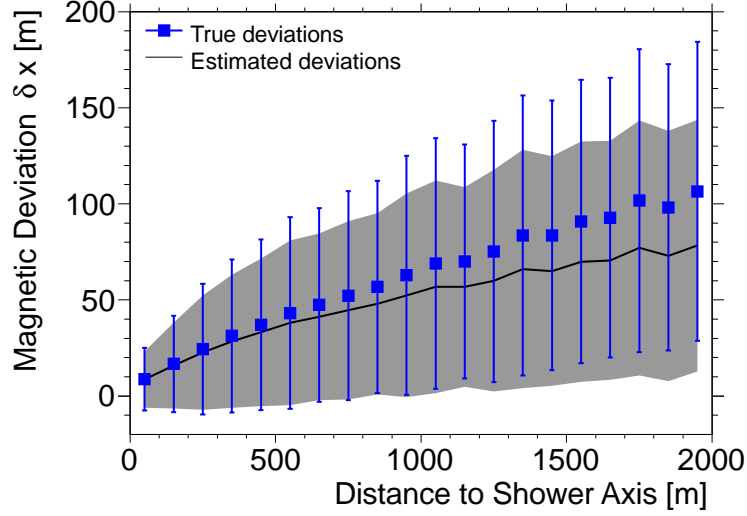


Figure 2: Magnetic deviations as a function of the distance to the shower axis observed on a simulated vertical shower (points). Superimposed are the deviations expected from Eq. (3) (line). The shaded region and the error bars give the corresponding dispersion.

99 Here, the term “muon density” refers to the time-integrated muon flux through the trans-  
 100 verse shower front plane associated to the air shower, and the barred coordinates represent  
 101 the positions of the muons in the transverse plane in the presence of the geomagnetic field:

$$\begin{aligned}\bar{x} &= x + \delta x_{\pm}(x, y), \\ \bar{y} &= y.\end{aligned}\quad (5)$$

102 Since Eq. (4) induces changes of the shower size  $S(1000)$ , it is of particular interest to get  
 103 an approximate relationship between  $\rho$  and  $\bar{\rho}$  around 1000 m. From Fig. 2, it is apparent  
 104 that around 1000 m the mean magnetic deviation is approximately constant over a distance  
 105 range larger than the size of the deviation. When focusing on the changes of density at  
 106 1000 m from the shower core, it is thus reasonable to neglect the  $x$  and  $y$  dependence of the  
 107 deviation  $\delta x_{\pm}$ , which allows an approximation of the density  $\bar{\rho}_{\mu}(\bar{x}, \bar{y})$  around 1000 m as

$$\begin{aligned}\bar{\rho}_{\mu}(\bar{x}, \bar{y}) &\simeq \rho_{\mu_+}(\bar{x} - \delta x_+, \bar{y}) + \rho_{\mu_-}(\bar{x} - \delta x_-, \bar{y}) \\ &\simeq \rho_{\mu}(\bar{x}, \bar{y}) + \frac{(\delta x)^2}{2} \frac{\partial^2 \rho_{\mu}}{\partial \bar{x}^2}(\bar{x}, \bar{y}),\end{aligned}\quad (6)$$

108 where we assumed  $\rho_{\mu_-} = \rho_{\mu_+} = \rho_{\mu}/2$ . The two opposite muon charges cancel out the  
 109 linear term in  $\delta x$  and we see that magnetic effects change the muon density around 1000 m  
 110 by a factor proportional to  $(\delta x)^2 \propto B_T^2 \propto \sin^2(\widehat{\mathbf{u}, \mathbf{b}})$ , where  $\mathbf{u}$  and  $\mathbf{b} = \mathbf{B}/|B|$  denote the  
 111 unit vectors in the shower direction and the magnetic field direction, respectively. This is  
 112 particularly important with regard to the azimuthal behaviour of the effect, as the azimuthal  
 113 dependence is contained *only* in the  $B_T^2(\theta, \varphi)$  term. This dependency is therefore a generic  
 114 expectation outlined by this toy model. The model will be verified in Section 4 by making  
 115 use of complete simulation of showers. On the other hand, the zenith angle dependence  
 116 relies on other ingredients that we will probe in an accurate way in Section 4, such as the  
 117 altitude distribution of the muon production and the muon energy distribution.

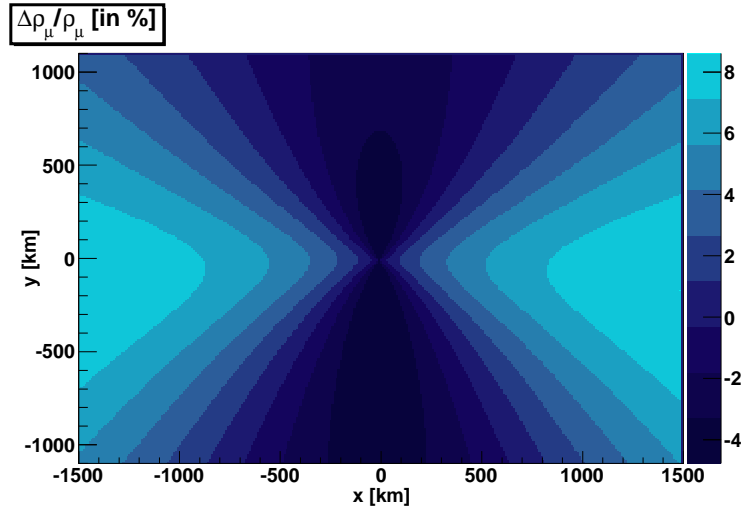


Figure 3: Relative changes of  $\Delta\rho_\mu/\rho_\mu$  in the transverse shower front plane due to the presence of the geomagnetic field, obtained at zenith angle  $\theta = 60^\circ$  and azimuthal angle aligned along  $D_B + 180^\circ$ .

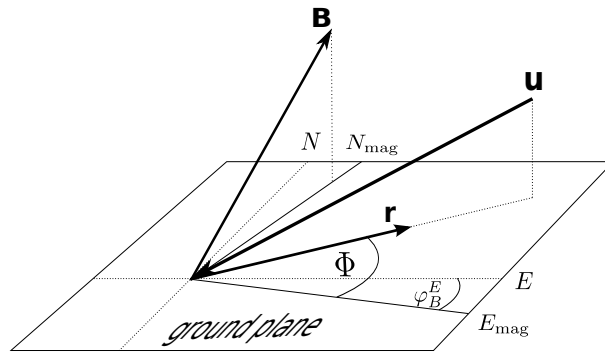


Figure 4: Definition of angle  $\Phi$  with respect to the magnetic East  $E_{\text{mag}}$  and the shower core for a given shower direction  $\mathbf{u}$  and a surface detector at  $\mathbf{r}$ . The azimuthal angle of the magnetic field vector  $\mathbf{B}$  defines the magnetic North  $N_{\text{mag}}$ .

### 118 3. Observation of geomagnetic effects in the Pierre Auger Observatory data

119 To illustrate the differences between  $\bar{\rho}_\mu$  and  $\rho_\mu$  described in Eq. (4), the relative changes  
 120  $\Delta\rho_\mu/\rho_\mu$  are shown in Fig. 3 in the transverse shower front plane by producing muon maps  
 121 from simulations at zenith angle  $\theta = 60^\circ$  and azimuthal angle aligned along  $D_B + 180^\circ$   
 122 in the presence and in the absence of the geomagnetic field. A predominant quadrupolar  
 123 asymmetry at the few percent level is visible, corresponding to the separation of positive  
 124 and negative charges in the direction of the Lorentz force.

125 This quadrupolar asymmetry is expected to induce to some extent a quadrupolar modu-  
 126 lation of the surface detector signals as a function of the *polar angle on the ground*, defined  
 127 here as the angle between the axis given by the shower core and the surface detector,  
 128 and the magnetic East  $\varphi_B^E = -D_B = -2.6^\circ$  (Fig. 4). The use of this particular angle, instead  
 129 of the polar angle  $\psi$  which is defined in the *shower front plane* (see Fig. 1), allows us to  
 130 remove dipolar asymmetries in the surface detector signals, the origin of which is related

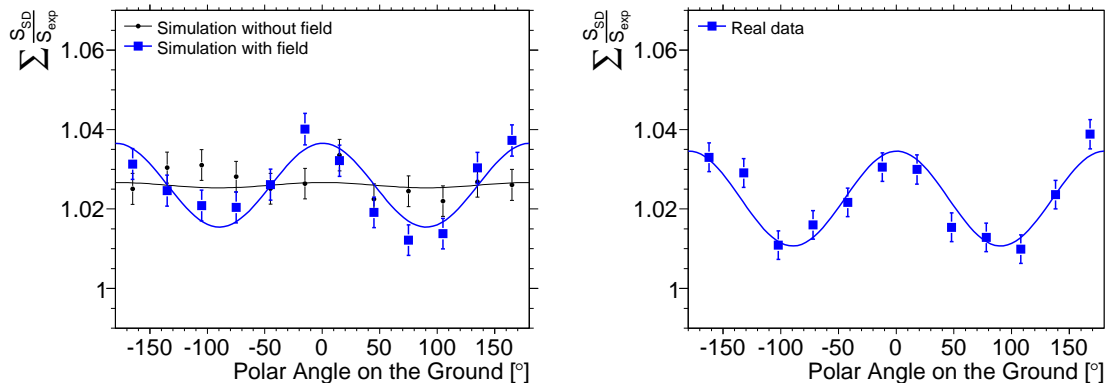


Figure 5: Average ratio of the true signal in each surface detector with respect to the expected one as a function of the polar angle on the ground. Left panel: using simulated showers in the presence (thick points) and in the absence (thin points) of the geomagnetic field. Right panel: using real data above 4 EeV. The solid lines give the fit of a quadrupolar modulation to the corresponding points.

131 to the radial divergence of particles from the shower axis. Such asymmetries cancel out in  
 132 this analysis, due to the isotropic distribution of the cosmic rays. To demonstrate the geo-  
 133 magnetic effect, we produced a realistic Monte-Carlo simulation using 30 000 isotropically  
 134 distributed showers (with zenith angles less than  $60^\circ$ ) with random core positions within  
 135 the array. The injected primary energies were chosen to be greater than 4 EeV (safely  
 136 excluding angle dependent trigger probability) and distributed according to a power law  
 137 energy spectrum  $dN/dE \propto E^{-\gamma}$  with power index  $\gamma = 2.7$ , so that this shower library is  
 138 as close as possible to the real data set. To each shower we apply the reconstruction pro-  
 139 cedure of the surface detector, leading to a fit of the lateral distribution function [10]. The  
 140 lateral distribution function parametrizes the signal strength in the shower plane, assuming  
 141 circular shower symmetry. By evaluating the lateral distribution function at the position of  
 142 the surface detector, we obtain the expected signal  $S_{\text{exp}}$ . This signal can be compared to  
 143 the true signal in the surface detector  $S_{\text{SD}}$ . The ratio between the observed and expected  
 144 signals as a function of the polar angle on the ground in simulated showers is shown in  
 145 the left panel of Fig. 5, with (thick points) and without (thin points) the geomagnetic field.  
 146 While a significant quadrupolar modulation with a fixed phase along  $D_B$  and amplitude  
 147  $\simeq (1.1 \pm 0.2)\%$  is observed when the field is on, no such modulation is observed when the  
 148 field is off ( $\simeq (0.1 \pm 0.2)\%$ ), as expected. In the right panel, the same analysis is performed  
 149 on the real data above 4 EeV, including again about 30 000 showers. A significant mod-  
 150 ulation of  $\simeq (1.2 \pm 0.2)\%$  is observed, agreeing both in amplitude and phase within the  
 151 uncertainties with the simulations performed *in the presence* of the geomagnetic field. This  
 152 provides clear hints of the influence of the geomagnetic field in the Auger data.

153 Note that this analysis is restricted to surface detectors that are more than 1000 m away  
 154 from the shower core. This cut is motivated by Fig. 3, showing that the quadrupolar am-  
 155 plitude is larger at large distances from the shower core. We further require the surface  
 156 detectors to have signals larger than  $4 \text{ VEM}^2$ . This cut is a compromise between keeping

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<sup>2</sup>VEM - Vertical Equivalent Muon - is the average charge corresponding to the Cherenkov light produced

157 good statistics and keeping trigger effects small. Above 4 VEM the measured amplitude  
 158 does not depend systematically on the signal strength cut. However a cut in the surface  
 159 detector signals induces a statistical trigger bias because showers with upward signal fluc-  
 160 tuations will trigger more readily. This explains the small discrepancy between real and  
 161 Monte-Carlo data in terms of the global normalisation in Fig. 5 which differs from 1 by  
 162  $\sim 3\%$ . Cutting at larger signals reduces this discrepancy.

163 Most importantly, depending on the incoming direction, the quadrupolar asymmetry  
 164 is also expected to affect the shower size  $S(1000)$  and thus the energy estimator as qual-  
 165 itatively described in Eq. (6). Consequently, these effects are expected to modulate the  
 166 estimated cosmic ray event rate at a given energy as a function of the incoming direction,  
 167 and in particular to generate a North/South asymmetry in the azimuthal distribution<sup>3</sup>. Such  
 168 an asymmetry is also expected in the case of a *genuine* large scale modulation of the flux  
 169 of cosmic rays. However related analyses of the azimuthal distribution are out of the scope  
 170 of this paper, and we restrict ourselves in the rest of this article to present a comprehensive  
 171 study of the geomagnetic distortions of the energy estimator. This will allow us to apply the  
 172 corresponding corrections in a forthcoming publication aimed at searching for large scale  
 173 anisotropies.

## 174 4. Geomagnetic distortions of the energy estimator

### 175 4.1. Geomagnetic distortions of the shower size $S(1000)$

176 The toy model presented in Section 2 allows us to understand the main features of  
 177 the influence of the geomagnetic field on the muonic component of extensive air showers.  
 178 To get an accurate estimation of the distortions induced by the field on the shower size  
 179  $S(1000)$  as a function of both the zenith and the azimuthal angles, we present here the  
 180 results obtained by means of end-to-end simulations of proton-initiated showers generated  
 181 with the AIRES program [13] and with the hadronic interaction model QGSJET [14]. We  
 182 have checked that the results obtained with the CORSIKA program [15] are compatible. We  
 183 consider a fixed energy  $E = 5$  EeV and seven fixed zenith angles between  $\theta = 0^\circ$  and  $\theta =$   
 184  $60^\circ$ . The dependency of the effect in terms of the primary mass and of the number of muons  
 185 in showers as well as its evolution with energy are sources of systematic uncertainties. The  
 186 influence of such systematics will be quantified in Section 6. Within our convention for the  
 187 azimuthal angle, the azimuthal direction of the magnetic North is  $\varphi_B^N = 90^\circ - D_B = 87.4^\circ$ .  
 188 The zenith direction of the field is  $\theta_B = 90^\circ - |I_B| = 54.8^\circ$ .

189 To verify the predicted behaviour of the shower size shift in terms of  $B_T^2$ , we first show  
 190 the results of the simulations of 1000 showers at a zenith angle  $\theta = \theta_B$  and for two distinct  
 191 azimuthal angles  $\varphi = \varphi_B^N$  and  $\varphi = \varphi_B^N + 90^\circ$ . Each shower is then thrown 10 times at the  
 192 surface detector array with random core positions and reconstructed using exactly the same  
 193 reconstruction procedure as the one applied to real data. For this specific zenith angle  $\theta_B$ ,  
 194 no shift is expected in the North direction  $\varphi_B^N$  as the transverse component of the magnetic

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by a vertical and central through-going muon in the surface detector. It is the unit used in the evaluation of the signal recorded by the detectors [12].

<sup>3</sup>The convention we use for the azimuthal angle  $\varphi$  is to define it relative to the East direction, counter-clockwise.

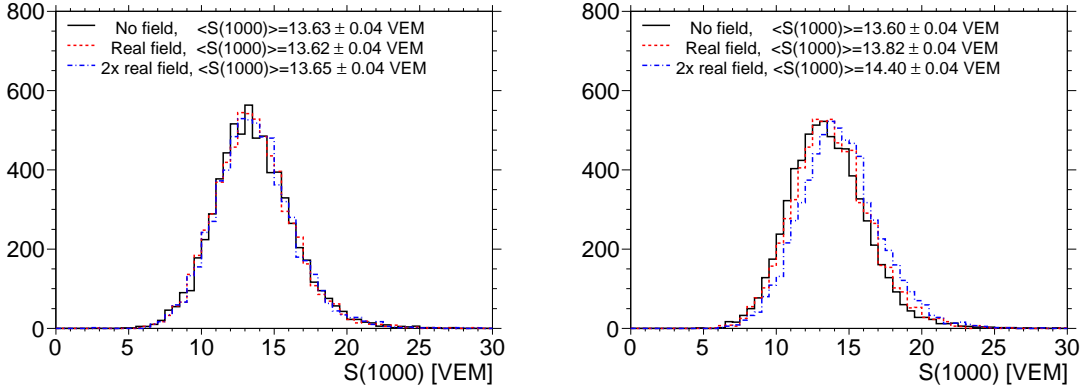


Figure 6: Distributions of shower size  $S(1000)$  obtained by simulating showers at zenith angle  $\theta = \theta_B$  and azimuthal angle  $\varphi_B^N$  (left) and  $\varphi_B^N + 90^\circ$  (right). Thick histogram: no magnetic field. Dotted histogram: real magnetic field in Malargüe. Dashed histogram: twice the real magnetic field in Malargüe.

195 field is zero. This is indeed the case as illustrated in the left panel of Fig. 6, showing  
 196 the distribution of reconstructed  $S(1000)$  for three different configurations of the magnetic  
 197 field: no field, real field in Malargüe, and twice the real field in Malargüe. It can be seen  
 198 that on average all histograms are – within the statistical uncertainties on the average –  
 199 centered on the same value. In the right panel of Fig. 6 we repeat the same analysis with  
 200 the showers generated in the direction  $\varphi_B^N + 90^\circ$ . Since the transverse component of the field  
 201 is now different from zero, a clear relative shift in terms of  $\Delta S(1000)/S(1000)$  is observed  
 202 between the three distributions: the shift is  $\simeq 1.6\%$  between the configurations with and  
 203 without the field, leading to a discrimination with a significance of  $\simeq 5.5\sigma$ , while the shift  
 204 is  $\simeq 6\%$  between the configurations with twice the real field and without the field leading  
 205 to a discrimination with a significance of  $\simeq 20\sigma$ . It can be noticed that the strength of the  
 206 shift is thus in overall agreement with the expected scaling  $B_T^2$ .

207 For the zenith angle  $\theta = \theta_B$ , in Fig. 7 we show the shift of the mean  $S(1000)$  ob-  
 208 tained by simulating 1000 showers in the same way as previously for eight different values  
 209 of the azimuth angle. Again, the results are displayed for configurations with the real  
 210 field (bottom) and with twice the real field (top). The expected behaviours in terms of  
 211  $\Delta S(1000)/S(1000) = G(\theta_B) \sin^2(\widehat{\mathbf{u}, \mathbf{b}})$  are shown by the continuous curves, where the nor-  
 212 malisation factor  $G$  is tuned by hand. Clearly, the shape of the curves agrees remarkably  
 213 well with the Monte Carlo data within the uncertainties. Hence, this study supports the  
 214 claim that the azimuthal dependence of the shift in  $S(1000)$  induced by the magnetic field  
 215 is proportional to  $B_T^2(\theta, \varphi)$ , in agreement with the expectations provided by general consid-  
 216 erations expressed in the previous section on the muonic component of the showers.

217 The  $B_T^2$  term encompassing the overall azimuthal dependence at each zenith angle, the  
 218 remaining shift  $G(\theta) = \Delta S(1000)/S(1000)/\sin^2(\widehat{\mathbf{u}, \mathbf{b}})$  depends on the zenith angle through  
 219 the altitude distribution of the muon production, the muon energy distribution, and the  
 220 weight of the muonic contribution to the shower size  $S(1000)$ . Repeating the simulations  
 221 at different zenith angles, we plot  $G$  as a function of the zenith angle in Fig. 8. Due to  
 222 the increased travel lengths of the muons and due to their larger relative contribution to

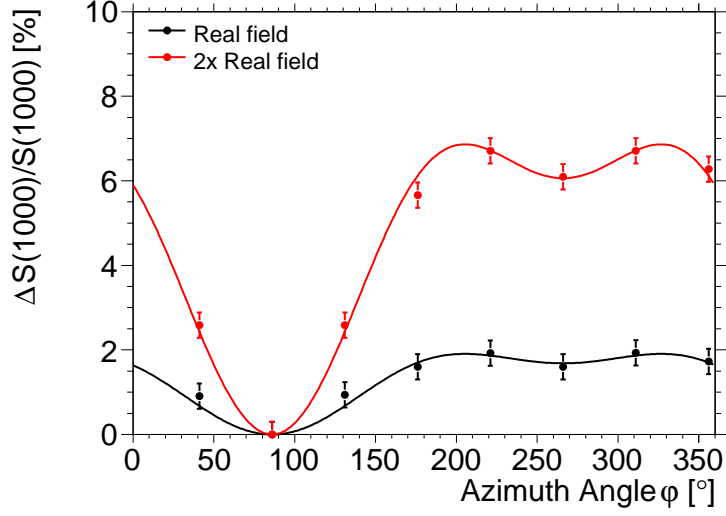


Figure 7:  $\Delta S(1000)/S(1000)$  (in %) as a function of the azimuthal angle  $\varphi$ , at zenith angle  $\theta = \theta_B$  for two different field strengths. Points are obtained by Monte Carlo shower simulation, lines are the expected behavior (see Section 2).

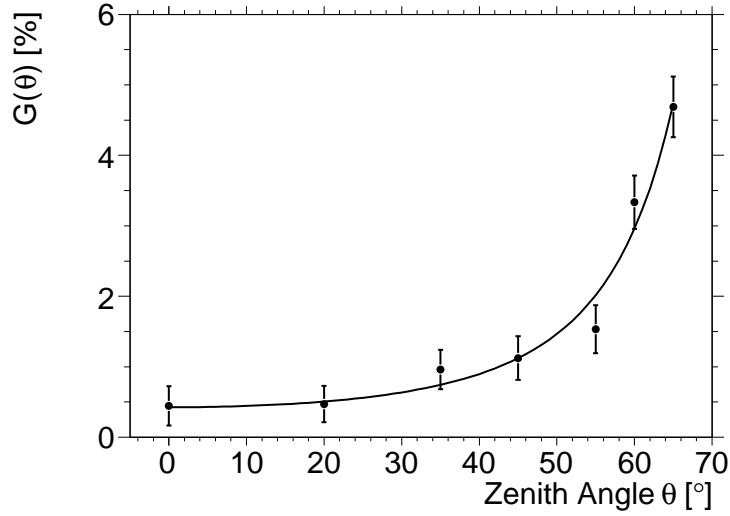


Figure 8:  $G(\theta) = \Delta S(1000)/S(1000)/\sin^2(\widehat{\mathbf{u}, \mathbf{b}})$  as a function of the zenith angle  $\theta$ .

223  $S(1000)$  at high zenith angles, the value of  $G$  rises rapidly for angles above  $\simeq 40^\circ$ . The  
 224 superimposed curve is an *empirical* fit, allowing us to get the following parametrisation of  
 225 the shower size distortions induced by the geomagnetic field,

$$\frac{\Delta S(1000)}{S(1000)}(\theta, \varphi) = 4.2 \cdot 10^{-3} \cos^{-2.8} \theta \sin^2(\widehat{\mathbf{u}, \mathbf{b}}). \quad (7)$$

#### 226 4.2. From shower size to energy

227 At the Pierre Auger Observatory, the shower size  $S(1000)$  is converted into energy  $E$   
 228 using a two-step procedure [10]. First, the evolution of  $S(1000)$  with zenith angle arising

229 from the attenuation of the shower with increasing atmospheric thickness is quantified by  
 230 applying the *Constant Intensity Cut* (CIC) method that is based on the (at least approximate)  
 231 isotropy of incoming cosmic rays. The CIC relates relates  $S(1000)$  in vertical and inclined  
 232 showers through a line of equal intensity in spectra at different zenith angles. This allows  
 233 us to correct the value of  $S(1000)$  for attenuation by computing its value had the shower ar-  
 234 rived from a fixed zenith angle, here 38 degrees (corresponding to the median of the angular  
 235 distribution of events for energies greater than 3 EeV). This zenith angle independent esti-  
 236 mator  $S_{38}$  is defined as  $S_{38} = S(1000)/CIC(\theta)$ . The calibration of  $S_{38}$  with energy  $E$  is then  
 237 achieved using a relation of the form  $E = AS_{38}^B$ , where  $A = 1.49 \pm 0.06(\text{stat}) \pm 0.12(\text{syst})$   
 238 and  $B = 1.08 \pm 0.01(\text{stat}) \pm 0.04(\text{syst})$  were estimated from the correlation between  $S_{38}$   
 239 and  $E$  in a subset of high quality "hybrid events" measured simultaneously by the surface  
 240 detector (SD) and the fluorescence detector (FD) [10]. In such a sample,  $S_{38}$  and  $E$  are  
 241 independently measured, with  $S_{38}$  from the SD and  $E$  from the FD.

242 This two-step procedure has an important consequence on the implementation of the  
 243 energy corrections for the geomagnetic effects. The CIC curve is constructed assuming  
 244 that the shower size estimator  $S(1000)$  does not depend on the azimuthal angle. The in-  
 245 duced azimuthal variation of  $S(1000)$  due to the geomagnetic effect is thus averaged while  
 246 the zenith angle dependence of the geomagnetic effects is absorbed when the CIC is imple-  
 247 mented. To illustrate this in a simplified way, consider the case in which the magnetic field  
 248 were directed along the zenith direction (*i.e.* in the case of a virtual Observatory located at  
 249 the Southern magnetic pole) so that the transverse component of the magnetic field would  
 250 not depend on the azimuthal direction of any incoming shower. Then the shift in  $S(1000)$   
 251 would depend *only* on the zenith angle in such a way that the Constant Intensity Cut method  
 252 would by construction absorb the shift induced by  $G(\theta)$  into the empirical  $CIC(\theta)$  curve,  
 253 while the empirical relationship  $E = AS_{38}^B$  would calibrate  $S_{38}$  into energy with no need for  
 254 any additional corrections.

255 This leads us to implement the energy corrections for geomagnetic effects, relating the  
 256 energy  $E_0$  reconstructed ignoring the geomagnetic effects to the *corrected* energy  $E$  by

$$E = \frac{E_0}{(1 + \Delta(\theta, \varphi))^B}, \quad (8)$$

257 with

$$\Delta(\theta, \varphi) = G(\theta) \left[ \sin^2(\widehat{\mathbf{u}, \mathbf{b}}) - \langle \sin^2(\widehat{\mathbf{u}, \mathbf{b}}) \rangle_\varphi \right] \quad (9)$$

258 where  $\langle \cdot \rangle_\varphi$  denotes the average over  $\varphi$  and where  $B$  is one of the parameters used in the  $S_{38}$   
 259 to  $E$  conversion described above. This expression implies that energies are *under-estimated*  
 260 preferentially for showers coming from the northern directions of the array, while they  
 261 are *over-estimated* for showers coming from the southern directions, the size of the effect  
 262 increasing with the zenith angle.

## 263 5. Consequences for large scale anisotropy searches

### 264 5.1. Impact on the estimated event rate

265 To provide an illustration of the impact of the energy corrections for geomagnetic ef-  
 266 fects, we calculate here, as a function of declination  $\delta$ , the deviation of the event rate  $N_0(\delta)$ ,

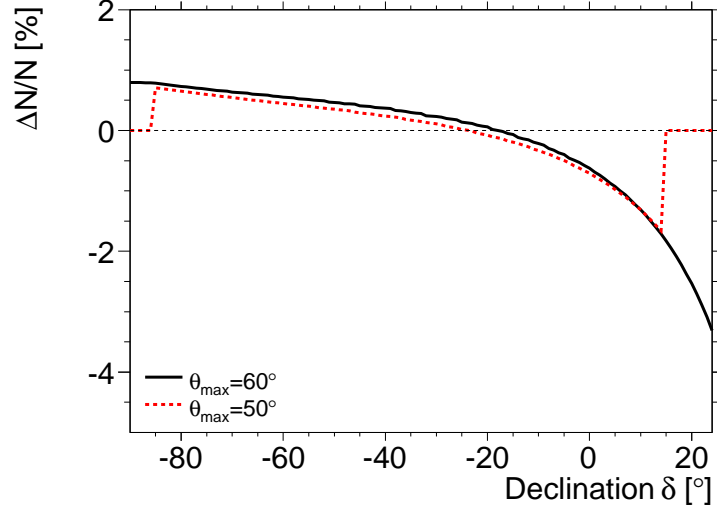


Figure 9: Relative differences  $\Delta N/N$  as a function of the declination, for 2 different values of  $\theta_{\max}$ .

267 measured if we were not to implement the corrections of the energy estimator by Eq. (8),  
 268 to the event rate  $N(\delta)$  expected from an isotropic background distribution.

269 The “canonical exposure” [16] holds for a full-time operation of the surface detector  
 270 array above the energy at which the detection efficiency is saturated over the considered  
 271 zenith range. In such a case, the directional detection efficiency is simply proportional to  
 272  $\cos \theta$ ,

$$\omega(\theta) \propto \cos(\theta) H(\theta - \theta_{\max}) \quad (10)$$

273 where  $H$  is the Heaviside function and  $\theta_{\max}$  is the maximal zenith angle considered. The  
 274 zenith angle is related to the declination  $\delta$  and the right ascension  $\alpha$  through

$$\cos \theta = \sin \ell_{\text{site}} \sin \delta + \cos \ell_{\text{site}} \cos \delta \cos \alpha \quad (11)$$

275 where  $\ell_{\text{site}}$  is the Earth’s latitude of the Observatory. The event rate at a given declination  $\delta$   
 276 and above an energy threshold  $E_{\text{th}}$  is obtained by integrating in energy and right ascension  
 277  $\alpha$ ,

$$N(\delta) \propto \int_{E_{\text{th}}}^{\infty} dE \int_0^{2\pi} d\alpha \omega(\theta) \frac{dN(\theta, \varphi, E)}{dE} \quad (12)$$

278 Note that at lower energies this integral acquires an additional energy and angle dependent  
 279 detection efficiency term  $\epsilon(E, \theta, \phi)$ . Hereafter we assume that the cosmic ray spectrum  
 280 is a power law, *i.e.*  $dN/dE \propto E^{-\gamma}$ . From Eq. (8) it follows that if the effect of the  
 281 geomagnetic field were not accounted for, the measured energy spectrum would have a  
 282 directional modulation given by

$$\frac{dN}{dE_0} \propto [1 + \Delta(\theta, \varphi)]^{B(\gamma-1)} E_0^{-\gamma}. \quad (13)$$

283 This leads to the following measured event rate above a given uncorrected energy  $E_{\text{th}}$ ,

$$N_0(\delta) \propto \int_{E_{\text{th}}}^{\infty} dE_0 \int_0^{2\pi} d\alpha H(\cos \theta - \cos \theta_{\max}) \cos \theta [1 + \Delta(\theta, \varphi)]^{B(\gamma-1)} E_0^{-\gamma}, \quad (14)$$



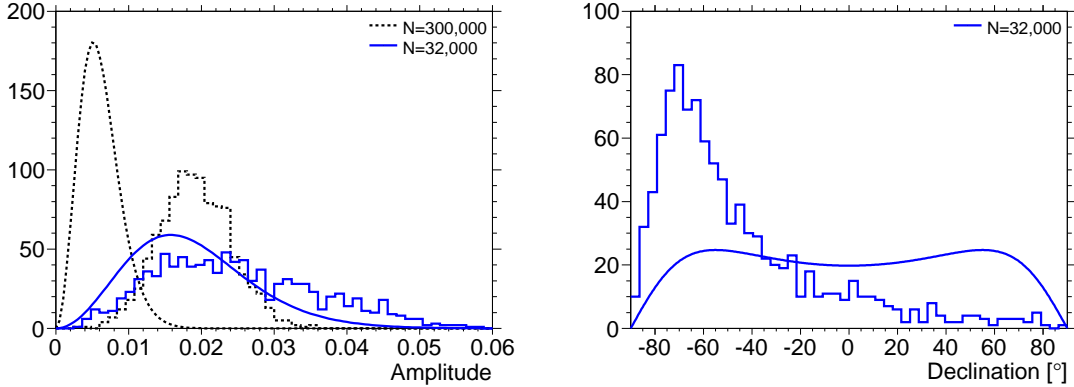


Figure 10: Dipolar reconstruction of arrival directions of mock data sets with event rates distorted by the geomagnetic effects. Left: distributions of amplitudes. Right: distributions of declinations. The smooth lines give the expected distribution in the case of isotropy.

284 where  $\varphi$  is related to  $\alpha$  and  $\delta$  through

$$\tan \varphi = \frac{\sin \delta \cos \ell_{\text{site}} - \cos \delta \cos \alpha \sin \ell_{\text{site}}}{\cos \delta \sin \alpha}. \quad (15)$$

285 The event rate  $N_0(\delta)$  as a function of declination is then calculated using Eq. (13) in Eq.  
 286 (12). The relative difference  $\Delta N/N$  is shown in Fig. 9 as a function of the declination, with  
 287 spectral index  $\gamma = 2.7$ . The energy over-estimation (under-estimation) of events coming  
 288 preferentially from the Southern (Northern) azimuthal directions, as described in Eq. (8),  
 289 leads to an effective excess (deficit) of the event rate for  $\delta \lesssim -20^\circ$  ( $\delta \gtrsim -20^\circ$ ), with an  
 290 amplitude of  $\simeq 2\%$  when considering  $\theta_{\text{max}} = 60^\circ$ . It is worth noting that this amplitude is  
 291 reduced to within 1% when considering  $\theta_{\text{max}} = 50^\circ$ , as shown by the dotted line.

### 292 5.2. Impact on dipolar modulation searches

293 The pattern displayed in Fig. 9 roughly imitates a dipole with an amplitude at the per-  
 294 cent level. To evaluate precisely the impact of this pattern on the assessment of a dipole  
 295 moment in the reconstructed arrival directions and to probe the statistics needed for the  
 296 sensitivity to such a spurious pattern, we apply the multipolar reconstruction adapted to the  
 297 case of a partial sky coverage [17] to mock data sets by limiting the maximum bound of the  
 298 expansion  $L_{\text{max}}$  to 1 (pure dipolar reconstruction). Since the distortions are axisymmetric  
 299 around the axis defined by the North and South celestial poles, only the multipolar coef-  
 300 ficient related to this particular axis is expected to be affected (here:  $a_{10}$ ). Consequently,  
 301 this particular coefficient has impacts on both the amplitude of the reconstructed dipole and  
 302 its direction with respect to the axis defined by the North and South celestial poles (the  
 303 technical details of relating the estimation of the multipolar coefficients to the spherical  
 304 coordinates of a dipole are given in the Appendix).

305 To simulate the directional distortions induced by Eq. (8), each mock data set is drawn  
 306 from the event rate  $N_0(\delta)$  corresponding to the uncorrected energies, and is reconstructed  
 307 using the canonical exposure in Eq. (10). The results of this procedure applied to 1000  
 308 samples are shown in Fig. 10. In the left panel, the distribution of the reconstructed am-  
 309 plitudes  $r$  using  $N = 300\,000$  events is shown by the dotted histogram. It clearly deviates

310 from the expected isotropic distribution displayed as the dotted curve which corresponds to  
 311 (see Appendix)

$$p_R(r) = \frac{r}{\sigma \sqrt{\sigma_z^2 - \sigma^2}} \operatorname{erfi}\left(\frac{\sqrt{\sigma_z^2 - \sigma^2}}{\sigma \sigma_z} \frac{r}{\sqrt{2}}\right) \exp\left(-\frac{r^2}{2\sigma^2}\right), \quad (16)$$

312 where  $\operatorname{erfi}(z) = \operatorname{erf}(iz)/i$ , and where the width parameters  $\sigma$  and  $\sigma_z$  can be calculated  
 313 from the exposure function [17]. With the particular exposure function used here, it turns  
 314 out that  $\sigma \simeq 1.02 \sqrt{3/N}$  and  $\sigma_z \simeq 1.59 \sqrt{3/N}$ . This allows us to estimate the spurious  
 315 dipolar amplitude<sup>4</sup> to be of the order of the mean of the dotted histogram, about  $\simeq 1.9\%$ .  
 316 Consequently, we can estimate that the spurious effect becomes predominant as soon as the  
 317 mean noise amplitude  $\langle r \rangle$  deduced from Eq. (16) is of the order of 1.9%,

$$\langle r \rangle = \sqrt{\frac{2}{\pi}} \left( \sigma_z + \frac{\sigma^2 \operatorname{arctanh}(\sqrt{1 - \sigma^2/\sigma_z^2})}{\sqrt{\sigma_z^2 - \sigma^2}} \right) \simeq 1.9\%. \quad (17)$$

318 This translates into the condition  $N \simeq 32\,000$  (solid histogram). Using such a number of  
 319 events, the bias induced on the amplitude reconstruction is illustrated in the same graph  
 320 by the longer tail of the full histogram with respect to the expected one, and is even more  
 321 evident in the right panel of Fig. 10, showing the distribution of the reconstructed decli-  
 322 nation direction of the dipole which already deviates to a large extent from the expected  
 323 distribution.

## 324 6. Systematic uncertainties

325 The parametrisation of  $G(\theta)$  in Eq. (7) was obtained by means of simulations of proton  
 326 showers at a fixed energy. The height of the first interaction influences the production  
 327 altitude of muons detected at 1000 m from the shower core at the ground level. Moreover,  
 328 as muons are produced at the end of the hadronic cascade, when the energy of the charged  
 329 mesons is diminished so much that their decay length becomes smaller than their interaction  
 330 length (which is inversely proportional to the air density), the energy distribution of muons  
 331 is also affected by the height of the first interaction. Because the air density is lower in  
 332 the upper atmosphere, this mechanism results in an increase of the energy of muons. The  
 333 muonic contribution to  $S(1000)$  depends also on both the primary mass and primary energy.  
 334 For all these reasons, the parametrisation of  $G(\theta)$  is expected to depend on both the primary  
 335 mass and primary energy.

336 To probe these influences, we repeat the same chain of end-to-end simulations using  
 337 proton showers at energies of 50 EeV and iron showers at 5 EeV. Results in terms of the  
 338 distortions of the observed event rate  $N(\delta)$  are shown in Fig. 11. We also display in the  
 339 same graph the results obtained using the hadronic interaction model QGSJETII [18]. The

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<sup>4</sup>Due to the partial sky exposure considered here, the estimate of the dipolar amplitude is biased by the higher multipolar orders needed to fully describe  $\Delta N/N$  shown in Fig. 10 [17]. The aim of this calculation is only to provide a quantitative illustration of the spurious measurement which would be performed due to the geomagnetic effects when reconstructing a pure dipolar pattern.

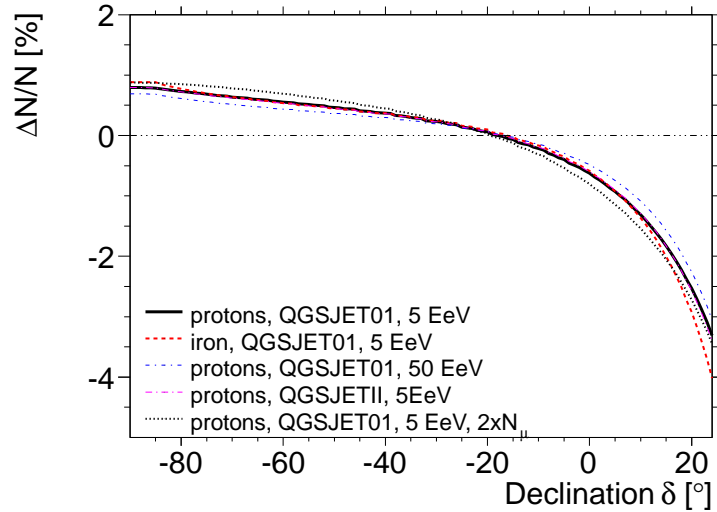


Figure 11: Relative differences  $\Delta N/N$  as a function of the declination, for different primary masses, different primary energies, different hadronic models and for increased number of muons in showers.

340 differences with respect to the reference model are small, so that the consequences on large  
 341 scale anisotropy searches presented in Section 5 remain unchanged within the statistics  
 342 available at the Pierre Auger Observatory.

343 In addition, there are discrepancies in the hadronic interaction model predictions re-  
 344 garding the number of muons in shower simulations and what is found in our data [19].  
 345 Higher number of muons influences the weight of the muonic contribution to  $S(1000)$ . The  
 346 consequences of increasing the number of muons by a factor of 2 on the distortions of the  
 347 observed event rate are also shown in Fig. 11. As the muonic contribution to  $S(1000)$  is  
 348 already large at high zenith angles in the reference model, this increase of the number of  
 349 muons does not lead to large differences.

## 350 7. Conclusion

351 In this work, we have identified and quantified a systematic uncertainty affecting the  
 352 energy determination of cosmic rays detected by the surface detector array of the Pierre  
 353 Auger Observatory. This systematic uncertainty, induced by the influence of the geomag-  
 354 netic field on the shower development, has a strength which depends on both the zenith  
 355 and the azimuthal angles. Consequently, we have shown that it induces distortions of the  
 356 estimated cosmic ray event rate at a given energy at the percent level in both the azimuthal  
 357 and the declination distributions, the latter of which mimics an almost dipolar pattern.

358 We have also shown that the induced distortions are already at the level of the statistical  
 359 uncertainties for a number of events  $N \approx 32\,000$  (we note that the full Auger surface  
 360 detector array collects about 6500 events per year with energies above 3 EeV). Accounting  
 361 for these effects is thus essential with regard to the correct interpretation of large scale  
 362 anisotropy measurements taking explicitly profit from the declination distribution.

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## 398 Appendix

399 The p.d.f. of the first harmonic amplitude for a data set of  $N$  points drawn at random  
400 over a circle is known to be the Rayleigh distribution. In this appendix, we generalise  
401 this distribution to the case of  $N$  points being drawn at random on the sphere over the  
402 exposure  $\omega(\delta)$  of the Pierre Auger Observatory. Assuming the underlying arrival direction  
403 distribution to be of the form  $\Phi(\alpha, \delta) = \Phi_0(1 + \mathbf{D} \cdot \mathbf{u})$ , the components of the dipolar vector

404 **D** are related to the multipolar coefficients through

$$D_x = \sqrt{3} \frac{a_{11}}{a_{00}}, \quad D_y = \sqrt{3} \frac{a_{1-1}}{a_{00}}, \quad D_z = \sqrt{3} \frac{a_{10}}{a_{00}}. \quad (18)$$

405 Denoting by  $x, y, z$  the estimates of  $D_x, D_y, D_z$ , the joint p.d.f.  $p_{X,Y,Z}(x, y, z)$  can be factorised  
 406 in the limit of large number of events in terms of three centered Gaussian distributions  
 407  $N(0, \sigma)$ ,

$$p_{X,Y,Z}(x, y, z) = p_X(x)p_Y(y)p_Z(z) = N(0, \sigma_x)N(0, \sigma_y)N(0, \sigma_z), \quad (19)$$

408 where the standard deviation parameters can be calculated from the exposure function [17].  
 409 With the particular exposure function used here, it turns out that numerical integrations lead  
 410 to  $\sigma \simeq 1.02 \sqrt{3/N}$  and  $\sigma_z \simeq 1.59 \sqrt{3/N}$ . The joint p.d.f.  $p_{R,\Delta,A}(r, \delta, \alpha)$  expressing the dipole  
 411 components in spherical coordinates is obtained from Eq. (19) by performing the Jacobian  
 412 transformation

$$\begin{aligned} p_{R,\Delta,A}(r, \delta, \alpha) &= \left| \frac{\partial(x, y, z)}{\partial(r, \delta, \alpha)} \right| p_{X,Y,Z}(x(r, \delta, \alpha), y(r, \delta, \alpha), z(r, \delta, \alpha)) \\ &= \frac{r^2 \cos \delta}{(2\pi)^{3/2} \sigma^2 \sigma_z} \exp \left[ -\frac{r^2 \cos^2 \delta}{2\sigma^2} - \frac{r^2 \sin^2 \delta}{2\sigma_z^2} \right]. \end{aligned} \quad (20)$$

413 From this joint p.d.f., the p.d.f. of the dipole amplitude (declination) is finally obtained by  
 414 marginalising over the other variables, yielding

$$\begin{aligned} p_R(r) &= \frac{r}{\sigma \sqrt{\sigma_z^2 - \sigma^2}} \operatorname{erfi} \left( \frac{\sqrt{\sigma_z^2 - \sigma^2}}{\sigma \sigma_z} \frac{r}{\sqrt{2}} \right) \exp \left( -\frac{r^2}{2\sigma^2} \right), \\ p_\Delta(\delta) &= \frac{\sigma \sigma_z^2 \cos \delta}{2 (\sigma_z^2 \cos^2 \delta + \sigma^2 \sin^2 \delta)^{3/2}}. \end{aligned} \quad (21)$$

415 Finally, one can derive from  $p_R$  quantities of interest, such as the expected mean noise  $\langle r \rangle$ ,  
 416 the RMS  $\sigma_r$  and the probability of obtaining an amplitude greater than  $r$ :

$$\langle r \rangle = \sqrt{\frac{2}{\pi}} \left( \sigma_z + \frac{\sigma^2 \operatorname{arctanh}(\sqrt{1 - \sigma^2/\sigma_z^2})}{\sqrt{\sigma_z^2 - \sigma^2}} \right), \quad (22)$$

$$\sigma_r = \sqrt{2\sigma^2 + \sigma_z^2 - \langle r \rangle^2}, \quad (23)$$

$$\operatorname{Prob}( > r ) = \operatorname{erfc} \left( \frac{r}{\sqrt{2}\sigma_z} \right) + \frac{\sigma}{\sqrt{\sigma_z^2 - \sigma^2}} \operatorname{erfi} \left( \frac{\sqrt{\sigma_z^2 - \sigma^2}}{\sqrt{2}\sigma \sigma_z} r \right) \exp \left( -\frac{r^2}{2\sigma^2} \right), \quad (24)$$

417 which are the equivalent to the well known Rayleigh formulas  $\langle r \rangle = \sqrt{\pi/N}$ ,  $\sigma_r = \sqrt{(4 - \pi)/N}$   
 418 and  $\operatorname{Prob}( > r ) = \exp(-Nr^2/4)$  when dealing with  $N$  points drawn at random over a cir-  
 419 cle [20].

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