**Interpretation of MINOS data in terms of non-standard neutrino interactions**

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(Dated: Aug 30, 2010)

The MINOS experiment at Fermilab has recently reported a tension between the oscillation results for neutrinos and anti-neutrinos. We show that this tension, if it persists, can be understood in the framework of non-standard neutrino interactions (NSI). While neutral current NSI (non-standard matter effects) are disfavored by atmospheric neutrinos, a new charged current coupling between tau neutrinos and nucleons can fit the MINOS data without violating other constraints. In particular, we show that loop-level contributions to flavor-violating τ decays are sufficiently suppressed. However, conflicts with existing bounds could arise once the effective theory considered here is embedded into a complete renormalizable model. We predict the future sensitivity of the T2K and NOνA experiments to the NSI parameter region favored by the MINOS fit, and show that both experiments are excellent tools to test the NSI interpretation of the MINOS data.

PACS numbers: 14.60.Pq, 12.60.-i, 13.15.+g

1. INTRODUCTION

Recently the Fermilab MINOS experiment has reported new results on $\bar{\nu}_\mu$ disappearance [1]. Interestingly, the values of the neutrino oscillation parameters $\Delta m^2_{32}$ and $\sin^2 2\theta_{23}$ preferred by this anti-neutrino measurement are in tension, at the 90% confidence level, with the preferred region in the $\Delta m^2_{32}$ versus $\sin^2 2\theta_{23}$ plane for the neutrino $\nu_\mu$ disappearance. A likely explanation of this tension is lack of statistics especially in the anti-neutrino channel where the number of observed events in the far detector is approximately 100 whereas for the neutrino channel the number of events in the far detector is of order 20 times larger. One could speculate that the tension between neutrino and anti-neutrino disappearance might be the first hint of CPT violation in the neutrino sector [2–5]. However, given that CPT conservation is such an important tenet of modern quantum field theory, it is important to explore other possibilities for new physics before giving up CPT conservation. Since the MINOS experiment is performed not in vacuum but with more than 700 kilometers of Earth matter between the source and the detector, there is the possibility of Wolfenstein type matter effects [6] leading to apparent CPT violation. While matter effects are not relevant to $\nu_\mu$ disappearance in MINOS in the standard three-flavor framework, they may become important in scenarios with sterile neutrinos [7], or if new non-standard interactions are contributing to the potential that the neutrinos experience when traveling through matter. The latter option will be explored in this paper. In addition, we will also consider non-standard interactions modifying the neutrino detection process in a CP non-conserving way.

Non-Standard Interactions (NSI) in the neutrino sector have been introduced first as an alternative to standard oscillations [6, 8, 9], and later as a possible addition [10, 11]. Many authors have in-
vestigated the phenomenology of such subdominant NSI, both model-independently [10–15] and in concrete models [7, 10, 16–24]. In particular, NSI have been considered in the context of solar neutrino observations [6, 9, 10, 25–31], reactor neutrino experiments [32], atmospheric neutrinos [33–39], conventional and upgraded neutrino beams [36, 38, 40–49], neutrino factories [13, 41, 50–63], beta beams [64], supernova neutrinos [65, 66], cosmological relic neutrinos [67], high energy astrophysical neutrinos [68], \( e^+e^- \) colliders [69, 70], low-energy flavor violation searches in the charged lepton sector [71, 72], neutrino-electron scattering [73, 74] and neutrino-nucleus scattering [74, 75]. Model-independent experimental bounds on NSI, derived from oscillation and non-oscillation data, are reviewed in [39, 71, 76]. In the context of the latest MINOS results, NSI have been brought up in [77, 78], and a concrete model has been proposed in [79].

In the following, we will first show analytically how NSI affect neutrino oscillations in the two flavor limit (sec. 2), and then perform fits to the MINOS \( \nu_\mu \) and \( \bar{\nu}_\mu \) data including different types of NSI in a three-flavor framework (sec. 3). In sec. 4, we will consider the potential of the T2K [80] and NO\( \nu \)A [81] experiments to test the NSI interpretation of MINOS. Finally, we will discuss our results from the model-building point of view and draw our conclusions in sec. 5.

## 2. ANALYTICAL FRAMEWORK OF NON-STANDARD INTERACTIONS

### 2.1. Neutral current NSI

At \( \mathcal{O}(\text{GeV}) \) energies relevant to neutrino oscillation experiments, non-standard interactions can be introduced in the Lagrangian as effective dimension 6 operators coupling neutrinos and charged fermions. We will first discuss new neutral current couplings of \( \mu \) and \( \tau \) neutrinos to normal matter, i.e. electrons, up-quarks, and down-quarks. Phenomenologically, such operators will result in non-standard matter effects, so that the Hamiltonian governing neutrino propagation in the \( \mu-\tau \) sector will read

\[
H = \frac{1}{2E} \left[ U \begin{pmatrix} 0 & \Delta m^2_{32} \end{pmatrix} U^\dagger + A \begin{pmatrix} \epsilon_{\mu\mu}^m & \epsilon_{\mu\tau}^m \\ \epsilon_{\mu\tau}^m & \epsilon_{\tau\tau}^m \end{pmatrix} \right],
\]

where

\[
U = \begin{pmatrix} \cos \theta_{23} & \sin \theta_{23} \\ -\sin \theta_{23} & \cos \theta_{23} \end{pmatrix}
\]

is the leptonic mixing matrix with the mixing angle \( \theta_{23} \), \( E \) is the neutrino energy, and \( A = 2\sqrt{2}G_FN_eE \) is the matter potential depending on the electron number density \( N_e \) along the neutrino trajectory. The parameters \( \epsilon_{\mu\mu}^m, \epsilon_{\mu\tau}^m, \) and \( \epsilon_{\tau\tau}^m \) give the relative strength of the non-standard interactions compared to Standard Model weak interactions. The superscript \( m \) indicates that these parameters describe non-standard neutrino matter effects. \( \epsilon_{\mu\tau}^m \) can in general be complex, while \( \epsilon_{\mu\mu}^m \) and \( \epsilon_{\tau\tau}^m \) have to be real in order to preserve the hermiticity of the Hamiltonian. In the following, we will set \( \epsilon_{\mu\mu}^m = 0 \) since terms proportional to the identity matrix do not affect the neutrino oscillation probability, implying that oscillation experiments are only sensitive to the combination \( \epsilon_{\tau\tau}^m - \epsilon_{\mu\mu}^m \).

The disappearance probability for \( \nu_\mu \) in matter of constant density is given by

\[
P(\nu_\mu \rightarrow \nu_\mu) = 1 - \sin^2 2\theta_N \sin^2 \left( \frac{\Delta m^2_{32} L}{4E} \right).
\]
where $\theta_N$ and $\Delta m_N^2$ are the mixing angle and mass squared difference in matter. These matter oscillation parameters can be found by solving the following set of coupled equations,

\begin{align}
\Delta m_N^2 \cos 2\theta_N &= \Delta m_{32}^2 \cos 2\theta_{23} + \epsilon_{\mu\tau}^m A, \\
\Delta m_N^2 \sin 2\theta_N \epsilon_{\mu\tau}^N &= \Delta m_{32}^2 \sin 2\theta_{23} + 2\epsilon_{\mu\tau}^m A.
\end{align}

The phase $\phi_N$ is unobservable in oscillations and can be phased away. The solution is trivial to find and is given by

$$
\Delta m_N^2 = \sqrt{(\Delta m_{32}^2 \cos 2\theta_{23} + \epsilon_{\mu\tau}^m A)^2 + (\Delta m_{32}^2 \sin 2\theta_{23} + 2\epsilon_{\mu\tau}^m A)^2} \quad (6)
$$

and

$$
\sin 2\theta_N = |\Delta m_{32}^2 \sin 2\theta_{23} + 2\epsilon_{\mu\tau}^m A|^2 / \Delta m_N^4. \quad (7)
$$

Thus, the disappearance probability can be written as

$$
P(\nu_\mu \rightarrow \nu_\mu) = 1 - \frac{|\Delta m_{32}^2 \sin 2\theta_{23} + 2\epsilon_{\mu\tau}^m A|^2}{\Delta m_N^4} \sin^2 \left( \frac{\Delta m_N^2 L}{4E} \right). \quad (8)
$$

with $\Delta m_N^2$ given by eq. (6). This expression agrees with that found in ref. [46] when expanded to first order in $\epsilon_{\mu\tau}^m$ and $\epsilon_{\mu\tau}^m$.

In vacuum, $A = 0$, and eq. (8) reduces to the standard two flavor disappearance probability. In the small $L/E$ limit, i.e. when $\sin^2(\Delta m_N^2 L/4E) \approx (\Delta m_N^2 L/4E)^2$,

$$
P(\nu_\mu \rightarrow \nu_\mu) \approx 1 - \left| \sin 2\theta_{23} + \frac{2\epsilon_{\mu\tau}^m A}{\Delta m_{32}^2} \right|^2 \left( \frac{\Delta m_{32}^2 L}{4E} \right)^2, \quad (9)
$$

so $\epsilon_{\mu\tau}^m$ modifies the disappearance probability in this limit whereas $\epsilon_{\mu\tau}^m$ does not.

For anti-neutrinos, $\epsilon_{\mu\tau}^m \rightarrow -\epsilon_{\mu\tau}^m$ and $A \rightarrow -A$, so that in matter

$$
P(\nu_\mu \rightarrow \nu_\mu) \neq P(\bar{\nu}_\mu \rightarrow \bar{\nu}_\mu) \quad (10)
$$

without CPT violation.

We note three interesting symmetries in eq. (8): First, the expression depends only on $\cos(\arg(\epsilon_{\mu\tau}^m))$, not on $\sin(\arg(\epsilon_{\mu\tau}^m))$. Therefore, it is invariant under the replacement

$$
\arg(\epsilon_{\mu\tau}^m) \rightarrow 2\pi n - \arg(\epsilon_{\mu\tau}^m) \quad (11)
$$

for arbitrary integer $n$. Moreover, it is easy to verify that $P(\nu_\mu \rightarrow \nu_\mu)$ is also invariant under the simultaneous replacements

$$
\epsilon_{\mu\tau}^m \rightarrow -\epsilon_{\mu\tau}^m, \quad \epsilon_{\mu\tau}^m \rightarrow -\epsilon_{\mu\tau}^m, \quad \Delta m_{32}^2 \rightarrow -\Delta m_{32}^2. \quad (12)
$$

as well as under the transformation

$$
\epsilon_{\mu\tau}^m \rightarrow -\epsilon_{\mu\tau}^m, \quad \theta_{23} \rightarrow \frac{\pi}{2} - \theta_{23}. \quad (13)
$$

These symmetries will generate an eightfold degeneracy. For fixed $E$, $P(\nu_\mu \rightarrow \nu_\mu)$ has an additional, continuous, symmetry: It is invariant under any simultaneous variation of $\Delta m_{32}^2$, $\theta_{23}$, $\epsilon_{\mu\tau}^m$, $|\epsilon_{\mu\tau}^m|$, and $\arg(\epsilon_{\mu\tau}^m)$ that leaves $\Delta m_N^2$ and $\sin^2 2\theta_N$ invariant. If we demand this invariance for neutrinos and anti-neutrinos, we obtain 4 equations for 5 free parameters, implying that the symmetry is continuous. However, since $A$ is energy-dependent, this symmetry will not be manifest in the
Figure 1: The survival probability for MINOS ($L = 735$ km) with a variety of neutral current NSI (non-standard matter effects) turned on as indicated in the plot. The solid (blue) lines are the neutrino survival probabilities whereas the dashed (red) lines are for anti-neutrinos. The dotted (black) lines are the vacuum survival probabilities. For the standard oscillation parameters, we have assumed $\Delta m^2_{32} = +2.86 \times 10^{-3}$ eV$^2$ and $\sin^2 \theta_{23} = 0.38$.

MINOS data, which covers a broad range of energies, and we will therefore not consider it further in this paper.

Note that these symmetries are exact only in the two-flavor framework. In the three-flavor case with large $\theta_{13}$, it is for example possible to determine the mass hierarchy by observing standard matter effects either in the $\nu_\mu \rightarrow \nu_e$ channel or directly in the $\nu_\mu \rightarrow \nu_\mu$ channel [82].

In fig. 1 we have plotted the disappearance probabilities with all three combinations of non-zero $\epsilon_{\mu\tau}^m$ and/or $\epsilon_{\tau\tau}^m$ for representative values of the $\epsilon^m$'s for the MINOS experiment. One can see that non-zero $\epsilon_{\mu\tau}^m$ changes the disappearance probability most notably at large energies and shifts the position of the minimum in energy. Whereas non-zero $\epsilon_{\tau\tau}^m$ changes the disappearance probability most notably near the first oscillation minimum, especially in the depth of the minimum. Since the tension between MINOS neutrino and anti-neutrino data is both in the position of the minimum and in its depth, one requires non-zero $\epsilon_{\mu\tau}^m$ and non-zero $\epsilon_{\tau\tau}^m$ in order to lift the tension in the optimal way.

2.2. Charged current NSI

As an alternative to neutral current NSI, we also discuss non-standard charged current interactions affecting the neutrino production and/or detection processes as an explanation for the MINOS results. If the Wilson coefficients of the corresponding effective operators are complex, the
interference term between the standard and non-standard Feynman amplitudes can be different for neutrinos and anti-neutrinos and CP-violating phenomena can emerge. The modifications to the far detector event spectra observed in MINOS can be induced by (i) operators leading to a modified flux of \( \nu_\mu \) at the far detector, but not at the near detector, and (ii) by operators leading to the production of muons in interactions of \( \nu_\tau \). (We neglect the possibility of non-standard interactions of \( \nu_\tau \) since their flux at the far detector is between one and two order of magnitude smaller than that of \( \nu_\mu \) because of the low \( \nu_\tau \) contamination of the NuMI beam and the smallness of the mixing angle \( \theta_{13} \).) The only way of realizing case (i) in a three-flavor framework is to postulate a \( \nu_\tau \) contamination in the NuMI beam, which would be invisible to the near detector, but would have partly oscillated into \( \nu_\mu \) when reaching the far detector. However, results from the NOMAD experiment [83] constrain the \( \nu_\tau \) contamination of a NuMI-like neutrino beam at short baseline to be less than \( 1.7 \times 10^{-4} \) at 90\% confidence level, much too small to be relevant to MINOS. We therefore neglect case (i) in the following, and focus on case (ii), a non-standard interaction of the type

\[
\nu_\tau + N \to X + \mu, \tag{14}
\]

where \( N \) is a nucleon and \( X \) stands for the hadronic interaction products. The operator generating this process has the structure

\[
\mathcal{L}_{\text{NSI}} \supset -2\sqrt{2}G_F \epsilon_{\tau\mu}^d V_{ud} [\bar{u} \gamma^\rho d] [\bar{\mu} \gamma_\rho P_L \nu_\tau] + h.c., \tag{15}
\]

where \( u \) and \( d \) denote the up- and down-quark fields, \( G_F \) is the Fermi constant, \( P_L = (1 - \gamma^5)/2 \), and \( \epsilon_{\tau\mu}^d \) gives the strength of the non-standard interaction compared to Standard Model weak interactions. (In the notation from ref. [76], this coefficient would be called \( \epsilon_{u\tau}^{dV} \).) In principle, one could also consider the axial-vector operator \([\bar{u} \gamma^\rho \gamma^5 d] [\bar{\mu} \gamma_\rho P_L \nu_\tau]\), but, being parity-odd, this operator would lead to the decay \( \pi \to \mu \nu_\tau \), which is strongly constrained by NOMAD, as discussed above. We will therefore neglect axial-vector NSI. We will also not consider scalar, pseudo-scalar, and tensor operators, since they could lead to the required interference between standard- and non-standard Feynman amplitudes only if the outgoing muon in the detector undergoes a helicity flip [46]. For \( \mathcal{O}(\text{GeV}) \) muons, this would correspond to a suppression of the non-standard interaction rate by \( m_\mu/E \sim 0.1 \).

If \( \epsilon_{\tau\mu}^d \) is non-zero, the counting rate in MINOS is no longer proportional to the standard survival probability \( \bar{P}(\nu_\mu \to \nu_\mu) \), but rather to an apparent \( \nu_\mu \) survival probability \( \bar{P}(\nu_\mu \to \nu_\mu) \), defined by the number of muons produced in the detector in interactions of neutrinos with a given energy \( E \), divided by the number of muons that would be produced in the absence of neutrino oscillations and non-standard interactions. Thus, \( \bar{P}(\nu_\mu \to \nu_\mu) \) includes the possibility that the neutrino flavor has changed into \( \nu_\tau \) during propagation, but non-standard interactions still lead to a final state muon, normally associated with \( \nu_\mu \) interactions. Since the amplitudes for the processes \( \nu_\mu + N \to X + \mu \) and \( \nu_\mu \to \tau \to \nu_\tau + N \to X + \mu \) can interfere, \( \bar{P}(\nu_\mu \to \nu_\mu) \) can be larger than unity and is therefore not a survival probability in the usual sense.

In the two-flavor approximation we find for \( \bar{P}(\nu_\mu \to \nu_\mu) \) [46]:

\[
\bar{P}(\nu_\mu \to \nu_\mu) = 1 - \left[ 1 + 2 |\epsilon_{\tau\mu}^d| \cot 2\theta_{23} \cos \left[ \frac{\Delta m_{32}^2}{4E} L \right] \right] \sin^2 2\theta_{23} \sin^2 \left( \frac{\Delta m_{32}^2}{4E} L \right) \cos \left( \frac{\Delta m_{32}^2}{4E} L \right) \cos \left( \frac{\Delta m_{32}^2}{4E} L \right). \tag{16}
\]

For anti-neutrinos, the sign of \( \arg(\epsilon_{\tau\mu}^d) \) has to be reversed, and thus

\[
\bar{P}(\bar{\nu}_\mu \to \bar{\nu}_\mu) = \bar{P}(\bar{\nu}_\mu \to \nu_\mu). \tag{17}
\]
Figure 2: The apparent survival probability for MINOS ($L = 735$ km) without NSI (black dotted curve), and in a scenario with non-zero $\epsilon^d_{\tau\mu}$. The solid (blue) line in this case is for neutrinos, while the red (dashed) line is for anti-neutrinos. For the standard oscillation parameters, we have assumed $\Delta m^2_{32} = 2.74 \times 10^{-3} \text{eV}^2$ and $\sin^2 \theta_{23} = 0.41$.

Note that the non-standard terms proportional to $|\epsilon^d_{\tau\mu}|$ and $|\epsilon^d_{\tau\mu}|^2$ in the first line of eq. (16) have the same energy dependence as the standard oscillation term and can therefore change only the depth of the oscillation dip, but not its position. They can thus only change the apparent value of $\sin^2 2\theta_{23}$ that would be reconstructed in a standard oscillation analysis neglecting NSI. Moreover, these terms do not depend on the sign of $\arg(\epsilon^d_{\tau\mu})$ and can therefore not introduce an asymmetry between neutrinos and anti-neutrinos. The interference term between standard and non-standard amplitudes in the second line of eq. (16), on the other hand, can be different for neutrinos and anti-neutrinos. Since eq. (16) is invariant under the simultaneous replacements $\arg(\epsilon^d_{\tau\mu}) \to -\arg(\epsilon^d_{\tau\mu})$ and $\Delta m^2_{32}/4E \to \pi - \Delta m^2_{32}/4E$, the depth of the oscillation minimum will be the same for neutrinos and anti-neutrinos, and only its position will be different. This is also illustrated in fig. 2, where we plot $\hat{P}(\nu_\mu \to \nu_\mu)$ including non-zero $\epsilon^d_{\tau\mu}$ for neutrinos and anti-neutrinos.

Like eq. (8) for neutral current NSI, also eq. (16) exhibits several symmetries. In particular, the expression is invariant under the simultaneous replacements

$$\arg(\epsilon^d_{\tau\mu}) \to 2\pi n - \arg(\epsilon^d_{\tau\mu}), \quad \Delta m^2_{32} \to -\Delta m^2_{32}$$

and under the transformation

$$\arg(\epsilon^d_{\tau\mu}) \to (2n + 1)\pi - \arg(\epsilon^d_{\tau\mu}), \quad \theta_{23} \to \frac{\pi}{2} - \theta_{23}$$

for arbitrary integer $n$. Actually, the second of these symmetries can be generalized to a continuous symmetry. To see this, note that eq. (16) is invariant under simultaneous changes of $|\epsilon^d_{\tau\mu}|$, $\arg(\epsilon^d_{\tau\mu})$, and $\theta_{23}$, provided that the coefficients of the energy dependent factors $\sin^2[\Delta m^2_{32} L/4E]$ and $\sin[\Delta m^2_{32} L/4E] \cos[\Delta m^2_{32} L/4E]$ remain invariant. This requirement imposes two constraints on the three parameters $|\epsilon^d_{\tau\mu}|$, $\arg(\epsilon^d_{\tau\mu})$, and $\theta_{23}$, so that there will be an infinite set of solutions.
Note that the symmetries of eq. (16), like those of eq. (8), are exact only in the two-flavor framework.

3. FIT TO MINOS DATA

To test the compatibility of MINOS data with the hypothesis of non-standard neutrino interactions and to extract the allowed values for the NSI parameters, we have performed fits using a modified version of GLoBES [84, 85] (see appendix A for details). We have first checked that we were able to reproduce the results of the two-flavor standard oscillation fits performed by the MINOS collaboration. We find that the fit to only $\nu_\mu$ ($\bar{\nu}_\mu$) data yields $\chi^2$/dof = 12.3/12 (2.3/3), while a combined fit to both data sets results in $\chi^2$/dof = 20.1/17. According to the parameter goodness-of-fit test described in ref. [86], this means that the probability for the apparent inconsistency between the $\nu_\mu$ and $\bar{\nu}_\mu$ data sets to be merely a statistical fluctuation is about 6%. Even though this probability is still relatively large, let us now study how the fit improves if the possibility of non-standard interactions are included.

3.1. Neutral current NSI

We begin by assuming only neutral current NSI (see sec. 2.1). Using a three-flavor fit including the NSI parameters $|\epsilon_{\mu\tau}^m|$, arg($\epsilon_{\mu\tau}^m$), and $\epsilon_{\tau\tau}^m$, we find the following parameter values for the best fit point:

$$
\epsilon_{\mu\tau}^m = -0.40 = 0.40 e^{1.0 i \pi} \quad \sin^2 \theta_{23} = 0.38 \\
\epsilon_{\tau\tau}^m = -2.16 \quad \Delta m_{32}^2 = +2.86 \times 10^{-3} \text{ eV}^2
$$

(20)

The $\chi^2$ at the best fit point is 12.6, and the number of degrees of freedom is 14. Note that the best fit value for $\epsilon_{\mu\tau}^m$ is essentially real and negative. An equally good fit is obtained when the sign of arg($\epsilon_{\mu\tau}^m$) is inverted (eq. (11)), when the signs of $\epsilon_{\mu\tau}^m$, $\epsilon_{\tau\tau}^m$, and $\Delta m_{32}^2$ are flipped simultaneously (eq. (12)), or when the sign of $\epsilon_{\tau\tau}^m$ and the octant of $\theta_{23}$ are changed (eq. (13)). This means that three-flavor effects are not large enough to spoil the symmetries of the two-flavor survival probability eq. (8). Thus, the best-fit point is eightfold degenerate. In fig. 3, we compare the theoretically predicted event spectrum at the parameter point eq. (20) (blue dotted histograms) to the MINOS data and to the spectra obtained from a two-flavor standard oscillation fit.

Our fit is in qualitative agreement with the one in ref. [77], but points to somewhat larger values of $|\epsilon_{\mu\tau}^m| \sim 0.4$ compared to $|\epsilon_{\mu\tau}^m| \sim 0.1$ in ref. [77]. We surmise that the main reason for this disagreement is that our fit is based on a simulation of the MINOS event spectrum, while, to our understanding, the authors of ref. [77] have directly fitted the oscillation probability to the ratio of observed to expected event numbers, as computed by the MINOS collaboration. A fit at the level of the oscillation probability, however, cannot fully include experimental energy resolution effects. The fit to the spectrum used in our work is able to reproduce the MINOS best fit points as well as the allowed regions in the $\Delta m_{32}^2$–$\sin^2 2\theta_{23}$ plane rather well (see fig. 8 in appendix A).

The allowed regions and best fit values for $|\epsilon_{\mu\tau}^m|$, arg($\epsilon_{\mu\tau}^m$), and $|\epsilon_{\tau\tau}^m|$ are shown in the three panels of fig. 4. We find that, in spite of the large best fit value for $|\epsilon_{\tau\tau}^m|$, MINOS is compatible with $\epsilon_{\tau\tau}^m = 0$ at the 90% confidence level. The case of pure standard oscillations, on the other hand, is ruled out at 90% confidence level. Note that only a fourfold rather than eightfold degeneracy is visible in fig. 4 because the transformation (13) only changes the sign of $\epsilon_{\tau\tau}^m$ and the octant of $\theta_{23}$, which are not displayed here.
The compatibility of the parameter region favored by MINOS with existing constraints depends strongly on whether atmospheric neutrino bounds are taken into account. Atmospheric neutrinos are very sensitive to non-standard matter effects since they can travel over very long distances inside the Earth before reaching the detector. Therefore, the MINOS-favored region of parameter space is in conflict with the limits derived in refs. [35, 39] from MACRO and Super-Kamiokande data. While these bounds are numerically very strong, they have been derived in a two-flavor framework. It has been called into question whether they would still hold when standard and non-standard three-flavor effects are taken into account [38, 48]. While we consider it highly unlikely that three-flavor effects would weaken the atmospheric neutrino bounds on NC NSI by the amount required to restore compatibility with the MINOS-favored parameter region, we cannot definitely rule out NC NSI as an explanation for the MINOS data at this time.

3.2. Charged current NSI

Let us now turn to the investigation of the charged current NSI introduced in sec. 2.2. We fit the MINOS data using a three-flavor analysis including the NSI parameters $|\epsilon_{\tau\mu}^d|$ and $\arg(\epsilon_{\tau\mu}^d)$, but, as for the neutral current case, we find that three-flavor effects are small. In particular, the symmetries of the $\nu_\mu$ survival probability found in sec. 2.2 for the two-flavor case are present also in the three-flavor framework. In particular, this means that there is a continuous family of best fit points (see eqs. (18) and (19), and the continuous generalization of (19) at the end of sec. 2.2). For reference, we here give one representative point from this family that is most consistent with existing bounds on $|\epsilon_{\tau\mu}^d|$: 

$\epsilon_{\tau\mu}^d = 0.12 i = 0.12 e^{0.5i\pi}$, \hspace{1cm} $\sin^2 \theta_{23} = 0.41$, \hspace{1cm} $\Delta m_{32}^2 = +2.74 \times 10^{-3}$ eV$^2$, \hspace{1cm} (21)
Figure 4: Constraints on the parameter space of neutral current non-standard interactions in the $\mu-\tau$ sector from MINOS $\nu_\mu$ and $\bar{\nu}_\mu$ data. Each panel corresponds to a projection of the three-dimensional NSI parameter space along one of its axes. This means that in each panel we have marginalized over one of the NSI parameters (as well as the six standard oscillation parameters). The best fit points are indicated by the colored dots. The symmetries from eqs. (11) and (12) are clearly visible in the plot, while the additional two-fold ambiguity eq. (13) is implicit. In the bottom panels, we explicitly indicate the parameter regions corresponding to a normal mass hierarchy (NH) and to an inverted mass hierarchy (IH), while in the top panel, NH and IH contours lie on top of each other. Exclusion limits from other experiments [39, 76] are shown in gray. See text for caveats pertaining to bounds from atmospheric neutrino measurements.

with $\chi^2$/dof = 15.9/15. The predicted MINOS event spectra at this point are shown as the solid blue histograms in fig. 3.

In fig. 5, we show the allowed regions in the $|\epsilon_{\tau\mu}^d| - \arg(\epsilon_{\tau\mu}^d)$ plane determined by our fit. The continuous family of best fit points is indicated by the thin dash-dotted black curves. To the best of our knowledge, no bound on the vector-type $\nu_\tau-\mu$ CC NSI considered here exists in the literature. (The bounds of order 0.1 derived in ref. [76] apply only to axial-vector operators.) However, using the methods developed in refs. [71, 72], the experimental limit on the branching ratio of the flavor-violating decay $\tau^\pm \rightarrow \mu^\pm \pi^0$ [87] can be translated into a bound of order 0.2 on $|\epsilon_{\tau\mu}^d|$ (and, in fact, also on the related coefficient $|\epsilon_{\mu\tau}^d|$) for vector and axial-vector type interactions (see appendix B for details and caveats). We expect that a bound could also be derived from
Figure 5: Constraints on the parameter space of charged current non-standard interactions between $\nu_\tau$ and muons from MINOS $\nu_\mu$ and $\bar{\nu}_\mu$ data. We have used a full three-flavor fit, marginalizing over the standard oscillation parameters. The discrete and continuous symmetries from eqs. (18) and (19) are clearly visible in the plot. The thin dash-dotted black curves indicate the positions of the approximately degenerate best fit points. We explicitly indicate the parameter regions corresponding to a normal mass hierarchy (NH, blue) and to an inverted mass hierarchy (IH, red). See text for comments on existing constraints on CC NSI.

lepton universality considerations in weak decays of parity-even hadrons, but that it would not be stronger than $O(0.1)$. Also, atmospheric neutrinos are sensitive to $|c_{\tau\mu}|$, but since CC NSI are not enhanced by the long baselines of atmospheric neutrinos, we expect these constraints to be relatively weak as well.

4. TESTING THE NSI INTERPRETATION OF MINOS DATA IN FUTURE EXPERIMENTS

To corroborate or refute the hypothesis of large non-standard interactions as an explanation for the apparent discrepancy between neutrino and anti-neutrino results in MINOS, it will be mandatory to gather more statistics in MINOS itself, and to look for possible NSI signals in future experiments like T2K and NO$\nu$A. In the following, we will neglect neutral-current NSI since we have seen in sec. 3.1 that they are disfavored as an explanation for the MINOS data by atmospheric neutrinos. Instead, we will focus on CC NSI. We have computed the expected event spectrum in MINOS, T2K, and NO$\nu$A assuming the values of the standard and non-standard oscillation parameters to be given by the MINOS best fit point eq. (21). We have then attempted a standard oscillation (no NSI) fit to this simulated data. If this fit is incompatible with the simulated data at a given confidence level, we say that the existence of a non-standard effect can be established experimentally at this confidence level. Our simulation of T2K follows [88–90], while that of NO$\nu$A is based on [81, 91]. We include only the $\nu_\mu$ and $\bar{\nu}_\mu$ disappearance channels.

In fig. 6, we show the predicted discovery potential in MINOS, T2K, and NO$\nu$A as a function of the integrated luminosity in neutrino mode and the integrated luminosity in anti-neutrino mode. We also indicate how the number of protons on target (pot) translates into a time of running at nominal luminosity ($2.5 \times 10^{20}$ pot/year for MINOS, $6 \times 10^{20}$ pot/year for NO$\nu$A, and $10^{21}$ pot/year for T2K). We see that optimal sensitivity is achieved if slightly more time is spent on running in anti-neutrino mode than on running in neutrino mode. This is easily understandable since the assumed
NSI effect manifests itself mainly as an apparent discrepancy between neutrino and anti-neutrino results, while each data sample individually appears to be consistent with standard oscillations. Thus, to optimally probe the non-standard effect, the event numbers in the $\nu_\mu$ and $\bar{\nu}_\mu$ samples should not be too different. On the other hand, anti-neutrino cross sections are about a factor of 3 smaller than neutrino cross section, so more time has to be devoted to $\bar{\nu}$ running to achieve this goal. Fig. 6 also shows that in order to improve the statistical significance of the anomalous effect in MINOS itself, more anti-neutrino running is desirable since the experiment has already taken a lot of data in neutrino mode. By comparing the three panels of fig. 6, we see that, as expected, the discovery potential of T2K is better than that of MINOS, while the best sensitivity is achieved in NO$\nu$A. After one year of nominal running in neutrino mode, NO$\nu$A could confirm the existence of the non-standard effect at the 90% confidence level, while in anti-neutrino mode, even a few months would be sufficient to achieve that sensitivity. This can be understood by noting that, for the parameter values favored by MINOS, eq. (21), the two $O(|\epsilon_{\tau\mu}|)$ NSI terms in eq. (16) have opposite signs for neutrinos, but the same sign for anti-neutrinos. Therefore, the non-standard effect is stronger for anti-neutrinos. To achieve a 3$\sigma$ discovery in T2K or NO$\nu$A, neutrino and anti-running are required, with at least one year spent in each mode for T2K, or half a year for NO$\nu$A.

If the true values of the NSI parameters are different from the best fit point eq. (21), the discovery reach in future experiments can be altered significantly. This is illustrated in fig. 7, where we plot the $\chi^2$ of a standard oscillation fit to simulated data affected by CC NSI as a function of the running time at nominal luminosity. The widths of the colored bands correspond to the 1$\sigma$ uncertainty in the NSI parameters from fig. 5. Fig. 7 shows that if nature has chosen unfavorable NSI parameters, it will be very hard for T2K and NO$\nu$A to announce a discovery. On the other hand, for favorable parameter values a 3$\sigma$ effect could be detected after less than one year of nominal running even in T2K.

5. DISCUSSION AND CONCLUSIONS

We have seen that, in order to explain the tension between the $\nu_\mu$ and $\bar{\nu}_\mu$ event samples in MINOS using NSI, the NSI couplings would have to be rather large, almost of the same order as Standard Model weak interactions. While we have shown in sec. 3 that there are regions of parameter space still consistent with MINOS data and with constraints from other experiments, one should keep in mind that the effective operators generating the NSI should ultimately arise...
Figure 7: \( \chi^2 \) as a function of exposure time for a standard oscillation fit to (simulated) data affected by charged current NSI. The widths of the colored bands corresponds to varying the NSI parameters within the 1\( \sigma \) allowed region preferred by MINOS (fig. 5). We have assumed running at the indicated nominal luminosities, and we have assumed the running time to be equally divided into neutrino and anti-neutrino running.

from an underlying renormalizable model. Model-dependent constraints, however, are usually much stronger than the model-independent bounds we have considered.

For example, the most straightforward implementations of dimension 6 NSI operators, based on the introduction of new heavy tree-level mediator fields, are phenomenologically not viable because \( SU(2) \) invariance would dictate that large neutrino NSI realized that way would have to be accompanied by large non-standard effects in the charged lepton sector [20, 21]. Therefore, such models are usually tightly constrained by rare decay searches [87]. NSI might arise from dimension 8 operators involving two Standard Model Higgs fields contracted with lepton doublets, so that after electroweak symmetry breaking, \( SU(2) \) breaking 4-fermion couplings can arise. However, dimension 8 operators of this type are typically accompanied by phenomenologically problematic dimension 6 operators unless the coefficients of different operators obey certain cancellation conditions [20]. Further model-dependent constraints on neutrino NSI operators can come from electroweak precision tests such as muon \( g-2 \) measurements, and from direct searches for possible mediators. All these constraints will typically force the mediators to be heavy (at least a few hundred GeV) or very weakly coupled.

The latter possibility—neutrino NSI mediated by light (\( \ll M_W \)), weakly coupled particles—is less well explored in the literature, so a scenario of this type could be responsible for the effects seen by MINOS. This is particularly interesting as models containing light new particles have recently received a lot of attention in the context of Dark Matter searches (see e.g. refs. [92–97]).

In conclusion, we have shown that the tension between the \( \nu_\mu \) and \( \bar{\nu}_\mu \) disappearance data in MINOS—if it persists—could be explained by non-standard neutrino interactions. While neutral current interactions (non-standard matter effects) of the required magnitude are most likely ruled
out by atmospheric neutrino constraints, a charged current operator leading to flavor-violating couplings between $\tau$-neutrinos and muons is not excluded. We have shown that such NSI can be tested in T2K and NO$\nu$A, provided that the experiments are operated in neutrino and anti-neutrino mode. It remains an open question if and how NSI large enough to explain the MINOS results can arise from a renormalizable model, especially if one imposes $SU(2)$ gauge invariance. Along the way, we have derived the new constraints $|\epsilon^d_{\tau\mu}| < 0.20$ and $|\epsilon^d_{\mu\tau}| < 0.20$ on flavor-violating vector or axial-vector type charged current non-standard interactions in the $\mu$–$\tau$ sector by considering their loop-level contributions to the flavor-violating decay $\tau^{\pm} \rightarrow \mu^{\pm}\pi^0$.

Acknowledgments

We are indebted to Mary Bishai for providing the simulated NuMI $\nu_\mu$ fluxes used as input to our simulation. We are also grateful to Bogdan Dobrescu, Thomas Schwetz-Mangold, and Jure Zupan for several insightful discussions on model-building aspects of non-standard neutrino interactions, and to Carla Biggio, Mattias Blennow, Enrique Fernandez-Martinez, Belen Gavela, and Bill Marciano for very helpful comments and discussions on NSI constraints from existing data. Finally, it is a pleasure to thank the organizers of the Neutrino 2010 conference, at which we learned about the new MINOS results, and the organizers of the INT Program on Long-Baseline Neutrino Physics and Astrophysics in Seattle, Washington, during which part of this work was completed, for two very enjoyable and fruitful meetings. PANM would like to thank the Fundação de Amparo à Pesquisa do Estado de São Paulo for financial support of his PhD project, and Fermilab for kind hospitality and support during his visit. PANM and JK have been partially supported by the US Department of Energy’s Institute for Nuclear Theory (INT) at the University of Washington. Fermilab is operated by Fermi Research Alliance, LLC under Contract No. DE-AC02-07CH11359 with the US Department of Energy.

Appendix A: GLOBES SIMULATION OF MINOS

In this appendix, we provide details on the parameters of our MINOS simulation. We have used GLoBES [84, 85], with an implementation of NSI developed in refs. [46, 56], and with a MINOS experiment description based on [1, 98–100]. The neutrino fluxes are taken from Monte Carlo simulations of the NuMI beam [101]. We use the same binning as the MINOS collaboration, but restrict our analysis to an energy window from 1–5 GeV for neutrinos, and from 1–8 GeV for anti-neutrinos. The reason for not including higher-energy neutrino data is that the $\nu_\mu$ fluxes available to us did not include the effect of unfocused high energy pions in the secondary beam. Detection efficiencies for $\nu_\mu$, neutral current backgrounds, and the actual MINOS data were taken from [1]. The neutrino–nucleon scattering cross sections are based on the cross sections for water targets in [102, 103], and to account for the difference between the cross sections for water and those for iron, we adjust the overall normalization factor in our simulation in such a way that we optimally reproduce the predictions of the MINOS Monte Carlo simulation. The matter density along the neutrino trajectory is assumed to be 2.8 g/cm$^3$. We have checked that the results of two-flavor standard oscillation fits to $\nu_\mu$ and $\bar{\nu}_\mu$ data agree well with the allowed regions obtained by the MINOS collaboration (see fig. 8).

In our fits, we use Gaussian prior terms to impose the 1$\sigma$ constraints $\sin^2 2\theta_{13} < 0.1$, $\sin^2 \theta_{12} = 0.319 \pm 0.023$, and $\Delta m^2_{21} = (7.59 \pm 0.30) \times 10^{-5} \text{ eV}^2$ [104, 105]. As systematic errors, we include
Figure 8: Allowed regions in the $\sin^2 2\theta_{23}$–$\Delta m_{32}^2$ plane from separate two-flavor standard oscillation fits to MINOS $\nu_\mu$ and $\bar{\nu}_\mu$ data. The thick contours and filled circles represent the 68% and 90% contours and best fit points from our fit, while thin lines and empty diamonds show the fit results obtained by the MINOS collaboration [1].

Figure 9: (a) and (b) The loop diagrams used to constrain $\epsilon_{\tau\mu}^d$ and $\epsilon_{\mu\tau}^d$, respectively. (c) A similar diagram that does not have a logarithmic divergence.

independent 4% (3%) normalization uncertainties on the signal (background) rates for neutrinos, and 5% (5%) normalization uncertainties on the signal (background) rates for anti-neutrinos.

Appendix B: NEW LOOP BOUNDS ON NON-STANDARD INTERACTIONS

Here, we outline the calculation of the loop diagram fig. 9 (a) which we use to translate the experimental bound on the rare decay $\tau^\pm \rightarrow \mu^\pm \pi^0$ into a constraint on $|\epsilon_{\tau\mu}^d|$. Following refs. [71, 72, 76], we perform the calculation in unitary gauge and keep only the logarithmically divergent part. Indeed, it has been argued that the finite and quadratically divergent terms cannot be calculated in a model-independent way, while the logarithmic divergence corresponds to the model-independent
one-loop renormalization group running of the effective NSI operator from the UV completion scale down to $M_W$ [71, 72, 76]. (This argument is invalid if the UV complete theory leads to another effective operator having the same renormalization group running and exactly canceling the NSI operator. If that case is considered, no model-independent bound on $|\epsilon_{\tau\mu}^d|$ can be derived.) Neglecting fermion masses, we find that the logarithmically divergent part of fig. 9 (a) is

$$\frac{3\sqrt{2}G_F\epsilon_{\tau\mu}^d V_{ud} \alpha}{2\pi s_w^2} \log \frac{\Lambda}{M_W} \left[ \bar{\tau} \gamma^\mu P_L \mu \right] \left[ \bar{u} \gamma^\mu P_L u \right],$$

(B1)

where $\alpha$ is the electromagnetic fine structure constant, $s_w$ is the sine of the weak mixing angle, $V_{ud}$ is a CKM matrix element, $P_L = (1 - \gamma^5)/2$, and $\Lambda$ is the UV completion scale. Comparing eq. (B1) to the operator $2\sqrt{2}G_F V_{ud} \left[ \bar{\tau} \gamma^\mu P_L \nu_\tau \right] \left[ \bar{u} \gamma^\mu P_L d \right]$ responsible for the standard decay $\tau^\pm \rightarrow \pi^\pm \nu_\tau$, using the QCD isospin symmetry between $\pi^0$ and $\pi^\pm$, and assuming $\log \Lambda/M_W \sim O(1)$, we find

$$|\epsilon_{\tau\mu}^d| \simeq \sqrt{2} \frac{\text{BR}(\tau^\pm \rightarrow \mu^\pm \pi^0)}{\text{BR}(\tau^\pm \rightarrow \pi^\pm \nu_\tau)} \frac{4\pi s_w^2}{3\alpha}. \quad (B2)$$

The factor 2 below the square root is due to the fact that the $\pi^0$ can be interpreted as a state $(\bar{u}u - \bar{d}d)/\sqrt{2}$ whose overlap with the final state $\bar{u}u$ from fig. 9 (a) is only $1/\sqrt{2}$. With the experimental constraint $\text{BR}(\tau^\pm \rightarrow \mu^\pm \pi^0) < 1.1 \times 10^{-7}$ [87], eq. (B2) leads to the new bound

$$|\epsilon_{\tau\mu}^d| \lesssim 0.20. \quad (B3)$$

In full analogy to the above, we can also use fig. 9 (b) to set a bound on $\epsilon_{\mu\tau}^d$:

$$|\epsilon_{\mu\tau}^d| \lesssim 0.20. \quad (B4)$$

Note that diagrams like the one in fig. 9 (c) with $\bar{d}d$ in the final state do not contribute to these bounds because they do not have logarithmic divergences.

While we have derived eqs. (B3) and (B4) by assuming vector-type NSI, our calculations can be easily generalized to set equivalent bounds on axial-vector operators and on $V - A$ operators, while no constraints can be derived for $V + A$ type interactions since the diagrams in fig. 9 are suppressed by a neutrino mass insertion in this case.
