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Semileptonic form factor ratio $B_s \to D_s/B \to D$ and its application to $BR(B_s^0 \to \mu^+\mu^-)$

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We present a (2+1)-flavor lattice QCD calculation of the form factor ratio between the semileptonic decays $\overline{B}{}_s^0 \to D_s^+ l^- \overline{\nu}$ and $\overline{B}{}^0 \to D^+ l^- \overline{\nu}$. This ratio is an important theoretical input to the hadronic determination of the B meson fragmentation fraction ratio f_s/f_d which enters in the measurement of $\mathrm{BR}(B_s^0 \to \mu^+\mu^-)$. Small lattice spacings and high statistics enable us to simulate the decays with a dynamic final D meson of small momentum and reliably extract the hadronic matrix elements at nonzero recoil. We report our preliminary result for the form factor ratio at the corresponding momentum transfer of the two decays $f_0^{(s)}(M_\pi^2)/f_0^{(d)}(M_K^2)$.

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1. Introduction

The rare decay $B_s^0 \to \mu^+\mu^-$ is a process that is potentially sensitive to physics beyond the standard model (SM). In the SM, the decay can go only through penguin or box topologies at the loop level. Thus, a small branching fraction has been predicted, with the aid of lattice QCD, to be $3.2(2)\times 10^{-9}$ [1, 2]. Recently, LHC*b* [3] and CDF [4] reported bounds on the branching fraction, to be followed by upcoming results from CMS. It is likely that a 5σ measurement will be made, even at the SM branching ratio, in the near future.

At LHC*b*, the extraction of the branching fraction relies on the normalization channels $B_u^+ \to J/\psi K^+$, $B_d^0 \to K^+\pi^-$ and $B_s^0 \to J/\psi\phi$ [5], through the following relation

$$\mathrm{BR}(B_s^0 \to \mu^+\mu^-) = \mathrm{BR}(B_q \to X)\frac{f_q}{f_s}\frac{\varepsilon_X}{\varepsilon_{\mu\mu}}\frac{N_{\mu\mu}}{N_X}, \tag{1.1}$$

where ε and N are the detector efficiencies and number of events. The fragmentation fractions, f_q ($q = u,d,s$ or Λ), denote the probability of a b quark hadronizing into a B_q meson or a b-flavored (e.g., Λ_b) baryon. The fragmentation fraction ratio f_s/f_d is crucial in the extraction of $\mathrm{BR}(B_s^0 \to \mu^+\mu^-)$. Currently, the uncertainty in f_s/f_d is the major source of uncertainty. Traditionally, f_s/f_d was measured using the ratio of the corresponding semileptonic decays. Fleischer, Serra and Tuning proposed [6] that the ratio can also be measured using the non-leptonic decays $\bar{B}_s^0 \to D_s^+\pi^-$ and $\bar{B}_d^0 \to D^+K^-$, which has the advantages of a cleaner background, similar reconstruction of final states, *etc*. The approach is based on factorization of the nonleptonic amplitudes into f_π or f_K and corresponding semileptonic form factors. The ratio f_s/f_d is related to $\mathrm{BR}(\bar{B}_s \to D\pi)/\mathrm{BR}(\bar{B} \to DK)$ in a way similar to Eq. (1.1). With the efficiencies and event counts combined with the factorization approximation, we have

$$\frac{f_s}{f_d} = \frac{|V_{us}|^2 f_K^2}{|V_{ud}|^2 f_\pi^2}\times\frac{\tau_{B^0}}{\tau_{B_s^0}}\times\left[\frac{\varepsilon_{DK}}{\varepsilon_{D_s\pi}}\frac{N_{D_s\pi}}{N_{DK}}\right]\times\frac{1}{\mathcal{N}_a\mathcal{N}_F}, \tag{1.2}$$

where τ is the lifetime and $\mathcal{N}_a \approx 1$ with corrections of a few percent due to nonfactorizable effects [6]. The semileptonic form factor ratio $\mathcal{N}_F = [f_0^{(s)}(M_\pi^2)/f_0^{(d)}(M_K^2)]^2$ is currently the decisive contributor to the theoretical error. The value currently used at LHC*b* is an estimate from QCD sum rules, $\mathcal{N}_F = 1.24(8)$ [7, 6]. However, this theoretical input and the size of its error need to be validated by a nonperturbative method such as lattice QCD. This paper is devoted to such a calculation.

The matrix elements of the $B \to D$ semileptonic decay (and similarly for $B_s \to D_s$) can be written as

$$\langle D(p')|\mathcal{V}^\mu|B(p)\rangle = f_+(q^2)\left[(p+p')^\mu - \frac{M_B^2-M_D^2}{q^2}q^\mu\right] + f_0(q^2)\,\frac{M_B^2-M_D^2}{q^2}q^\mu. \tag{1.3}$$

However, for heavy quarks it is convenient to use the variables $h_\pm$, defined by

$$\frac{\langle D(p')|\mathcal{V}^\mu|B(p)\rangle}{\sqrt{M_B M_D}} = h_+(w)\,(v+v')^\mu + h_-(w)\,(v-v')^\mu, \tag{1.4}$$

where $v = p/M$ and the recoil variable is $w = v\cdot v'$. We will use the form factors $h_\pm$ in our entire analysis and convert them in the end to f_+, f_0 using Eqs. (1.3) and (1.4).

In these proceedings, we report a preliminary result of the form factor ratio $f_0^{(s)}(M_\pi^2)/f_0^{(d)}(M_K^2)$ by analyzing the semileptonic decays $\bar{B}_s^0 \to D_s^+ l^-\bar{\nu}$ and $\bar{B}^0 \to D^+ l^-\bar{\nu}$ on the lattice. We use an identical subset of the MILC gauge configurations for both of the $B_s \to D_s$ and $B \to D$ processes. To reduce the statistical errors effectively, we construct a set of ratios at small recoil, from which we extract the lattice form factors $h_\pm$. The extrapolation to physical light quark masses and to the continuum is performed using root staggered chiral perturbation theory (rSχPT). The results are extrapolated to maximum recoil by employing a model-independent parametrization. In Sec.4, we report our lattice result.

2. Numerical details

2.1 Data setup

Our calculation uses four ensembles of the MILC's (2+1)-flavor gauge configurations [8], two at each of the lattice spacings $a \approx 0.12$ fm and ≈ 0.09 fm. The ensembles as well as the parameters used are summarized in Table 1. The strange and light sea quarks were simulated using the asqtad-improved staggered action [9]. The action is also used in our strange and light valence quarks. The heavy quarks (charm and bottom) are simulated using the Sheikholeslami-Wohlert (SW) clover action with the Fermilab interpretation [10]. For the $B \to D$ decay, the spectator light quark is degenerate with the light sea quark (full QCD). While for the $B_s \to D_s$ decay, the strange quark is set close to its physical value. The charm and bottom quarks in our calculation are tuned to their physical values up to a tuning uncertainty. The corresponding bare hopping parameter $\kappa_{b(c)}$, as well as the coefficient for the clover term c_{SW} are given explicitly in Table 1.

2.2 Lattice extraction

In this work, we are interested only in the vector current operator. On the lattice we define $V^\mu = \sqrt{Z^{cc}_{V^4} Z^{bb}_{V^4}}\, \overline{\Psi}_c i\gamma^\mu \Psi_b$, where Z^{hh} are normalization factors. The vector current in the continuum is $\mathscr{V}^\mu = \rho_{V^\mu} V^\mu$, where $\rho^2_{V^\mu} = Z^{bc}_{V^\mu} Z^{cb}_{V^\mu} / Z^{bb}_{V^4} Z^{cc}_{V^4}$. The factor ρ_{V^μ} can be calculated perturbatively and has been found to be very close to one [11, 12]. We expect the ρ_Vs to largely cancel in the ratio of the form factors. Hence this correction is negligible compared with other systematic errors and we take $\rho_V = 1$ for this analysis.

We employ the following three-point functions in our analysis,

$$C^{DV^\mu B}_{3pt}(0,t,T;\mathbf{p}_D) = \sum_{\mathbf{x},\mathbf{y}} \langle 0|\mathscr{O}_D(0,\mathbf{0})\overline{\Psi}_c i\gamma^\mu \Psi_b(t,\mathbf{y})\mathscr{O}^+_B(0,\mathbf{x})|0\rangle\, e^{i\mathbf{p}_D\cdot\mathbf{y}}, \tag{2.1}$$

$$C^{DV^\mu D}_{3pt}(0,t,T;\mathbf{p}_D) = \sum_{\mathbf{x},\mathbf{y}} \langle 0|\mathscr{O}_D(0,\mathbf{0})\overline{\Psi}_c i\gamma^\mu \Psi_c(t,\mathbf{y})\mathscr{O}^+_D(0,\mathbf{x})|0\rangle\, e^{i\mathbf{p}_D\cdot\mathbf{y}}, \tag{2.2}$$

$$C^{BV^4 B}_{3pt}(0,t,T;\mathbf{0}) = \sum_{\mathbf{x},\mathbf{y}} \langle 0|\mathscr{O}_B(0,\mathbf{0})\overline{\Psi}_b i\gamma^4 \Psi_b(t,\mathbf{y})\mathscr{O}^+_B(0,\mathbf{x})|0\rangle. \tag{2.3}$$

The B meson is at rest. To obtain the dependence of the form factors at small recoil w, we simulate the final state D meson at a few small momenta, *i.e.*, $\mathbf{p} = 2\pi(1,0,0)/L$, $2\pi(1,1,0)/L$, $2\pi(1,1,1)/L$

a (fm)	am_l/am_s	N_{confs}	c_{SW}	κ_c	κ_b	$am_x(B \to D)$	$am_x(B_s \to D_s)$
≈0.12	0.020/0.050	2052	1.525	0.1259	0.0918	0.020	0.0349
≈0.12	0.007/0.050	2110	1.530	0.1254	0.0901	0.007	0.0349
≈0.09	0.0124/0.031	1996	1.473	0.1277	0.0982	0.0124	0.0261
≈0.09	0.0062/0.031	1931	1.476	0.1276	0.0979	0.0062	0.0261

Table 1: MILC ensembles of configurations used in this analysis.

and $2\pi(2,0,0)/L$. The correlation functions for $D \to D$ and $B \to B$ serve as normalization. The $D \to D$ correlation function with a non-zero final state momentum is used to extract the recoil w to alleviate the need of renormalizing the four velocity.

From these correlation functions we construct three different ratios, fits to which include contributions from the lowest-order excited states. Explicitly,

$$\frac{C_{3pt}^{DV^iD}(0,t,T;\mathbf{p})}{C_{3pt}^{DV^4D}(0,t,T;\mathbf{p})} = d^i\,(1+\mathscr{D}_{02}\,e^{-\Delta E(\mathbf{0})(T-t)}+\mathscr{D}_{20}\,e^{-\Delta E(\mathbf{p})t}), \tag{2.4}$$

$$\frac{C_{3pt}^{DV^iB}(0,t,T;\mathbf{p})}{C_{3pt}^{DV^4B}(0,t,T;\mathbf{p})} = b^i\,(1+\mathscr{B}_{02}\,e^{-\Delta m(T-t)}+\mathscr{B}_{20}\,e^{-\Delta E(\mathbf{p})t}), \tag{2.5}$$

$$\frac{C_{3pt}^{DV^iB}(0,t,T;\mathbf{p})}{C_{3pt}^{DV^4B}(0,t,T;\mathbf{0})} \times \left[\frac{Z_0(\mathbf{0})}{Z_0(\mathbf{p})}\sqrt{\frac{E_0(\mathbf{p})}{E_0(\mathbf{0})}}e^{(E_0(\mathbf{p})-E_0(\mathbf{0}))t}\right] = a^i\,(1+\mathscr{A}_{02}\,e^{-\Delta m(T-t)}+\mathscr{A}_{20}\,e^{-\Delta E(\mathbf{p})t}+\mathscr{A}'_{20}\,e^{-\Delta E_2(\mathbf{0})t})e^{\delta t}. \tag{2.6}$$

The factor in the square brackets of Eq. (2.6) cancels the time dependence of the ratio, stemming from the fact that the numerator and denominator have final state D mesons with different momenta. ΔE and Δm denote the lowest splittings and δ is a parameter that accounts for the imprecise $E(\mathbf{p})-E(\mathbf{0})$ in the bracket in Eq. (2.6). In the fits the lowest-lying energy splittings ΔE, Δm are treated as fit parameters. The splittings can be extracted from the two-point functions. So, we employ a multi-channel fitting procedure, combining the two-point functions and the ratios of the three-point functions. We find that such a treatment results in more robust fits and more precise splittings. From d_i, b_i and a_i we can easily recover the form factors $h_\pm$ at small recoil w,

$$w = \frac{1+\mathbf{d}\cdot\mathbf{d}}{1-\mathbf{d}\cdot\mathbf{d}}, \tag{2.7}$$

$$h_+(w) = h_+(1)(a_i/b_i-\mathbf{a}\cdot\mathbf{d}), \tag{2.8}$$

$$h_-(w) = h_+(1)(a_i/b_i-a_i/d_i). \tag{2.9}$$

3. Results

The extrapolation of our lattice results to the physical quark masses and the continuum is guided by rSχPT [13, 14]. However, in the case of h_+ the light quark mass dependence is accompanied by a small recoil w dependence. Such a dependence was included in the continuum chiral perturbation theory in Ref. [15] and was extended to the NLO rSχPT in Ref. [16]. For h_-, the NLO correction is simply a constant which is inversely proportional to the charm quark mass. We follow the same setup, adding NNLO analytic terms and including a^2 dependence. The remaining recoil dependence of the form factors is fitted to a simple quadratic expansion at zero recoil.

The results of the chiral/continuum extrapolation are shown in Fig. 1. The form factor h_+ for both of the $B \to D$ and $B_s \to D_s$ decays shows a small dependence on the light quark masses and lattice spacings. The extrapolated physical values are very close to the lattice data points. This suggests that h_+ is insensitive to the light degrees of freedom. However, sizable light quark mass and lattice spacing dependence appears in the case of h_-, as indicated by the variation due to the sea quark masses and the differences between $h_-^{B\to D}$ and $h_-^{B_s\to D_s}$ (spectator mass). Note that the difference between $h_+^{B\to D}$ and $h_+^{B_s\to D_s}$ is minor. Considering the subleading role that h_- plays in contributing to f_+, f_0, we expect the U-spin symmetry breaking effect to be smaller than what was expected in [6, 7]. Such an observation is bolstered by the recent lattice calculations on f_+, f_0 of the $D_{(s)} \to \pi(K)$ decays [17].

With the physical values of $h_\pm$, we can easily calculate f_0, f_+ using the physical masses of the D and B mesons. However, to evaluate the form factors at a small momentum transfer ($q^2 = M_\pi^2, M_K^2$), we need to extrapolate the results near maximum recoil. We use the model-independent z-parametrization [18] with the constraint $f_0(0) = f_+(0)$. We take five synthetic points in the recoil range where we have lattice data points. We take the values of f_+, f_0 by evaluating our chiral/continuum extrapolation result at these five recoil points and perform the z-parametrization. The result is shown in Figure 2. We study the effect of a pole at a vector B_c meson in the Blascke factor of the z-expansion of f_+. We find that the shapes of the form factors are only weakly affected by the inclusion of such a pole.

By expanding the form factors at the respective momentum transfers, we finally arrive at

$$f_0^{(s)}(M_\pi^2)/f_0^{(d)}(M_K^2) = 1.035(39)(20). \tag{3.1}$$

The first error is from statistics. The second error is the systematical error due only to the uncertainty on $g_{DD^*\pi}$ and to the variation of fits in the z-parametrization. We are in the process of building a full systematic error budget.

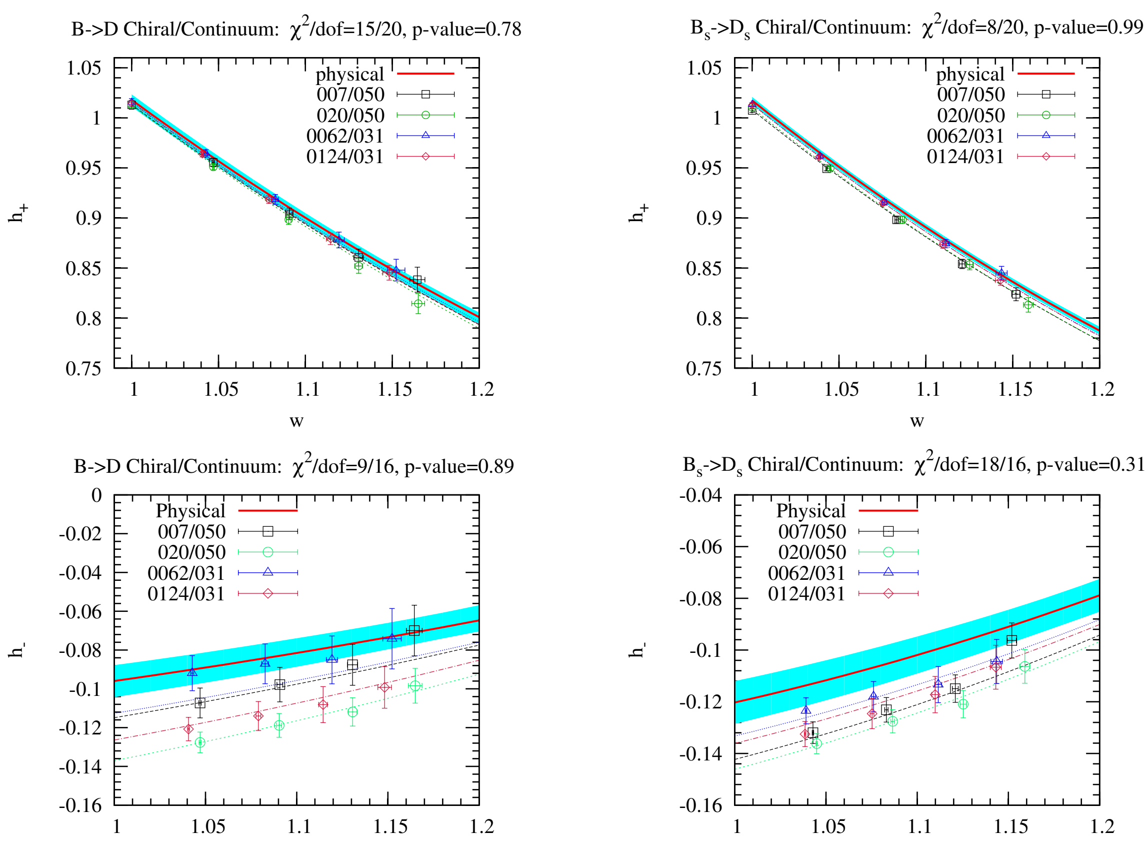

Figure 1: Chiral/continuum extrapolation of $h_\pm(w)$ for the $B \to D$ (left) and $B_s \to D_s$ decays (right).

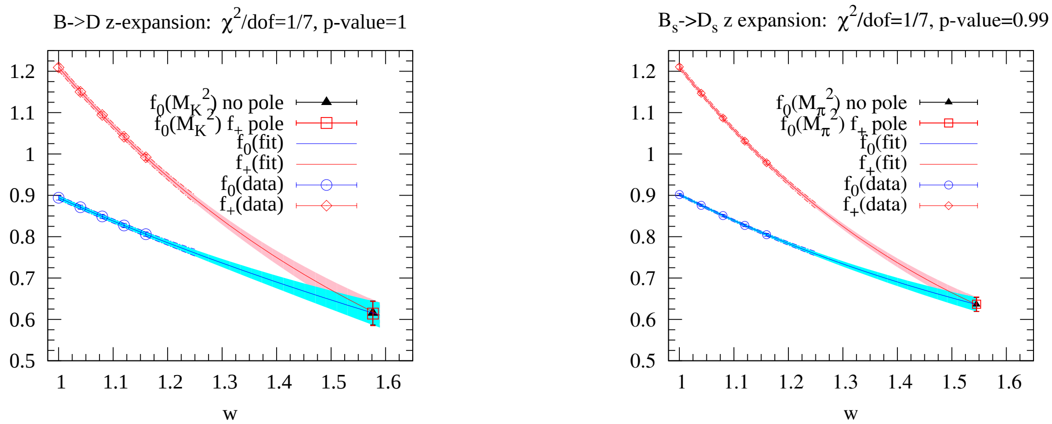

Figure 2: The z-expansion of form factors f_0, f_+. The points that we include in the z-expansion fits are shown explicitly. The dashed curves indicate the result of chiral/continuum extrapolation.

4. Conclusions

In summary, we present a (2+1)-flavor lattice QCD calculation of the form factor ratio $f_0^{(s)}(M_\pi^2)/f_0^{(d)}(M_K^2)$, which is a major theoretical input for the extraction of the fragmentation fraction ratio f_s/f_d. The essential part of our calculation is to extract the $B \to D$ and $B_s \to D_s$ semileptonic form factors at non-zero recoil. We reduce the systematic uncertainty by fitting the lowest-order excited states, and we employ a simultaneous multi-channel fit procedure to address correlations and reduce the statistical uncertainty. Our chiral/continuum results show that the corrections to the finite lattice spacings and finite light quark masses are small. Our preliminary result is $f_0^{(s)}(M_\pi^2)/f_0^{(d)}(M_K^2) = 1.035(39)(20)$, with a partial systematic error budget. As a consequence, we obtain $\mathscr{N}_F = 1.071(78)(40)$ which implies a smaller U-spin breaking effect than that suggested in [7], $\mathscr{N}_F = 1.24(8)$. A more comprehensive analysis with a detailed error budget is still in progress and will be reported in a forthcoming paper.

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