

# Raising the Higgs mass with Yukawa couplings for isotriplets in vector-like extensions of minimal supersymmetry

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Extra vector-like matter with both electroweak-singlet masses and large Yukawa couplings can significantly raise the lightest Higgs boson mass in supersymmetry through radiative corrections. I consider models of this type that involve a large Yukawa coupling between weak isotriplet and isodoublet chiral supermultiplets. The particle content can be completed to provide perturbative gauge coupling unification, in several different ways. The impact on precision electroweak observables is shown to be acceptably small, even if the new particles are as light as the current experimental bounds of order 100 GeV. I study the corrections to the lightest Higgs boson mass, and discuss the general features of the collider signatures for the new fermions in these models.

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## I. INTRODUCTION

Supersymmetry as an extension of the Standard Model addresses the hierarchy problem associated with the small ratio of the electroweak breaking scale to the Planck scale or other very high energy scales. However, the lack of a signal for the lightest neutral scalar boson,  $h^0$ , at the CERN LEP2  $e^+e^-$  collider imposes some tension on the minimal supersymmetric standard model (MSSM) parameter space, motivating an examination of further extensions that can increase the theoretical prediction of the mass of  $h^0$ .

In minimal supersymmetry, the biggest radiative corrections to  $m_{h^0}^2$  come from one-loop diagrams with top quarks and squarks, and are proportional to the fourth power of the top Yukawa coupling. This suggests that one could improve the situation by introducing new supermultiplets with large Yukawa couplings that would raise the  $h^0$  mass still further. This has been considered for the case of a fourth chiral family [1, 2]. However, in supersymmetry, the Yukawa couplings of a fourth chiral family would have to be so large (in order to evade discovery by LEP2 and the Tevatron) that perturbation theory would break down not far above the electroweak scale [1]. This would mean that the apparent success of gauge coupling unification in the MSSM is merely an illusion. Even accepting this, the corrections to precision electroweak physics would be too large in most of the parameter space, unless there are rather specific splittings of fermion masses [3].

Instead, one can consider models with extra matter in chiral supermultiplets comprised of vector-like representations of the Standard Model gauge group  $SU(3)_c \times SU(2)_L \times U(1)_Y$ , i.e., those that allow tree-level superpotential mass terms before spontaneous electroweak symmetry breaking. These bare mass terms are responsible for most of the vector-like fermion masses. However, if the extra vector-like matter includes appropriate representations differing by 1/2 unit of weak isospin, then they can also have Yukawa couplings to the MSSM Higgs supermultiplets. If large enough, these new Yukawa couplings can yield a significant enhancement of  $m_{h^0}^2$  through one-loop effects, helping to explain why  $h^0$  was not kinematically accessible to LEP2.

Earlier model-building work [4–8] along these lines has considered extra vector-like matter transforming in gauge representations of the types already present in the MSSM, and their conjugates. Under  $SU(3)_C \times SU(2)_L \times U(1)_Y$ , these candidate extra superfields transform like:

$$\begin{aligned}
 Q &= (\mathbf{3}, \mathbf{2}, 1/6), & \bar{Q} &= (\bar{\mathbf{3}}, \mathbf{2}, -1/6), & U &= (\mathbf{3}, \mathbf{1}, 2/3), & \bar{U} &= (\bar{\mathbf{3}}, \mathbf{1}, -2/3), \\
 D &= (\mathbf{3}, \mathbf{1}, -1/3), & \bar{D} &= (\bar{\mathbf{3}}, \mathbf{1}, 1/3), & L &= (\mathbf{1}, \mathbf{2}, -1/2), & \bar{L} &= (\mathbf{1}, \mathbf{2}, 1/2), \\
 E &= (\mathbf{1}, \mathbf{1}, -1), & \bar{E} &= (\mathbf{1}, \mathbf{1}, 1), & N, \bar{N} &= (\mathbf{1}, \mathbf{1}, 0).
 \end{aligned}
 \tag{1.1}$$

(Each bar appearing here is part of the name of the field, and does not denote any kind of conjugation.) Requiring that these new particles are not much heavier than 1 TeV, and that the gauge couplings still unify perturbatively, there are three types of models, with new (non-MSSM) chiral supermultiplets:

$$\text{LND}^n \text{ models:} \quad n \times (L, \bar{L}, N, \bar{N}, D, \bar{D}) \quad \text{for } (n = 1, 2, 3), \tag{1.2}$$

$$\text{QUE model:} \quad Q, \bar{Q}, U, \bar{U}, E, \bar{E}, \tag{1.3}$$

$$\text{QDEE model:} \quad Q, \bar{Q}, D, \bar{D}, E, \bar{E}, E, \bar{E}. \tag{1.4}$$

In each case, the number of singlets  $N$  or  $\bar{N}$  is actually arbitrary, since they do not directly affect

the running of the gauge couplings, but including the  $N, \bar{N}$  in the  $(\text{LND})^n$  models allows new Yukawa couplings. There is also a possible model with new supermultiplets:

$$\text{QUDLE model: } \quad Q, \bar{Q}, U, \bar{U}, D, \bar{D}, L, \bar{L}, E, \bar{E}. \quad (1.5)$$

However, to avoid the gauge couplings become non-perturbative in the ultraviolet before they have a chance to unify,<sup>†</sup> the average of the new particle masses in the QUDLE model would have to be at least about 2.5 TeV. This does not rule out the QUDLE model, but it goes strongly against the motivation of avoiding fine tuning. (If the large masses of the new fermions are due mostly to supersymmetric mass terms, then one cannot have a large enough hierarchy between scalar and fermion masses to increase  $m_{h^0}^2$  appreciably, unless the soft supersymmetry breaking scalar masses are much larger still.) Up to the inclusion of singlets, the LND model content corresponds to a  $\mathbf{5} + \bar{\mathbf{5}}$  of  $SU(5)$ , the QUE model to a  $\mathbf{10} + \bar{\mathbf{10}}$  of  $SU(5)$ , and the QUDLE model to a  $\mathbf{16} + \bar{\mathbf{16}}$  of  $SO(10)$ , although one need not subscribe to a belief in those groups as grand unified gauge symmetries.

In ref. [7], I showed that the LND, QUE and QDEE models are compatible with precision electroweak constraints, even if the new Yukawa couplings are as large as their quasi-fixed-point values and the new quarks and leptons are approximately as light as their present direct search limits from Tevatron and LEP2.

However, the new vector-like matter may include other representations not listed in eq. (1.1). Let us denote possible  $SU(2)_L$  triplet and  $SU(3)_C$  octet chiral supermultiplets by:

$$T = (\mathbf{1}, \mathbf{3}, 0), \quad O = (\mathbf{8}, \mathbf{1}, 0). \quad (1.6)$$

These are real representations of the gauge group, and so can have Majorana-type superpotential mass terms by themselves. If we denote by  $n_Q$  the number of  $Q, \bar{Q}$  pairs, and similarly for  $n_U, n_D, n_L$ , and  $n_E$ , and denote by  $n_T$  and  $n_O$  the number of  $T$  and  $O$  supermultiplets respectively, then the one-loop beta functions for the gauge couplings (with a GUT normalization  $g_1 = \sqrt{5/3}g'$ ) are:

$$Q \frac{dg_1}{dQ} = \beta_{g_1} = \frac{g_1^3}{16\pi^2} (33 + n_Q + 8n_U + 2n_D + 3n_L + 6n_E)/5, \quad (1.7)$$

$$Q \frac{dg_2}{dQ} = \beta_{g_2} = \frac{g_2^3}{16\pi^2} (1 + 3n_Q + n_L + 2n_T), \quad (1.8)$$

$$Q \frac{dg_3}{dQ} = \beta_{g_3} = \frac{g_3^3}{16\pi^2} (-3 + 2n_Q + n_U + n_D + 3n_O), \quad (1.9)$$

where  $Q$  is the renormalization scale. Perturbative unification requires that the one-loop contributions to the beta functions from the new fields are equal and not too large, so that

$$(n_Q + 8n_U + 2n_D + 3n_L + 6n_E)/5 = 3n_Q + n_L + 2n_T = 2n_Q + n_U + n_D + 3n_O \equiv N, \quad (1.10)$$

where  $N$  is 1, 2, or 3. (The details and precise quality of the unification depend also on 2-loop

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<sup>†</sup> To correctly implement this perturbativity requirement, it is mandatory to use 2-loop (or higher) beta functions. The numerical results in this paper always use 2-loop beta functions for all parameters. These can be obtained straightforwardly from the general results listed in [9, 10], and so are not listed explicitly here.

effects, including the effects of new Yukawa couplings. However, these effects do not make a dramatic difference, provided that  $N \leq 3$ .) This allows us to recognize some model possibilities different from those in eqs. (1.2)-(1.4). Consider models with extra chiral supermultiplets beyond the MSSM:

$$\text{TUD model:} \quad T, U, \bar{U}, D, \bar{D}, \quad (1.11)$$

$$\text{TEDD model:} \quad T, E, \bar{E}, D, \bar{D}, D, \bar{D}, \quad (1.12)$$

$$\text{OLLLE model:} \quad O, L, \bar{L}, L, \bar{L}, L, \bar{L}, E, \bar{E}, \quad (1.13)$$

$$\text{OTLEE model:} \quad O, T, L, \bar{L}, E, \bar{E}, E, \bar{E}, \quad (1.14)$$

$$\text{TLUDD model:} \quad T, L, \bar{L}, U, \bar{U}, D, \bar{D}, D, \bar{D}, \quad (1.15)$$

$$\text{TLEDDD model:} \quad T, L, \bar{L}, E, \bar{E}, D, \bar{D}, D, \bar{D}, D, \bar{D}. \quad (1.16)$$

The first two have  $N = 2$ , and the last four have  $N = 3$ . As before, these models can be augmented by any number of gauge singlet supermultiplets,<sup>‡</sup> which do not affect the gauge coupling running.

There are other possible representations of  $SU(3)_c \times SU(2)_L \times U(1)_Y$  that one could try to include. However, if one requires no unconfined fractional electric charges, then all such vector-like combinations, which include for example  $(\mathbf{1}, \mathbf{5}, 0)$  or  $(\mathbf{1}, \mathbf{3}, 1) + (\mathbf{1}, \mathbf{3}, -1)$  or  $(\mathbf{1}, \mathbf{2}, 3/2) + (\mathbf{1}, \mathbf{2}, -3/2)$  or  $(\mathbf{3}, \mathbf{2}, -5/6) + (\bar{\mathbf{3}}, \mathbf{2}, 5/6)$  or  $(\mathbf{3}, \mathbf{1}, -4/3) + (\bar{\mathbf{3}}, \mathbf{1}, 4/3)$  or  $(\mathbf{6}, \mathbf{1}, 1/3) + (\bar{\mathbf{6}}, \mathbf{1}, -1/3)$  or  $(\mathbf{1}, \mathbf{1}, 2) + (\mathbf{1}, \mathbf{1}, -2)$ , would contribute too much to  $N$  and can not be consistent with perturbative gauge coupling unification, unless the average of the new particle masses is at least in the multi-TeV range. So with these requirements,  $T$  and  $O$  are the only new possibilities beyond eq. (1.1). Restricting the new supermultiplets to those in eqs. (1.1) and (1.6) assures that small mixings with the MSSM quark and lepton or gaugino and higgsino fields can eliminate stable exotic particles, which otherwise could be disastrous relics of the early universe. For some other recent discussions of vector-like supermultiplets in supersymmetry, see [11]-[14].

In this paper, I will reserve the capital letters as above for new extra supermultiplets, and use lowercase letters for the usual chiral MSSM quark and lepton supermultiplets:

$$\begin{aligned} q_i &= (\mathbf{3}, \mathbf{2}, 1/6), & \bar{u}_i &= (\bar{\mathbf{3}}, \mathbf{1}, -2/3), & \bar{d}_i &= (\bar{\mathbf{3}}, \mathbf{1}, 1/3), & \ell_i &= (\mathbf{1}, \mathbf{2}, -1/2), \\ \bar{e}_i &= (\mathbf{1}, \mathbf{1}, 1), & H_u &= (\mathbf{1}, \mathbf{2}, 1/2), & H_d &= (\mathbf{1}, \mathbf{2}, -1/2), \end{aligned} \quad (1.17)$$

with  $i = 1, 2, 3$  denoting the three families. The MSSM part of the superpotential, in the approximation that only third-family Yukawa couplings are included, is:

$$W = \mu H_u H_d + y_t \bar{u}_3 q_3 H_u - y_b \bar{d}_3 q_3 H_d - y_\tau \bar{e}_3 \ell_3 H_d. \quad (1.18)$$

[Products of weak isospin doublet fields implicitly have their  $SU(2)_L$  indices contracted with an antisymmetric tensor  $\epsilon^{12} = -\epsilon^{21}$ , with the first component of every doublet having  $T_3 = 1/2$  and the second component having  $T_3 = -1/2$ . So, for example  $q_3 H_d = t H_d^- - b H_d^0$ , with the minus signs working out to give positive masses after the neutral components of the Higgs fields get vacuum

<sup>‡</sup> For example, the OTLEE model augmented by five singlets would correspond to an adjoint representation of the GUT group  $SU(3)_c \times SU(3)_L \times SU(3)_R \subset E_6$ .

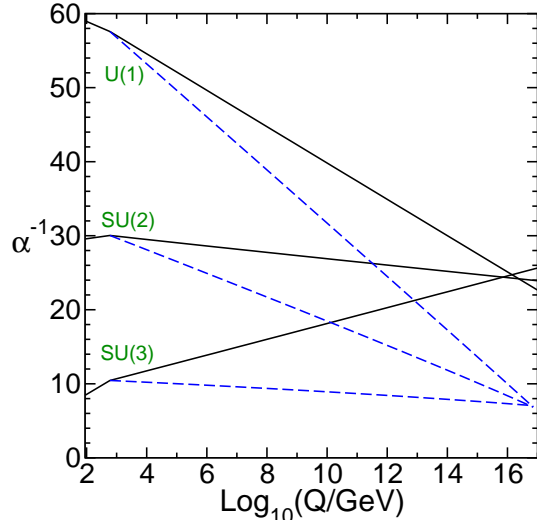


FIG. 1: Gauge coupling unification in the MSSM (solid lines) and in the OTLEE model of eq. (1.14) (dashed blue lines). The running is performed with 2-loop beta functions, with all particles beyond the Standard Model taken to decouple at  $Q = 600$  GeV, and  $m_t = 173.1$  GeV with  $\tan\beta = 10$ , with the extra Yukawa couplings set to 0 for simplicity.

expectation values (VEVs).]

Because of their vector-like representations, any Yukawa coupling-induced mixing between the new fields  $Q, \bar{Q}, U, \bar{U}, D, \bar{D}, L, \bar{L}, E, \bar{E}$  and their MSSM counterparts will not be governed by a GIM mechanism, and so must be highly suppressed. Therefore, to first approximation one can consider only Yukawa couplings that connect pairs of new fields. This can be enforced by an (approximate) symmetry, for example a  $Z_2$  under which the new superfields are odd and the MSSM quark and lepton superfields are even, or vice versa. The TUD and TEDD models do not have any allowed Yukawa couplings between pairs of new fields, and the OLLLE model allows only Yukawa couplings of the form  $H_u \bar{L} E$  and  $H_d \bar{L} \bar{E}$  (and  $H_u L \bar{N}$  and  $H_d \bar{L} N$  if singlets are present), which are qualitatively similar to the ones in the LND model already studied in refs. [6, 7], with fixed points that are not large enough to raise the  $h^0$  mass by a very significant amount.

In contrast, the OTLEE, TLUDD, and TLEDDD models all allow<sup>§</sup> the qualitatively new possibility of (doublet)-(triplet)-(doublet) superpotential Yukawa couplings  $k$  and  $k'$  involving the MSSM Higgs fields  $H_u, H_d$  and the weak isotriplet  $T$  field and the new vector-like isodoublet fields  $L$  and  $\bar{L}$ . Including also the relevant gauge-singlet mass terms, the superpotential is:

$$W = k H_u T L + k' H_d T \bar{L} + \frac{1}{2} M_T T^2 + M_L \bar{L} L. \quad (1.19)$$

In this paper, I will examine the features of models that include this structure. In particular, when  $k$  is large, it can induce a significant positive correction to  $m_{h^0}$ . The infrared quasi-fixed point for  $k$  is not too small to do so, in part because of the larger  $SU(2)_L$  Casimir invariant for the triplet  $T$  compared to a doublet (2 compared to  $3/4$ ). In the following, I will use the OTLEE model as an example, but many of the results apply also to the TLUDD and TLEDDD models with only small numerical changes. The unification of the gauge couplings in the OTLEE model is shown in Figure 1, with  $k = k' = 0$  for simplicity. Although the  $SU(3)_c$  gauge coupling would not run according to the one-loop renormalization group (RG) equations, two-loop effects are seen to cause it to get

<sup>§</sup> The OTLEE and TLEDDD models can also have Yukawa couplings  $H_u \bar{L} E$ ,  $H_d \bar{L} \bar{E}$  (and  $H_u L \bar{N}$  and  $H_d \bar{L} N$  if singlets are present), but I will assume these are absent or negligible for simplicity. If present, they would reduce the quasi-fixed point values of  $k, k'$ .

stronger in the ultraviolet, but not enough to become non-perturbative before unification takes place. The runnings in the TLUDD and TLEDDD models are only slightly different; all three of these models have  $N = 3$  from eq. (1.10).

## II. THE NEW PARTICLES AND THEIR MASSES

In this section, I consider the fermion and scalar content of the  $T = (T^+, T^0, T^-)$ ,  $L = (L^0, L^-)$ , and  $\bar{L} = (\bar{L}^+, \bar{L}^0)$  supermultiplets. After the mixing implied by the Yukawa couplings  $k$  and  $k'$  in eq. (1.19), the fermions will consist of three neutral Majorana fermion mass eigenstates, and two charged Dirac mass eigenstates denoted here by  $\psi_i^0$  for  $i = 1, 2, 3$ , and  $\psi_i^\pm$  for  $i = 1, 2$ , respectively. To find the mass eigenstates and their mixing angles, the superpotential eq. (1.19) can be written explicitly in terms of the different electric charge components of the gauge eigenstate fields as:

$$\begin{aligned} W = & M_T(T^+T^- + \frac{1}{2}T^0T^0) + M_L(L^-\bar{L}^+ - L^0\bar{L}^0) \\ & + k(T^0L^-H_u^+ + T^0L^0H_u^0 + \sqrt{2}T^+L^-H_u^0 - \sqrt{2}T^-L^0H_u^+) \\ & + k'(T^0\bar{L}^+H_d^- + T^0\bar{L}^0H_d^0 + \sqrt{2}T^+\bar{L}^0H_d^- - \sqrt{2}T^-\bar{L}^+H_d^0). \end{aligned} \quad (2.1)$$

Therefore, the mass matrices after electroweak symmetry breaking are, in two-component fermion notation [15]:

$$\mathcal{L} = -\frac{1}{2} \begin{pmatrix} T^0 & L^0 & \bar{L}^0 \end{pmatrix} \mathcal{M}_0 \begin{pmatrix} T^0 \\ L^0 \\ \bar{L}^0 \end{pmatrix} - \begin{pmatrix} T^- & L^- \end{pmatrix} \mathcal{M}_\pm \begin{pmatrix} T^+ \\ \bar{L}^+ \end{pmatrix} + \text{c.c.}, \quad (2.2)$$

$$\mathcal{M}_0 = \begin{pmatrix} M_T & kv_u & k'v_d \\ kv_u & 0 & -M_L \\ k'v_d & -M_L & 0 \end{pmatrix}, \quad \mathcal{M}_\pm = \begin{pmatrix} M_T & -\sqrt{2}k'v_d \\ \sqrt{2}kv_u & M_L \end{pmatrix}, \quad (2.3)$$

where  $v_u$  and  $v_d$  are the VEVs of the Higgs fields  $H_u^0$  and  $H_d^0$ , with  $v_u/v_d = \tan\beta$ . The real positive fermion mass eigenvalues and unitary mixing matrices  $N$ ,  $U$ , and  $V$  are defined by

$$N^* \mathcal{M}_0 N^\dagger = \text{diag}(m_{\psi_1^0}, m_{\psi_2^0}, m_{\psi_3^0}), \quad (2.4)$$

$$U^* \mathcal{M}_\pm V^\dagger = \text{diag}(m_{\psi_1^\pm}, m_{\psi_2^\pm}), \quad (2.5)$$

with  $(T^0, L^0, \bar{L}^0)_j = N_{ij}^* \psi_i^0$  and  $(T^-, L^-)_j = U_{ij}^* \psi_i^-$  and  $(T^+, \bar{L}^+)_j = V_{ij}^* \psi_i^+$ .

The scalar components of the  $T$ ,  $L$ ,  $\bar{L}$  supermultiplets mix to form four complex charged scalars  $\phi_i^\pm$  for  $i = 1, \dots, 4$ , and six real neutral scalars  $\phi_i^0$  for  $i = 1, \dots, 6$ . The general form of the soft supersymmetry-breaking Lagrangian is:

$$\begin{aligned} -\mathcal{L}_{\text{soft}} = & a_k(T^0L^-H_u^+ + T^0L^0H_u^0 + \sqrt{2}T^+L^-H_u^0 - \sqrt{2}T^-L^0H_u^+) \\ & + a_{k'}(T^0\bar{L}^+H_d^- + T^0\bar{L}^0H_d^0 + \sqrt{2}T^+\bar{L}^0H_d^- - \sqrt{2}T^-\bar{L}^+H_d^0) \end{aligned}$$

$$\begin{aligned}
& +b_T(T^+T^- + \frac{1}{2}T^0T^0) + b_L(L^-\bar{L}^+ - L^0\bar{L}^0) + \text{c.c.} \\
& +m_T^2(|T^0|^2 + |T^+|^2 + |T^-|^2) + m_L^2(|L^0|^2 + |L^-|^2) + m_{\bar{L}}^2(|\bar{L}^+|^2 + |\bar{L}^0|^2). \quad (2.6)
\end{aligned}$$

It follows that the  $6 \times 6$  gauge-eigenstate squared-mass matrix for the neutral scalars is

$$\begin{pmatrix} C & D^\dagger \\ D & C \end{pmatrix}, \quad (2.7)$$

in  $3 \times 3$  blocks, where

$$C = \mathcal{M}_0^\dagger \mathcal{M}_0 + \text{diag}(m_T^2, m_L^2 + \Delta_{\frac{1}{2},0}, m_{\bar{L}}^2 + \Delta_{-\frac{1}{2},0}) \quad (2.8)$$

with electroweak  $D$ -term contributions defined by  $\Delta_{T_3,q} = \frac{1}{2}[T_3 g^2 + (T_3 - q)g'^2](v_d^2 - v_u^2)$ , and

$$D = \begin{pmatrix} b_T & a_k v_u - k\mu^* v_d & a_{k'} v_d - k'\mu^* v_u \\ a_k v_u - k\mu^* v_d & 0 & -b_L \\ a_{k'} v_d - k'\mu^* v_u & -b_L & 0 \end{pmatrix}. \quad (2.9)$$

For the charged scalars the  $4 \times 4$  gauge-eigenstate squared-mass matrix is:

$$\begin{pmatrix} E & G^\dagger \\ G & F \end{pmatrix}, \quad (2.10)$$

where the  $2 \times 2$  blocks are

$$E = \mathcal{M}_\pm^\dagger \mathcal{M}_\pm + \text{diag}(m_T^2 + \Delta_{1,1}, m_{\bar{L}}^2 + \Delta_{\frac{1}{2},1}), \quad (2.11)$$

$$F = \mathcal{M}_\pm \mathcal{M}_\pm^\dagger + \text{diag}(m_T^2 + \Delta_{-1,-1}, m_L^2 + \Delta_{-\frac{1}{2},-1}), \quad (2.12)$$

$$G = \begin{pmatrix} b_T & \sqrt{2}(-a_{k'} v_d + k'\mu^* v_u) \\ \sqrt{2}(a_k v_u - k\mu^* v_d) & b_L \end{pmatrix}. \quad (2.13)$$

The tree-level scalar squared masses  $m_{\phi_i^0}^2$  and  $m_{\phi_i^\pm}^2$  are the eigenvalues of eqs. (2.7) and (2.10). I will assume that, as usual in phenomenologically viable supersymmetric models, the soft terms  $m_T^2$ ,  $m_L^2$ , and  $m_{\bar{L}}^2$  are large enough to make the scalar mass eigenstates  $\phi_i^0$  and  $\phi_i^\pm$  much heavier than their fermion counterparts  $\psi_i^0$  and  $\psi_i^\pm$ .

An important feature of these models is that infrared quasi-fixed points [16] govern the new Yukawa couplings. This can be seen qualitatively from the one-loop parts of the RG equations:

$$Q \frac{dk}{dQ} = \beta_k = \frac{k}{16\pi^2} (8k^2 + 2k'^2 + 3y_t^2 - 7g_2^2 - \frac{3}{5}g_1^2), \quad (2.14)$$

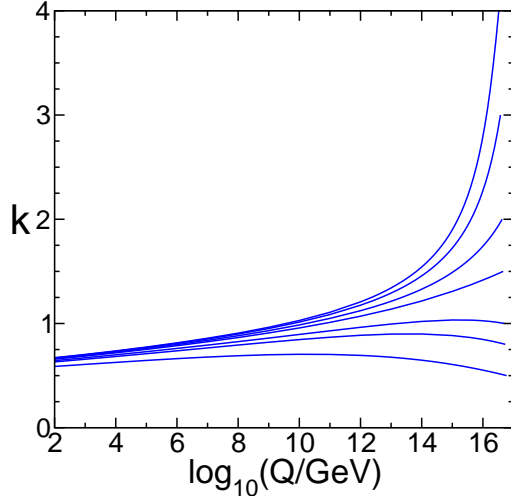


FIG. 2: The running of the Yukawa coupling  $k$  for various different input values at the unification scale, with  $k' = 0$  and  $\tan\beta = 10$ . This illustrates the quasi-fixed point structure, leading to  $k \approx 0.69$  at  $Q = 500$  GeV.

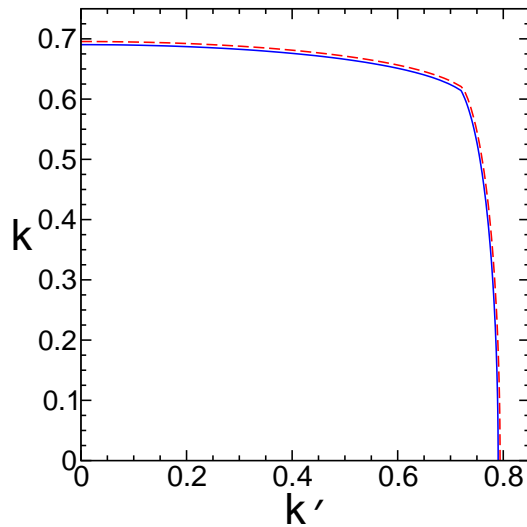


FIG. 3: The infrared quasi-fixed point contours for the Yukawa couplings  $k$  and  $k'$  evaluated at a renormalization scale of  $Q = 500$  GeV (solid line) and  $Q = 1000$  GeV (dashed line), with  $\tan\beta = 10$ . The regions below and to the left of the contours are allowed by  $k, k' < 3$  at the unification scale.

$$Q \frac{dk'}{dQ} = \beta_{k'} = \frac{k'}{16\pi^2} (8k'^2 + 2k^2 + 3y_b^2 + y_\tau^2 - 7g_2^2 - \frac{3}{5}g_1^2). \quad (2.15)$$

The infrared quasi-fixed points occur when the positive contributions from Yukawa couplings nearly cancel the negative contributions from gauge couplings. In the following, we will be most interested in the case that  $k$  is as large as possible, because when  $\tan\beta > 1$  this leads to the largest possible contribution to the mass of  $h^0$ ; this is obtained when  $k' = 0$ . The two-loop RG running of  $k$  for various different input values at the unification scale is shown in Figure 2. More generally, the contour of quasi-fixed points in the  $(k', k)$  plane is shown in Figure 3, obtained by requiring the perturbativity condition<sup>†</sup>  $k, k' < 3$  at the unification scale. Although there is coupling between  $k$  and  $k'$  in their RG equations, the quasi-fixed point value of  $k$  does not vary much as long as  $k'$  is not too large. In the following, I will use  $k = 0.69$  as the fixed point value, motivated by the fact that a wide range of input values at the unification scale will end up close to this fixed point.

The phenomenology of these models will depend strongly on the fermion masses. These masses are shown in Figure 4 for  $(k, k') = (0.69, 0)$  and  $\tan\beta = 10$  and varying superpotential mass

<sup>†</sup> This criterion is somewhat arbitrary, but the fixed point values are not very sensitive to it.



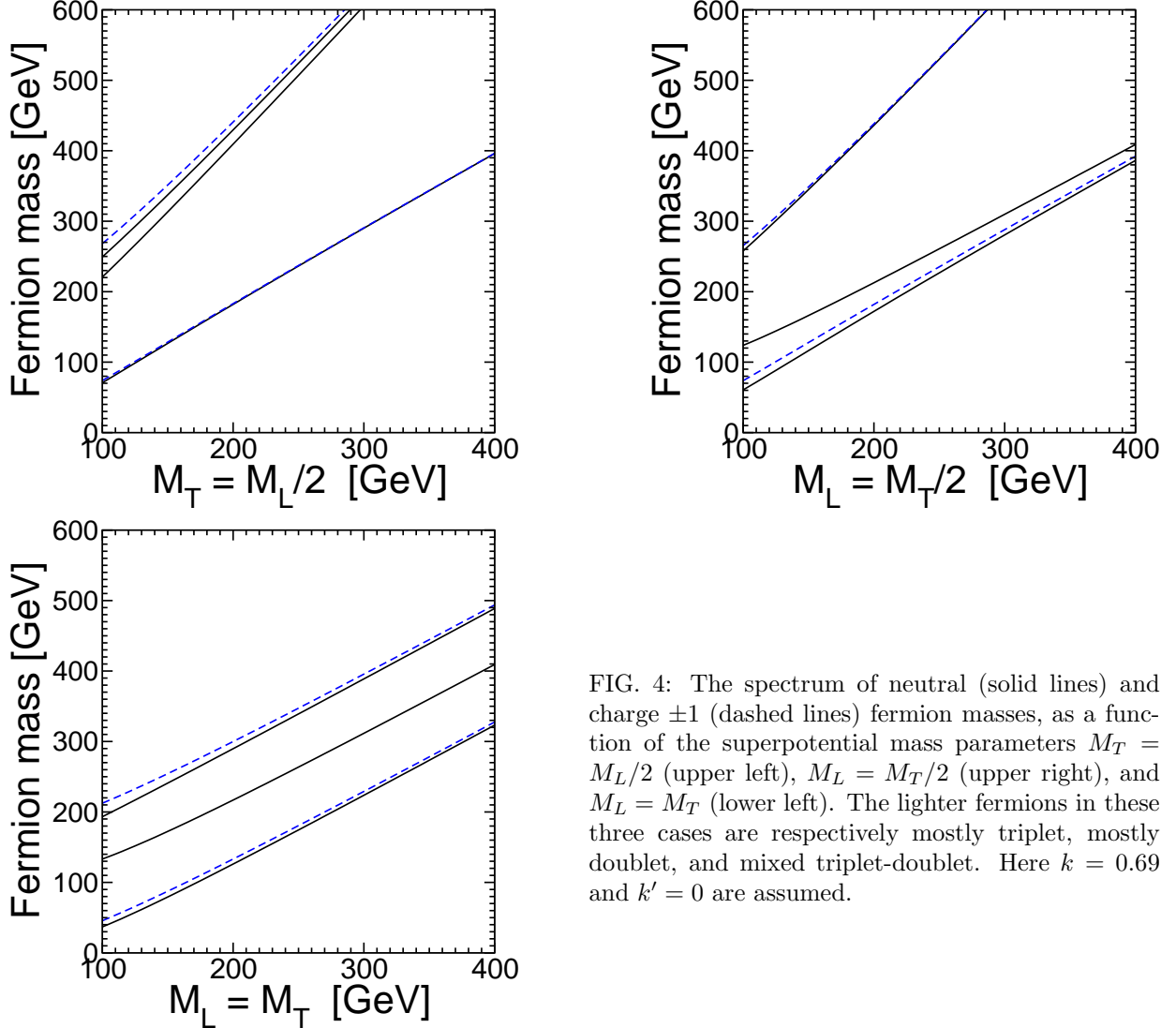


FIG. 4: The spectrum of neutral (solid lines) and charge  $\pm 1$  (dashed lines) fermion masses, as a function of the superpotential mass parameters  $M_T = M_L/2$  (upper left),  $M_L = M_T/2$  (upper right), and  $M_L = M_T$  (lower left). The lighter fermions in these three cases are respectively mostly triplet, mostly doublet, and mixed triplet-doublet. Here  $k = 0.69$  and  $k' = 0$  are assumed.

parameters  $M_T$  and  $M_L$ , for three different fixed ratios  $M_T/M_L = 0.5, 1$ , and  $2$ . One-loop radiative corrections to the masses are potentially important, and so are included using the results of Appendix A. In all cases, the lightest of the new fermions turns out to be the neutral  $\psi_1^0$ .

When  $M_T < M_L$ , the lightest fermions  $\psi_1^0$  and  $\psi_1^\pm$  form a very nearly degenerate triplet, but the presence of the Yukawa coupling  $k$  and one-loop radiative corrections ensures a non-zero splitting. When  $M_L < M_T$ , the lightest fermions  $\psi_1^0, \psi_2^0, \psi_1^\pm$  are mostly a Dirac pair of doublets, with a much larger mass splitting than the light triplet case. When  $M_L \sim M_T$ , there is significant mixing between the doublets and the triplet, although the splitting between  $m_{\psi_1^\pm}$  and  $m_{\psi_1^0}$  can be seen to remain fairly small. The mass splitting between the lowest-lying states

$$\Delta m \equiv m_{\psi_1^\pm} - m_{\psi_1^0} \quad (2.16)$$

plays an important role in collider signals, and so is shown in Figure 5 for cases with the lightest fermions mostly doublets ( $M_T = 2M_L$ ), mixed ( $M_T = M_L$ ), and mostly triplet ( $M_L = 2M_T$  and  $M_L = 3M_T$ ). The one-loop radiative corrections always increase  $\Delta m$ . The mass splitting is

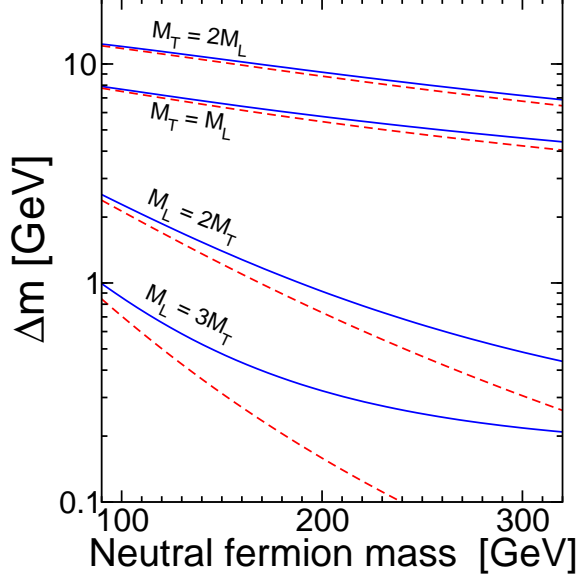


FIG. 5: The mass difference  $\Delta m = m_{\psi_1^\pm} - m_{\psi_1^0}$  between the next-lightest (charged) and lightest (neutral) new fermion masses, as a function of  $m_{\psi_1^0}$ . The four solid lines correspond to the cases, from top to bottom,  $M_T = 2M_L$ ,  $M_T = M_L$ ,  $M_L = 2M_T$ , and  $M_L = 3M_T$ , including one-loop radiative corrections. For comparison, the dashed lines show what the results would be with the one-loop corrections omitted. Here  $k = 0.69$  and  $k' = 0$  and  $\tan\beta = 10$  are assumed. Note that  $\Delta m$  is always positive, and decreases as the lightest fermions become more triplet-like, but is prevented from becoming too small by the radiative corrections.

smallest in the extreme limit of pure winos ( $kv_u \ll M_T \ll M_L$ ) where it asymptotically approaches  $\Delta m = 0.16$  GeV [17–19]. However, in most cases the mass splitting is considerably larger because of the Yukawa coupling, and it is always larger than the charged pion mass.

The RG running of the soft supersymmetry breaking terms has several interesting features that are comparable to those found in the LND, QUE and QDEE models studied in [7]. [To be concrete, here I use  $\tan\beta = 10$ ,  $m_t = 173.1$ , and  $(k, h) = (3, 0)$  at the unification scale. It cannot be under-emphasized that working to only one-loop order would yield very misleading results, because of the large values of the gauge couplings and non-trivial running of  $g_3$  at high scales.] First, if one assumes that the gaugino masses are unified with a value  $m_{1/2}$  at the same scale as the gauge couplings, then one finds that the RG running leads to quite different ratios than in the MSSM,

$$(M_1, M_2, M_3) = (0.13, 0.23, 0.47)m_{1/2}, \quad (\text{OTLEE model}) \quad (2.17)$$

$$= (0.13, 0.24, 0.62)m_{1/2}, \quad (\text{TLUDD, TLEDDD models}) \quad (2.18)$$

$$= (0.41, 0.77, 2.28)m_{1/2}, \quad (\text{MSSM}), \quad (2.19)$$

evaluated at  $Q = 1$  TeV. In particular, the ratios  $M_3/M_2$  and  $M_3/M_1$  are both much smaller in the extended models than in the MSSM. The extended models therefore predict a more compressed spectrum of superpartners than is found in the MSSM with unified gaugino masses.

If one takes the soft scalar squared masses and (scalar)<sup>3</sup> terms to vanish at the unification scale, corresponding to the “no-scale” or “gaugino-mediated” boundary conditions  $m_0^2 = 0$  and  $A_0 = 0$ , then one finds for the ordinary first- and second-family squark and slepton mass parameters at  $Q = 1$  TeV:

$$(m_{\tilde{q}_1}, m_{\tilde{u}_1}, m_{\tilde{d}_1}, m_{\tilde{\ell}_1}, m_{\tilde{e}_1}) = (1.15, 1.08, 1.08, 0.50, 0.30)m_{1/2} \quad (\text{OTLEE model}), \quad (2.20)$$

$$= (1.29, 1.23, 1.22, 0.51, 0.30)m_{1/2} \quad (\text{TLUDD, TLEDDD models}), \quad (2.21)$$

$$= (2.08, 2.01, 2.00, 0.67, 0.37)m_{1/2} \quad (\text{MSSM}). \quad (2.22)$$

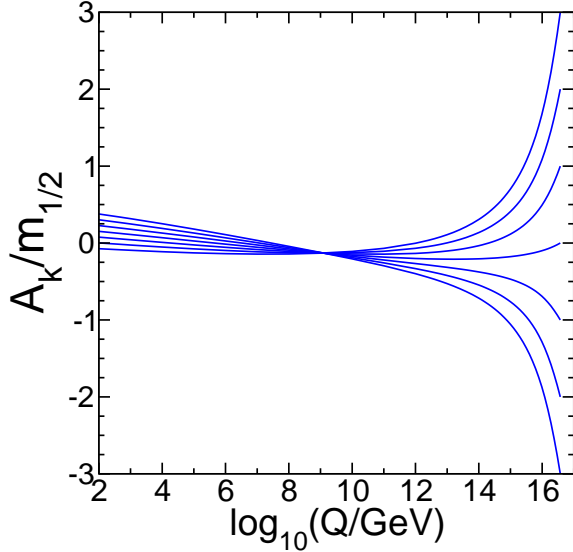


FIG. 6: Renormalization group running of (scalar)<sup>3</sup> coupling parameter  $A_k = a_k/k$ , normalized by  $m_{1/2}$ , in the OTLEE model with  $k$  near its quasi-fixed point trajectory (defined by  $k = 3$  at the gauge coupling unification scale). This shows a strongly attractive quasi-fixed point behavior for  $A_k$ . Very similar results obtain for the TLUDD and TLEDDD models.

Again, one sees a compression of the mass spectrum for the extended models compared to the MSSM. The soft masses for the new scalars in the OTLEE, TLUDD, and TLEDDD models are, respectively:

$$(m_{\tilde{T}}, m_{\tilde{L}}, m_{\tilde{\bar{L}}}, m_{\tilde{E}}, m_{\tilde{\bar{E}}}, m_{\tilde{O}}) = (0.73, 0.29, 0.51, 0.29, 0.30, 1.51)m_{1/2}, \quad (2.23)$$

$$(m_{\tilde{T}}, m_{\tilde{L}}, m_{\tilde{\bar{L}}}, m_{\tilde{U}}, m_{\tilde{\bar{U}}}, m_{\tilde{D}}, m_{\tilde{\bar{D}}}) = (0.74, 0.33, 0.51, 1.23, 1.23, 1.22, 1.22)m_{1/2}, \quad (2.24)$$

$$(m_{\tilde{T}}, m_{\tilde{L}}, m_{\tilde{\bar{L}}}, m_{\tilde{E}}, m_{\tilde{\bar{E}}}, m_{\tilde{D}}, m_{\tilde{\bar{D}}}) = (0.75, 0.33, 0.51, 0.29, 0.30, 1.23, 1.23)m_{1/2}. \quad (2.25)$$

Comparing eqs. (2.20)-(2.25) to eqs. (2.17)-(2.19) shows that, unlike the MSSM, the extended models considered here permit gaugino mass domination for the soft supersymmetry breaking terms at the unification scale while still having a bino-like neutralino as the LSP. (This feature was also observed in the QUE and QDEE models in ref. [7].)

Another important consideration is the running of the (scalar)<sup>3</sup> coupling  $a_k$ . The coupling  $a_k$  will play an important role in the corrections to  $m_{h^0}$  to be discussed below. It turns out that when  $k$  is near its quasi-fixed point trajectory, then the quantity

$$A_k \equiv a_k/k \quad (2.26)$$

itself has a strongly attractive quasi-fixed point near small multiples of  $m_{1/2}$ , as shown in Figure 6 for the OTLEE model. I have checked that the TLUDD and TLEDDD models give very similar results, and that this behavior is not very sensitive to the assumption of gaugino mass unification, if  $m_{1/2}$  is replaced by the value of  $M_2$  at the unification scale.

### III. CORRECTIONS TO THE LIGHTEST HIGGS SCALAR BOSON MASS

In this section, I consider the contribution of the new doublet and triplet supermultiplets  $L$ ,  $\bar{L}$ , and  $T$  to the lightest Higgs scalar boson mass. The effective potential approximation provides a simple way to estimate this contribution, and is equivalent to neglecting non-zero external momen-

tum effects in  $h^0$  self-energy diagrams. (This is an accurate approximation since  $m_{h^0}^2 < 4m_{\psi_1^0}^2$ .) The one-loop contribution to the effective potential due to the particles in the  $L$ ,  $\bar{L}$ , and  $T$  supermultiplets is:

$$\Delta V = \sum_{i=1}^6 F(m_{\phi_i^0}^2) - 2 \sum_{i=1}^3 F(m_{\psi_i^0}^2) + 2 \sum_{i=1}^4 F(m_{\phi_i^\pm}^2) - 4 \sum_{i=1}^2 F(m_{\psi_i^\pm}^2). \quad (3.1)$$

Here  $m_{\phi_i^0}^2$ ,  $m_{\psi_i^0}^2$ ,  $m_{\phi_i^\pm}^2$ , and  $m_{\psi_i^\pm}^2$  are the VEV-dependent tree-level squared-mass eigenvalues from eqs. (2.3)-(2.5), (2.7), and (2.10), and  $F(x) = x^2[\ln(x/Q^2) - 3/2]/64\pi^2$ , with  $Q$  the renormalization scale. I will assume the decoupling approximation that the neutral Higgs mixing angle (in the standard convention, described e.g. in [20]) is  $\alpha = \beta - \pi/2$ , which is valid if  $m_{A^0}^2 \gg m_{h^0}^2$ . Then the correction to  $m_{h^0}^2$  is

$$\Delta m_{h^0}^2 = \left\{ \frac{\sin^2 \beta}{2} \left[ \frac{\partial^2}{\partial v_u^2} - \frac{1}{v_u} \frac{\partial}{\partial v_u} \right] + \frac{\cos^2 \beta}{2} \left[ \frac{\partial^2}{\partial v_d^2} - \frac{1}{v_d} \frac{\partial}{\partial v_d} \right] + \sin \beta \cos \beta \frac{\partial^2}{\partial v_u \partial v_d} \right\} \Delta V. \quad (3.2)$$

In the OTLEE, TLUDD, and TLEDDD models, the other new fields ( $O, E, \bar{E}, U, \bar{U}, D, \bar{D}$ ) do not make a significant radiative contribution to the Higgs mass, as they do not have Yukawa couplings to  $H_u$  and  $H_d$ .

Before obtaining numerical results in a realistic model, it is useful to first consider a relatively simple analytical result for the case that the superpotential mass parameters are equal ( $M_T = M_L \equiv M_F$ ) and the non-holomorphic soft supersymmetry-breaking squared masses are also equal ( $m_T^2 = m_L^2 = m_{\bar{L}}^2 \equiv m^2$ ), and neglecting the holomorphic terms  $b_T$  and  $b_L$ . Then, writing

$$x = M_S^2/M_F^2, \quad M_S^2 \equiv M_F^2 + m^2 = \text{average scalar mass} \quad (3.3)$$

$$X_k = A_k - \mu \cot \beta, \quad (3.4)$$

and, expanding in  $k$ , I find

$$\Delta m_{h^0}^2 = \frac{v^2}{4\pi^2} \left\{ k^4 \sin^4 \beta \left[ f(x) + \frac{X_k^2}{M_S^2} \left( 5 - \frac{2}{x} \right) - \frac{5X_k^4}{12M_S^4} \right] + \frac{3}{4} (g^2 + g'^2) k^2 \sin^2 \beta \cos(2\beta) \left[ \ln(M_S^2/Q^2) + X_k^2/2M_S^2 \right] \right\} \quad (3.5)$$

where

$$f(x) = 5 \ln(x) - \frac{9}{2} + \frac{11}{2x} - \frac{1}{x^2}. \quad (3.6)$$

Note that  $x$  is approximately the ratio of the mean squared masses of the scalars to the fermions and is therefore assumed greater than 1, while the mixing between the new triplet and the new doublet scalars is parameterized by  $X_k$ . Similar to the models discussed in [4-8], the contribution to  $\Delta m_{h^0}^2$  does not decouple with the overall new particle mass scale, provided that there is a hierarchy maintained between the scalars and the fermions. The electroweak  $D$ -term contribution

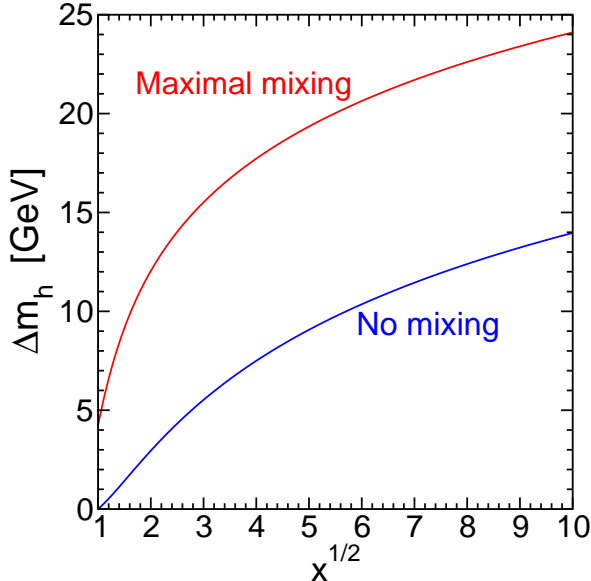


FIG. 7: Estimates for the corrections to the  $h^0$  mass as a function of  $\sqrt{x} = M_S/M_F$ , where  $M_S$  and  $M_F$  are the mean scalar and fermion masses in the supermultiplets  $L, \bar{L}, T$ , in the simplified model framework used in eq. (3.5) of the text, using  $v^2 k^4 \sin^4 \beta = (83 \text{ GeV})^2$ , corresponding to the quasi-fixed point with reasonably large  $\tan \beta$ . The lower line is the no-mixing case  $X_k = 0$ , and the upper line is the “maximal mixing” case  $X_k^2 = 6M_S^2(1 - 2/5x)$ . The  $h^0$  mass before the correction is taken to be 110 GeV.

involving  $g, g'$  is quite small, provided one chooses a RG scale  $Q \sim M_S$ , and is neglected below. The maximum possible contribution to  $\Delta m_{h^0}^2$  occurs when  $X_k^2 = 6M_S^2(1 - 2/5x)$ , leading to a “maximal mixing” result given by  $\Delta m_{h^0}^2 = \frac{v^2}{4\pi^2} k^4 \sin^4 \beta f_{\text{max}}(x)$ , where

$$f_{\text{max}}(x) = f(x) + \frac{3}{5}(5 - 2/x)^2. \quad (3.7)$$

In Figure 7, I show an estimate of these corrections to  $\Delta m_{h^0}$ , using  $v^2 k^4 \sin^4 \beta = (83 \text{ GeV})^2$ , corresponding to the quasi-fixed point with reasonably large  $\tan \beta$ , and assuming that the predicted  $h^0$  mass before the correction is 110 GeV, so that  $\Delta m_{h^0} = \sqrt{(110 \text{ GeV})^2 + \Delta m_{h^0}^2} - 110 \text{ GeV}$ .

As found in the previous section, the quasi-fixed point behavior of the running of the scalar trilinear coupling  $a_k$  implies that the mixing parameter  $X_k$  is probably actually much smaller than in the “maximal mixing” case. Also, the soft supersymmetry breaking squared masses  $m_T^2$ ,  $m_L^2$ , and  $m_{\bar{L}}^2$  need not be degenerate. A perhaps better-motivated scenario is therefore the gaugino mass dominated case shown in Figure 8, where I take  $(m_T, m_L, m_{\bar{L}}) = (0.73, 0.29, 0.51)m_{1/2}$  and  $A_k = 0.11m_{1/2}$  [see eq. (2.23) and Figure 6], with  $M_T = M_L = M_F$  adjusted so that the lightest new charged fermion mass is  $m_{\psi_1^\pm} = 100, 125, 150, 200, 250$ , and 400 GeV. These results were computed using the complete expressions in eqs. (2.3), (2.7)-(2.13) and (3.1), (3.2), not from the simplified expansion in  $k$ . The correction to  $m_{h^0}$  turns out to be not dramatically sensitive to  $b_T$  and  $b_L$  (taken to be 0 here), or  $\mu$  (set to  $m_{1/2}$  here) or  $\tan \beta$  (set to 10 here) provided it is not too small. For a given value of  $m_{\psi_1^\pm}$ , the upper bound on corrections to  $m_{h^0}$  is nearly saturated when  $M_T = M_L$ .

Figure 8 shows that the corrections to  $m_{h^0}$  are moderate, but can easily exceed 5 GeV for an average new scalar mass  $M_S$  less than 1 TeV, provided that at least one new charged fermion is lighter than about 200 GeV. However, it should be kept in mind that the actual corrections can be larger or smaller than indicated in Figure 8, depending on the details of the new particle spectrum. If the fixed point behavior for  $a_k$  noted above is evaded somehow, then the corrections to  $m_{h^0}$  could be substantially larger. The contribution to  $\Delta m_{h^0}$  also monotonically increases with the scalar masses (for fixed fermion masses), and so in principle could be much larger, subject to

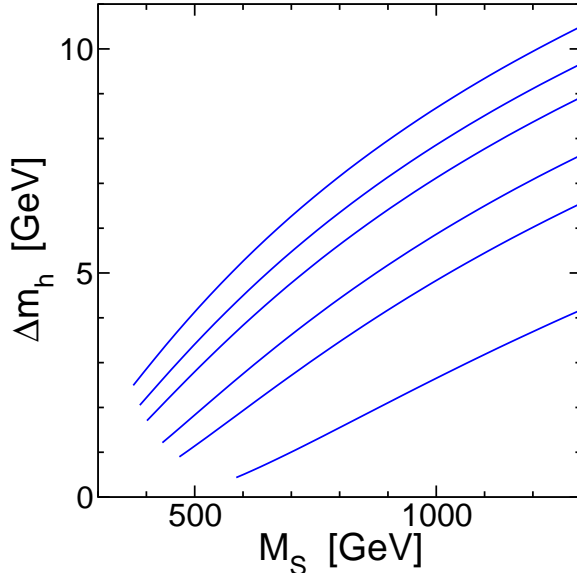


FIG. 8: Corrections to  $m_{h^0}$ , for  $k = 0.69$  and  $M_T = M_L \equiv M_F$  in the gaugino mass dominated scenario  $(m_T, m_L, m_{\overline{L}}) = (0.73, 0.29, 0.51)m_{1/2}$  and  $A_k = 0.11m_{1/2}$ , with varying  $m_{1/2}$ , and other parameters as described in the text. The lines correspond to, from top to bottom,  $m_{\psi_1^\pm} = 100, 125, 150, 200, 250,$  and  $400$  GeV (corresponding to  $M_F = 165, 192, 219, 272, 324,$  and  $478$  GeV respectively). The quantity  $M_S$  on the horizontal axis is the geometric mean of the new scalar masses. The value of  $m_{h^0}$  before the corrections is taken to be  $110$  GeV.

considerations of fine-tuning that intuitively should get worse with larger supersymmetry breaking. Due to the impossibility of defining an objective measure of fine tuning, I will not attempt to quantify the merits of this trade-off, but simply note that that even a contribution of a few GeV to  $m_{h^0}$  is quite significant in the context of the supersymmetric little hierarchy problem. Smaller fermion masses  $m_{\psi_i^0}, m_{\psi_i^\pm}$  may be considered preferred in the sense that this maximizes  $\Delta m_{h^0}$ .

#### IV. PRECISION ELECTROWEAK EFFECTS

The Yukawa couplings  $k$  and  $k'$  break the custodial symmetry of the Higgs sector, and therefore contribute to virtual corrections to  $W^\pm$ ,  $Z$ , and photon self-energies, of the type that are constrained by precision electroweak observables. Similarly to the cases analyzed in [7], these corrections are actually benign, at least if one uses  $M_t$ ,  $M_W$ , and  $Z$ -peak observables as in the LEP Electroweak Working Group analyses [21, 22]. (A different set of observables are used in [23], leading to a worse fit.) This is because the corrections decouple with larger vector-like masses  $M_T$  and  $M_L$ , even if the Yukawa couplings are large and soft supersymmetry breaking effects including  $m_T^2$ ,  $m_L^2$  and  $m_{\overline{L}}^2$  produce a large scalar-fermion hierarchy. In particular, they decouple even when the corrections to  $m_{h^0}^2$  do not.

The most important new physics contributions to the precision electroweak observables can be summarized in terms of the Peskin-Takeuchi  $S$  and  $T$  parameters [24] (similar parameterizations of oblique electroweak observables were discussed in [25]). In this paper, I will use the updated experimental values

$$s_{\text{eff}}^2 = 0.23153 \pm 0.00016 \quad \text{ref. [21]} \quad (4.1)$$

$$M_W = 80.399 \pm 0.025 \text{ GeV} \quad \text{ref. [22, 26]} \quad (4.2)$$

$$\Gamma_\ell = 83.985 \pm 0.086 \text{ MeV} \quad \text{ref. [21]} \quad (4.3)$$

$$\Delta\alpha_h^{(5)}(M_Z) = 0.02758 \pm 0.00035 \quad \text{ref. [21]} \quad (4.4)$$

$$M_t = 173.1 \pm 1.3 \text{ GeV} \quad \text{ref. [27]} \quad (4.5)$$

$$\alpha_s(M_Z) = 0.1187 \pm 0.0020 \quad \text{ref. [23]} \quad (4.6)$$

with  $M_Z = 91.1875$  GeV held fixed. For the Standard Model predictions for  $s_{\text{eff}}^2$ ,  $M_W$ , and  $\Gamma_\ell$  in terms of the other parameters, I use refs. [28], [29], and [30], respectively. These values are then used to determine the best experimental fit values and the 68% and 95% confidence level (CL) ellipses for  $S$  and  $T$ , relative to a Standard Model template with  $M_t = 173.1$  GeV and  $M_h = 115$  GeV, using

$$\frac{s_{\text{eff}}^2}{(s_{\text{eff}}^2)_{\text{SM}}} = 1 + \frac{\alpha}{4s_W^2 c_{2W}} S - \frac{\alpha c_W^2}{c_{2W}} T, \quad (4.7)$$

$$\frac{M_W^2}{(M_W^2)_{\text{SM}}} = 1 - \frac{\alpha}{2c_{2W}} S + \frac{\alpha c_W^2}{c_{2W}} T, \quad (4.8)$$

$$\frac{\Gamma_\ell}{(\Gamma_\ell)_{\text{SM}}} = 1 - \alpha d_W S + \alpha(1 + s_{2W}^2 d_W) T, \quad (4.9)$$

where  $s_W = \sin \theta_W$ ,  $c_W = \cos \theta_W$ ,  $s_{2W} = \sin(2\theta_W)$ ,  $c_{2W} = \cos(2\theta_W)$ , and  $d_W = (1 - 4s_W^2)/(1 - 4s_W^2 + 8s_W^4)c_{2W}$ . The best fit is found to be  $\Delta S = 0.057$  and  $\Delta T = 0.080$ , relative to the Standard Model template with  $M_t = 173.1$  GeV and  $M_h = 115$  GeV. The new physics contributions are given at one-loop order in terms of electroweak vector boson self-energy functions  $\Pi_{WW}$ ,  $\Pi_{ZZ}$ ,  $\Pi_{\gamma\gamma}$ , and  $\Pi_{Z\gamma}$ , which are computed for the fermions of the  $T$ ,  $L$ ,  $\bar{L}$  sector in Appendix B. The contributions to  $S$  and  $T$  from scalars will be much smaller when they are much heavier than the fermions, due to decoupling, because most of the scalar masses comes from vector-like soft supersymmetry breaking terms. I will therefore neglect those contributions here. I have also neglected the contributions from the ordinary MSSM superpartners, which are typically not very large and which also decouple quadratically with large soft supersymmetry breaking terms. Note also that in the OTLEE, TLUDD, and TLEDDD models, the fields that do not have Yukawa couplings do not contribute to the  $S$  and  $T$  parameters.

It is useful to first consider the simple case that electroweak symmetry breaking is treated as a perturbation in the vector-like  $T$ ,  $L$ ,  $\bar{L}$  sector. For  $kv_u, k'v_d \ll M_T, M_L$ , I find:

$$\Delta T = \frac{3(k^2 v_u^2 - k'^2 v_d^2)^2}{32\pi s_W^2 m_W^2 M_L M_T} f_T(r) \quad (4.10)$$

$$\Delta S = \frac{2(k^2 v_u^2 + k'^2 v_d^2)}{5\pi M_L M_T} f_{S1}(r) - \frac{19kk'v_u v_d}{30\pi M_T M_L} f_{S2}(r) \quad (4.11)$$

where  $r = M_L^2/M_T^2$ , and

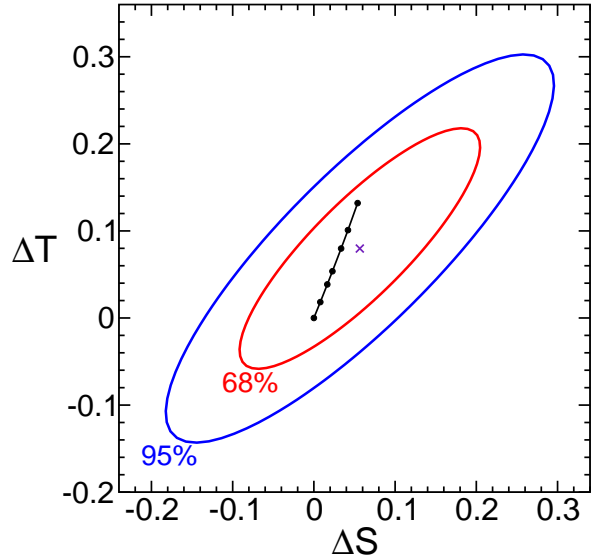
$$f_T(r) = \frac{2\sqrt{r}}{9(1-r)^4} [(6 + 36r - 36r^2) \ln(r) + 29 - 36r - 9r^2 + 16r^3], \quad (4.12)$$

$$f_{S1}(r) = \frac{5\sqrt{r}}{6(r-1)^5} [(1 - 2r + 9r^2 - 4r^3 + 2r^4) \ln(r) + 5r - 6r^2 + 3r^3 - 2r^4], \quad (4.13)$$

$$f_{S2}(r) = \frac{10}{19(r-1)^5} [(4r + 6r^2 - 6r^3 + 8r^4) \ln(r) - 2 + 21r - 39r^2 + 35r^3 - 15r^4], \quad (4.14)$$

normalized so that  $f_T(1) = f_{S1}(1) = f_{S2}(1) = 1$ . Despite the appearances of denominators

FIG. 9: Corrections to electroweak precision observables  $S, T$  from the new fermions in the  $T, L, \bar{L}$  multiplets, at the fixed point  $(k, k') = (0.69, 0)$ , for varying  $M_T = M_L$ . The corrections are evaluated using eqs. (B.1), (B.2) and (B.3)-(B.6). The seven dots on the line segment correspond to, from top to bottom,  $m_{\psi_1^\pm} = 100, 125, 150, 200, 250, 400$  GeV and  $\infty$ . The experimental best fit is shown as the  $\times$  at  $(\Delta S, \Delta T) = (0.057, 0.080)$ . Also shown are the 68% and 95% CL ellipses. The point  $\Delta S = \Delta T = 0$  is defined to be the Standard Model prediction for  $m_t = 173.1$  GeV and  $m_{h^0} = 115$  GeV. Results for  $M_T \neq M_L$  are very similar; the corrections to  $S$  and  $T$  are slightly smaller than shown here, for the same values of  $m_{\psi_1^\pm}$ .



singular at  $r = 1$ , these functions are actually quite slowly varying. For the case  $k' = 0$ , there follow numerical estimates (for  $r \approx 1$ , which very nearly saturates the maximum corrections for a given value of  $M_L M_T$ ):

$$\Delta T = 0.42 \left( \frac{k}{0.69} \right)^4 \sin^4 \beta \frac{(100 \text{ GeV})^2}{M_L M_T}, \quad (4.15)$$

$$\Delta S = 0.18 \left( \frac{k}{0.69} \right)^2 \sin^2 \beta \frac{(100 \text{ GeV})^2}{M_L M_T}. \quad (4.16)$$

These contributions decouple quadratically with increasing fermion masses, in contrast to the corrections to  $m_{h^0}^2$ . However, it should be noted that this expansion in small  $kv_u$  and  $k'v_d$  is not extremely accurate, unless both  $M_L$  and  $M_T$  are much larger than 100 GeV, and overestimates the true corrections to  $S, T$ .

A more accurate evaluation using the complete formulas of Appendix B is shown in Figure 9, for the case that  $M_T = M_L$  and  $k = 0.69$ , the quasi-fixed point value. The seven dots on the line segment correspond to lighter new charged fermion masses  $m_{\psi_1^\pm} = 100, 125, 150, 200, 250, 400$  GeV and  $\infty$ , from top to bottom. (For reference, the first six points correspond to  $M_T = M_L = 165, 192, 219, 272, 324, \text{ and } 478$  GeV respectively.) Figure 9 shows that for charged fermion masses not excluded by the CERN LEP2  $e^+e^-$  collider, the  $S$  and  $T$  parameters remain within the current 68% confidence level ellipse, and can even give a better (but not significantly so) fit than the Standard Model. Results for  $M_T \neq M_L$  are very similar, with corrections to  $S$  and  $T$  that are slightly smaller than shown here, for the same values of  $m_{\psi_1^\pm}$ . As a caveat, it is important to keep in mind that the results above are sensitive to my choice of following the LEP Electroweak Working Group [21, 22] in the choice of fit observables; if one chose instead the set of observables used in [23], the fits to  $S$  and  $T$  would be worse.



## V. COLLIDER PHENOMENOLOGY OF THE EXTRA FERMIONS

The collider phenomenology of the models discussed in this paper depends on the decay modes of the new fermions. First consider the fermions  $\psi_i^\pm$  and  $\psi_i^0$  in the  $T$ ,  $L$ , and  $\bar{L}$  multiplets that are common to the OTLEE, TLUDD, and TLEDDD models. As discussed in section II, the lightest of these is always the neutral fermion  $\psi_1^0$ . This particle can only decay by virtue of mixing with other MSSM fermions. The simplest possibility is that all such mixing is forbidden. Then  $\psi_1^0$  would be absolutely stable, and in principle could be a component of the dark matter. However, the thermal relic density would be very low due to an unsuppressed annihilation rate, similar to the familiar cases of almost pure wino or higgsino LSPs in the MSSM.

If  $T$ ,  $L$  and  $\bar{L}$  are assigned<sup>†</sup> even matter parity, then the new fermions all have odd R-parity and can mix with the MSSM charginos and neutralinos through supersymmetric terms involving the higgsinos and a supersymmetry-breaking term involving the winos:

$$W = \mu_L L H_u + \mu_{\bar{L}} \bar{L} H_d + \lambda H_u T H_d, \quad (5.1)$$

$$-\mathcal{L} = M'_2 T \widetilde{W} + \text{c.c.} \quad (5.2)$$

These mixing terms enable the decays  $\psi_1^0 \rightarrow h^0 \tilde{N}_1$  and  $\psi_1^0 \rightarrow Z^0 \tilde{N}_1$ , and may also enable decays to heavier ordinary charginos and neutralinos, depending on the kinematics. The  $M'_2$  term is not one of the usual type of soft supersymmetry breaking terms, but could arise from a non-renormalizable interaction. With this even matter parity assignment, the supersymmetric mixing terms will introduce terms in the scalar potential that will cause  $L$ ,  $\bar{L}$  and  $T$  to obtain VEVs (see e.g. [33]), a possibility not covered in the preceding sections and not pursued further here.

If instead  $T$ ,  $L$  and  $\bar{L}$  are assigned odd matter parity, then  $\psi_1^0$  has even R-parity and can only decay through a small mixing with the MSSM leptons, via the superpotential terms:

$$W = \epsilon_T H_u T \ell_i + \epsilon_L H_d L \bar{e}_i. \quad (5.3)$$

In general this implies lepton family number violation, so it is necessary to assume that either the couplings only involve a single lepton family, or are very small; in addition, lepton universality constraints suggest that mixing with the tau lepton may be most important. In the  $(T^0, L^0, \bar{L}^0, \nu_\ell)$  basis, the neutral fermion mass matrix is:

$$\widehat{\mathcal{M}}_0 = \begin{pmatrix} M_T & k v_u & k' v_d & \epsilon_T v_u \\ k v_u & 0 & -M_L & 0 \\ k' v_d & -M_L & 0 & 0 \\ \epsilon_T v_u & 0 & 0 & 0 \end{pmatrix}. \quad (5.4)$$

<sup>†</sup> Matter parity is trivially related to R-parity by an extra factor of  $-1$  for fermions, and is assumed here to be exactly conserved. Each of  $T$  and  $O$ , and each of the pairs of new superfields such as  $(L, \bar{L})$  that share a superpotential mass term, can be independently assigned either even or odd matter parity, consistently with the requirements of gauged discrete symmetries [31, 32]. However, allowing the Yukawa couplings  $k$  and/or  $k'$  requires that  $T$ ,  $L$  and  $\bar{L}$  all have the same matter parity.

The smallest eigenvalue of this will be (a contribution to) the squared mass of the Standard Model neutrino  $\nu_\ell$ , and should be approximately  $\epsilon_T^2 v_u^2 / M_T$ . Interpreting the bound  $\Delta m_{23}^2 < 3 \times 10^{-3} \text{ eV}^2$  [23] as applying to the  $\nu_\ell$  alone (although there could be degeneracy in the neutrino sector, so this is not a strict bound), one would arrive at the rough estimate  $\epsilon_T < 4 \times 10^{-7} \sqrt{M_T / 100 \text{ GeV}}$ . The coupling  $\epsilon_L$  is not so constrained, only contributing to mixing between the new fermions and the Standard Model lepton  $\ell$  through the charged fermion mass matrix:

$$\widehat{\mathcal{M}}_\pm = \begin{pmatrix} M_T & -\sqrt{2}k'v_d & 0 \\ \sqrt{2}kv_u & M_L & \epsilon_L v_d \\ \sqrt{2}\epsilon_T v_u & 0 & y_\ell v_d \end{pmatrix}. \quad (5.5)$$

Below I will assume  $\epsilon_L, \epsilon_T \ll y_\ell$ , so that their effects can be treated as small perturbations. The decays of  $\psi_1^0$  due to  $\epsilon_T$  and  $\epsilon_L$  are:  $\psi_1^0 \rightarrow Z\nu_\ell$  and  $\psi_1^0 \rightarrow Z\bar{\nu}_\ell$  (combined below by an abuse of notation as  $\psi_1^0 \rightarrow Z\nu$ ), and  $\psi_1^0 \rightarrow h^0\nu_\ell$  and  $\psi_1^0 \rightarrow h^0\bar{\nu}_\ell$  (similarly combined below as  $\psi_1^0 \rightarrow h\nu$ ), and  $\psi_1^0 \rightarrow W^\pm\ell^\mp$  (hereafter combined by writing  $\psi_1^0 \rightarrow W\ell$ ). The corresponding decay widths are computed in Appendix C. If  $\epsilon_L \gg \epsilon_T$ , then the decay will be entirely charged current, with  $\text{BR}(\psi_1^0 \rightarrow W\ell) = 1$ . Otherwise, the three final states  $W\ell$ ,  $Z\nu$ , and  $h\nu$  can have comparable branching ratios, as shown in Figure 10 for the case  $\epsilon_T \gg \epsilon_L$ . For  $m_{\psi_1^0} \lesssim m_Z$ , the  $W\ell$  final state essentially always dominates, due to kinematics. It is notable that in the limit that  $\psi_1^0$  is mostly doublet, the branching ratio to  $W\ell$  is suppressed for masses above 100 GeV. The  $\psi_1^0$  decay lengths are necessarily macroscopic if  $\epsilon_T < 4 \times 10^{-7} \sqrt{M_T / 100 \text{ GeV}}$  (as suggested by the observed neutrino mass splitting) and  $\epsilon_L = 0$ , if  $m_{\psi_1^0} \lesssim 100 \text{ GeV}$ . However, the minimum decay length allowed by this condition on  $\epsilon_T$  rapidly becomes smaller for larger  $m_{\psi_1^0}$ , with typically  $c\tau_{\min} \sim 1 \text{ cm}$  for  $m_{\psi_1^0} = 100 \text{ GeV}$  and  $c\tau_{\min} \sim 1 \text{ mm}$  for  $m_{\psi_1^0} = 125 \text{ GeV}$ .

The heavier new fermions ( $\psi_i^\pm$  for  $i = 1, 2$  and  $\psi_i^0$  for  $i = 2, 3, 4$ ) can decay to the lightest one  $\psi_1^0$  via emission of real or virtual  $W^\pm, Z^0, h^0$  bosons. As can be seen in Figure 5, the available phase space for the decay of  $\psi_1^\pm$  to  $\psi_1^0$  can be quite small. Depending on  $\Delta m$ , the soft decay products could include one or more pions, a soft jet, or more rarely a lepton. These may be hard to detect (and certainly to trigger on), especially in a hadron collider environment and especially when the lightest fermions are mostly triplet. Extensive studies [18, 19, 34–43] have been made of the somewhat similar case of collider production of nearly degenerate MSSM winos or higgsinos. When the mass difference for light winos or higgsinos is sufficiently small, the  $\psi_1^\pm$  decay could manifest itself as a “stub”, a stiff, highly ionizing track that ends in the vertex detector or in the tracker before making it to the calorimeters, or else has a kink to a very soft charged track. It could also be seen as a pion with a measurable non-zero impact parameter. These features could be seen in an off-line analysis, if the event is triggered by other means.

However, in the present context there are two major differences from the MSSM wino and higgsino cases. First, because a large Yukawa coupling  $k$  increases the mass splitting  $\Delta m$ , making it larger than  $m_{\pi^\pm}$ , the decay of  $\psi_1^\pm$  to  $\psi_1^0$  will almost always occur at a distance scale much smaller than the size of the vertex detector or innermost tracker. The critical case of smallest  $\Delta m$  occurs when the lightest fermions are mostly triplets, where one may take over the results of ref. [34], which show that  $c\tau$  is of order 1 cm (1 mm) for  $\Delta m = 0.3$  (0.5) GeV. Comparison with Figure 5 above shows that, for example,  $c\tau \gtrsim 1 \text{ cm}$  for  $\psi_1^\pm \rightarrow \psi_1^0\pi^\pm$  with  $m_{\psi_1^0} < 200 \text{ GeV}$  will only occur if  $M_T \lesssim M_L/3$  (assuming that  $k$  is at its fixed point).

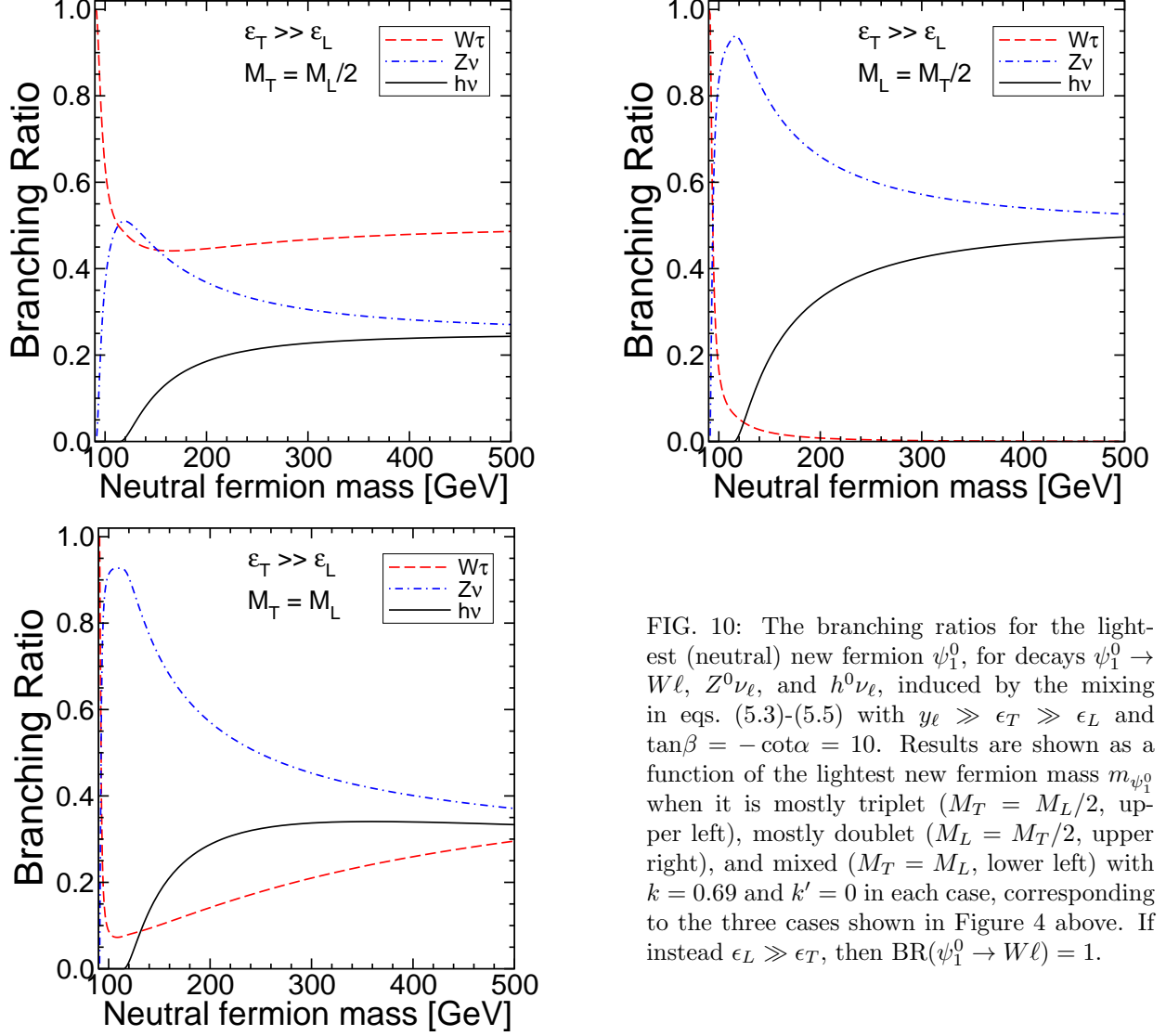


FIG. 10: The branching ratios for the lightest (neutral) new fermion  $\psi_1^0$ , for decays  $\psi_1^0 \rightarrow W\ell$ ,  $Z^0\nu_\ell$ , and  $h^0\nu_\ell$ , induced by the mixing in eqs. (5.3)-(5.5) with  $y_\ell \gg \epsilon_T \gg \epsilon_L$  and  $\tan\beta = -\cot\alpha = 10$ . Results are shown as a function of the lightest new fermion mass  $m_{\psi_1^0}$  when it is mostly triplet ( $M_T = M_L/2$ , upper left), mostly doublet ( $M_L = M_T/2$ , upper right), and mixed ( $M_T = M_L$ , lower left) with  $k = 0.69$  and  $k' = 0$  in each case, corresponding to the three cases shown in Figure 4 above. If instead  $\epsilon_L \gg \epsilon_T$ , then  $\text{BR}(\psi_1^0 \rightarrow W\ell) = 1$ .

The second major difference applies to hadron collider searches. Unlike the case of nearly degenerate higgsinos or winos, the new fermions described in the present paper are unlikely to occur in significant numbers in cascade decays of heavier gluinos, squarks or sleptons, because they lack the couplings to MSSM fermion-sfermion pairs that are implied by supersymmetry for gauginos and higgsinos. This makes the discovery of the new fermions in the case of stable  $\psi_1^0$  at hadron colliders much more of a challenge in the present case than in those studies [19, 34, 36, 37, 39, 41, 43] that make use of gluon, squark or slepton production followed by cascade decays to degenerate wino-like or higgsino-like states.

The best existing limits on the new fermions  $\psi_i^\pm$  and  $\psi_i^0$  come from the LEP2 experiment searches for exotic leptons and wino-like and higgsino-like charginos and neutralinos. The L3 experiment has produced 95% confidence level limits [44] on the mass of a neutral vector-like weak doublet fermion  $\psi_1^0$ , assuming it decays by  $\psi_1^0 \rightarrow \ell W$  with a mean decay length of less than 1 cm:

$$m_{\psi_1^0} > 99.3, 102.7, \text{ or } 102.6 \text{ GeV} \quad (\text{for } \ell = \tau, \mu, \text{ or } e). \quad (5.6)$$

L3 also found a limit for the case of a new charged vector-like doublet fermion which decays to a stable neutral partner  $\psi_1^0$  according to  $\psi_1^\pm \rightarrow W^* \psi_1^0$

$$m_{\psi_1^\pm} > 102.1 \text{ GeV} \quad (\text{for } 5 \text{ GeV} < \Delta m < 60 \text{ GeV}), \quad (5.7)$$

where  $m_{\psi_1^0} > 40 \text{ GeV}$  is also assumed. As can be seen from Figure 5 above, the assumption on  $\Delta m$  in eq. (5.7) is indeed satisfied when the lighter fermions are mostly doublet and  $k$  is at its fixed point value.

However, the preceding limits do not apply to the case where the lighter fermions are mostly triplet. In particular, the production cross-section of  $e^-e^+ \rightarrow \psi_1^0\psi_1^0$  (mediated by the  $Z$  boson) vanishes in the extreme limit of a pure triplet. In that limit,  $\Delta m$  also becomes very small (see Figure 5 above), and a different strategy is needed. ALEPH and OPAL have searched [45, 46] for nearly degenerate charged and neutral higgsinos and winos in the heavy-sneutrino limit, corresponding to the cases  $M_T \ll M_L$  and  $M_L \ll M_T$  considered here (see Figure 4 and 5 above). They used  $e^-e^+ \rightarrow \psi_1^+\psi_1^-\gamma$  and triggered on a hard isolated photon, as suggested in refs. [19, 34, 35]. The relevant ALEPH limits [45] for vector-like triplet and doublet fermions are:

$$m_{\psi_1^\pm} > 94 \text{ GeV} \quad (\text{for triplet with } 0.2 \text{ GeV} < \Delta m < 5 \text{ GeV}), \quad (5.8)$$

$$m_{\psi_1^\pm} > 90 \text{ GeV} \quad (\text{for doublet with } 0.2 \text{ GeV} < \Delta m < 5 \text{ GeV}) \quad (5.9)$$

where  $\psi_1^0$  is assumed stable on time scales relevant for collider detectors. Comparison with Figure 5 above shows that only the triplet limit (5.8) is directly relevant for our case with  $k$  large. ALEPH also gave different limits for  $\Delta m < m_\pi$  (long-lived charged particle),  $\Delta m \approx m_\pi$ , and  $m_\pi < \Delta m < 0.2 \text{ GeV}$ , but the presence of a quasi-fixed point Yukawa coupling  $k$  increases  $\Delta m$  well above these ranges. The limits obtained by OPAL [46] are similar but slightly weaker.

The pair production rates of the new fermions for hadron colliders are depicted in Figure 11 as a function of  $m_{\psi_1^\pm}$ , for the Tevatron and for four possible LHC energies. The three panels correspond to the scenarios depicted in Figure 4. The total production cross-sections shown in Figure 11 for the new fermions at hadron colliders are dominated by the processes:

$$p\bar{p} \text{ or } pp \rightarrow \psi_1^+\psi_1^- \text{ or } \psi_1^\pm\psi_1^0, \quad (5.10)$$

with  $\psi_1^\pm\psi_2^0$  and  $\psi_1^0\psi_2^0$  production making smaller but appreciable contributions, especially when the lightest states have significant doublet content. There is a larger production cross section for  $\psi_1^+\psi_1^0$  than for  $\psi_1^-\psi_1^0$  at the LHC, because there are more  $u$  quarks than  $d$  quarks in the proton. There are evidently no published Tevatron limits that are directly applicable to new weakly interacting fermions  $\psi_i^\pm, \psi_i^0$  of the type discussed in this paper, despite the fact that the production cross-section should yield (before any cuts or efficiencies) over 1000 events with the current  $8.5 \text{ fb}^{-1}$  of integrated luminosity if  $m_{\psi_1^\pm} < 150 \text{ GeV}$ .

If the  $\psi_1^0$  is stable (or quasi-stable on detector length scales), then there is a troublesome issue of triggering on the signal events at hadron colliders, since the products of  $\psi_1^\pm$  decays to  $\psi_1^0$  will not carry much energy. One possible strategy [18, 40, 42, 43] is to sacrifice some cross-section and

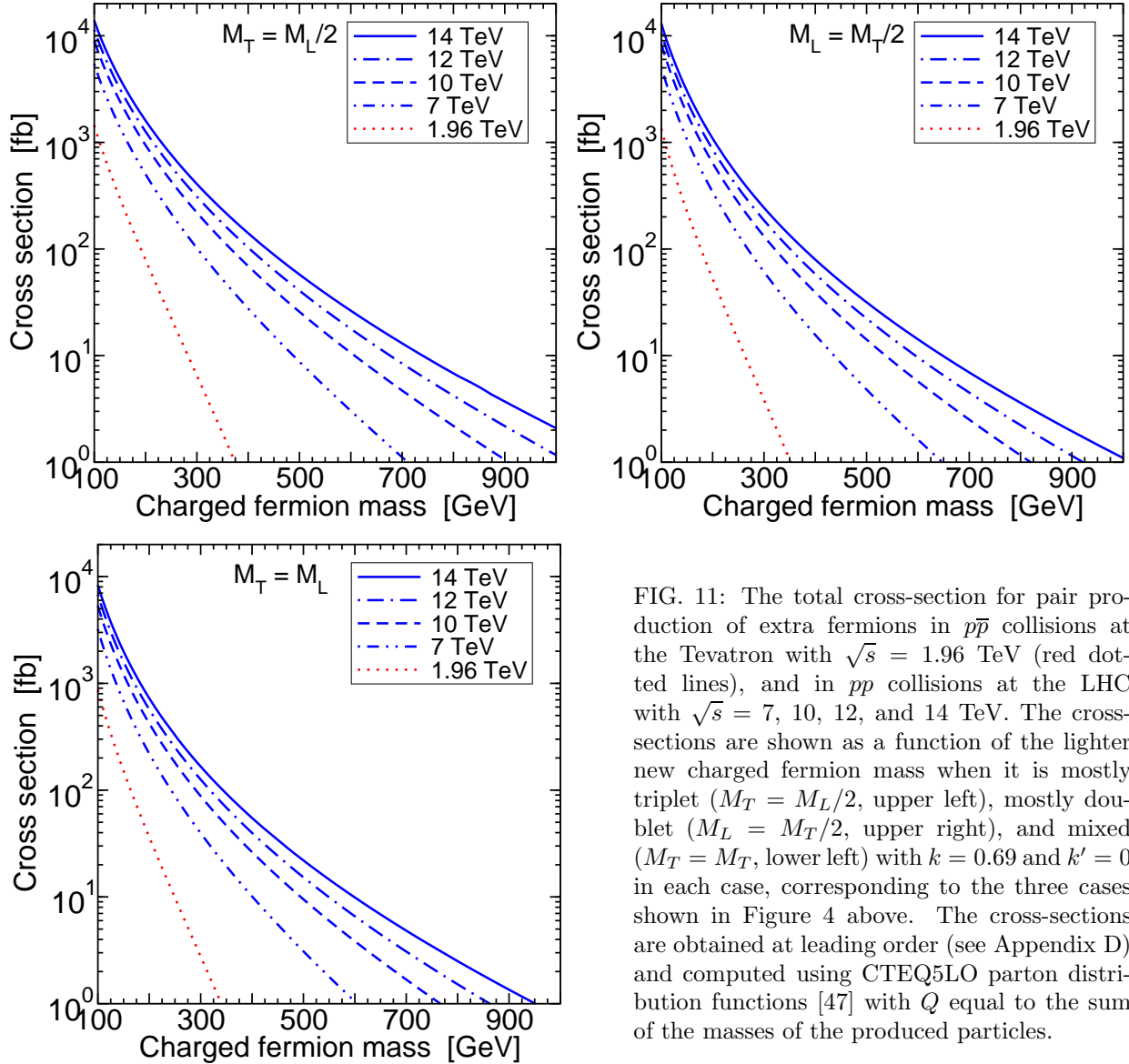


FIG. 11: The total cross-section for pair production of extra fermions in  $p\bar{p}$  collisions at the Tevatron with  $\sqrt{s} = 1.96$  TeV (red dotted lines), and in  $pp$  collisions at the LHC with  $\sqrt{s} = 7, 10, 12,$  and  $14$  TeV. The cross-sections are shown as a function of the lighter new charged fermion mass when it is mostly triplet ( $M_T = M_L/2$ , upper left), mostly doublet ( $M_L = M_T/2$ , upper right), and mixed ( $M_T = M_T$ , lower left) with  $k = 0.69$  and  $k' = 0$  in each case, corresponding to the three cases shown in Figure 4 above. The cross-sections are obtained at leading order (see Appendix D) and computed using CTEQ5LO parton distribution functions [47] with  $Q$  equal to the sum of the masses of the produced particles.

rely on events with one extra hard central jet, for example from the parton-level processes

$$q\bar{q} \rightarrow \psi\psi g, \quad gg \rightarrow \psi\psi q, \quad (5.11)$$

and then to search for evidence of the  $\psi_1^\pm$  track and/or the decay  $\psi_1^\pm \rightarrow \psi_1^0 \pi^\pm$  in off-line analysis, once the events are triggered on using the hard mono-jet. However, in the present context, the most striking stub signature requires  $\Delta m \lesssim 0.6$  GeV so that  $c\tau$  is large enough to give some events with  $\psi_1^0$  making it through at least part of the vertex detector or tracker. This can occur, but only in the  $M_T \ll M_L$  case of mostly triplet  $\psi_1^\pm, \psi_1^0$ . For larger  $\Delta m$ , things will probably be much more difficult. The severe difficulties involving triggering and backgrounds are discussed in detail for the similar situation of a chargino and neutralino with  $\Delta m =$  a few GeV in the case of the Tevatron in ref. [36].

Another attack [38, 43] on the problem of nearly degenerate winos that should also be applicable

to the present case is to rely on vector boson fusion, at parton level:

$$qq' \rightarrow qq'\psi\psi, \quad (5.12)$$

and then to trigger on the two hard forward quark jets, requiring  $E_T^{\text{miss}}$  but no (or very soft) hadronic activity in the central region from the escaping  $\psi_1^0$  or the  $\psi_1^\pm$  decays. One can also look for soft muons, which occur in 12% to 20% of the  $\psi_1^\pm$  decays for  $\Delta m > 0.6$  GeV [34]. This method can be applied to the cases of larger  $\Delta m$  from a few GeV up to tens of GeV. Ref. [43] finds a reach of perhaps up to 200 GeV at the LHC for  $300 \text{ fb}^{-1}$  at  $\sqrt{s} = 14$  TeV for the comparable case of MSSM wino-like fermions, depending on how well the backgrounds can be understood.

On the other hand, if  $\psi_1^0$  decays promptly to  $W\ell$ ,  $Z\nu$ , or  $h^0\nu$  as discussed above, then one can search at hadron colliders for the following triggerable final states:

$$W^+W^-\ell^+\ell^-, \quad W^-W^-\ell^+\ell^+, \quad W^+W^+\ell^-\ell^-, \quad (5.13)$$

$$W^\pm\ell^\mp Z^0 + E_T^{\text{miss}}, \quad W^\pm\ell^\mp h^0 + E_T^{\text{miss}}, \quad (5.14)$$

$$Z^0 Z^0 + E_T^{\text{miss}}, \quad Z^0 h^0 + E_T^{\text{miss}}, \quad h^0 h^0 + E_T^{\text{miss}}, \quad (5.15)$$

with additional pions or soft jets (or possibly even stubs and high impact parameter pions) from the  $\psi_1^\pm$  decays to  $\psi_1^0$ . Here  $\ell$  could mean any one of  $\tau$ ,  $e$ , or  $\mu$ . As discussed above, the decays of  $\psi_1^0$  are such that only eq. (5.13) is relevant if  $\epsilon_L \gg \epsilon_T$ , while otherwise all of (5.13)-(5.15) are possibilities. The same-sign dilepton events in eq. (5.13) will occur half of the time, due to the Majorana nature of  $\psi_1^0$ , providing a low-background signature.<sup>‡</sup> However, the leptons  $\ell$  may well be all (or mostly) taus. In that case, one can still look for same-sign dilepton events from the leptonic decays of  $W$ 's and  $\tau$ 's. The discovery potential of the current LHC run at  $\sqrt{s} = 7$  TeV is limited by the aimed-for luminosity of  $1 \text{ fb}^{-1}$ , but from Figure 11 above there should be several hundred events, before cuts and efficiencies, for  $m_{\psi_1^0}$  up to 200 GeV. There is also an intermediate case in which  $\psi_1^0$  decays could happen at a measurable distance from the beam line inside the tracker or the vertex detector, giving an interesting signal of two or more charged tracks emanating from a displaced vertex. A study of the difficulties and opportunities for discovering the new fermions at the Tevatron (where one might hazard a guess that it should be possible to set a limit with existing data, for  $\psi_1^0$  decaying promptly) and LHC in this scenario would be interesting, but is beyond the scope of the present paper.

In the OTLEE, TLUDD, and TLEDDD models there are additional fields whose presence ensures perturbative gauge coupling unification, which can be searched for at the LHC. They include  $SU(2)_L$ -singlet Dirac quarks and leptons,  $t'$ ,  $b'$ , and  $\tau'$ , which can decay only by mixing with their Standard Model counterparts. The absence of a GIM-type mechanism suggests that the mixing with the Standard Model third family is most likely to be important, and so one expects decays:

$$t' \rightarrow Wb, \quad Zt, \quad h^0 t, \quad (\text{TLUDD model}), \quad (5.16)$$

$$b' \rightarrow Wt, \quad Zb, \quad h^0 b, \quad (\text{TLUDD and TLEDDD models}), \quad (5.17)$$

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<sup>‡</sup> For recent detailed studies of the somewhat similar case of same-sign dileptons from pair production of heavy fourth-family neutrinos at the Tevatron and LHC, see refs. [48, 49].

$$\tau' \rightarrow W\nu_\tau, \quad Z\tau, \quad h^0\tau, \quad (\text{OTLEE and TLEDDD models}). \quad (5.18)$$

The branching ratios will depend only on the mass of the new fermion, as discussed in Appendix B of ref. [7], and are depicted in the left panel of Figure 12, the left panel of Figure 13, and in Figure 14 of that reference. In the limit of large masses, the  $W$ ,  $Z$ , and  $h^0$  final state branching ratios asymptote to 0.5, 0.25, and 0.25 respectively, in each case. The present Tevatron constraints and LHC search signatures are also discussed in ref. [7] (see also ref. [50]). In the OTLEE model, there is also a color octet fermion  $O$ , which can only decay if it has odd R-parity, by virtue of mixing with the MSSM gluino through a supersymmetry-breaking Dirac mass term  $-\mathcal{L} = M_3\tilde{g}O + \text{c.c.}$  (see e.g. [51]). If this mixing is large enough, then one can have prompt decays of the  $O$  fermion that are similar to those of the gluino, but with different kinematics:

$$O \rightarrow q\tilde{q}, \quad q\bar{q}\tilde{N}_i, \quad q\bar{q}\tilde{C}_i. \quad (5.19)$$

Distinguishing the production and decay of the  $O$  fermion from that of the MSSM gluino could be quite a challenge. If the mixing is small or absent (if  $O$  is assigned even R-parity), then the color octet  $O$  fermion could be quasi-stable or stable, and form  $R$ -hadron-like bound states, with signals that have been well-studied for Tevatron and LHC; see [52].

## VI. OUTLOOK

In general, the constraints implied by measurements of precision electroweak observables severely limit the types of new physics that can be added to the Standard Model. Minimal supersymmetry gives only small corrections to these observables, because it only introduces new particles with vector-like (electroweak singlet) masses. Given the necessary presence of the vector-like pair  $H_u, H_d$  with a bare electroweak singlet mass  $\mu$  in the MSSM, it is natural to consider extensions that contain additional vector-like supermultiplets. In this paper, I have studied models with the novel feature of a large Yukawa coupling between a new weak triplet and doublet. A motivation for this is that, like the models studied in [4–8], it can raise the lightest Higgs boson mass. If the new triplet-doublet Yukawa coupling and the associated (scalar)<sup>3</sup> coupling are at their infrared quasi-fixed points, then  $m_{h^0}$  can be increased by 5 to 10 GeV, for an average new scalar mass less than 1 TeV and a lightest new charged fermion mass of order 100 to 250 GeV. This increase is significant because many otherwise attractive supersymmetric models predict that, without such a correction,  $m_{h^0}$  would be below the LEP2 limit by a similar margin. If the couplings are not governed by the fixed points, or if the new scalars are heavier, then even larger corrections to  $m_{h^0}$  are possible.

The new fermions  $\psi_i^0$  and  $\psi_i^\pm$  in the triplet and doublet chiral supermultiplets are subject to direct search limits from LEP2, which at most limit their masses to be greater than about 100 GeV. Tevatron limits evidently do not yet exist, but may be possible with existing data. In principle, there are also indirect constraints from precision electroweak observables  $S, T$ , but these turn out to be easily satisfied even if the lightest new charged fermion is as light as 100 GeV and the new Yukawa coupling  $k$  is as large as its fixed point value. The lightest of the new fermions  $\psi_1^0$  could be stable, or could decay through mixing with MSSM gauginos, or through mixing with Standard Model leptons. In the latter case, the new fermions can be considered as extra vector-like leptons, and one can have decays  $\psi_1^0 \rightarrow W\ell$  or  $Z\nu$  or  $h^0\nu$ , with branching ratios and decay lengths that

depend on the nature of the mixing couplings. At hadron colliders, the production of  $\psi_1^+\psi_1^-$  and  $\psi_1^\pm\psi_1^0$  should dominate. If  $\psi_1^0$  is stable on collider detector length scales, then discovery at the LHC may be difficult, because of small visible energy in events and moderate cross sections. If the decay  $\psi_1^0$  occurs promptly, then one may see same-sign dilepton events with low hadronic activity in the central region. In any case, observation of the new particles and distinguishing them from supersymmetric backgrounds should provide an interesting challenge for the Tevatron and LHC.

### Appendix A: Radiative corrections to new fermion masses

The one-loop radiative corrections to the new fermion masses, due to diagrams involving electroweak gauge bosons and Higgs bosons, are:

$$\begin{aligned} \Delta m_{\psi_i^0} &= \frac{1}{16\pi^2 m_{\psi_i^0}} \left\{ |g_{\psi_i^0\psi_j^0\psi_j^0}^Z|^2 B_{FV}(\psi_j^0, Z) - \text{Re}[(g_{\psi_i^0\psi_j^0\psi_j^0}^Z)^2] m_{\psi_i^0} m_{\psi_j^0} B_{\overline{FV}}(\psi_j^0, Z) \right. \\ &+ (|g_{\psi_i^0\psi_k^+\psi_k^+}^W|^2 + |g_{\psi_i^0\psi_k^-\psi_k^-}^W|^2) B_{FV}(\psi_k^+, W) - 2\text{Re}[g_{\psi_i^0\psi_k^+\psi_k^+}^W g_{\psi_i^0\psi_k^-\psi_k^-}^W] m_{\psi_i^0} m_{\psi_k^+} B_{\overline{FV}}(\psi_k^+, W) \\ &+ (|Y_{\psi_i^0\psi_k^+}^{\Phi^+}|^2 + |Y_{\psi_i^0\psi_k^+}^{\Phi^-}|^2) B_{FS}(\psi_k^+, \Phi^+) + 2\text{Re}[Y_{\psi_i^0\psi_k^+}^{\Phi^+} Y_{\psi_i^0\psi_k^+}^{\Phi^-}] m_{\psi_i^0} m_{\psi_k^+} B_{\overline{FS}}(\psi_k^+, \Phi^+) \\ &+ |Y_{\psi_i^0\psi_j^0}^{\Phi^0}|^2 B_{FS}(\psi_j^0, \Phi^0) + \text{Re}[(Y_{\psi_i^0\psi_j^0}^{\Phi^0})^2] m_{\psi_i^0} m_{\psi_j^0} B_{\overline{FS}}(\psi_j^0, \Phi^0) \left. \right\}, \end{aligned} \quad (\text{A.1})$$

$$\begin{aligned} \Delta m_{\psi_i^+} &= \frac{1}{32\pi^2 m_{\psi_i^+}} \left\{ e^2 m_{\psi_i^+}^2 [10 - 6 \ln(m_{\psi_i^+}^2/Q^2)] \right. \\ &+ (|g_{\psi_j^0\psi_i^+\psi_i^+}^W|^2 + |g_{\psi_j^0\psi_i^-\psi_i^-}^W|^2) B_{FV}(\psi_j^0, W) - 2\text{Re}[g_{\psi_j^0\psi_i^+\psi_i^+}^W g_{\psi_j^0\psi_i^-\psi_i^-}^W] m_{\psi_i^+} m_{\psi_j^0} B_{\overline{FV}}(\psi_j^0, W) \\ &+ (|g_{\psi_i^+\psi_k^+\psi_k^+}^Z|^2 + |g_{\psi_i^+\psi_k^-\psi_k^-}^Z|^2) B_{FV}(\psi_k^+, Z) - 2\text{Re}[g_{\psi_i^+\psi_k^+\psi_k^+}^Z g_{\psi_i^+\psi_k^-\psi_k^-}^Z] m_{\psi_i^+} m_{\psi_k^+} B_{\overline{FV}}(\psi_k^+, Z) \\ &+ (|Y_{\psi_i^+\psi_k^+}^{\Phi^0}|^2 + |Y_{\psi_i^+\psi_k^+}^{\Phi^-}|^2) B_{FS}(\psi_k^+, \Phi^0) + 2\text{Re}[Y_{\psi_i^+\psi_k^+}^{\Phi^0} Y_{\psi_i^+\psi_k^+}^{\Phi^-}] m_{\psi_i^+} m_{\psi_k^+} B_{\overline{FS}}(\psi_k^+, \Phi^0) \\ &+ (|Y_{\psi_j^0\psi_i^+}^{\Phi^+}|^2 + |Y_{\psi_j^0\psi_i^+}^{\Phi^-}|^2) B_{FS}(\psi_j^0, \Phi^+) + 2\text{Re}[Y_{\psi_j^0\psi_i^+}^{\Phi^+} Y_{\psi_j^0\psi_i^+}^{\Phi^-}] m_{\psi_i^+} m_{\psi_j^0} B_{\overline{FS}}(\psi_j^0, \Phi^+) \left. \right\}. \end{aligned} \quad (\text{A.2})$$

Here the virtual particle labels  $j = 1, 2, 3$  and  $k = 1, 2$  and  $\Phi^0 = h^0, H^0, A^0, G^0$  and  $\Phi^+ = H^+, G^+$  are implicitly summed over where they appear. The couplings are given in terms of the mixing matrices defined in eqs. (2.4) and (2.5) by:

$$g_{\psi_i^+\psi_j^+\psi_j^+}^Z = \frac{1}{\sqrt{g^2 + g'^2}} \left[ \frac{1}{2}(g^2 - g'^2)V_{i2}^*V_{j2} + g^2V_{i1}^*V_{j1} \right], \quad (\text{A.3})$$

$$g_{\psi_i^-\psi_j^-\psi_j^-}^Z = \frac{1}{\sqrt{g^2 + g'^2}} \left[ \frac{1}{2}(-g^2 + g'^2)U_{i2}^*U_{j2} - g^2U_{i1}^*U_{j1} \right], \quad (\text{A.4})$$

$$g_{\psi_i^0\psi_j^0\psi_j^0}^Z = \frac{1}{2}\sqrt{g^2 + g'^2} [N_{i2}^*N_{j2} - N_{i3}^*N_{j3}], \quad (\text{A.5})$$

$$g_{\psi_i^0\psi_j^+\psi_j^+}^W = g \left( \frac{1}{\sqrt{2}}N_{i3}^*V_{j2} - N_{i1}^*V_{j1} \right), \quad (\text{A.6})$$

$$g_{\psi_i^0\psi_j^-\psi_j^-}^W = g \left( \frac{1}{\sqrt{2}}N_{i2}^*U_{j2} + N_{i1}^*U_{j1} \right), \quad (\text{A.7})$$

$$Y_{\psi_i^0\psi_j^0}^{\Phi^0} = -\frac{k}{\sqrt{2}}w_{\Phi^0}(N_{i1}^*N_{j2}^* + N_{i2}^*N_{j1}^*) - \frac{k'}{\sqrt{2}}x_{\Phi^0}(N_{i1}^*N_{j3}^* + N_{i3}^*N_{j1}^*), \quad (\text{A.8})$$



$$Y_{\psi_i^+ \psi_j^-}^{\Phi^0} = -kw_{\Phi^0} V_{i1}^* U_{j2}^* + k' x_{\Phi^0} V_{i2}^* U_{j1}^*, \quad (\text{A.9})$$

$$Y_{\psi_i^0 \psi_j^-}^{\Phi^+} = kw_{\Phi^+} (-N_{i1}^* U_{j2}^* + \sqrt{2} N_{i2}^* U_{j1}^*), \quad (\text{A.10})$$

$$Y_{\psi_i^0 \psi_j^+}^{\Phi^-} = -k' x_{\Phi^+} (N_{i1}^* V_{j2}^* + \sqrt{2} N_{i3}^* V_{j1}^*), \quad (\text{A.11})$$

with  $w_{\Phi^0} = (\cos\alpha, \sin\alpha, i\cos\beta, i\sin\beta)$  and  $x_{\Phi^0} = (-\sin\alpha, \cos\alpha, i\sin\beta, -i\cos\beta)$  for  $\Phi^0 = (h^0, H^0, A^0, G^0)$ , and  $w_{\Phi^+} = (\cos\beta, \sin\beta)$  and  $x_{\Phi^+} = (\sin\beta, -\cos\beta)$  for  $\Phi^+ = (H^+, G^+)$ . The one-loop self-energy integral functions are, in Feynman gauge and following the notation of [53],

$$B_{FV}(x, y) = 2B_{FS}(x, y) = (y - x - s)B(x, y) + A(y) - A(x), \quad (\text{A.12})$$

$$B_{\overline{FV}}(x, y) = -4B_{\overline{FS}}(x, y) = 4B(x, y), \quad (\text{A.13})$$

where there is an implicit argument  $s$  set equal to the squared mass of the particle whose mass correction is being computed, and

$$A(x) = x \ln(x/Q^2) - x, \quad (\text{A.14})$$

$$B(x, y) = - \int_0^1 dt \ln([tx + (1-t)y - t(1-t)s - i\epsilon]/Q^2) \quad (\text{A.15})$$

with  $Q$  the renormalization scale. By convention, the name of a particle appearing as an argument of one of these functions stands for the squared mass of the particle. In Feynman gauge,  $m_{G^0} = m_Z$  and  $m_{G^+} = m_W$ . The result above is similar to that for MSSM charginos and neutralinos in Appendix D of [54] and section V.C of [53]. In the numerical results shown in Figures 4 and 5, I took  $m_{H^0}, m_{A^0}, m_{H^+}$  to be large enough to decouple, with  $\alpha = \beta - \pi/2$ , and  $k' = 0$ , and  $m_{h^0} = 115$  GeV.

## Appendix B: Contributions to precision electroweak parameters

This Appendix gives formulas for the contributions of the fermions in the new chiral supermultiplets  $T, L, \overline{L}$  to the Peskin-Takeuchi precision electroweak parameters [24]. For convenience I will follow the notations and conventions of [55], which were also followed in [7]. The oblique parameters  $S$  and  $T$  are defined in terms of electroweak vector boson self-energies by

$$\frac{\alpha S}{4s_W^2 c_W^2} = \left[ \Pi_{ZZ}(M_Z^2) - \Pi_{ZZ}(0) - \frac{c_{2W}}{c_W s_W} \Pi_{Z\gamma}(M_Z^2) - \Pi_{\gamma\gamma}(M_Z^2) \right] / M_Z^2, \quad (\text{B.1})$$

$$\alpha T = \Pi_{WW}(0)/M_W^2 - \Pi_{ZZ}(0)/M_Z^2. \quad (\text{B.2})$$

The new fermion contributions to the electroweak vector boson self-energies are:

$$\Delta\Pi_{\gamma\gamma}(s) = -\frac{1}{16\pi^2} 2g^2 s_W^2 \sum_{i=1,2} G(\psi_i^+), \quad (\text{B.3})$$

$$\Delta\Pi_{Z\gamma}(s) = -\frac{1}{16\pi^2} g s_W \sum_{i=1,2} (g_{\psi_i^+ \psi_i^{++}}^Z - g_{\psi_i^- \psi_i^{-+}}^Z) G(\psi_i^+), \quad (\text{B.4})$$

$$\begin{aligned}
\Delta\Pi_{ZZ}(s) = & -\frac{1}{16\pi^2} \left[ \sum_{i,j=1}^3 \left\{ |g_{\psi_i^0 \psi_j^{0\dagger}}^Z|^2 H(\psi_i^0, \psi_j^0) - 2\text{Re}[(g_{\psi_i^0 \psi_j^{0\dagger}}^Z)^2] m_{\psi_i^0} m_{\psi_j^0} B(\psi_i^0, \psi_j^0) \right\} \right. \\
& + \sum_{i,j=1}^2 \left\{ (|g_{\psi_i^+ \psi_j^{+\dagger}}^Z|^2 + |g_{\psi_i^- \psi_j^{-\dagger}}^Z|^2) H(\psi_i^+, \psi_j^+) \right. \\
& \left. \left. - 4\text{Re}[g_{\psi_i^+ \psi_j^{+\dagger}}^Z g_{\psi_i^- \psi_j^{-\dagger}}^Z] m_{\psi_i^+} m_{\psi_j^+} B(\psi_i^+, \psi_j^+) \right\} \right], \tag{B.5}
\end{aligned}$$

$$\begin{aligned}
\Delta\Pi_{WW}(s) = & -\frac{1}{16\pi^2} \sum_{i=1}^3 \sum_{j=1}^2 \left\{ (|g_{\psi_i^0 \psi_j^{+\dagger}}^W|^2 + |g_{\psi_i^0 \psi_j^{-\dagger}}^W|^2) H(\psi_i^0, \psi_j^+) \right. \\
& \left. - 4\text{Re}[g_{\psi_i^0 \psi_j^{+\dagger}}^W g_{\psi_i^0 \psi_j^{-\dagger}}^W] m_{\psi_i^0} m_{\psi_j^+} B(\psi_i^0, \psi_j^+) \right\}, \tag{B.6}
\end{aligned}$$

In these expressions, the couplings are found in eqs. (A.3)-(A.7) of Appendix A, and the one-loop integral functions are given by eqs. (A.14), (A.15) and

$$\begin{aligned}
H(x, y) = & \{ [2s - x - y - (x - y)^2/s] B(x, y) + 2A(x) + 2A(y) + 2x + 2y - 2s/3 \\
& + (y - x)[A(x) - A(y)]/s \}/3, \tag{B.7}
\end{aligned}$$

$$G(x) = H(x, x) + 2xB(x, x), \tag{B.8}$$

as in ref. [55]. Particle names should be understood to stand for the squared mass when used as an argument of one of these functions, and there are implicit arguments  $s$  and  $Q$  for  $B(x, y)$ ,  $H(x, y)$ , and  $G(x)$  which are identified with the invariant squared mass argument of the self-energy function in which they appear and the RG scale.

### Appendix C: Decay widths of new fermions

This Appendix gives formulas for the decay widths of the lightest new fermion in the  $T$ ,  $L$ ,  $\bar{L}$  multiplets,  $\psi_1^0$ , to Standard Model states. These decays are mediated by the Yukawa couplings  $\epsilon_T, \epsilon_L$  in eqs. (5.3)-(5.5), which provide small mass mixings that can be treated as perturbations compared to the other entries in the mass matrices. For simplicity, I assume that only one Standard Model lepton family  $\ell$  is involved. Define unitary mixing matrices  $\widehat{N}$  ( $4 \times 4$ ) and  $\widehat{U}, \widehat{V}$  ( $3 \times 3$ ) by

$$\widehat{N}^* \widehat{\mathcal{M}}_0 \widehat{N}^\dagger = \text{diag}(m_{\nu_\ell}, m_{\psi_1^0}, m_{\psi_2^0}, m_{\psi_3^0}), \tag{C.1}$$

$$\widehat{U}^* \widehat{\mathcal{M}}_\pm \widehat{V}^\dagger = \text{diag}(m_\ell, m_{\psi_1^+}, m_{\psi_2^+}). \tag{C.2}$$

Then the relevant couplings of  $\psi_1^0$  to Standard Model particles are:

$$g_{\psi_1^0 \ell^\dagger}^W = g(\widehat{N}_{21}^* \widehat{U}_{11} + \frac{1}{\sqrt{2}} \widehat{N}_{22}^* \widehat{U}_{12} + \frac{1}{\sqrt{2}} \widehat{N}_{24}^* \widehat{U}_{13}), \tag{C.3}$$

$$g_{\psi_1^0 \bar{\ell}^\dagger}^W = g(-\widehat{N}_{21}^* \widehat{V}_{11} + \frac{1}{\sqrt{2}} \widehat{N}_{23}^* \widehat{V}_{12}), \tag{C.4}$$

$$g_{\psi_1^0 \nu_\ell^\dagger}^Z = \frac{1}{2} \sqrt{g^2 + g'^2} (\widehat{N}_{22}^* \widehat{N}_{12} - \widehat{N}_{23}^* \widehat{N}_{13} + \widehat{N}_{24}^* \widehat{N}_{14}), \tag{C.5}$$

$$\begin{aligned}
Y_{\psi_1^0 \nu_\ell}^{h^0} &= \frac{\cos\alpha}{\sqrt{2}} [k(\widehat{N}_{21}^* \widehat{N}_{12}^* + \widehat{N}_{11}^* \widehat{N}_{22}^*) + \epsilon_T(\widehat{N}_{21}^* \widehat{N}_{14}^* + \widehat{N}_{11}^* \widehat{N}_{24}^*)] \\
&\quad - \frac{\sin\alpha}{\sqrt{2}} k'(\widehat{N}_{21}^* \widehat{N}_{13}^* + \widehat{N}_{11}^* \widehat{N}_{23}^*).
\end{aligned} \tag{C.6}$$

It follows that

$$\Gamma(\psi_1^0 \rightarrow W^+ \ell^-) = \Gamma(\psi_1^0 \rightarrow W^- \ell^+) = \frac{m_{\psi_1^0}}{32\pi} (|g_{\psi_1^0 \ell^+}^W|^2 + |g_{\psi_1^0 \ell^-}^W|^2) (1 - r_W)^2 (2 + 1/r_W), \tag{C.7}$$

$$\Gamma(\psi_1^0 \rightarrow Z^0 \nu_\ell) = \Gamma(\psi_1^0 \rightarrow Z^0 \bar{\nu}_\ell) = \frac{m_{\psi_1^0}}{32\pi} |g_{\psi_1^0 \nu_\ell}^Z|^2 (1 - r_Z)^2 (2 + 1/r_Z), \tag{C.8}$$

$$\Gamma(\psi_1^0 \rightarrow h^0 \nu_\ell) = \Gamma(\psi_1^0 \rightarrow h^0 \bar{\nu}_\ell) = \frac{m_{\psi_1^0}}{32\pi} |Y_{\psi_1^0 \nu_\ell}^{h^0}|^2 (1 - r_{h^0})^2. \tag{C.9}$$

where  $r_X = m_X^2/m_{\psi_1^0}^2$  for  $X = W, Z, h^0$ , and  $m_\ell$  and  $m_{\nu_\ell}$  are neglected for kinematic purposes. In the limit of small  $\epsilon_T$ , the  $Z^0 \nu_\ell$  and  $h^0 \nu_\ell$  partial widths go to 0 because  $\widehat{N}_{11}$ ,  $\widehat{N}_{12}$ ,  $\widehat{N}_{13}$  and  $\widehat{N}_{24}$  become small. The dominant decay in that case is  $W\ell$ , through the last term in  $g_{\psi_1^0 \ell^\pm}^W$ .

#### Appendix D: Collider production of new fermions

This Appendix contains formulas for the parton-level differential cross sections for<sup>†</sup>  $u\bar{d} \rightarrow \psi_i^+ \psi_j^0$  and  $f\bar{f} \rightarrow \psi_i^- \psi_j^+$  and  $f\bar{f} \rightarrow \psi_i^0 \psi_j^0$ . The general form of the result is

$$\frac{d\sigma}{dt} = \frac{1}{64N_c \pi s^2} \sum |\mathcal{M}|^2 \tag{D.1}$$

with  $\sum |\mathcal{M}|^2$  to be given below for each process, and

$$t = [m_i^2 + m_j^2 - s + \lambda^{1/2}(s, m_i^2, m_j^2) \cos\theta]/2 \tag{D.2}$$

where  $\theta$  is the angle between the first initial-state fermion and the outgoing fermion labeled  $i$ , and  $\sqrt{s}$  is the center-of-momentum energy, and  $\lambda(x, y, z) = x^2 + y^2 + z^2 - 2xy - 2xz - 2yz$ . Also  $s + t + u = m_i^2 + m_j^2$ . Couplings needed below are found in Appendix A. For quarks (leptons) in the initial state,  $N_c = 3$  (1). The computations and presentations here are similar to those in sections 6.12, 6.14, and 6.15 of [15].

For  $u\bar{d} \rightarrow \psi_i^+ \psi_j^0$ , the result is

$$\sum |\mathcal{M}|^2 = |c_1|^2 (u - m_{\psi_i^+}^2)(u - m_{\psi_j^0}^2) + |c_2|^2 (t - m_{\psi_i^+}^2)(t - m_{\psi_j^0}^2) + 2\text{Re}[c_1 c_2^*] s m_{\psi_i^+} m_{\psi_j^0}, \tag{D.3}$$

<sup>†</sup> Other parton-level processes with smaller cross sections, such as those in eqs. (5.11) and (5.12), may turn out to be more relevant for observable signals, especially if  $\psi_1^0$  is stable. Gluon fusion contributions to the production, which could be significant for a new chiral family of leptons [56], should be very small in the vector-like case.

where

$$c_1 = \frac{\sqrt{2}g}{s - m_W^2} g_{\psi_j^0 \psi_i^{+\dagger}}^W, \quad c_2 = -\frac{\sqrt{2}g}{s - m_W^2} (g_{\psi_j^0 \psi_i^{-\dagger}}^W)^*. \quad (\text{D.4})$$

For  $f\bar{f} \rightarrow \psi_i^- \psi_j^+$ , the result is

$$\begin{aligned} \sum |\mathcal{M}|^2 &= (|c_1|^2 + |c_4|^2)(u - m_{\psi_i^-}^2)(u - m_{\psi_j^+}^2) + (|c_2|^2 + |c_3|^2)(t - m_{\psi_i^-}^2)(t - m_{\psi_j^+}^2) \\ &\quad + 2\text{Re}[c_1 c_2^* + c_3 c_4^*] s m_{\psi_i^-} m_{\psi_j^+}, \end{aligned} \quad (\text{D.5})$$

where

$$c_1 = -\frac{2Q_f e^2}{s} \delta_{ij} + \frac{2(T_f^3 - Q_f s_W^2)g}{c_W(s - m_Z^2)} g_{\psi_j^- \psi_i^{-\dagger}}^Z, \quad (\text{D.6})$$

$$c_2 = -\frac{2Q_f e^2}{s} \delta_{ij} - \frac{2(T_f^3 - Q_f s_W^2)g}{c_W(s - m_Z^2)} g_{\psi_i^+ \psi_j^{+\dagger}}^Z, \quad (\text{D.7})$$

$$c_3 = -\frac{2Q_f e^2}{s} \delta_{ij} - \frac{2Q_f s_W^2 g}{c_W(s - m_Z^2)} g_{\psi_j^- \psi_i^{-\dagger}}^Z, \quad (\text{D.8})$$

$$c_4 = -\frac{2Q_f e^2}{s} \delta_{ij} + \frac{2Q_f s_W^2 g}{c_W(s - m_Z^2)} g_{\psi_i^+ \psi_j^{+\dagger}}^Z, \quad (\text{D.9})$$

with  $(Q_f, T_f^3) = (2/3, 1/2)$  for  $f = u$ , and  $(-1/3, -1/2)$  for  $f = d$ , and  $(-1, -1/2)$  for  $f = e$ .

For  $f\bar{f} \rightarrow \psi_i^0 \psi_j^0$ ,

$$\begin{aligned} \sum |\mathcal{M}|^2 &= (|c_1|^2 + |c_2|^2)[(u - m_{\psi_i^0}^2)(u - m_{\psi_j^0}^2) + (t - m_{\psi_i^0}^2)(t - m_{\psi_j^0}^2)] \\ &\quad - 2\text{Re}[c_1^2 + c_2^2] s m_{\psi_i^0} m_{\psi_j^0}, \end{aligned} \quad (\text{D.10})$$

where

$$c_1 = \frac{2(T_f^3 - Q_f s_W^2)g}{c_W(s - m_Z^2)} g_{\psi_i^0 \psi_j^{0\dagger}}^Z, \quad c_2 = -\frac{2Q_f s_W^2 g}{c_W(s - m_Z^2)} g_{\psi_i^0 \psi_j^{0\dagger}}^Z. \quad (\text{D.11})$$

When  $i = j$ , one must also include an additional factor of  $1/2$  in the total cross section for identical final state particles.

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