FERMILAB-PUB-10-040-T

Resource Letter QCD-1: Quantum Chromodynamics

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(Dated: May 20, 2010)

This Resource Letter provides a guide to the literature on Quantum Chromodynamics (QCD), the relativistic quantum field theory of the strong interactions. Journal articles, books, and other documents are cited for the following topics: quarks and color, the parton model, Yang-Mills theory, experimental evidence for color, QCD as a color gauge theory, asymptotic freedom, QCD for heavy hadrons, QCD on the lattice, the QCD vacuum, pictures of quark confinement, early and modern applications of perturbative QCD, the determination of the strong coupling and quark masses, QCD and the hadron spectrum, hadron decays, the quark-gluon plasma, the strong nuclear interaction, and QCD's role in nuclear physics.

The letter E after an item indicates elementary level or material of general interest to persons becoming informed in the field. The letter I, for intermediate level, indicates material of a somewhat more specialized nature, and the letter A indicates rather specialized or advanced material.

PACS numbers: 12.38.-t, 24.85.+p FERMILAB-PUB-10/040-T

Contents

I. INTRODUCTION

Quantum chromodynamics (QCD) is a remarkably simple, successful, and rich theory of the strong interactions. The theory provides a dynamical basis for the *quark-model* description of the *hadrons*, the strongly interacting particles such as protons and pions that are accessible for direct laboratory study. Interactions among the quarks are mediated by vector force particles called *gluons*, which themselves experience strong interactions. The nuclear force that binds protons and neutrons together in atomic nuclei emerges from the interactions among quarks and gluons.

QCD describes a wealth of physical phenomena, from the structure of nuclei to the inner workings of neutron stars and the cross sections for the highest-energy elementary-particle collisions. The QCD literature is correspondingly extraordinarily vast. To arrive at a manageable number of cited papers in this Resource Letter, we have chosen works that should be useful to professors and students planning a course for classroom or independent study. We have included some classic contributions and all elementary presentations of which we are aware. For more advanced material we favor works that provide ambitious students an entrée to the contemporary literature. These include well-established highly-cited review articles (because a search of literature citing a review is a gateway to newer topics) and more modern treatments that portray the state of the art and document the preceding literature well. Wherever possible, we give links to digital versions of the articles we cite. Many published articles are available in electronic form through the World Wide Web sites of individual journals, or through the e-print archive; see Appendix A.

From the time of its development in the 1950s, quantum electrodynamics (QED), the relativistic quantum field theory of photons and electrons, was viewed as exemplary. In the late 1960s and early 1970s, with the development of the electroweak theory, it became increasingly attractive to look to relativistic quantum field theories—specifically gauge theories—for the description of all the fundamental interactions.

1. "Quantum field theory," F. Wilczek, *Rev. Mod. Phys.* **71**, S85–S95 (1999) [`hep-th/9803075`]. (I)

QCD represents the culmination of that search for the strong interactions. In some respects, it has supplanted QED as our "most perfect" theory.

2. "What QCD tells us about nature—and why we should listen," F. Wilczek, *Nucl. Phys.* **A663-664**, 3–20 (2000) [`hep-ph/9907340`]. (I)

An excellent summary of the foundations and implications of QCD is

3. "Asymptotic freedom and quantum chromodynamics: the key to the understanding of the strong nuclear forces," Advanced Information on the Nobel Prize in Physics, `http://nobelprize.org/nobel_prizes/physics/laureates/2004/phyadv04.pdf`. (E–I)

Some encyclopedia articles on QCD are

4. "Quantum chromodynamics," A. S. Kronfeld in **Macmillan Encyclopedia of Physics**, vol. 3, edited by J. S. Rigden, pp. 1260–1264 (Macmillan, New York, 1996). [doi: `10.1223/0028973593`]. (E–I)

5. "Quantum chromodynamics," G. Sterman in **Encyclopedia of Mathematical Physics**, edited by J.-P. Françoise, G. L. Naber, and Tsou Sheung Tsun, pp. 144–153 (Elsevier, Amsterdam, 2006). [`hep-ph/0512344`]. (I–A)

6. "Quantum chromodynamics," C. Quigg in **McGraw-Hill Encyclopedia of Science & Technology**, vol. 14, pp. 670–676 (McGraw-Hill, New York, 2007), 10th ed. [doi: `10.1036/1097-8542.562500`]. (E–I)

For a book-length exposition of the wonders of QCD, see

7. The Lightness of Being: Mass, Ether, and the Unification of Forces, F. Wilczek (Basic Books, New York, 2008). (E)

The rest of this Resource Letter is organized as follows. We begin in Sec. II by reviewing the basics of the theory of QCD, giving its Lagrangian, some essential aspects of its dynamics, and providing a connection to earlier ideas. In Sec. III we cover literature on theoretical tools for deriving physical consquences of the QCD Lagrangian. Section IV covers the most salient aspects of the confrontation of QCD with experimental observations and measurements. Section V situates QCD within the broader framework of the standard model of particle physics. We conclude in Sec. VI with a brief essay on frontier problems in QCD. Appendix A gives links to basic online resources.

II. QCD

As a theory of the strong interactions, QCD describes the properties of hadrons. In QCD, the familiar mesons (the pion, kaon, etc.) are bound states of *quarks* and *antiquarks*; the familiar baryons (the proton, neutron, $\Delta(1232)$ resonance, etc.) are bound states of three quarks. Just as the photon binds electric charges into atoms, the binding agent is the quantum of a gauge field, called the gluon. Hadrons made of exclusively of gluons, with no need for valence quarks, may also exist and are called glueballs. Properties of hadrons are tabulated in

8. "Review of particle physics," C. Amsler *et al.*, Particle Data Group, *Phys. Lett.* **B667**, 1 (2008) [doi: `10.1016/j.physletb.2008.07.018`] [`http://pdg.lbl.gov`]. (E–I–A)

In this section we begin with the Lagrangian formulation of QCD. Readers who are not yet familiar with the Dirac equation may wish to skip this mathematical discussion and head straight to Sec. II B for a résumé of the main themes of QCD, to Sec. II C for a list of textbooks, or to Sec. II D for resources on the ideas out of which the quantum field theory QCD emerged in the early 1970s.

A. A gauge theory for the strong interactions

Quantum chromodynamics is the theory of strong interactions among quarks derived from the color gauge symmetry $\mathrm{SU(3)_c}$. It is advantageous for many purposes to express the theory in Lagrangian form. As in a classical theory, one can easily derive the equations of motion. In a quantum theory, the Lagrangian also provides a convenient framework for quantization and the development of perturbation theory, via Feynman rules, in a Lorentz-covariant fashion. The Lagrangian formalism lends itself particularly to the consideration of symmetry principles and their consequences. Invariance of the Lagrangian under a *global*, *i.e.*, position-independent, symmetry operation implies a conservation law through Noether's theorem. Requiring the Lagrangian to be invariant under *local*, *i.e.*, position-dependent, transformations demands an interacting theory, in which spin-one force particles couple minimally to the conserved current of the global symmetry. Thus a global U(1) phase symmetry is related to conservation of the electromagnetic current, and local U(1) phase symmetry underlies QED. A symmetry used to derive a theory of interactions is called a *gauge symmetry.*

In nature, we find six *flavors* of quarks, "up," "down," "charm," "strange," "top," and "bottom." The electric charges of the quarks are $2e/3$ for the up, charm, and top flavors, and $-e/3$ for the down, strange, and bottom flavors, where $-e$ is the electon's charge. The essence of the QCD Lagrangian is captured for a single flavor:

$$\mathcal{L} = \bar{\psi}(i\gamma^\mu \mathcal{D}_\mu - m)\psi - \tfrac{1}{2}\,\mathrm{tr}(G_{\mu\nu}G^{\mu\nu}). \qquad (1)$$

The composite spinor for color-triplet quarks of mass m is

$$\psi = \begin{pmatrix} q_{\mathrm{red}} \\ q_{\mathrm{green}} \\ q_{\mathrm{blue}} \end{pmatrix}, \qquad (2)$$

where each element q_i is a four-component Dirac spinor, acted upon by the Dirac matrices γ^μ. The gauge-covariant derivative is

$$\mathcal{D}_\mu = \partial_\mu + igB_\mu\,, \qquad (3)$$

where g is the strong coupling constant and the object B_μ is a three-by-three matrix in color space formed from the eight (gluon) color gauge fields B^l_μ and the generators $\frac{1}{2}\lambda^l$ of $\mathrm{SU(3)_c}$ as

$$B_\mu = \tfrac{1}{2}\boldsymbol{B}_\mu \cdot \boldsymbol{\lambda} = \tfrac{1}{2}B^l_\mu \lambda^l\,. \qquad (4)$$

The gluon field-strength tensor is

$$\begin{aligned} G_{\mu\nu} &= \tfrac{1}{2}\boldsymbol{G}_{\mu\nu}\cdot\boldsymbol{\lambda} = \tfrac{1}{2}G^l_{\mu\nu}\lambda^l \qquad (5)\\ &= (ig)^{-1}\left[\mathcal{D}_\nu, \mathcal{D}_\mu\right] = \partial_\nu B_\mu - \partial_\mu B_\nu + ig\left[B_\nu, B_\mu\right]. \end{aligned}$$

The λ-matrices satisfy

$$\begin{aligned} \mathrm{tr}(\lambda^l) &= 0, \qquad (6)\\ \mathrm{tr}(\lambda^k\lambda^l) &= 2\delta^{kl}, \qquad (7)\\ \left[\lambda^j, \lambda^k\right] &= 2if^{jkl}\lambda^l\,, \qquad (8) \end{aligned}$$

and the *structure constants* f^{jkl} can be expressed as

$$f^{jkl} = (4i)^{-1}\ \mathrm{tr}\left\{\lambda^l\left[\lambda^j, \lambda^k\right]\right\}\,. \qquad (9)$$

The nonvanishing structure constants distinguish QCD from QED: QCD is a *non-Abelian* gauge theory. The gluon field-strength can be expressed in component form as

$$G^l_{\mu\nu} = \partial_\nu B^l_\mu - \partial_\mu B^l_\nu + gf^{jkl}B^j_\mu B^k_\nu, \qquad (10)$$

and the last term marks a fundamental dynamical difference between QCD and QED. Via $\mathrm{tr}(G_{\mu\nu}G^{\mu\nu})$ in Eq. (1), it leads to three-gluon and four-gluon interactions that have no counterpart in QED. The gluon carries color charge and, thus, experiences strong interactions, whereas the neutral photon does not couple directly to other photons.

The color matrices for the fundamental (quark) representation satisfy

$$\sum_i \lambda^i_{ab}\lambda^i_{bc} = 4C_F\delta_{ac}, \quad C_F = \frac{N^2-1}{2N}, \qquad (11)$$

while the color matrices for the adjoint (gluon) representation, $T^k_{ij} = -if^{ijk}$, obey

$$\mathrm{tr}(T^kT^l) = \sum_{ij} f^{ijk}f^{ij\ell} = C_A\delta^{k\ell}, \quad C_A = N. \qquad (12)$$

For QCD based on $\mathrm{SU}(N=3)$ gauge symmetry, the quark and gluon color factors are

$$C_F = \tfrac{4}{3}, \quad C_A = 3. \qquad (13)$$

It is sometimes advantageous to carry out calculations for general values of C_F and C_A, to test the non-Abelian structure of QCD (see Sec. IV D).

Physical arguments in favor of the $\mathrm{SU(3)_c}$ gauge theory are collected in

9. "Advantages of the color-octet gluon picture," H. Fritzsch, M. Gell-Mann, and H. Leutwyler, *Phys. Lett.* **B47**, 365–368 (1973) [doi: `10.1016/0370-2693(73)90625-4`]. (I–A)

The Lagrangian $\mathcal{L}$ in Eq. (1) is invariant under the transformations

$$\begin{aligned} \psi(x) &\mapsto e^{i\omega(x)}\psi(x), \qquad (14)\\ \bar{\psi}(x) &\mapsto \bar{\psi}(x)e^{-i\omega(x)}, \qquad (15)\\ B_\mu(x) &\mapsto e^{i\omega(x)}[B_\mu + (ig)^{-1}\partial_\mu]e^{-i\omega(x)}, \qquad (16) \end{aligned}$$

where the matrix $\omega(x) = \frac{1}{2}\omega^l(x)\lambda^l$ depends on the space-time coordinate x. Generically the matrices $\omega(x)$ and $B_\mu(x)$ do not commute, a feature that again distinguishes QCD from QED.

If we recast the matter term in the Lagrangian (1) in terms of left-handed and right-handed fermion fields,

$$\mathcal{L}_q = \bar{\psi}_{\mathrm{L}} i\gamma^\mu \mathcal{D}_\mu \psi_{\mathrm{L}} + \bar{\psi}_{\mathrm{R}} i\gamma^\mu \mathcal{D}_\mu \psi_{\mathrm{R}} - m\left(\bar{\psi}_{\mathrm{L}}\psi_{\mathrm{R}} + \bar{\psi}_{\mathrm{R}}\psi_{\mathrm{L}}\right), \qquad (17)$$

we see that it becomes highly symmetrical in the limit of vanishing quark mass, $m \to 0$. Absent the mass term, there is no coupling between the left-handed and right-handed quark fields, $\psi_{\mathrm{L,R}} \equiv \frac{1}{2}(1 \mp \gamma_5)\psi$, and so the Lagrangian is invariant under separate global phase transformations on the left-handed and right-handed fields. Generalizing to the case of n_{f} flavors of massless quarks, we find that the QCD Lagrangian displays an $\mathrm{SU}(n_{\mathrm{f}})_{\mathrm{L}} \times \mathrm{SU}(n_{\mathrm{f}})_{\mathrm{R}} \times \mathrm{U}(1)_{\mathrm{L}} \times \mathrm{U}(1)_{\mathrm{R}}$ *chiral symmetry.* In nature, the up- and down-quark masses are very small (compared to the proton mass), and the strange-quark mass is also small. Therefore, $n_{\mathrm{f}} = 2$ (isospin) and $n_{\mathrm{f}} = 3$ (flavor SU(3)) chiral symmetries are approximate. We return to chiral symmetries in Sec. III A 1.

The U(1) factors may be rewritten $\mathrm{U}(1)_V \times \mathrm{U}(1)_A$. The vector ($V$) symmetry applies the same phase factor to left- and right-handed fields; it leads via Noether's theorem to a conserved charge, namely baryon number. The axial-vector (A) symmetry applies opposite phase factors to left- and right-handed fields; it is broken by certain quantum-mechanical effects, called anomalies, discussed in Sec. III A 2.

B. First consequences

As mentioned above, the three- and four-gluon interactions make the physics of QCD essentially different from the mathematically similar QED. In quantum electrodynamics, an electron's charge is partially screened by vacuum polarization of the surrounding cloud of virtual electron-positron pairs. The effect can be measured with a probe of wavelength $1/Q$, and described by a scale dependence, or running, of the fine structure constant $\alpha \equiv e^2/4\pi$. Omitting charged particles other than the electron, a first-order calculation of the running yields

$$\frac{1}{\alpha(Q)} = \frac{1}{\alpha(m_e)} - \frac{2}{3\pi}\log\left(\frac{Q}{m_e}\right) , \tag{18}$$

where m_e is the electron's mass, and the formula holds for $Q > m_e$. Note the sign of the logarithm: at larger values of Q, which is to say shorter distances, the effective charge increases.

In quantum chromodynamics, gluons can fluctuate into further quark-antiquark pairs, and this vacuum polarization exerts a similar screening effect, tending to increase the effective color charge at short distances. But this tendency is overcome by *antiscreening* effects that arise from the contributions of gluon loops to the vacuum polarization. The gluon loops are present because of the three-gluon and four-gluon vertices that arise from the non-Abelian nature of the $\mathrm{SU(3)_c}$ symmetry. To one-loop approximation, the strong-interaction analogue of the fine structure constant, $\alpha_{\mathrm{s}} \equiv g^2/4\pi$, evolves as

$$\frac{1}{\alpha_{\mathrm{s}}(Q)} = \frac{1}{\alpha_{\mathrm{s}}(\mu)} + \frac{33 - 2n_{\mathrm{f}}}{6\pi}\log\left(\frac{Q}{\mu}\right) , \tag{19}$$

where μ defines the reference, or renormalization, scale.

If the number of quark flavors $n_{\mathrm{f}} \le 16$, as it is in our six-flavor world, then the coefficient of the $\log(Q/\mu)$ term is *positive,* and α_{s} decreases at large values of Q or short distances. This is the celebrated property of *asymptotic freedom,* announced in

10. "Ultraviolet behavior of non-Abelian gauge theories," D. J. Gross and F. Wilczek, *Phys. Rev. Lett.* **30**, 1343–1346 (1973) [doi: `10.1103/PhysRevLett.30.1343`]. (A)

11. "Reliable perturbative results for strong interactions," H. D. Politzer, *Phys. Rev. Lett.* **30**, 1346–1349 (1973) [doi: `10.1103/PhysRevLett.30.1346`]. (A)

Asymptotic freedom points to the existence of a domain in which the strong interactions become sufficiently weak that scattering processes can be treated reliably in perturbation theory using techniques based on the evaluation of Feynman diagrams. The path to asymptotic freedom is described in the Nobel Lectures,

12. "The discovery of asymptotic freedom and the emergence of QCD," D. J. Gross, *Rev. Mod. Phys.* **77**, 837–849 (2005) [doi: `10.1073/pnas.0503831102`]. (I)

13. "The dilemma of attribution," H. D. Politzer, *Rev. Mod. Phys.* **77**, 851–856 (2005) [doi: `10.1073/pnas.0501644102`]. (I)

14. "Asymptotic freedom: from paradox to paradigm," F. Wilczek, *Rev. Mod. Phys.* **77**, 857–870 (2005) [`hep-ph/0502113`]. (I)

For another view of the historical setting, see

15. "When was asymptotic freedom discovered? Or the rehabilitation of quantum field theory," G. 't Hooft, *Nucl. Phys. Proc. Suppl.* **74**, 413 (1999) [`hep-th/9808154`]. (A)

Asymptotically free theories are of special interest because they predict behavior very close to Bjorken scaling in deeply inelastic scattering (see Secs. II D 4 and IV E). No renormalizable field theory without non-Abelian gauge fields can be asymptotically free:

16. "Price of asymptotic freedom," S. Coleman and D. J. Gross, *Phys. Rev. Lett.* **31**, 851–854 (1973) [doi: `10.1103/PhysRevLett.31.851`]. (A)

Asymptotic freedom is thoroughly established in laboratory scattering experiments, as discussed in Sec. IV A.

The complementary behavior of QCD in the long-distance limit, known as *infrared slavery*, points to the confinement of quarks into color-singlet hadrons, as explained in

17. "The confinement of quarks," Y. Nambu, *Sci. Am.* **235**, 48–70 (November, 1976). (E)

This picture leads to the crucial insight that most of the mass of hadrons such as the proton arises not from the masses of their constituents, the quarks, but from the quarks' kinetic energy and the energy stored in the gluon field,

18. "Mass without mass I: most of matter," F. Wilczek, *Phys. Today* **52**, 11–13 (November, 1999) [doi: `10.1063/1.882879`]. (E)

19. "Mass without mass II: the medium is the mass-age," F. Wilczek, *Phys. Today* **53**, 13–14 (January, 2000) [doi: `10.1063/1.882927`]. (E)

20. "The origin of mass," F. Wilczek, *Mod. Phys. Lett.* **A21**, 701–712 (2006) [doi: `10.1142/S0217732306020135`]. (I)

21. "Spontaneous symmetry breaking as a basis of particle mass," C. Quigg, *Rept. Prog. Phys.* **70**, 1019–1054 (2007) [`arXiv:0704.2232 [hep-ph]`]. (I–A)

The development of *lattice gauge theory* has made possible a quantitative understanding of how these phenomena emerge at the low-energy scale associated with confinement.

22. "Confinement of quarks," K. G. Wilson, *Phys. Rev.* **D10**, 2445–2459 (1974) [doi: `10.1103/PhysRevD.10.2445`]. (A)

The essential ideas are described in

23. "The lattice theory of quark confinement," C. Rebbi, *Sci. Am.* **248**, 54–65 (February, 1983). (E)

24. "Quarks by computer," D. H. Weingarten, *Sci. Am.* **274**, 116–120 (February, 1996). (E)

and how it all began is recalled in

25. "The origins of lattice gauge theory," K. G. Wilson, *Nucl. Phys. Proc. Suppl.* **140**, 3–19 (2005) [`hep-lat/0412043`]. (I)

Visualizations of the QCD vacuum, the structure of the proton, and other insights from lattice QCD are presented and explained at

26. "Visualizations of QCD," D. B. Leinweber, `http://www.physics.adelaide.edu.au/~dleinweb/VisualQCD/Nobel/`. (E–I–A)

An example is shown in Fig. 1, depicting the process $p \leftrightarrow \Lambda K^+$ on a background of the gluonic ground state.

Lattice gauge theory is yielding a growing range of nonperturbative computations of hadron properties that are needed to interpret experiments and observations in particle physics, nuclear physics, and astrophysics:

27. "Quantum chromodynamics with advanced computing," A. S. Kronfeld, USQCD Collaboration, *J. Phys. Conf. Ser.* **125**, 012067 (2008) [`arXiv:0807.2220 [physics.comp-ph]`]. (E–I)

By 2008 lattice-QCD calculations of the hadron masses had been carried out with an accuracy of a few per cent, as discussed in Sec. IV B.

For particle physics, an important structural feature of QCD is that it can be married successfully to theories of the weak interaction, even though quarks carry color, but leptons do not.

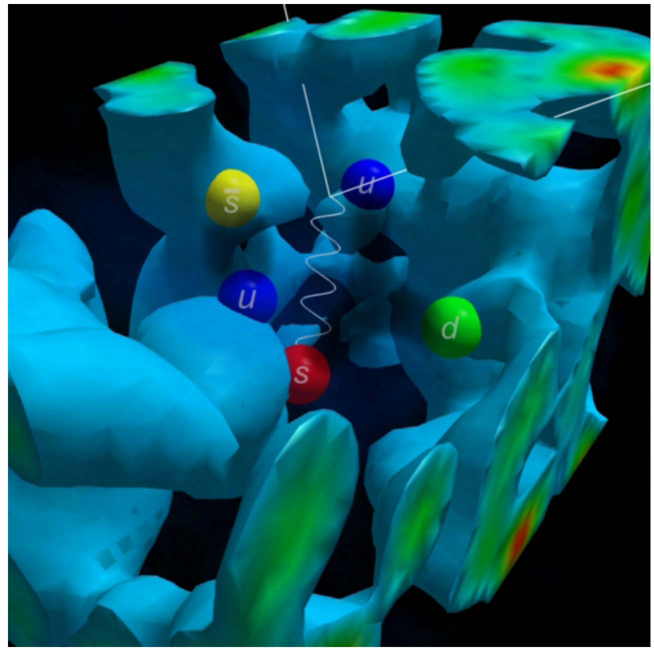

FIG. 1: Artist's depiction of a proton (two u and one d quarks) fluctuating into a Λ baryon (uds) and a K^+ meson ($\bar{s}u$), in the background of vacuum fluctations of gluons. The blob's false color scale denotes the intensity of the gluon field at a given moment, proceeding through the rainbow from blue to red (most intense). From Derek Leinweber, CSSM, University of Adelaide (Ref. 26).

28. "Non-Abelian gauge theories of the strong interactions," S. Weinberg, *Phys. Rev. Lett.* **31**, 494–497 (1973) [doi: `10.1103/PhysRevLett.31.494`]. (I–A)

C. Textbooks

Many books treat quantum chromodynamics, in whole or in part, from a modern point of view. Among books addressed to general readers and undergraduate students, see Ref. 7 and

29. In Search of the Ultimate Building Blocks, G. 't Hooft (Cambridge University Press, Cambridge & New York, 1997). (E)

30. Quarks: Frontiers in Elementary Particle Physics, Y. Nambu (World Scientific, Singapore, 1985). (E)

31. The Quantum Quark, A. Watson (Cambridge University Press, Cambridge, 2004). (E–I)

Several excellent textbooks are addressed to graduate students and researchers:

32. QCD and Collider Physics, R. K. Ellis, W. J. Stirling, and B. R. Webber (Cambridge University Press, Cambridge, 1996). (I–A)

33. Quantum Chromodynamics: High Energy Ex-

periments and Theory, G. Dissertori, I. G. Knowles, and M. Schmelling (Oxford University Press, Oxford & New York, 2003). (I–A)

34. The Theory of Quark and Gluon Interactions, F. J. Ynduráin (Springer, Berlin & New York, 2006), 4th ed. (I–A)

35. Quantum Chromodynamics, W. Greiner, S. Schramm, and E. Stein (Springer, Berlin, 2007), 3rd ed. (I–A)

36. Foundations of Quantum Chromodynamics: an Introduction to Perturbative Methods in Gauge Theories, T. Muta (World Scientific, Singapore & River Edge, NJ, 2009), 3rd ed. (I–A)

37. Quantum Chromodynamics: Perturbative and Nonperturbative Aspects, B. L. Ioffe, V. S. Fadin, and L. N. Lipatov (Cambridge University Press, Cambridge & New York, 2010). (I–A)

Among many fine field-theory textbooks,

38. An Introduction to Quantum Field Theory, G. Sterman (Cambridge University Press, Cambridge & New York, 1993). (I–A)

is particularly inclined toward QCD and the issue of factorization.

The biannual Review of Particle Physics (Ref. 8) contains several concise reviews of QCD and other topics in particle physics. For a well-chosen collection of longer review articles providing an encyclopedic treatment of QCD, see

39. At the Frontier of Particle Physics: Handbook of QCD, edited by M. Shifman, Boris Ioffe Festschrift in four volumes (World Scientific, Singapore, 2001, 2002). (I–A)

D. Antecedent physical theories

The modern gauge theory is the synthesis of several ideas. For completeness we provide some historical and review references here.

1. *Flavor symmetry and current algebra*

The idea of flavor symmetries underlying hadron masses and decay amplitudes predates QCD:

40. "Derivation of strong interactions from a gauge invariance," Y. Ne'eman, *Nucl. Phys.* **26**, 222–229 (1961) [doi: `10.1016/0029-5582(61)90134-1`]. (A)

41. "The eightfold way: a theory of strong interaction symmetry," M. Gell-Mann, *Caltech Synchrotron Report CTSL-20* (1961) [`http://www.osti.gov/accomplishments/pdf/DE04008239/DE04008239.pdf`]. (I)

42. "Symmetries of baryons and mesons," M. Gell-Mann, *Phys. Rev.* **125**, 1067–1084 (1962) [doi: `10.1103/PhysRev.125.1067`]. (I–A)

Early papers are collected in

43. The Eightfold Way, M. Gell-Mann and Y. Ne'eman (W. A. Benjamin, New York, 1964). (I)

Both weak and electromagnetic interactions of the strongly interacting particles (hadrons) are described by currents. The $\mathrm{SU}(3)_{\text{flavor}}$ classification symmetry relates properties of the weak and electromagnetic interactions of hadrons. Gell-Mann proposed that the charges associated with weak-interaction currents could be identified with $\mathrm{SU}(3)_{\text{flavor}}$ symmetry operators:

44. "The symmetry group of vector and axial vector currents," M. Gell-Mann, *Physics* **1**, 63–75 (1964). (I–A)

The current-algebra hypothesis states that the time components of the vector and axial-vector matrix elements satisfy quark-model equal-time commutation relations. Current algebra fixes the strength of the leptonic and hadronic parts of the weak current, and it proved immensely fruitful for interactions involving pseudoscalar mesons. In QCD, an $\mathrm{SU}(3)_{\text{flavor}}$ symmetry appears in the limit that the quark masses can be neglected.

A useful early review of current algebra can be found in

45. "Current algebra," J. D. Bjorken and M. Nauenberg, *Ann. Rev. Nucl. Part. Sci.* **18**, 229–264 (1968) [doi: `10.1146/annurev.ns.18.120168.001305`]. (I–A)

and an early reprint volume with explanatory text is

46. Current Algebras and Applications to Particle Physics, S. L. Adler and R. F. Dashen (W. A. Benjamin, New York, 1968). (I–A)

A later, somewhat more mature assessment is

47. Lectures on Current Algebra and Its Applications, S. B. Treiman, R. W. Jackiw, and D. Gross (Princeton University Press, Princeton, N.J., 1972). (I–A)

2. *The original quark model*

The notion of fractionally-charged quarks was introduced in

48. "A schematic model of baryons and mesons," M. Gell-Mann, *Phys. Lett.* **8**, 214–215 (1964) [doi: `10.1016/S0031-9163(64)92001-3`]. (E–I)

49. "An SU(3) model for strong interaction symmetry and its breaking," G. Zweig CERN-TH-401 (1964) [`http://cdsweb.cern.ch/search.py?recid=352337`]. (E–I)

50. "An SU(3) model for strong interaction symmetry and its breaking 2," G. Zweig CERN-TH-412 (1964)

[http://cdsweb.cern.ch/search.py?recid=570209]. (E–I)

Zweig used the term "aces" for quarks. An early review of the quark model is in

51. "Quarks," M. Gell-Mann, *Acta Phys. Austriaca Suppl.* **9**, 733–761 (1972). (I–A)

and helpful compilations of references on the quark model appear in

52. "Resource letter Q-1: quarks," O. W. Greenberg, *Am. J. Phys.* **50**, 1074–1089 (1982) [doi: 10.1119/1.12922]. (E–I–A)

53. "Hadron spectra and quarks," S. Gasiorowicz and J. L. Rosner, *Am. J. Phys.* **49**, 954–984 (1981) [doi: 10.1119/1.12597]. (I–A)

A challenge to these ideas came from the non-observation of free, fractionally-charged particles. The current limits are collected in Ref. 8, and descriptions of the techniques may be found in

54. "Quark search experiments at accelerators and in cosmic rays," L. Lyons, *Phys. Rept.* **129**, 225 (1985) [doi: 10.1016/0370-1573(85)90011-0]. (I)

55. "Searches for fractional electric charge in terrestrial materials," P. F. Smith, *Ann. Rev. Nucl. Part. Sci.* **39**, 73–111 (1989) [doi: 10.1146/annurev.ns.39.120189.000445]. (I)

56. "Searches for fractionally charged particles," M. L. Perl, E. R. Lee, and D. Loomba, *Ann. Rev. Nucl. Part. Sci.* **59**, 47–65 (2009) [doi: 10.1146/annurev-nucl-121908-122035]. (I)

With confinement in QCD, however, the search for isolatable fractional charges is a somewhat more subtle subject, perhaps explaining why searches for fractionally-charged particles have been to no avail.

3. Quarks with color

A second challenge to the quark model lay in the spin-and-statistics puzzle for the baryons. If the baryon $J = \frac{1}{2}$ octet and $J = \frac{3}{2}$ decuplet are taken to be composites of three quarks, all in relative s-waves, then the wave functions of the decuplet states appear to be symmetric in space×spin×isospin, in conflict with the Pauli exclusion principle. As explicit examples, consider the Ω^-, formed of three (presumably) identical strange quarks, sss, or the Δ^{++}, an isospin-$\frac{3}{2}$ state made of three up quarks, uuu. To reconcile the successes of the quark model with the requirement that fermion wave functions be antisymmetric, it is necessary to hypothesize that each quark flavor comes in three distinguishable species, which we label by the primary colors red, green, and blue. Baryon wave functions may then be antisymmetrized in color. For a review of the role of color in models of hadrons, see

57. "Color models of hadrons," O. W. Greenberg and C. A. Nelson, *Phys. Rept.* **32**, 69–121 (1977) [doi: 10.1016/0370-1573(77)90035-7]. (I–A)

Further observational evidence in favor of the color-triplet quark model is marshaled in

58. "Light-cone current algebra, π^0 decay, and e^+e^- annihilation," W. A. Bardeen, H. Fritzsch, and M. Gell-Mann in **Scale and Conformal Symmetry in Hadron Physics**, edited by R. Gatto (Wiley, New York, 1973), pp. 139–151, [hep-ph/0211388]. (A)

For a critical look at circumstances under which the number of colors can be determined in $\pi^0 \to \gamma\gamma$ decay, see

59. "Can one see the number of colors?," O. Bär and U. J. Wiese, *Nucl. Phys.* **B609**, 225–246 (2001) [hep-ph/0105258]. (A)

The cross section for the reaction $e^+e^- \to$ hadrons (*cf.* Sec. IV C) provides independent evidence that quarks are color triplets. As discussed above, color attains a deeper dynamical meaning in QCD.

4. Partons

Meanwhile, high-energy scattering experiments showed signs of nucleon substructure in the SLAC-MIT experiments on deeply inelastic electron-nucleon scattering. The structure functions that describe the internal structure of the target nucleon as seen by a virtual-photon probe depend in principle on two kinematic variables: the energy $\nu = E - E'$ lost by the scattered electron and the four-momentum transfer, Q^2. At large values of ν and Q^2 the structure functions depend, to good approximation, only on the single dimensionless variable, $x = Q^2/2M\nu$ (where M is the nucleon mass), as anticipated by

60. "Asymptotic sum rules at infinite momentum," J. D. Bjorken, *Phys. Rev.* **179**, 1547–1553 (1969) [doi: 10.1103/PhysRev.179.1547]. (A)

Bjorken scaling implies that the virtual photon scatters off pointlike constituents; otherwise large values of Q^2 would resolve the size of the constituents. An early overview is

61. "The structure of the proton and the neutron," H. W. Kendall and W. K. H. Panofsky, *Sci. Am.* **224**, 60–75 (June, 1971). (E)

The first observations are reported in

62. "High-energy inelastic *e*-*p* scattering at 6° and 10°," E. D. Bloom *et al.*, *Phys. Rev. Lett.* **23**, 930–934 (1969) [doi: 10.1103/PhysRevLett.23.930]. (I–A)

63. "Observed behavior of highly inelastic electron-proton scattering," M. Breidenbach *et al.*, *Phys. Rev. Lett.* **23**, 935–939 (1969) [doi:

`10.1103/PhysRevLett.23.935`]. (I–A)

The experiments and their interpretation in terms of the parton model, which regards the nucleon as a collection of quasifree charged scattering centers, are reviewed in the Nobel Lectures,

64. "Deep inelastic scattering: the early years," R. E. Taylor, *Rev. Mod. Phys.* **63**, 573–595 (1991) [doi: `10.1103/RevModPhys.63.573`]. (I)

65. "Deep inelastic scattering: experiments on the proton and the observation," H. W. Kendall, *Rev. Mod. Phys.* **63**, 597–628 (1991) [doi: `10.1103/RevModPhys.63.597`]. (I)

66. "Deep inelastic scattering: comparisons with the quark model," J. I. Friedman, *Rev. Mod. Phys.* **63**, 615–629 (1991) [doi: `10.1103/RevModPhys.63.615`]. (I)

and in the narrative,

67. The Hunting of the Quark, M. Riordan (Simon & Schuster, New York, 1987). (E–I)

A theoretical framework called the "parton model" based on pointlike constituents of unknown properties was developed in

68. Photon-hadron Interactions, R. P. Feynman, Frontiers in physics. (W. A. Benjamin, Reading, Mass., 1972). (I)

69. "Inelastic electron-proton and γ-proton scattering and the structure of the nucleon," J. D. Bjorken and E. A. Paschos, *Phys. Rev.* **185**, 1975–1982 (1969) [doi: `10.1103/PhysRev.185.1975`]. (I–A)

70. An Introduction to Quarks and Partons, F. E. Close (Academic Press, London & New York, 1979). (I)

Complementary experiments in high-energy neutrino beams soon sealed the identification of the charged partons as quarks, and pointed to the importance of neutral partons later identified as the gluons of QCD.

71. "Measurement of the neutrino-nucleon and antineutrino-nucleon total cross-sections," T. Eichten *et al.*, *Phys. Lett.* **B46**, 274–280 (1973) [doi: `10.1016/0370-2693(73)90702-8`]. (I)

See Sec. IV E for references to works covering the recent experiments.

The parton-model interpretation of high-transverse-momentum scattering in hadron collisions was pioneered by

72. "Inclusive processes at high transverse momentum," S. M. Berman, J. D. Bjorken, and J. B. Kogut, *Phys. Rev.* **D4**, 3388 (1971) [doi: `10.1103/PhysRevD.4.3388`]. (I)

73. "Can we measure parton-parton cross sections?," J. D. Bjorken, *Phys. Rev.* **D8**, 4098–4106 (1973) [doi: `10.1103/PhysRevD.8.4098`]. (I)

74. "Implications of parton model concepts for large transverse momentum production of hadrons," S. D. Ellis and M. B. Kislinger, *Phys. Rev.* **D9**, 2027–2051 (1974) [doi: `10.1103/PhysRevD.9.2027`]. (I)

and implemented in practical terms in

75. "Quark elastic scattering as a source of high transverse momentum mesons," R. D. Field and R. P. Feynman, *Phys. Rev.* **D15**, 2590–2616 (1977) [doi: `10.1103/PhysRevD.15.2590`]. (I)

76. "A parametrization of the properties of quark jets," R. D. Field and R. P. Feynman, *Nucl. Phys.* **B136**, 1–76 (1978) [doi: `10.1016/0550-3213(78)90015-9`]. (I)

77. "Quantum-chromodynamic approach for the large-transverse-momentum production of particles and jets," R. P. Feynman, R. D. Field, and G. C. Fox, *Phys. Rev.* **D18**, 3320 (1978) [doi: `10.1103/PhysRevD.18.3320`]. (I–A)

5. *Gauge invariance and Yang-Mills theory*

The idea that a theory of the strong nuclear interactions could be derived from a non-Abelian symmetry such as isospin dates to the work of

78. "Conservation of isotopic spin and isotopic gauge invariance," C. N. Yang and R. L. Mills, *Phys. Rev.* **96**, 191–195 (1954) [doi: `10.1103/PhysRev.96.191`]. (I–A)

79. "The problem of particle types and other contributions to the theory of elementary particles," R. Shaw, PhD thesis, Cambridge University, 1955. (I–A)

The development of the notions of gauge invariance is detailed in

80. "Historical roots of gauge invariance," J. D. Jackson and L. B. Okun, *Rev. Mod. Phys.* **73**, 663–680 (2001) [`hep-ph/0012061`]. (E–I)

and many useful readings are compiled in

81. "Resource letter GI-1: gauge invariance," T. P. Cheng and L.-F. Li, *Am. J. Phys.* **56**, 586–600, 1048(E) (1988) [doi: `10.1119/1.15522`]. (E–I–A)

The concepts and consequences of local gauge invariance are recalled in

82. "Gauge fields," R. Mills, *Am. J. Phys.* **57**, 493–507 (1989) [doi: `10.1119/1.15984`]. (I)

and the history of gauge theories is explored in

83. The Dawning of Gauge Theory, L. O'Raifeartaigh, Princeton Series in Physics (Princeton University Press, Princeton, N.J., 1997). (E–I)

84. "Gauge theory: historical origins and some modern developments," L. O'Raifeartaigh and N. Straumann, *Rev. Mod. Phys.* **72**, 1–23 (2000) [doi: `10.1103/RevModPhys.72.1`]. (I–A)

In particular, Shaw's thesis is reprinted and discussed in Chapter 9 of Ref. 83. For an assessment of a half-century's development of gauge symmetry, see

85. Fifty Years of Yang-Mills Theory, edited by G. 't Hooft (World Scientific, Singapore & Hackensack, N.J., 2005). (A)

For early attempts to build a realistic gauge theory of the strong interactions, see Ref. 40 and

86. "Theory of strong interactions," J. J. Sakurai, *Ann. Phys.* **11**, 1–48 (1960) [doi: 10.1016/0003-4916(60)90126-3]. (A)

The idea of a vector gluon theory may be found in

87. "A systematics of hadrons in subnuclear physics," Y. Nambu in **Preludes in Theoretical Physics in Honor of V. F. Weisskopf**, edited by A. De Shalit, H. Feshbach, and L. Van Hove (North-Holland, Amsterdam, 1966), pp. 133–142. (A)

and the path from currents to a gauge theory of the strong interactions is laid out in

88. "Current algebra: quarks and what else?," H. Fritzsch and M. Gell-Mann in **Proceedings of the XVI International Conference on High Energy Physics**, vol. 2, edited by J. D. Jackson and A. Roberts (National Accelerator Laboratory, Batavia, Illinois, 1972), pp. 135–165, [hep-ph/0208010]. (I–A)

III. THEORETICAL TOOLS

Like many a realistic physical theory, Yang-Mills theories defy exact solution. To gain a theoretical understanding and, hence, to see whether QCD mirrors nature, several lines of attack are necessary. We describe some of the literature behind several theoretical tools, ordered by increasing complexity. Readers concerned mainly with the physical consequences of QCD may wish to pass first to Sec. IV.

A. Symmetries

1. Light quarks

The spontaneous breaking of chiral symmetries was studied before the advent of QCD, to explain why the mass of the pions is so much smaller than that of the nucleons.

89. "Axial vector current conservation in weak interactions," Y. Nambu, *Phys. Rev. Lett.* **4**, 380–382 (1960) [doi: 10.1103/PhysRevLett.4.380]. (I–A)

A prominent feature of spontaneously broken symmetries in quantum field theories is the appearance of a massless particle:

90. "Field theories with superconductor solutions," J. Goldstone, *Nuovo Cim.* **19**, 154–164 (1961) [doi: 10.1007/BF02812722]. (A)

91. "Broken symmetries," J. Goldstone, A. Salam, and S. Weinberg, *Phys. Rev.* **127**, 965–970 (1962) [doi: 10.1103/PhysRev.127.965]. (A)

The massless states are called Nambu-Goldstone particles. When a small amount of explicit symmetry breaking arises, as with pions, these states acquire a small mass and are called pseudo-Nambu–Goldstone particles.

An informative toy model in which the nucleon mass arises essentially as a self-energy in analogy with the appearance of the mass gap in superconductivity was presented in

92. "Dynamical model of elementary particles based on an analogy with superconductivity I," Y. Nambu and G. Jona-Lasinio, *Phys. Rev.* **122**, 345–358 (1961) [doi: 10.1103/PhysRev.122.345]. (A)

93. "Dynamical model of elementary particles based on an analogy with superconductivity. II," Y. Nambu and G. Jona-Lasinio, *Phys. Rev.* **124**, 246–254 (1961) [doi: 10.1103/PhysRev.124.246]. (A)

This construction, three years before the invention of quarks, prefigured our current understanding of the masses of strongly interacting particles in quantum chromodynamics. The pions arose as light nucleon-antinucleon bound states, following the introduction of a tiny "bare" nucleon mass and spontaneous chiral-symmetry breaking.

Spontaneous symmetry breaking is common in physics, and parallels to condensed-matter physics are drawn in

94. "Spontaneous symmetry breaking in particle physics: a case of cross fertilization," Y. Nambu, *Rev. Mod. Phys.* **81**, 1015–1018 (2009) [doi: 10.1103/RevModPhys.81.1015]. (E)

Meanwhile, QCD explains the origin of chiral symmetry via the smallness of the up-, down-, and strange-quark masses—recall the discussion following Eq. (17). The spontaneous breaking is driven by the formation of a condensate of the light quarks, measured by the vacuum expectation value $\langle 0|\bar{q}q|0\rangle$. For a careful calculation see

95. "Determination of the chiral condensate from 2+1-flavor lattice QCD," H. Fukaya *et al.*, JLQCD Collaboration, *Phys. Rev. Lett.* **104**, 122002 (2010) [arXiv:0911.5555 [hep-lat]]. (A)

yielding (in the $\overline{\mathrm{MS}}$ scheme at 2 GeV)

$$\langle 0|\bar{q}q|0\rangle = \left[242 \pm 4^{+19}_{-18}\ \mathrm{MeV}\right]^3, \tag{20}$$

where the first error stems from Monte Carlo statistics and the second encompasses systematic effects, such as extrapolation to vanishingly small up- and down-quark masses.

2. Anomalous chiral symmetries

Among the chiral symmetries of light quarks, the flavor-singlet symmetry is special, because a quantum-mechanical effect, called the anomaly, breaks the classical conservation law. This effect implies that the η', unlike the pions and kaons, should not be a pseudo-Nambu–Goldstone particle with small mass.

96. "A global fit to determine the pseudoscalar mixing angle and the gluonium content of the η' meson," F. Ambrosino *et al.*, *JHEP* **07**, 105 (2009) [`arXiv:0906.3819 [hep-ph]`]. (I)

The details of how this arises are connected to the non-trivial vacuum structure of QCD (discussed in Sec. III G):

97. "How instantons solve the U(1) problem," G. 't Hooft, *Phys. Rept.* **142**, 357–387 (1986) [doi: `10.1016/0370-1573(86)90117-1`]. (I–A)

98. "Chiral dynamics in the large N limit," P. Di Vecchia and G. Veneziano, *Nucl. Phys.* **B171**, 253 (1980) [doi: `10.1016/0550-3213(80)90370-3`]. (I–A)

99. "Anomalies and low-energy theorems of quantum chromodynamics," M. A. Shifman, *Phys. Rept.* **209**, 341–378 (1991) [doi: `10.1016/0370-1573(91)90020-M`]. (I–A)

The phase of the quark mass matrix combines with the coefficient of $\varepsilon_{\mu\nu\rho\sigma}\,\mathrm{tr}(G^{\mu\nu}G^{\rho\sigma})$ in the Lagrangian to cause effects that violate CP symmetry. Curiously, this combination—the difference of two quantities with starkly distinct origins—is constrained by the neutron electric dipole moment to be $\sim 10^{-11}$. The *strong CP problem* was clearly posed, and a still-popular resolution proposed in

100. "CP conservation in the presence of pseudoparticles," R. D. Peccei and H. R. Quinn, *Phys. Rev. Lett.* **38**, 1440 (1977) [doi: `10.1103/PhysRevLett.38.1440`]. (I–A)

Further possible resolutions are explained in

101. "The strong CP problem," M. Dine in **Flavor Physics for the Millenium**, edited by J. L. Rosner (World Scientific, Singapore, 2000), [`hep-ph/0011376`]. (E–I–A)

The Peccei-Quinn solution requires a new particle, the axion, with several implications for particle physics and, possibly, cosmology. These connections, and the status of axion searches, are reviewed in

102. "Axions and the strong CP problem," J. E. Kim and G. Carosi, *Rev. Mod. Phys.* **82**, 557–601 (2010) [`arXiv:0807.3125 [hep-ph]`]. (I–A)

3. Heavy quarks

Hadrons containing heavy quarks exhibit simplifying features. In a bound state with one heavy quark, and any number of light quarks and gluons, the identity (flavor or spin) of the heavy quark alters the dynamics very little, because the heavy quark sits essentially at rest inside the hadron:

103. "Hadrons containing a heavy quark and QCD sum rules," E. V. Shuryak, *Nucl. Phys.* **B198**, 83 (1982) [doi: `10.1016/0550-3213(82)90546-6`]. (I–A)

104. "On annihilation of mesons built from heavy and light quark and $\bar{B}^0$-B^0 oscillations," M. A. Shifman and M. B. Voloshin, *Sov. J. Nucl. Phys.* **45**, 292 (1987). (I–A)

The center of mass of the hadron and the heavy quark are essentially the same, with the light degrees of freedom in orbit around the heavy quark. A set of approximate symmetries emerge, the heavy-quark flavor and spin symmetries.

105. "Weak decays of heavy mesons in the static quark approximation," N. Isgur and M. B. Wise, *Phys. Lett.* **B232**, 113 (1989) [doi: `10.1016/0370-2693(89)90566-2`]. (I–A)

106. "Weak transition form-factors between heavy mesons," N. Isgur and M. B. Wise, *Phys. Lett.* **B237**, 527 (1990) [doi: `10.1016/0370-2693(90)91219-2`]. (I–A)

In a meson with a heavy quark and corresponding antiquark, the two orbit each other. The velocity depends on the heavy-quark mass, but the spin decouples (to leading order), in analogy with QED applied to atomic physics.

107. "Effective Lagrangians for bound state problems in QED, QCD, and other field theories," W. E. Caswell and G. P. Lepage, *Phys. Lett.* **B167**, 437 (1986) [doi: `10.1016/0370-2693(86)91297-9`]. (I–A)

B. Potential models

The observation that asymptotic freedom suggests nonrelativistic atoms of heavy quarks and antiquarks is due to

108. "Heavy quarks and e^+e^- annihilation," T. Appelquist and H. D. Politzer, *Phys. Rev. Lett.* **34**, 43–45 (1975) [doi: `10.1103/PhysRevLett.34.43`]. (A)

The nonrelativistic description was elaborated in

109. "Charmonium: the model," E. Eichten, K. Gottfried, T. Kinoshita, K. D. Lane, and T.-M. Yan, *Phys. Rev.* **D17**, 3090 (1978) [doi: `10.1103/PhysRevD.17.3090`]. (I–A)

110. "Charmonium: comparison with experiment," E. Eichten, K. Gottfried, T. Kinoshita, K. D. Lane, and T.-M. Yan, *Phys. Rev.* **D21**, 203 (1980) [doi: `10.1103/PhysRevD.21.203`]. (I–A)

A midterm review of potential models can be found in

111. "Heavy quark systems," W. Kwong, J. L. Rosner, and C. Quigg, *Ann. Rev. Nucl. Part. Sci.* **37**, 325–382 (1987). (I–A)

and newer reviews include

112. "Quarkonia and their transitions," E. Eichten, S. Godfrey, H. Mahlke, and J. L. Rosner, *Rev. Mod. Phys.*

80, 1161–1193 (2008) [`hep-ph/0701208`]. (I–A)

More recently this line of research has been addressed further through effective field theories (see Sec. III E 2).

C. Renormalization and factorization

The renormalization group as a technique for summing to all orders in perturbation theory in electrodynamics was invented by

113. "Normalization of constants in the quanta theory," E. C. G. Stückelberg and A. Petermann, *Helv. Phys. Acta* **26**, 499–520 (1953). (A)

114. "Quantum electrodynamics at small distances," M. Gell-Mann and F. E. Low, *Phys. Rev.* **95**, 1300–1312 (1954) [doi: `10.1103/PhysRev.95.1300`]. (A)

A clear statement of the algorithm and a thorough review of early applications appears in

115. Introduction to the Theory of Quantized Fields, N. N. Bogolyubov and D. V. Shirkov (Wiley-Interscience, New York, 1959). (A)

The modern formulation of the renormalization group equations is due to

116. "Broken scale invariance in scalar field theory," C. G. Callan, Jr., *Phys. Rev.* **D2**, 1541–1547 (1970) [doi: `10.1103/PhysRevD.2.1541`]. (A)

117. "Small distance behavior in field theory and power counting," K. Symanzik, *Commun. Math. Phys.* **18**, 227–246 (1970) [doi: `10.1007/BF01649434`] [`http://projecteuclid.org/getRecord?id=euclid.cmp/1103842537`]. (A)

The power of renormalization group methods for a wide range of physical problems was recognized by

118. "The renormalization group and strong interactions," K. G. Wilson, *Phys. Rev.* **D3**, 1818 (1971) [doi: `10.1103/PhysRevD.3.1818`]. (A)

119. "Problems in physics with many scales of length," K. G. Wilson, *Sci. Am.* **241**, 140–157 (August, 1979). (E)

A fascinating survey with many references is

120. "The renormalization group and critical phenomena," K. G. Wilson, *Rev. Mod. Phys.* **55**, 583–600 (1983) [doi: `10.1103/RevModPhys.55.583`]. (I–A)

The theoretical apparatus required for a general analysis of quantum corrections and their implications for a running coupling constant is presented in

121. "Dilatations," S. R. Coleman in **Aspects of Symmetry** (Cambridge University Press, Cambridge, 1985), pp. 67–98. (I–A)

122. Methods in Field Theory [Méthodes en Théorie des Champs], edited by R. Balian and J. Zinn-Justin (North-Holland, Amsterdam & New York, 1981). (A)

123. "Renormalization group and the deep structure of the proton," A. Peterman, *Phys. Rept.* **53**, 157 (1979) [doi: `10.1016/0370-1573(79)90014-0`]. (A)

124. Renormalization: an Introduction to Renormalization, the Renormalization Group, and the Operator-product Expansion, J. C. Collins, Cambridge monographs on mathematical physics (Cambridge University Press, Cambridge & New York, 1984). (A)

The ability to predict characteristics of high-energy reactions rests on parton-hadron duality and on separating short-distance hard-scattering matrix elements described by perturbative QCD from long-distance (nonperturbative) effects related to hadronic structure. *Duality* refers to the observation that inclusive hadronic observables may be computed in terms of quark and gluon degrees of freedom. These ideas are reviewed and confronted with recent experimental data in

125. "Quark-hadron duality in electron scattering," W. Melnitchouk, R. Ent, and C. Keppel, *Phys. Rept.* **406**, 127–301 (2005) [`hep-ph/0501217`]. (I–A)

updating the classic reference:

126. "Similarity of parton and hadron spectra in QCD jets," Y. I. Azimov, Y. L. Dokshitzer, V. A. Khoze, and S. I. Troian, *Z. Phys.* **C27**, 65–72 (1985) [doi: `10.1007/BF01642482`]. (A)

The distinction between short-distance and long-distance (or short and long time scales) is reminiscent of the Born-Oppenheimer approximation in molecular physics. The *factorization* of amplitudes and cross sections into parton distribution functions, elementary scattering amplitudes, and fragmentation functions that describe how partons materialize into hadrons, was an element of the exploratory studies reported in Ref. 75. Within the framework of QCD, factorization has been proved in many settings:

127. "The theorems of perturbative QCD," J. C. Collins and D. E. Soper, *Ann. Rev. Nucl. Part. Sci.* **37**, 383–409 (1987) [doi: `10.1146/annurev.ns.37.120187.002123`]. (I–A)

128. "Factorization of hard processes in QCD," J. C. Collins, D. E. Soper, and G. Sterman in **Perturbative Quantum Chromodynamics**, edited by A. H. Mueller (World Scientific, Singapore, 1989), pp. 1–91, [`hep-ph/0409313`]. (I–A)

129. "Perturbation theory and the parton model in QCD," R. K. Ellis, H. Georgi, M. Machacek, H. D. Politzer, and G. G. Ross, *Nucl. Phys.* **B152**, 285 (1979) [doi: `10.1016/0550-3213(79)90105-6`]. (I–A)

The short-distance behavior of quantum field theories, including QCD, is clarified by the operator-product expansion:

130. "Operator product expansions and composite field operators in the general framework of quantum field theory," K. G. Wilson and W. Zimmermann, *Commun. Math. Phys.* **24**, 87–106 (1972) [doi: `10.1007/BF01878448`] [`http://projecteuclid.org/getRecord?id=euclid.cmp/1103857739`]. (A)

in which a product of operators is related to a series of local operators.

Factorization of amplitudes hinges on an understanding of universal behaviors of field theory when massless particles become soft or if two massless particles become collinear. These features already appear in QED for the scattering of a high-energy photon off an electron:

131. "Vertex parts at very high energies in quantum electrodynamics," V. V. Sudakov, *Sov. Phys. JETP* **3**, 65 (1956). (I–A)

and were generalized to quarks and QCD in

132. "Asymptotic behavior of the Sudakov form-factor in QCD," A. Sen, *Phys. Rev.* **D24**, 3281 (1981) [doi: `10.1103/PhysRevD.24.3281`]. (I–A)

133. "Analytic continuation of the Sudakov form-factor in QCD," L. Magnea and G. Sterman, *Phys. Rev.* **D42**, 4222–4227 (1990) [doi: `10.1103/PhysRevD.42.4222`]. (A)

For processes with four or more external particles, as in collisions, these aspects were further developed in

134. "The singular behaviour of QCD amplitudes at two-loop order," S. Catani, *Phys. Lett.* **B427**, 161–171 (1998) [`hep-ph/9802439`]. (A)

135. "Evolution of color exchange in QCD hard scattering," N. Kidonakis, G. Oderda, and G. Sterman, *Nucl. Phys.* **B531**, 365–402 (1998) [`hep-ph/9803241`]. (A)

136. "Multi-loop amplitudes and resummation," G. Sterman and M. E. Tejeda-Yeomans, *Phys. Lett.* **B552**, 48–56 (2003) [`hep-ph/0210130`]. (A)

Understanding the singularity structure of massless gauge-theory amplitudes continues to be germane, with application to high-energy collider physics:

137. "The two-loop anomalous dimension matrix for soft gluon exchange," S. M. Aybat, L. J. Dixon, and G. Sterman, *Phys. Rev. Lett.* **97**, 072001 (2006) [`hep-ph/0606254`]. (A)

138. "Factorization constraints for soft anomalous dimensions in QCD scattering amplitudes," E. Gardi and L. Magnea, *JHEP* **03**, 079 (2009) [`arXiv:0901.1091 [hep-ph]`]. (A)

139. "On soft singularities at three loops and beyond," L. J. Dixon, E. Gardi, and L. Magnea, *JHEP* **02**, 081 (2010) [`arXiv:0910.3653 [hep-ph]`]. (A)

For similar results derived using effective-field-theory techniques, see Sec. III E 3.

Techniques of factorization have been extended to exclusive processes in

140. "Exclusive processes in perturbative quantum chromodynamics," G. P. Lepage and S. J. Brodsky, *Phys. Rev.* **D22**, 2157 (1980) [doi: `10.1103/PhysRevD.22.2157`]. (I–A)

and adapted to decays of hadrons containing a heavy quark in

141. "QCD factorization for exclusive, non-leptonic B meson decays: general arguments and the case of heavy-light final states," M. Beneke, G. Buchalla, M. Neubert, and C. T. Sachrajda, *Nucl. Phys.* **B591**, 313–418 (2000) [`hep-ph/0006124`]. (I–A)

The study of higher orders in perturbation theory, particularly the renormalization parts, can anticipate the pattern of nonperturbative effects. A standard review is

142. "Renormalons," M. Beneke, *Phys. Rept.* **317**, 1–142 (1999) [`hep-ph/9807443`]. (A)

An intriguing feature of these effects makes the definition of quark masses somewhat subtle. The so-called "pole mass," which corresponds closely to the classical notion of mass, is well-defined in perturbation theory:

143. "The perturbative pole mass in QCD," A. S. Kronfeld, *Phys. Rev.* **D58**, 051501 (1998) [`hep-ph/9805215`]. (A)

yet the perturbative series signals the necessity of nonperturbative effects:

144. "Heavy quark effective theory beyond perturbation theory: renormalons, the pole mass and the residual mass term," M. Beneke and V. M. Braun, *Nucl. Phys.* **B426**, 301–343 (1994) [`hep-ph/9402364`]. (A)

145. "The pole mass of the heavy quark: perturbation theory and beyond," I. I. Bigi, M. A. Shifman, N. G. Uraltsev, and A. I. Vainshtein, *Phys. Rev.* **D50**, 2234–2246 (1994) [`hep-ph/9402360`]. (A)

As a consequence, quark masses reported below are renormalized Lagrangian masses.

D. Unitarity and analyticity

Underlying the notion of parton-hadron duality, which enters into many applications of factorization, are unitarity and analyticity. Unitarity means merely that quantum mechanics (and, hence, quantum field theory) preserves probability, thereby imposing limits on scattering amplitudes and related quantities. Analyticity means that scattering amplitudes are analytic functions of kinematic variables, apart from poles or branch cuts, which correspond to stable particles and resonances or multiparticle thresholds, respectively.

These ideas and the formalism of quantum field theory can be used to derive semi-quantitative and, sometimes, quantitative dynamical information. This approach goes under the name "QCD sum rules" and started with

146. "QCD and resonance physics: sum rules," M. A. Shifman, A. I. Vainshtein, and V. I. Zakharov, *Nucl. Phys.* **B147**, 385–447 (1979) [doi: 10.1016/0550-3213(79)90022-1]. (I–A)

147. "QCD and resonance physics: applications," M. A. Shifman, A. I. Vainshtein, and V. I. Zakharov, *Nucl. Phys.* **B147**, 448–518 (1979) [doi: 10.1016/0550-3213(79)90023-3]. (I–A)

An early, well-regarded review is

148. "Hadron properties from QCD sum rules," L. J. Reinders, H. Rubinstein, and S. Yazaki, *Phys. Rept.* **127**, 1–97 (1985) [doi: 10.1016/0370-1573(85)90065-1]. (I–A)

A review and reprint volume is

149. Vacuum Structure and QCD Sum Rules, edited by M. Shifman (North Holland, Amsterdam, 1992). (E–I–A)

A more recent monograph explaining QCD sum rules is

150. QCD as a Theory of Hadrons (from Partons to Confinement), S. Narison (Cambridge University Press, Cambridge, 2002). (E–I–A)

and several pedagogical reviews of applications can be found in Ref. 39.

E. Effective field theories

Effective field theories isolate important low-energy degrees of freedom, absorbing the effects of highly virtual processes, such as those of high-mass particles, into coupling strengths of interactions. For capsule reviews, see

151. "Effective field theory," H. Georgi, *Ann. Rev. Nucl. Part. Sci.* **43**, 209–252 (1993). (I–A)

152. "Effective field theories," G. Ecker in **Encyclopedia of Mathematical Physics**, edited by J.-P. Françoise, G. L. Naber, and Tsou Sheung Tsun (Elsevier, Amsterdam, 2006). [hep-ph/0507056]. (I–A)

153. "Introduction to effective field theory," C. P. Burgess, *Ann. Rev. Nucl. Part. Sci.* **57**, 329–362 (2007) [hep-th/0701053]. (I–A)

Two classes of effective field theories are employed to study QCD, one in which (light) quarks and gluons remain the basic degrees of freedom, and another treating hadrons as fundamental. In both cases, the power of the method is to retain and respect symmetry, renormalization, unitarity, analyticity, and cluster decomposition.

1. Chiral perturbation theory

The consequences of spontaneously broken symmetries are encoded in current algebra (see Sec. II D 1) and can be summarized in an effective Lagrangian for pions:

154. "Dynamical approach to current algebra," S. Weinberg, *Phys. Rev. Lett.* **18**, 188–191 (1967) [doi: 10.1103/PhysRevLett.18.188]. (I–A)

155. "Nonlinear realizations of chiral symmetry," S. Weinberg, *Phys. Rev.* **166**, 1568–1577 (1968) [doi: 10.1103/PhysRev.166.1568]. (I–A)

The formalism was extended to general patterns of spontaneous symmetry breaking in

156. "Structure of phenomenological Lagrangians. 1," S. R. Coleman, J. Wess, and B. Zumino, *Phys. Rev.* **177**, 2239–2247 (1969) [doi: 10.1103/PhysRev.177.2239]. (A)

157. "Structure of phenomenological Lagrangians. 2," C. G. Callan, Jr., S. R. Coleman, J. Wess, and B. Zumino, *Phys. Rev.* **177**, 2247–2250 (1969) [doi: 10.1103/PhysRev.177.2247]. (A)

For hadron dynamics chiral Lagrangians were developed further in

158. "Chiral $\mathrm{SU}(3)\times\mathrm{SU}(3)$ as a symmetry of the strong interactions," R. F. Dashen, *Phys. Rev.* **183**, 1245–1260 (1969) [doi: 10.1103/PhysRev.183.1245]. (I–A)

159. "Soft pions, chiral symmetry, and phenomenological Lagrangians," R. F. Dashen and M. Weinstein, *Phys. Rev.* **183**, 1261–1291 (1969) [doi: 10.1103/PhysRev.183.1261]. (I–A)

160. "Perturbation theory about a Goldstone symmetry," L.-F. Li and H. Pagels, *Phys. Rev. Lett.* **26**, 1204–1206 (1971) [doi: 10.1103/PhysRevLett.26.1204]. (I–A)

An early review is

161. "Departures from chiral symmetry: a review," H. Pagels, *Phys. Rept.* **16**, 219 (1975) [doi: 10.1016/0370-1573(75)90039-3]. (E–I–A)

The connection with the quark model is developed in

162. "Chiral quarks and the nonrelativistic quark model," A. Manohar and H. Georgi, *Nucl. Phys.* **B234**, 189 (1984) [doi: 10.1016/0550-3213(84)90231-1]. (A)

Chiral Lagrangians were then exploited to develop a systematic low-energy expansion, called chiral perturbation theory (χPT):

163. "Phenomenological Lagrangians," S. Weinberg, *Physica* **A96**, 327 (1979) [doi: 10.1016/0378-4371(79)90223-1]. (I–A)

164. "Chiral perturbation theory to one loop," J. Gasser and H. Leutwyler, *Ann. Phys.* **158**, 142 (1984) [doi:

10.1016/0003-4916(84)90242-2]. (I–A)

An excellent place to start learning the modern perspective is

165. "On the foundations of chiral perturbation theory," H. Leutwyler, *Ann. Phys.* **235**, 165–203 (1994) [hep-ph/9311274]. (E–I–A)

This is now a subject with broad applications, describing, for example, the pion and kaon clouds surrounding a nucleon. This material is pedagogically reviewed in

166. "Introduction to chiral perturbation theory," S. Scherer, *Adv. Nucl. Phys.* **27**, 277 (2003) [hep-ph/0210398]. (I–A)

167. "Chiral perturbation theory: introduction and recent results in the one-nucleon sector," S. Scherer, *Prog. Part. Nucl. Phys.* **64**, 1–60 (2010) [arXiv:0908.3425 [hep-ph]]. (I–A)

States with nucleonic properties can also arise from soliton configurations of the pion field, which was first noticed before the advent of QCD:

168. "A unified model of K and π mesons," T. H. R. Skyrme, *Proc. Roy. Soc. Lond.* **A252**, 236–245 (1959) [doi: 10.1098/rspa.1959.0149]. (I–A)

The so-called Skyrmion approach to the nucleon enjoyed a renaissance in the 1980s, reviewed in

169. "The Skyrme model," I. Zahed and G. E. Brown, *Phys. Rept.* **142**, 1–102 (1986) [doi: 10.1016/0370-1573(86)90142-0]. (A)

2. *Heavy-quark effective theory and nonrelativistic QCD*

The simpler dynamics of heavy-quark systems lend themselves to effective field theories. For heavy-light hadrons (those with one heavy quark), this insight led to the development of the heavy-quark effective theory (HQET) in

170. "Heavy quarks on the lattice," E. Eichten, *Nucl. Phys. Proc. Suppl.* **4**, 170 (1988) [doi: 10.1016/0920-5632(88)90097-7]. (A)

171. "An effective field theory for the calculation of matrix elements involving heavy quarks," E. Eichten and B. Hill, *Phys. Lett.* **B234**, 511 (1990) [doi: 10.1016/0370-2693(90)92049-O]. (A)

172. "Static effective field theory: $1/m$ corrections," E. Eichten and B. Hill, *Phys. Lett.* **B243**, 427–431 (1990) [doi: 10.1016/0370-2693(90)91408-4]. (A)

173. "An effective field theory for heavy quarks at low-energies," H. Georgi, *Phys. Lett.* **B240**, 447 (1990) [doi: 10.1016/0370-2693(90)91128-X]. (I–A)

174. "The static quark effective theory," B. Grinstein, *Nucl. Phys.* **B339**, 253–268 (1990) [doi: 10.1016/0550-3213(90)90349-I]. (A)

Some pedagogical reviews are

175. "Light quark, heavy quark systems," B. Grinstein, *Ann. Rev. Nucl. Part. Sci.* **42**, 101–145 (1992) [hep-ph/9310362]. (I–A)

176. "Heavy quark symmetry," M. Neubert, *Phys. Rept.* **245**, 259–396 (1994) [hep-ph/9306320]. (I–A)

177. "Aspects of heavy quark theory," I. Bigi, M. Shifman, and N. Uraltsev, *Ann. Rev. Nucl. Part. Sci.* **47**, 591–661 (1997) [hep-ph/9703290]. (I–A)

and a textbook is

178. Heavy Quark Physics, A. V. Manohar and M. B. Wise (Cambridge University Press, Cambridge & New York, 2000). (I–A)

In quarkonium, a heavy quark's velocity is larger than in a heavy-light hadron. The appropriate effective field theory has the same Lagrangian as HQET, but the relative importance of various interactions is different. This field theory is called nonrelativistic QCD (NRQCD) and was first developed for bound-state problems in Ref. 107 and

179. "Effective Lagrangians for simulating heavy quark systems," G. P. Lepage and B. A. Thacker, *Nucl. Phys. Proc. Suppl.* **4**, 199 (1988) [doi: 10.1016/0920-5632(88)90102-8]. (I–A)

180. "Heavy quark bound states in lattice QCD," B. A. Thacker and G. P. Lepage, *Phys. Rev.* **D43**, 196–208 (1991) [doi: 10.1103/PhysRevD.43.196]. (I–A)

The classification of NRQCD interactions, focusing on the quarkonium spectrum, was further elucidated in

181. "Improved nonrelativistic QCD for heavy quark physics," G. P. Lepage, L. Magnea, C. Nakhleh, U. Magnea, and K. Hornbostel, *Phys. Rev.* **D46**, 4052–4067 (1992) [hep-lat/9205007]. (I–A)

NRQCD was extended to encompass decay, production, and annihilation in

182. "Rigorous QCD predictions for decays of P-wave quarkonia," G. T. Bodwin, E. Braaten, and G. P. Lepage, *Phys. Rev.* **D46**, 1914–1918 (1992) [hep-lat/9205006]. (A)

183. "Rigorous QCD analysis of inclusive annihilation and production of heavy quarkonium," G. T. Bodwin, E. Braaten, and G. P. Lepage, *Phys. Rev.* **D51**, 1125–1171 (1995) [hep-ph/9407339]. (A)

In some applications, the QCD coupling α_s is small at both the heavy-quark mass and heavy-quark momentum scales:

184. "Two-loop correction to the leptonic decay of quarkonium," M. Beneke, A. Signer, and V. A. Smirnov, *Phys. Rev. Lett.* **80**, 2535–2538 (1998) [hep-ph/9712302].

(A)

Then the appropriate effective field theory is potential NRQCD (PNRQCD):

185. "Potential NRQCD: an effective theory for heavy quarkonium," N. Brambilla, A. Pineda, J. Soto, and A. Vairo, *Nucl. Phys.* **B566**, 275 (2000) [`hep-ph/9907240`]. (A)

PNRQCD provides a field-theoretic basis for understanding the success of the potential models of Sec. III B. For a review, consult

186. "Effective field theories for heavy quarkonium," N. Brambilla, A. Pineda, J. Soto, and A. Vairo, *Rev. Mod. Phys.* **77**, 1423 (2005) [`hep-ph/0410047`]. (I–A)

NRQCD and PNRQCD have also been used to understand top-quark pair production at threshold. Top quarks decay before toponium forms:

187. "Production and decay properties of ultraheavy quarks," I. I. Y. Bigi, Y. L. Dokshitzer, V. A. Khoze, J. H. Kühn, and P. M. Zerwas, *Phys. Lett.* **B181**, 157 (1986) [doi: `10.1016/0370-2693(86)91275-X`]. (I–A)

188. "Threshold behavior of heavy top production in e^+e^- collisions," V. S. Fadin and V. A. Khoze, *JETP Lett.* **46**, 525–529 (1987). (I–A)

189. "Production of a pair of heavy quarks in e^+e^- annihilation in the threshold region," V. S. Fadin and V. A. Khoze, *Sov. J. Nucl. Phys.* **48**, 309–313 (1988). (I–A)

but top and antitop still orbit each other during their fleeting existence. A useful review is

190. "Top-antitop pair production close to threshold: synopsis of recent NNLO results," A. H. Hoang *et al.*, *Eur. Phys. J. direct* **C2**, 1 (2000) [`hep-ph/0001286`]. (I–A)

3. *Soft collinear effective theory*

In high-energy amplitudes, one often considers a jet of particles, the details of which are not detected. The semi-inclusive nature of jets circumvents issues of infrared and collinear divergences, much like the Bloch-Nordsieck mechanism in QED:

191. "Note on the radiation field of the electron," F. Bloch and A. Nordsieck, *Phys. Rev.* **52**, 54–59 (1937) [doi: `10.1103/PhysRev.52.54`]. (I–A)

192. "Mass singularities of Feynman amplitudes," T. Kinoshita, *J. Math. Phys.* **3**, 650–677 (1962) [doi: `10.1063/1.1724268`]. (I–A)

193. "Degenerate systems and mass singularities," T. D. Lee and M. Nauenberg, *Phys. Rev.* **133**, B1549–B1562 (1964) [doi: `10.1103/PhysRev.133.B1549`]. (I–A)

The infrared and collinear degrees of freedom can be isolated in the soft collinear effective theory (SCET), first established for decays of B mesons:

194. "Summing Sudakov logarithms in $B \to X_s\gamma$ in effective field theory," C. W. Bauer, S. Fleming, and M. E. Luke, *Phys. Rev.* **D63**, 014006 (2000) [`hep-ph/0005275`]. (A)

195. "An effective field theory for collinear and soft gluons: heavy to light decays," C. W. Bauer, S. Fleming, D. Pirjol, and I. W. Stewart, *Phys. Rev.* **D63**, 114020 (2001) [`hep-ph/0011336`]. (A)

196. "Soft-collinear factorization in effective field theory," C. W. Bauer, D. Pirjol, and I. W. Stewart, *Phys. Rev.* **D65**, 054022 (2002) [`hep-ph/0109045`]. (A)

197. "Soft-collinear effective theory and heavy-to-light currents beyond leading power," M. Beneke, A. P. Chapovsky, M. Diehl, and T. Feldmann, *Nucl. Phys.* **B643**, 431–476 (2002) [`hep-ph/0206152`]. (A)

Meanwhile, SCET has been applied to many high-energy scattering processes, starting with

198. "Hard scattering factorization from effective field theory," C. W. Bauer, S. Fleming, D. Pirjol, I. Z. Rothstein, and I. W. Stewart, *Phys. Rev.* **D66**, 014017 (2002) [`hep-ph/0202088`]. (A)

and more recently to many aspects of jets:

199. "On the structure of infrared singularities of gauge-theory amplitudes," T. Becher and M. Neubert, *JHEP* **06**, 081 (2009) [`arXiv:0903.1126 [hep-ph]`]. (A)

200. "Soft radiation in heavy-particle pair production: all-order colour structure and two-loop anomalous dimension," M. Beneke, P. Falgari, and C. Schwinn, *Nucl. Phys.* **B828**, 69–101 (2010) [`arXiv:0907.1443 [hep-ph]`]. (A)

201. "Factorization structure of gauge theory amplitudes and application to hard scattering processes at the LHC," J.-y. Chiu, A. Fuhrer, R. Kelley, and A. V. Manohar, *Phys. Rev.* **D80**, 094013 (2009) [`arXiv:0909.0012 [hep-ph]`]. (A)

202. "Factorization at the LHC: from PDFs to initial state jets," I. W. Stewart, F. J. Tackmann, and W. J. Waalewijn, `arXiv:0910.0467 [hep-ph]`. (A)

203. "Factorization and resummation of Higgs boson differential distributions in soft-collinear effective theory," S. Mantry and F. Petriello, `arXiv:0911.4135 [hep-ph]`. (A)

204. "Consistent factorization of jet observables in exclusive multijet cross-sections," S. D. Ellis, A. Hornig, C. Lee, C. K. Vermilion, and J. R. Walsh, `arXiv:0912.0262 [hep-ph]`. (A)

F. Lattice gauge theory

With an explicit definition of its ultraviolet behavior, lattice gauge theory lends itself to computational methods, essentially integrating the functional integral of QCD numerically:

205. "Monte Carlo calculations for the lattice gauge theory," K. G. Wilson in **Recent Developments in Gauge Theories**, edited by G. 't Hooft *et al.* (Plenum, New York, 1980), pp. 363–402. (I–A)

The first study connecting the confining regime to asymptotic freedom appeared in

206. "Monte Carlo study of quantized SU(2) gauge theory," M. Creutz, *Phys. Rev.* **D21**, 2308–2315 (1980) [doi: `10.1103/PhysRevD.21.2308`]. (I–A)

207. "Asymptotic-freedom scales," M. Creutz, *Phys. Rev. Lett.* **45**, 313–316 (1980) [doi: `10.1103/PhysRevLett.45.313`]. (I–A)

A useful reprint collection of early work is

208. Lattice Gauge Theories and Monte Carlo Simulations, edited by C. Rebbi (World Scientific, Singapore, 1983). (E–I–A)

There are several good textbooks on lattice gauge theory, including

209. Quarks, Gluons, and Lattices, M. Creutz (Cambridge University Press, Cambridge, 1983). (E–I)

210. Quantum Fields on a Lattice, I. Montvay and G. Münster (Cambridge University Press, Cambridge, 1994). (I–A)

211. Introduction to Quantum Fields on a Lattice: A Robust Mate, J. Smit, Cambridge Lecture Notes in Physics (Cambridge University Press, Cambridge & New York, 2002). (I–A)

212. Lattice Gauge Theories: An Introduction, H. J. Rothe (World Scientific, Singapore, 2005). (I–A)

213. Lattice Methods for Quantum Chromodynamics, T. DeGrand and C. DeTar (World Scientific, Singapore, 2006). (I–A)

214. Quantum Chromodynamics on the Lattice, C. Gattringer and C. B. Lang (Springer, Berlin, 2010). (I–A)

Lattice gauge theory is also the foundation of attempts at rigorous construction of gauge theories:

215. Gauge Theories as a Problem of Constructive Quantum Field Theory and Statistical Mechanics, E. Seiler (Springer, Berlin, 1982). (I–A)

216. Quantum Physics: A Functional Integral Point of View, J. Glimm and A. M. Jaffe (Springer, New York, 1987), 2nd ed. (I–A)

For many years, numerical lattice-QCD calculations omitted the computationally very demanding contribution of sea quarks (quark-antiquark pairs that fluctuate out of the vacuum), leading to uncontrolled uncertainties. The first demonstration that incorporation of sea-quark effects brings a wide variety of computed hadron properties into agreement with experiment is

217. "High-precision lattice QCD confronts experiment," C. T. H. Davies *et al.*, HPQCD, MILC, and Fermilab Lattice Collaborations, *Phys. Rev. Lett.* **92**, 022001 (2004) [`hep-lat/0304004`]. (I–A)

The maturation of numerical lattice QCD is discussed in

218. "Lattice quantum chromodynamics comes of age," C. E. DeTar and S. Gottlieb, *Phys. Today* **57**, 45–51 (February, 2004) [doi: `10.1063/1.1688069`]. (E–I)

With these developments it is now possible to compute the hadron masses with a few percent precision:

219. "The weight of the world is quantum chromodynamics," A. S. Kronfeld, *Science* **322**, 1198–1199 (2008) [doi: `10.1126/science.1166844`]. (E)

220. "Mass by numbers," F. Wilczek, *Nature* **456**, 449–450 (2008) [doi: `10.1038/456449a`]. (E)

and make predictions of hadronic properties needed to interpret experiments:

221. "Predictions with lattice QCD," A. S. Kronfeld, Fermilab Lattice Collaboration, *J. Phys. Conf. Ser.* **46**, 147–151 (2006) [`hep-lat/0607011`]. (I)

A more detailed comparison of lattice-QCD calculations with experiment is given in Sec. IV.

Numerical lattice QCD is not merely a brute-force approach, but a synthesis of computation and effective field theories. Errors from nonzero lattice spacing are controlled with Symanzik's effective theory of cutoff effects:

222. "Continuum limit and improved action in lattice theories 1: principles and ϕ^4 theory," K. Symanzik, *Nucl. Phys.* **B226**, 187 (1983) [doi: `10.1016/0550-3213(83)90468-6`]. (A)

223. "Continuum limit and improved action in lattice theories 2: O(N) nonlinear σ model in perturbation theory," K. Symanzik, *Nucl. Phys.* **B226**, 205 (1983) [doi: `10.1016/0550-3213(83)90469-8`]. (A)

work that grew out of Ref. 117. Errors from finite volume can be controlled with general properties of massive field theories on a torus:

224. "Volume dependence of the energy spectrum in massive quantum field theories 1: stable particle states," M. Lüscher, *Commun. Math. Phys.* **104**, 177 (1986) [doi: `10.1007/BF01211589`]. (A)

225. "Spontaneously broken symmetries: effective Lagrangians at finite volume," J. Gasser and H. Leutwyler, *Nucl. Phys.* **B307**, 763 (1988) [doi: `10.1016/0550-3213(88)90107-1`]. (A)

226. "Two-particle states on a torus and their relation to the scattering matrix," M. Lüscher, *Nucl. Phys.* **B354**, 531–578 (1991) [doi: `10.1016/0550-3213(91)90366-6`]. (A)

The light quarks in computer simulations often have masses larger than those of the up and down quarks, but the extrapolation in quark mass can be guided by adapting chiral perturbation theory:

227. "Chiral perturbation theory at non-zero lattice spacing," O. Bär, *Nucl. Phys. Proc. Suppl.* **140**, 106–119 (2005) [`hep-lat/0409123`]. (A)

228. "Chiral perturbation theory," V. Bernard and U.-G. Meißner, *Ann. Rev. Nucl. Part. Sci.* **57**, 33–60 (2007) [`hep-ph/0611231`]. (I–A)

The charmed and bottom quarks often have masses close to the ultraviolet cutoff (introduced by the lattice), but the effects can be understood with HQET and NRQCD:

229. "Heavy quarks and lattice QCD," A. S. Kronfeld, *Nucl. Phys. Proc. Suppl.* **129**, 46–59 (2004) [`hep-lat/0310063`]. (A)

The idea that lattice QCD is a synthesis of computational and theoretical physics is explored in

230. "Uses of effective field theory in lattice QCD," A. S. Kronfeld, vol. 4 of Shifman, Ref. 39, ch. 39, pp. 2411–2477. [`hep-lat/0205021`]. (I)

Lattice gauge theory and chiral symmetry coexist uneasily:

231. "No-go theorem for regularizing chiral fermions," H. B. Nielsen and M. Ninomiya, *Phys. Lett.* **B105**, 219 (1981) [doi: `10.1016/0370-2693(81)91026-1`]. (A)

The efforts to understand and overcome these difficulties, for theories like QCD, is reviewed in

232. "Exact chiral symmetry on the lattice," H. Neuberger, *Ann. Rev. Nucl. Part. Sci.* **51**, 23–52 (2001) [`hep-lat/0101006`]. (A)

Lattice gauge theory, with its rigorous mathematical definition, is a suitable arena for deriving mass inequalities:

233. "Mass inequalities for QCD," D. Weingarten, *Phys. Rev. Lett.* **51**, 1830 (1983) [doi: `10.1103/PhysRevLett.51.1830`]. (A)

234. "Some inequalities among hadron masses," E. Witten, *Phys. Rev. Lett.* **51**, 2351 (1983) [doi: `10.1103/PhysRevLett.51.2351`]. (A)

These and related developments have been reviewed in

235. "QCD inequalities," S. Nussinov and M. A. Lampert, *Phys. Rept.* **362**, 193–301 (2002) [`hep-ph/9911532`]. (A)

G. The QCD vacuum and confinement

The space of all non-Abelian gauge fields is not simply connected, but consists of sectors labeled by an integer n. The sectors arise when trying to satisfy a *gauge condition*, namely to specify $\omega(x)$ in order to choose one representative field $B_\mu(x)$ among all those related by Eq. (16).

In some cases it is necessary to specify different conditions in different regions of spacetime, and then it turns out that $\omega(x)$ on the overlaps of the regions is an n-to-one mapping onto $\mathrm{SU}(N_\mathrm{c})$. In the quantum theory, tunneling can occur between the different sectors, and the tunneling events are called "instantons." A classic discussion can be found in

236. "The uses of instantons," S. R. Coleman in **Aspects of Symmetry** (Cambridge University Press, Cambridge, 1985), pp. 265–350. (I–A)

Some further features appear at nonzero temperature:

237. "QCD and instantons at finite temperature," D. J. Gross, R. D. Pisarski, and L. G. Yaffe, *Rev. Mod. Phys.* **53**, 43 (1981) [doi: `10.1103/RevModPhys.53.43`]. (I–A)

Because of these sectors, the QCD Lagrangian, Eq. (1), can contain a term proportional to $\varepsilon_{\mu\nu\rho\sigma}\,\mathrm{tr}(G^{\mu\nu}G^{\rho\sigma})$. The physical implication of this term is a possible violation of CP symmetry, as is discussed further in Sec. III A 2.

It is widely believed that the nontrivial vacuum structure is connected to the special features of QCD, notably confinement. Opinion is divided whether instantons, i.e., the tunneling events, play the principal role, or whether strong quantum fluctuations do. The case for instantons can be traced from

238. "Structure of the QCD vacuum and hadrons," E. Shuryak, *Phys. Rept.* **264**, 357–373 (1996) [doi: `10.1016/0370-1573(95)00048-8`]. (A)

239. "The QCD vacuum as an instanton liquid," E. Shuryak and T. Schäfer, *Ann. Rev. Nucl. Part. Sci.* **47**, 359–394 (1997) [doi: `10.1146/annurev.nucl.47.1.359`]. (I–A)

and the case for fluctuations from

240. "Instantons, the quark model, and the $1/N$ expansion," E. Witten, *Nucl. Phys.* **B149**, 285 (1979) [doi: `10.1016/0550-3213(79)90243-8`]. (I–A)

241. "Evidence against instanton dominance of topological charge fluctuations in QCD," I. Horvath, N. Isgur, J. McCune, and H. B. Thacker, *Phys. Rev.* **D65**, 014502 (2002) [`hep-lat/0102003`]. (A)

Another approach to confinement starts with the observation that any gauge condition has more than one solution:

242. "Quantization of non-Abelian gauge theories," V. N. Gribov, *Nucl. Phys.* **B139**, 1 (1978) [doi: `10.1016/0550-3213(78)90175-X`]. (A)

In the Coulomb gauge one demands $\boldsymbol{\nabla}\cdot\boldsymbol{A}=0$; further demanding a unique resolution of the Gribov ambiguity, one finds, with some assumptions, a confining potential:

243. "Renormalization in the Coulomb gauge and order parameter for confinement in QCD," D.

Zwanziger, *Nucl. Phys.* **B518**, 237–272 (1998) [doi: `10.1016/S0550-3213(98)00031-5`]. (A)

A string picture of confinement emerges naturally from the perturbative properties of QCD. The energy required to separate a quark and antiquark,

$$E = \sigma R, \tag{21}$$

is proportional to the string tension σ and the separation R. Furthermore, the property of asymptotic freedom means that the "dielectric constant" of the QCD vacuum is $\varepsilon_{\mathrm{QCD}} < 1$, in contrast to the familiar result for a dielectric substance, $\varepsilon > 1$. The QCD vacuum is thus a *dia-electric* medium. An electrostatic analogy leads to a heuristic understanding of confinement. It is energetically favorable for a test charge placed in a very effective dia-electric medium to carve out a bubble in which $\varepsilon = 1$. In the limit of a perfect dia-electric medium, the bubble radius and the energy stored in the electric field tend to infinity. In contrast, the radius of the bubble surrounding a test dipole placed in the medium occupies a finite volume, even in the perfect dia-electric limit, because the field lines need not extend to infinity.

244. "Vacuum polarization and the absence of free quarks in four dimensions," J. B. Kogut and L. Susskind, *Phys. Rev.* **D9**, 3501–3512 (1974) [doi: `10.1103/PhysRevD.9.3501`]. (I–A)

The dia-electric analogy is reviewed in Sec. 8.8 of

245. Gauge Theories of the Strong, Weak, and Electromagnetic Interactions, C. Quigg (Perseus Westview Press, Boulder, Colorado, 1997). (I–A)

The physical picture is highly similar to MIT bag model:

246. "A new extended model of hadrons," A. Chodos, R. L. Jaffe, K. Johnson, C. B. Thorn, and V. F. Weisskopf, *Phys. Rev.* **D9**, 3471–3495 (1974) [doi: `10.1103/PhysRevD.9.3471`]. (I)

The exclusion of chromoelectric flux from the QCD vacuum is reminiscent of the exclusion of magnetic flux from a type-II superconductor. In a dual version of the Meissner effect, with the roles of electric and magnetic properties swapped, the chromoelectric field between a separating quark and antiquark takes the form of an Abrikosov flux tube. For an introduction and tests of the picture, see

247. "Color confinement and dual superconductivity of the vacuum I," A. Di Giacomo, B. Lucini, L. Montesi, and G. Paffuti, *Phys. Rev.* **D61**, 034503 (2000) [`hep-lat/9906024`]. (I–A)

Lattice gauge theory (Ref. 22) was originally invented to understand confinement. Reviews of more recent analytical and numerical work can be found in

248. "The confinement problem in lattice gauge theory," J. Greensite, *Prog. Part. Nucl. Phys.* **51**, 1 (2003) [`hep-lat/0301023`]. (I–A)

249. "Quark confinement: the hard problem of hadron physics," R. Alkofer and J. Greensite, *J. Phys.* **G34**, S3 (2007) [`hep-ph/0610365`]. (I–A)

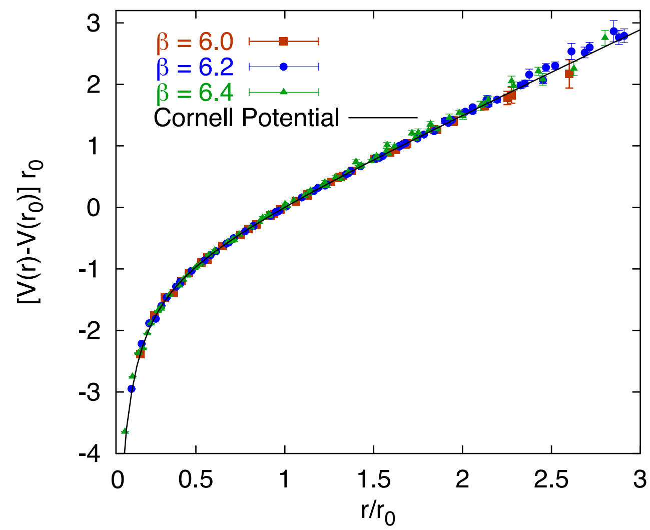

FIG. 2: The potential energy $V(r)$ between static sources of color (in an approximation without sea quarks). The zero of energy and the units are set by a conventional distance r_0, defined by $r_0^2\, dV/dr = 1.65$. The data points are from lattice QCD, generated at several values of $\beta = 6/g^2$, which—via dimensional transmutation—corresponds to varying the spacing between lattice sites. The black curve is a fit of these data to the potential model of Refs. 109,110. From Ref. 250.

The connection between QCD potentials, spectroscopy, and confinement is reviewed in

250. "QCD forces and heavy quark bound states," G. S. Bali, *Phys. Rept.* **343**, 1–136 (2001) [`hep-ph/0001312`]. (I–A)

An important theme in Ref. 250 is the lattice-QCD computation of the potential energy between static sources of color. As shown in Fig. 2, the potential looks Coulombic at short distances, in accord with asymptotic freedom, and linear at long distances, in accord with Eq. (21).

A series of conferences is devoted to the confinement problem. Their agendas and proceedings can be traced from

251. "Quark Confinement and the Hadron Spectrum 9," `http://teorica.fis.ucm.es/Confinement9/`. (A)

H. Dyson-Schwinger Equations

A fruitful continuum approach to nonperturbative dynamics is based on the infinite tower of Dyson-Schwinger equations, coupled integral equations that relate the Green functions of a field theory to each other. Solving these equations provides a solution of the theory, in that a field theory is completely defined by all of its n-point Green functions. A good starting point is

252. "Dyson-Schwinger equations and their application to hadronic physics," C. D. Roberts and A. G. Williams, *Prog. Part. Nucl. Phys.* **33**, 477–575 (1994) [hep-ph/9403224]. (I–A)

A newer review, focused on mesons is

253. "Dyson-Schwinger equations: a tool for hadron physics," P. Maris and C. D. Roberts, *Int. J. Mod. Phys.* **E12**, 297–365 (2003) [nucl-th/0301049]. (I–A)

and a broader survey of results from this approach can be found in

254. "Hadron physics: the essence of matter," L. Chang and C. D. Roberts, arXiv:1003.5006 [nucl-th]. (I–A)

Truncations and approximations of Dyson-Schwinger equations can shed light on confinement:

255. "A solution to coupled Dyson-Schwinger equations for gluons and ghosts in Landau gauge," L. von Smekal, A. Hauck, and R. Alkofer, *Ann. Phys.* **267**, 1 (1998) [hep-ph/9707327]. (A)

Finally, these techniques have been extended to QCD thermodynamics in

256. "Schwinger-Dyson approach to color superconductivity in dense QCD," D. K. Hong, V. A. Miransky, I. A. Shovkovy, and L. C. R. Wijewardhana, *Phys. Rev.* **D61**, 056001 (2000) [hep-ph/9906478]. (A)

257. "Dyson-Schwinger equations: density, temperature and continuum strong QCD," C. D. Roberts and S. M. Schmidt, *Prog. Part. Nucl. Phys.* **45**, S1–S103 (2000) [nucl-th/0005064]. (I–A)

I. Perturbative amplitudes

A key consequence of factorization is to relate amplitudes for (some) hadronic processes to underlying processes of quarks and gluons. Parton amplitudes can be computed via Feynman diagrams, as discussed in Refs. 32,34–38. As the complexity of the process increases, however, this approach becomes intractable. Remarkably, QCD amplitudes are simpler than the individual diagrams might suggest:

258. "An amplitude for n-gluon scattering," S. J. Parke and T. R. Taylor, *Phys. Rev. Lett.* **56**, 2459 (1986) [doi: 10.1103/PhysRevLett.56.2459]. (I)

Perturbative QCD amplitudes also are related by recursion in the number of scattered gluons:

259. "Recursive calculations for processes with n gluons," F. A. Berends and W. T. Giele, *Nucl. Phys.* **B306**, 759 (1988) [doi: 10.1016/0550-3213(88)90442-7]. (A)

For an older review that remains useful for graduate students, see

260. "Multiparton amplitudes in gauge theories," M. L. Mangano and S. J. Parke, *Phys. Rept.* **200**, 301–367 (1991) [doi: 10.1016/0370-1573(91)90091-Y]. (I–A)

The simplifications can be related to deep connections between Yang-Mills theories and string theories:

261. "The computation of loop amplitudes in gauge theories," Z. Bern and D. A. Kosower, *Nucl. Phys.* **B379**, 451–561 (1992) [doi: 10.1016/0550-3213(92)90134-W]. (A)

262. "Perturbative gauge theory as a string theory in twistor space," E. Witten, *Commun. Math. Phys.* **252**, 189–258 (2004) [hep-th/0312171]. (A)

A parallel, and perhaps, even more fruitful, alternative to Feynman diagrams starts with constraints of unitarity:

263. "One-loop n-point gauge theory amplitudes, unitarity and collinear limits," Z. Bern, L. J. Dixon, D. C. Dunbar, and D. A. Kosower, *Nucl. Phys.* **B425**, 217–260 (1994) [hep-ph/9403226]. (A)

The first decade of the 2000s witnessed rapid conceptual and technical development of these two sets of ideas, by many researchers, too many to list here. The review

264. "On-shell methods in perturbative QCD," Z. Bern, L. J. Dixon, and D. A. Kosower, *Ann. Phys.* **322**, 1587–1634 (2007) [arXiv:0704.2798 [hep-ph]]. (I–A)

contains a comprehensive set of references, and the most recent developments are discussed in

265. "Multi-parton scattering amplitudes via on-shell methods," C. F. Berger and D. Forde, *Ann. Rev. Nucl. Part. Sci.* **60**, in press (2010) [arXiv:0912.3534 [hep-ph]]. (I–A)

J. Parton-shower Monte Carlo programs

In a high-energy collision, although the parton-scattering can be factorized and computed in perturbation theory, a description of the full event is complicated first by radiation of gluons and $q\bar{q}$ pairs and later by the formation of hadrons. Several computer codes have been developed to automate the calculation of the initial parton scatter, treat the shower of partons, and model the hadronization. Useful reviews to the concepts can be found in

266. "Tools for the simulation of hard hadronic collisions," M. L. Mangano and T. J. Stelzer, *Ann. Rev. Nucl. Part. Sci.* **55**, 555–588 (2005) [doi: 10.1146/annurev.nucl.55.090704.151505]. (I–A)

267. "Monte Carlo tools," T. Sjöstrand, arXiv:0911.5286 [hep-ph]. (I–A)

and a hands-on guide is

268. "Les Houches guidebook to Monte Carlo generators for hadron collider physics," M. A. Dobbs *et al.*, hep-ph/0403045. (I–A)

K. Extensions of QCD

QCD belongs to a class of Yang-Mills theories, and further information can be gleaned by varying the number of colors, N_c, and the number of flavors, n_f. Some classic and useful references on QCD as $N_c \to \infty$ are

269. "A planar diagram theory for strong interactions," G. 't Hooft, *Nucl. Phys.* **B72**, 461 (1974) [doi: 10.1016/0550-3213(74)90154-0]. (I–A)

270. "Baryons in the $1/N$ expansion," E. Witten, *Nucl. Phys.* **B160**, 57 (1979) [doi: 10.1016/0550-3213(79)90232-3]. (A)

271. "Some aspects of large N theories," S. R. Das, *Rev. Mod. Phys.* **59**, 235–261 (1987) [doi: 10.1103/RevModPhys.59.235]. (A)

272. "Chiral and large-N_c limits of quantum chromodynamics and models of the baryon," T. D. Cohen, *Rev. Mod. Phys.* **68**, 599–608 (1996) [hep-ph/9512275]. (A)

273. "Large-N_c baryons," E. Jenkins, *Ann. Rev. Nucl. Part. Sci.* **48**, 81–119 (1998) [hep-ph/9803349]. (I–A)

Supersymmetry is a spacetime symmetry connecting bosonic and fermionic representations of the Poincaré group. Gauge theories with supersymmetry enjoy some simplifying features:

274. "Electric-magnetic duality, monopole condensation, and confinement in $N = 2$ supersymmetric Yang-Mills theory," N. Seiberg and E. Witten, *Nucl. Phys.* **B426**, 19–52 (1994) [hep-th/9407087]. (A)

275. "Anti-de Sitter space, thermal phase transition, and confinement in gauge theories," E. Witten, *Adv. Theor. Math. Phys.* **2**, 505–532 (1998) [hep-th/9803131]. (A)

leading to interesting relations between strongly-coupled gauge theories of certain (N_c, n_f) and weakly-coupled dual gauge theories with (N'_c, n'_f).

L. String theory

String theory is a mathematical description of particles as vibrational modes of one-dimensional objects, instead of as points. First developed as a model of hadrons, string theory fell out of favor after the rise of QCD. But it has enjoyed a tremendous interest as a unifying theory of quantum mechanics and gravity, spurring a vast literature in mathematical physics. Now string theory has come full circle, with string techniques applied to gauge theories, starting with

276. "The large N limit of superconformal field theories and supergravity," J. M. Maldacena, *Adv. Theor. Math. Phys.* **2**, 231–252 (1998) [hep-th/9711200]. (A)

An excellent pedagogical introduction is given in

277. "Introduction to the AdS/CFT correspondence," I. R. Klebanov in **Strings, Branes, and Gravity: TASI99**, edited by J. Harvey, S. Kachru, and E. Silverstein (World Scientific, Singapore, 2001), [hep-th/0009139]. (A)

Several developments address hadron properties, for example

278. "Glueball mass spectrum from supergravity," C. Csaki, H. Ooguri, Y. Oz, and J. Terning, *JHEP* **01**, 017 (1999) [hep-th/9806021]. (A)

279. "Glueball spectrum for QCD from AdS supergravity duality," R. C. Brower, S. D. Mathur, and C.-I. Tan, *Nucl. Phys.* **B587**, 249–276 (2000) [hep-th/0003115]. (A)

280. "The string dual of a confining four-dimensional gauge theory," J. Polchinski and M. J. Strassler, hep-th/0003136. (A)

281. "Hard scattering and gauge-string duality," J. Polchinski and M. J. Strassler, *Phys. Rev. Lett.* **88**, 031601 (2002) [hep-th/0109174]. (A)

282. "The Pomeron and gauge-string duality," R. C. Brower, J. Polchinski, M. J. Strassler, and C.-I. Tan, *JHEP* **12**, 005 (2007) [hep-th/0603115]. (I–A)

IV. CONFRONTING QCD WITH EXPERIMENT

A. Running of α_s

A fundamental consequence of QCD is the property of asymptotic freedom, the decrease of the strong coupling constant $\alpha_s(Q)$ with increasing values of the momentum scale, Q. In first approximation (see Sec. II B), we expect a linear increase of $1/\alpha_s(Q)$ with $\log Q$:

$$\frac{1}{\alpha_s(Q)} = \frac{1}{\alpha_s(\mu)} + \frac{33 - 2n_f}{6\pi} \log\left(\frac{Q}{\mu}\right), \tag{19}$$

so long as the number of active quark flavors, n_f, does not exceed 16. In fact, the scale dependence of α_s to be expected in QCD has been computed to order α_s^5:

283. "The four-loop beta function in quantum chromodynamics," T. Van Ritbergen, J. A. M. Vermaseren, and S. A. Larin, *Phys. Lett.* **B400**, 379–384 (1997) [hep-ph/9701390]. (A)

The decrease of α_s with Q has been demonstrated by measurements in many experimental settings (Ref. 8). Over the past decade, the precision of α_s determinations has improved dramatically, thanks to a plethora of results from various processes aided by improved calculations at higher orders in perturbation theory. The progress is reviewed, and critically evaluated, in

284. "The 2009 world average of $\alpha_s(M_Z)$," S. Bethke, *Eur. Phys. J.* **C64**, 689–703 (2009) [arXiv:0908.1135

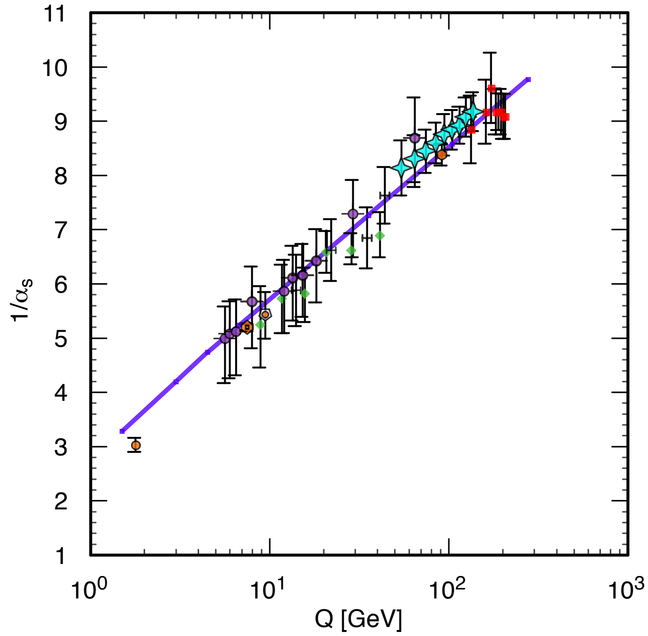

FIG. 3: Measurements of the strong coupling $1/\alpha_s(Q)$ as a function of the energy scale $\ln Q$. In addition to hadronic τ-decay, quarkonium, Υ decay, and Z^0-pole values from Ref. 284, we display black crosses: e^+e^- collisions (Ref. 285); red squares: e^+e^- collisions (Ref. 286); green diamonds: $e^\pm p$ collisions (Ref. 287); barred purple circles: $e^\pm p$ collisions (Refs. 288,289); cyan crosses: $\bar{p}p$ collisions (Ref. 290); average value of $\alpha_s(M_Z)$, in 4-loop approximation and using 3-loop threshold matching at the heavy-quark pole masses $m_c = 1.5$ GeV and $m_b = 4.7$ GeV.

`[hep-ph]`]. (A)

which draws particular attention to the high level of recent activity in the area of hadronic τ decays.

A representative selection of experimental determinations is shown, together with the evolution expected in QCD, in Fig. 3. We have drawn the displayed values from Ref. 284, together with determinations from e^+e^- event shapes reported in

285. "Studien zur Quantenchromodynamik und Messung der starken Kopplungskonstanten α_s bei $\sqrt{s} = 14$–44 GeV mit dem JADE-Detektor," P. A. Movilla Fernández [`http://darwin.bth.rwth-aachen.de/opus3/volltexte/2003/483/`]. (A; in German)

286. "Determination of α_s from hadronic event shapes in e^+e^- annihilation at 192 GeV $\leq \sqrt{s} \leq$ 208 GeV," P. Achard *et al.*, L3 Collaboration, *Phys. Lett.* **B536**, 217–228 (2002) [`hep-ex/0206052`]. (A)

from jet studies in $e^\pm p$ scattering reported in

287. "Jet-radius dependence of inclusive-jet cross sections in deep inelastic scattering at HERA," S. Chekanov *et al.*, ZEUS Collaboration, *Phys. Lett.* **B649**, 12–24 (2007) [`hep-ex/0701039`]. (A)

288. "Jet production in ep collisions at high Q^2 and determination of α_s," F. D. Aaron *et al.*, H1 Collaboration, *Eur. Phys. J.* **C65**, 363–383 (2010) [`arXiv:0904.3870 [hep-ex]`]. (A)

289. "Jet production in ep collisions at low Q^2 and determination of α_s," F. D. Aaron *et al.*, H1 Collaboration, `arXiv:0911.5678 [hep-ex]`. (A)

and the running coupling constant inferred from inclusive jet production in $\bar{p}p$ collisions,

290. "Determination of the strong coupling constant from the inclusive jet cross section in $p\bar{p}$ collisions at $\sqrt{s} = 1.96$ TeV," V. M. Abazov *et al.*, D0 Collaboration, *Phys. Rev.* **D80**, 111107 (2009) [`arXiv:0911.2710 [hep-ex]`]. (I–A)

The trend toward asymptotic freedom is clear, and the agreement with the predicted evolution is excellent, within the uncertainties in the measurements. An interesting challenge for the future will be to measure $\alpha_s(Q)$ with precision sufficient to detect the expected change of slope at the top-quark threshold.

It is conventional, and enlightening, to rewrite the evolution equation (19) in the form

$$\frac{1}{\alpha_s(Q)} = \frac{33 - 2n_f}{6\pi} \log\left(\frac{Q}{\Lambda_{\mathrm{QCD}}}\right), \tag{22}$$

where Λ_{QCD} is the QCD scale parameter, with dimensions of energy. (A generalization beyond leading order is given in Sec. 9 of Ref. 8.) Several subtleties attend this simple and useful parametrization. First, if we enforce the requirement that $\alpha_s(Q)$ be continuous at flavor thresholds, then Λ_{QCD} must depend on the number of active quark flavors. Second, the value of Λ_{QCD} depends on the renormalization scheme; the canonical choice is the modified minimal subtraction ($\overline{\mathrm{MS}}$) scheme introduced in

291. "Deep inelastic scattering beyond the leading order in asymptotically free gauge theories," W. A. Bardeen, A. J. Buras, D. W. Duke, and T. Muta, *Phys. Rev.* **D18**, 3998 (1978) [doi: `10.1103/PhysRevD.18.3998`]. (A)

The n_f and scheme dependence is given via labels on Λ, *e.g.*, $\Lambda^{(n_f)}_{\overline{\mathrm{MS}}}$. Representative estimates of the QCD scale are $\Lambda^{(5)}_{\overline{\mathrm{MS}}} = 213$ MeV, $\Lambda^{(4)}_{\overline{\mathrm{MS}}} = 296$ MeV, and $\Lambda^{(3)}_{\overline{\mathrm{MS}}} = 338$ MeV (Ref. 284). The appearance of a dimensional quantity to parametrize the running coupling is sometimes called "dimensional transmutation."

The theoretical underpinnings of high-precision determinations of α_s in deeply inelastic scattering are presented in

292. "Non-singlet QCD analysis of deep inelastic world data at $O(\alpha_s^3)$," J. Blümlein, H. Böttcher, and A. Guffanti, *Nucl. Phys.* **B774**, 182–207 (2007) [`hep-ph/0607200`]. (I–A)

which provides a detailed analysis of remaining uncertainties in the QCD scale.

For another critical assessment of the theoretical analyses that underlie determinations of α_s, see

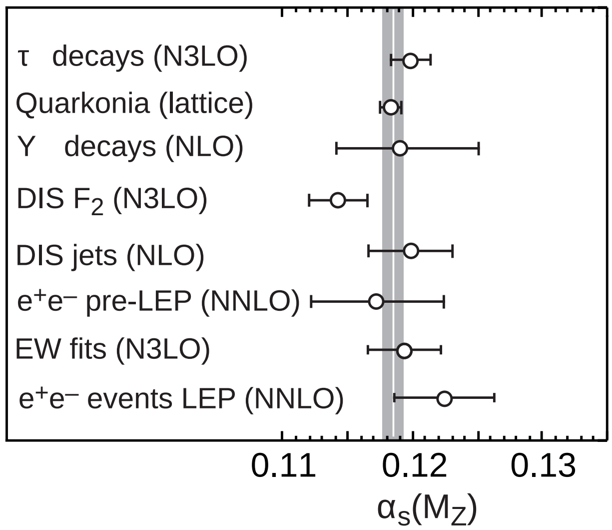

FIG. 4: Determinations of $\alpha_{\mathrm{s}}(M_Z)$ from several processes. In most cases, the value measured at a scale μ has been evolved to $\mu = M_Z$. Error bars include the theoretical uncertainties. Adapted from Ref. 284.

293. "On the running coupling constant in QCD," G. M. Prosperi, M. Raciti, and C. Simolo, *Prog. Part. Nucl. Phys.* **58**, 387–438 (2007) [hep-ph/0607209]. (I–A)

When evolved to a common scale $\mu = M_Z$, the various determinations of α_{s} lead to consistent values, as shown in Fig. 4. A representative mean value (Ref. 284) is

$$\alpha_{\mathrm{s}}(M_Z) = 0.1184 \pm 0.0007. \tag{23}$$

The agreement of the determination of α_{s} from the hadron spectrum, via lattice QCD, and from high-energy scattering processes, via perturbative QCD (and factorization for deeply inelastic scattering), indicates that QCD describes both hadron and partons. In other words, a single theory accounts for all facets of the strong interactions.

B. Hadron spectrum

Soon after the conception of quantum chromodynamics, theorists formulated QCD-inspired models to open a dialogue with experiment. Simple notions about the order of levels, augmented by an effective color-hyperfine interaction were put forward in

294. "Hadron masses in a gauge theory," A. De Rujula, H. Georgi, and S. L. Glashow, *Phys. Rev.* **D12**, 147–162 (1975) [doi: 10.1103/PhysRevD.12.147]. (I)

The extension to excited baryons was given by

295. "*P*-wave baryons in the quark model," N. Isgur and G. Karl, *Phys. Rev. D* **18**, 4187–4205 (1978) [doi: 10.1103/PhysRevD.18.4187]. (I)

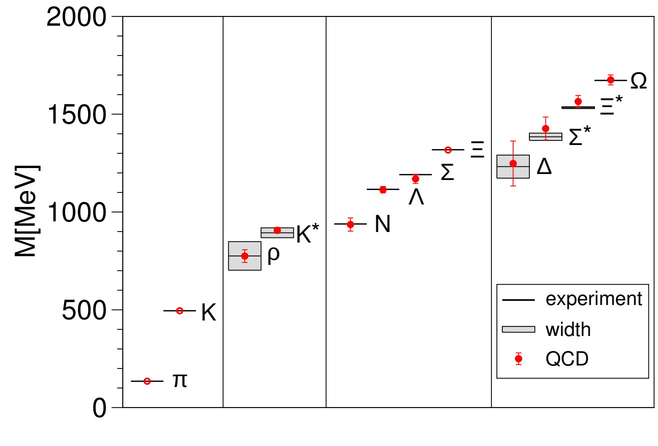

FIG. 5: Meson (π, K, ρ, K^*) and baryon masses (others) from lattice QCD, using the π, K, and Ξ masses to fix the three free parameters ($m_u + m_d$, m_s, Λ_{QCD}). From Ref. 299.

An extensive analysis of the meson spectrum in a QCD-inspired quark model is

296. "Mesons in a relativized quark model with chromodynamics," S. Godfrey and N. Isgur, *Phys. Rev.* **D32**, 189–231 (1985) [doi: 10.1103/PhysRevD.32.189]. (I)

Massless quarks were confined within a finite radius by fiat in the MIT bag model, which is explained in

297. "The bag model of quark confinement," K. A. Johnson, *Sci. Am.* **241**, 112–121 (July, 1979). (E)

298. "Bag models of hadrons," C. E. DeTar and J. F. Donoghue, *Ann. Rev. Nucl. Part. Sci.* **33**, 235–264 (1983) [doi: 10.1146/annurev.ns.33.120183.001315]. (I–A)

Lattice QCD provides a way to compute the hadron mass spectrum directly from the QCD Lagrangian. The state of the art for light hadrons is shown in Fig. 5 and described in

299. "*Ab-initio* determination of light hadron masses," S. Dürr *et al.*, BMW Collaboration, *Science* **322**, 1224–1227 (2008) [arXiv:0906.3599 [hep-lat]]. (I–A)

300. "2+1-flavor lattice QCD toward the physical point," S. Aoki *et al.*, PACS-CS Collaboration, *Phys. Rev.* **D79**, 034503 (2009) [arXiv:0807.1661 [hep-lat]]. (A)

301. "Full nonperturbative QCD simulations with 2+1 flavors of improved staggered quarks," A. Bazavov *et al.*, *Rev. Mod. Phys.* in press (2010) [arXiv:0903.3598 [hep-lat]]. (I–A)

With α_{s}, the quark masses are the fundamental parameters of QCD. Hadron masses depend on the quark masses, so these calculations yield as by-products the best estimates of the light-quark masses (Ref. 301)

$$\begin{aligned} m_u &= 1.9 \pm 0.2 \text{ MeV}, \\ m_d &= 4.6 \pm 0.3 \text{ MeV}, \end{aligned} \tag{24}$$

$$m_s = 88 \pm 5 \text{ MeV};$$

or, defining $\widehat{m} = (m_u + m_d)/2$,

$$\widehat{m} = 3.54^{+0.64}_{-0.35} \text{ MeV},$$
$$m_s = 91.1^{+14.6}_{-6.2} \text{ MeV}, \quad (25)$$

from

302. "Light quark masses from unquenched lattice QCD," T. Ishikawa *et al.*, CP-PACS and JLQCD Collaborations, *Phys. Rev.* **D78**, 011502 (2008) [`arXiv:0704.1937 [hep-lat]`]. (A)

Both groups use $2+1$ flavors of sea quarks. The quoted masses are in the $\overline{\text{MS}}$ scheme at 2 GeV. Ratios of these results agree with chiral perturbation theory:

303. "The problem of mass," S. Weinberg, *Trans. New York Acad. Sci.* **38**, 185–201 (1977) [`http://ccdb4fs.kek.jp/cgi-bin/img/allpdf?197711133`]. (I–A)

304. "Quark masses," J. Gasser and H. Leutwyler, *Phys. Rept.* **87**, 77–169 (1982) [doi: `10.1016/0370-1573(82)90035-7`]. (I–A)

305. "Light quark masses and chiral symmetry," J. F. Donoghue, *Ann. Rev. Nucl. Part. Sci.* **39**, 1–17 (1989) [doi: `10.1146/annurev.ns.39.120189.000245`]. (I–A)

The estimates Eqs. (24) and (25) show that the up- and down-quark masses account for only $3\widehat{m} \approx 10$ MeV out of the nucleon mass of 940 MeV. Accordingly, to percent-level accuracy, nearly all the mass of everyday matter arises from chromodynamic energy of gluons and the kinetic energy of the confined quarks.

In the elementary quark model, mesons are $q\bar{q}$ color singlets, whereas baryons are qqq color singlets. Although QCD favors these configurations as the states of lowest energy, it also admits other body plans: quarkless mesons called glueballs, $q\bar{q}g$ mesons called hybrids, $qq\bar{q}\bar{q}$ mesons called tetraquarks, $qqqq\bar{q}$ baryons called pentaquarks, etc. At this time, there are no credible reports of non–quark-model baryons. The rich body of experimental information on non–quark-model mesons is reviewed in

306. "Glueballs, hybrids, multiquarks. experimental facts versus QCD-inspired concepts," E. Klempt and A. Zaitsev, *Phys. Rept.* **454**, 1–202 (2007) [`arXiv:0708.4016 [hep-ph]`]. (I–A)

307. "The experimental status of glueballs," V. Crede and C. A. Meyer, *Prog. Part. Nucl. Phys.* **63**, 74–116 (2009) [`arXiv:0812.0600 [hep-ex]`]. (I–A)

Strong theoretical evidence for glueballs comes from lattice-QCD calculations in an approximation to QCD without quarks:

308. "The glueball spectrum from an anisotropic lattice study," C. J. Morningstar and M. J. Peardon, *Phys. Rev.* **D60**, 034509 (1999) [`hep-lat/9901004`]. (I–A)

309. "Numerical evidence for the observation of a scalar glueball," J. Sexton, A. Vaccarino, and D. Weingarten, *Phys. Rev. Lett.* **75**, 4563–4566 (1995) [`hep-lat/9510022`]. (I–A)

310. "Glueball Regge trajectories and the Pomeron: a lattice study," H. B. Meyer and M. J. Teper, *Phys. Lett.* **B605**, 344–354 (2005) [`hep-ph/0409183`]. (I–A)

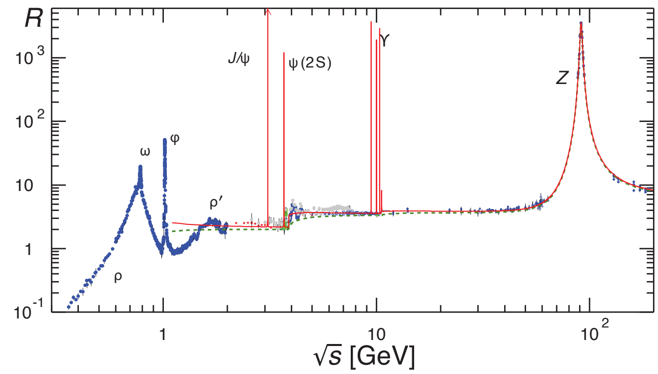

FIG. 6: World data on the ratio R from Eq. (28), compared with predictions of the quark-parton model (dashed curve) and perturbative QCD at three loops (solid line), from Ref. 8.

C. The reaction $e^+e^- \to$ hadrons

In the framework of the quark-parton model, the cross section for hadron production in electron-positron annihilations at center-of-momentum energy $\sqrt{s}$ is given by

$$\sigma_{\text{qpm}}(e^+e^- \to \text{hadrons}) = \frac{4\pi\alpha^2}{3s}\left[3\sum_q e_q^2\theta(s-4m_q^2)\right], \quad (26)$$

where e_q and m_q are the charge and mass of quark flavor q and the step function θ is a crude representation of kinematic threshold.

The factor 3 preceding the sum over active flavors is a consequence of quark color. The rough agreement between measurements of the ratio of hadron production to muon-pair production and the prediction (26), shown as the dashed line in Fig. 6, is powerful evidence that quarks are color triplets.

The parton-level prediction is modified by real and virtual emission of gluons, much as the quantum electrodynamics prediction for $\sigma(e^+e^- \to \mu^+\mu^-) = 4\pi\alpha^2/3s$ is changed by real and virtual emission of photons. To leading order in the running coupling $\alpha_s(s)$, the result is

$$\sigma_{\text{QCD}}(e^+e^- \to \text{hadrons}) = \sigma_{\text{qpm}}\left[1 + \frac{\alpha_s}{\pi} + \mathcal{O}(\alpha_s^2)\right]. \quad (27)$$

The QCD prediction for

$$R \equiv \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)}, \quad (28)$$

now known through order α_s^3, is shown as the solid line in Fig. 6.

The success of the perturbative prediction hangs on the validity of asymptotic freedom, to be sure, but also on the utility of quark-hadron duality and inclusive nature of the total hadronic cross section, such that potential infrared divergences cancel. The calculational technology, for the case of negligible quark masses, is reviewed in

311. "QCD corrections to the e^+e^- cross-section and the Z boson decay rate: concepts and results," K. G. Chetyrkin, J. H. Kühn, and A. Kwiatkowski, *Phys. Rept.* **277**, 189–281 (1996) [doi: `10.1016/S0370-1573(96)00012-9`]. (A)

Many studies of QCD in the reaction $e^+e^- \to Z$ are reviewed in

312. "Tests of perturbative QCD at LEP," S. Bethke and J. E. Pilcher, *Ann. Rev. Nucl. Part. Sci.* **42**, 251–289 (1992) [doi: `10.1146/annurev.ns.42.120192.001343`]. (I–A)

Moments of the cross section

$$\mathcal{M}_n \equiv \int_{4m_Q^2}^{\infty} ds\, s^{-(n+1)} R_Q(s), \tag{29}$$

where R_Q is the part of R [Eq. (28)] due to $Q\bar{Q}$, are useful for determining the masses of the charmed and bottom quarks. The most recent results are

$$m_c(3\text{ GeV}) = 986 \pm 13\text{ MeV}, \tag{30}$$

$$m_b(10\text{ GeV}) = 3610 \pm 16\text{ MeV}, \tag{31}$$

where the values are again in the $\overline{\text{MS}}$ scheme and the argument indicates the renormalization point. These results are taken from

313. "Charm and bottom quark masses: an update," K. G. Chetyrkin *et al.*, *Phys. Rev.* **D80**, 074010 (2009) [`arXiv:0907.2110 [hep-ph]`]. (A)

which also serves as a useful entrée to the literature.

D. Jets and event shapes in $e^+e^- \to$ hadrons

A *hadron jet* is a well-collimated cone of correlated particles produced by the hadronization of an energetic quark or gluon. Evidence that hadron jets produced in the electron-positron annihilation into hadrons follow the distributions calculated for $e^+e^- \to q\bar{q}$ was presented in

314. "Azimuthal asymmetry in inclusive hadron production by e^+e^- annihilation," R. Schwitters *et al.*, *Phys. Rev. Lett.* **35**, 1320–1322 (1975) [doi: `10.1103/PhysRevLett.35.1320`]. (I–A)

315. "Evidence for jet structure in hadron production by e^+e^- annihilation," G. Hanson *et al.*, *Phys. Rev. Lett.* **35**, 1609–1612 (1975) [doi: `10.1103/PhysRevLett.35.1609`]. (I–A)

The notion that gluon radiation should give rise to three-jet events characteristic of the final state $q\bar{q}g$ was made explicit by

316. "Search for gluons in e^+e^- annihilation," J. R. Ellis, M. K. Gaillard, and G. G. Ross, *Nucl. Phys.* **B111**, 253 (1976) [doi: `10.1016/0550-3213(76)90542-3`]. (I–A)

and confirmed in experiments at the PETRA storage ring at the DESY Laboratory in Hamburg:

317. "Evidence for planar events in e^+e^- annihilation at high energies," R. Brandelik *et al.*, TASSO Collaboration, *Phys. Lett.* **B86**, 243 (1979) [doi: `10.1016/0370-2693(79)90830-X`]. (I–A)

318. "Evidence for gluon bremsstrahlung in e^+e^- annihilations at high energies," C. Berger *et al.*, PLUTO Collaboration, *Phys. Lett.* **B86**, 418 (1979) [doi: `10.1016/0370-2693(79)90869-4`]. (I–A)

319. "Observation of planar three jet events in e^+e^- annihilation and evidence for gluon bremsstrahlung," W. Bartel *et al.*, JADE Collaboration, *Phys. Lett.* **B91**, 142 (1980) [doi: `10.1016/0370-2693(80)90680-2`]. (I–A)

320. "Discovery of three jet events and a test of quantum chromodynamics at PETRA energies," D. P. Barber *et al.*, Mark J Collaboration, *Phys. Rev. Lett.* **43**, 830 (1979) [doi: `10.1103/PhysRevLett.43.830`]. (I–A)

For a retrospective account of the discovery, see

321. "The first evidence for three-jet events in e^+e^- collisions at PETRA: first direct observation of the gluon," P. Söding, B. Wiik, G. Wolf, and S. L. Wu in **International Europhysics Conference on High Energy Physics (HEP 95)**, edited by J. Lemonne, C. Vander Velde, and F. Verbeure (World Scientific, Singapore, 1996), pp. 3–10 [`http://ccdb4fs.kek.jp/cgi-bin/img_index?9610251`]. (I)

The definition of a three-jet cross section corresponding to the quark-antiquark-gluon final state is plagued by infrared difficulties—as is the specification of any final state with a definite number of partons. It is, however, possible to define infrared-safe energy-weighted cross sections that are calculable within QCD, as shown in

322. "Jets from quantum chromodynamics," G. Sterman and S. Weinberg, *Phys. Rev. Lett.* **39**, 1436 (1977) [doi: `10.1103/PhysRevLett.39.1436`]. (I–A)

Modern definitions of jets—taking infrared safety, calculability, ease of measurement into account, and the extension to hadronic collisions—are surveyed in

323. "Towards jetography," G. P. Salam, `arXiv:0906.1833 [hep-ph]`. (I–A)

Definitions within SCET are discussed in Refs. 199–204.

Various observables are sensitive to different combinations of the quark and gluon color factors, C_F and C_A, and so an ensemble of measurements may serve to test the QCD group-theory structure via Eq. (13). The constraints from a number of studies at LEP are compiled in Fig. 7. The combined result, presented in

324. "Tests of quantum chromodynamics at e^+e^- colliders," S. Kluth, *Rept. Prog. Phys.* **69**, 1771–1846 (2006)

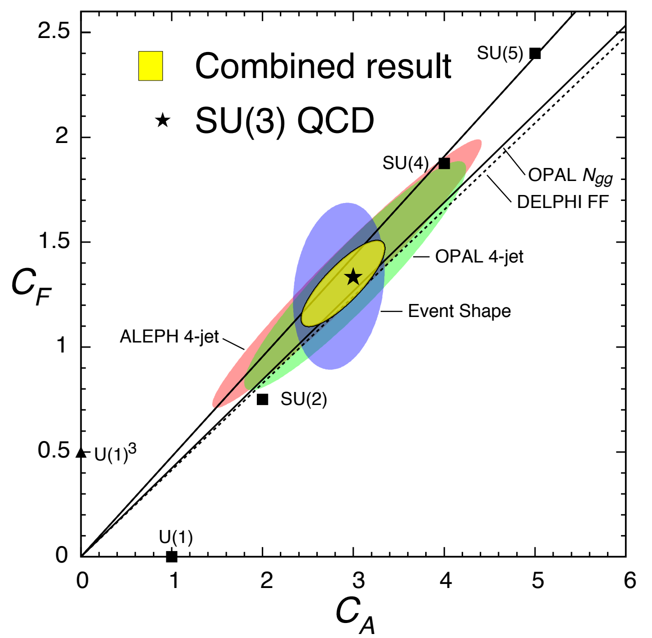

FIG. 7: Determinations of the color factors C_F and C_A from studies of event shapes, jet cross sections, and fragmentation functions at LEP (adapted from Ref. 324). The common fit, shown as the shaded ellipse, favors the QCD expectations indicated by ★, and discriminates against other hypotheses. The error ellipses show the 90% confidence level.

[hep-ex/0603011]. (I–A)

yields

$$\begin{aligned} C_F &= 1.30 \pm 0.01 \text{ (stat.)} \pm 0.09 \text{ (syst.)} \\ C_A &= 2.89 \pm 0.03 \text{ (stat.)} \pm 0.21 \text{ (syst.)}, \end{aligned} \tag{32}$$

in excellent agreement with the expectations $C_F = \frac{4}{3}$, $C_A = 3$.

E. Departures from Bjorken scaling in deeply inelastic scattering

At high resolution (high Q), the detailed composition of the nucleon is described by the parton distribution functions extracted in deeply inelastic lepton-nucleon scattering and other hard processes. The consequences for lepton-nucleon scattering were first worked out (to leading order) in

325. "Asymptotically free gauge theories I," D. J. Gross and F. Wilczek, *Phys. Rev.* **D8**, 3633–3652 (1973) [doi: 10.1103/PhysRevD.8.3633]. (A)

326. "Electroproduction scaling in an asymptotically free theory of strong interactions," H. Georgi and H. D. Politzer, *Phys. Rev.* **D9**, 416–420 (1974) [doi: 10.1103/PhysRevD.9.416]. (I–A)

According to the parton model, a hadron is a collection of quasifree quarks, antiquarks, and gluons. In the "infinite momentum frame," in which the longitudinal momentum of the hadron is very large, each parton carries a fraction x of the hadron's momentum. A parton distribution function $f_i(x_i)$ specifies the probability of finding a parton of species i with momentum fraction x_i. A highly intuitive formalism that generalizes the parton distributions to $f_i(x_i, Q^2)$ and stipulates the evolution of parton distributions with momentum transfer Q^2 was given in

327. "Asymptotic freedom in parton language," G. Altarelli and G. Parisi, *Nucl. Phys.* **B126**, 298–318 (1977) [doi: 10.1016/0550-3213(77)90384-4]. (A)

328. "Deep inelastic ep scattering in perturbation theory," V. N. Gribov and L. N. Lipatov, *Sov. J. Nucl. Phys.* **15**, 438–450 (1972). (I)

329. "Calculation of the structure functions for deep inelastic scattering and e^+e^- annihilation by perturbation theory in quantum chromodynamics," Y. L. Dokshitzer, *Sov. Phys. JETP* **46**, 641–653 (1977). (I)

The Altarelli-Parisi prescription is appropriate for moderate values of x and large values of Q^2. The extension to higher-order corrections in the $\overline{\text{MS}}$ scheme is presented in Ref. 291 and reviewed in

330. "Asymptotic freedom in deep inelastic processes in the leading order and beyond," A. J. Buras, *Rev. Mod. Phys.* **52**, 199–276 (1980) [doi: 10.1103/RevModPhys.52.199]. (I–A)

An early quantitative test appears in

331. "Tests of QCD and nonasymptotically free theories of the strong interaction by an analysis of the nucleon structure functions xF_3, F_2, and $\bar{q}$," H. Abramowicz *et al.*, *Z. Phys.* **C13**, 199 (1982) [doi: 10.1007/BF01575772]. (I–A)

Increasingly comprehensive data sets deepened the dialogue between theory and experiment. For an informative sequence of reviews, see

332. "Muon scattering," J. Drees and H. E. Montgomery, *Ann. Rev. Nucl. Part. Sci.* **33**, 383–452 (1983) [doi: 10.1146/annurev.ns.33.120183.002123]. (I–A)

333. "Deep inelastic lepton-nucleon scattering," S. R. Mishra and F. Sciulli, *Ann. Rev. Nucl. Part. Sci.* **39**, 259–310 (1989) [doi: 10.1146/annurev.ns.39.120189.001355]. (I–A)

334. The Structure of the Proton: Deep Inelastic Scattering, R. G. Roberts (Cambridge University Press, Cambridge & New York, 1990). (I–A)

335. "Precision measurements with high energy neutrino beams," J. M. Conrad, M. H. Shaevitz, and T. Bolton, *Rev. Mod. Phys.* **70**, 1341–1392 (1998) [hep-ex/9707015]. (I–A)

336. "HERA collider physics," H. Abramowicz and A. Caldwell, *Rev. Mod. Phys.* **71**, 1275–1410 (1999)

[hep-ex/9903037]. (I–A)

337. Deep Inelastic Scattering, R. Devenish and A. Cooper-Sarkar (Oxford University Press, Oxford & New York, 2004). (I–A)

338. "Collider physics at HERA," M. Klein and R. Yoshida, *Prog. Part. Nucl. Phys.* **61**, 343–393 (2008) [arXiv:0805.3334 [hep-ex]]. (I–A)

The series of annual workshops on deeply inelastic scattering and QCD may be traced from

339. Deep Inelastic Scattering: Proceedings of the XVII International Workshop, edited by C. Glasman and J. Terron (Science Wise Publishing, 2009) [http://www.sciwipub.com/index.php?doit=dis2009]. (A)

In addition to its "valence" components, a hadron contains quark-antiquark pairs and gluons, by virtue of quantum fluctuations. In the extreme limit $Q \to \infty$, for any hadron, the momentum fraction carried by gluons approaches 8/17, and that carried by any of the six species of quark or antiquark approaches 3/68. The asymptotic equilibrium partition reflects the relative strengths of the quark-antiquark-gluon and three-gluon couplings, as well as the number of flavors. The current state of the art for parton distributions (at finite Q) is comprehensively documented in

340. "Parton distributions," M. Dittmar *et al.*, arXiv:0901.2504 [hep-ph]. (A)

A library providing a common interface to many modern sets of parton distributions is

341. "LHAPDF: the Les Houches Accord parton distribution function interface," M. R. Whalley and A. Buckley, http://projects.hepforge.org/lhapdf. (A)

The sets of parton distributions currently in wide use may be traced from

342. "Implications of CTEQ global analysis for collider observables," P. M. Nadolsky *et al.*, *Phys. Rev.* **D78**, 013004 (2008) [arXiv:0802.0007 [hep-ph]]. (A)

343. "The CTEQ Meta-Page," CTEQ Collaboration, http://www.cteq.org. (A)

344. "Parton distributions for the LHC," A. D. Martin, W. J. Stirling, R. S. Thorne, and G. Watt, *Eur. Phys. J.* **C63**, 189–285 (2009) [arXiv:0901.0002 [hep-ph]]. (I–A)

345. "MSTW parton distribution functions," A. D. Martin, W. J. Stirling, R. S. Thorne, and G. Watt, http://projects.hepforge.org/mstwpdf/. (A)

346. "Dynamical NNLO parton distributions," P. Jimenez-Delgado and E. Reya, *Phys. Rev.* **D79**, 074023 (2009) [arXiv:0810.4274 [hep-ph]]. (A)

347. "Dynamical Parton Distribution Functions," P. Jimenez-Delgado *et al.*, http://doom.physik.uni-dortmund.de/pdfserver. (A)

348. "Combined measurement and QCD analysis of the inclusive ep scattering cross sections at HERA," F. D. Aaron *et al.*, H1 and ZEUS Collaborations, arXiv:0911.0884 [hep-ex]. (A)

349. "A first unbiased global NLO determination of parton distributions and their uncertainties," R. D. Ball *et al.*, NNPDF Collaboration, arXiv:1002.4407 [hep-ph]. (I–A)

350. "Fixed target Drell-Yan data and NNLO QCD fits of parton distribution functions," S. Alekhin, K. Melnikov, and F. Petriello, *Phys. Rev.* **D74**, 054033 (2006) [hep-ph/0606237] [http://mail.ihep.ru/~alekhin/pdfs.html]. (A)

It is conventional to separate quark (and antiquark) distributions into "valence" components that account for a hadron's net quantum numbers and "sea" contributions in which quarks balance antiquarks overall. Neither a symmetry nor QCD dynamics demand that $q_i(x) = \bar{q}_i(x)$ locally, and experiment has now revealed a flavor asymmetry in the light-quark sea of the proton.

351. "High-energy hadron-induced dilepton production from nucleons and nuclei," P. L. McGaughey, J. M. Moss, and J. C. Peng, *Ann. Rev. Nucl. Part. Sci.* **49**, 217–253 (1999) [hep-ph/9905409]. (I–A)

352. "Flavor asymmetry of antiquark distributions in the nucleon," S. Kumano, *Phys. Rept.* **303**, 183–257 (1998) [hep-ph/9702367]. (I–A)

Sum rules that parton distributions must respect in QCD are reviewed in

353. "Parton-model sum rules," I. Hinchliffe and A. Kwiatkowski, *Ann. Rev. Nucl. Part. Sci.* **46**, 609–645 (1996) [hep-ph/9604210]. (I–A)

The number densities $q(x, Q^2)$, $\bar{q}(x, Q^2)$, and $g(x, Q^2)$, of quarks, antiquarks, and gluons within a hadron can be calculated at large Q^2 by Altarelli-Parisi evolution (Ref. 327) from initial distributions determined at Q_0^2. However, at small values of the momentum fraction x, the resulting densities may become large enough that the partons overlap spatially, so that scattering and recombination may occur, as argued in

354. "Semihard processes in QCD," L. V. Gribov, E. M. Levin, and M. G. Ryskin, *Phys. Rept.* **100**, 1–150 (1983) [doi: 10.1016/0370-1573(83)90022-4]. (I–A)

Recombination probabilities were computed in

355. "Gluon recombination and shadowing at small values of x," A. H. Mueller and J.-W. Qiu, *Nucl. Phys.* **B268**, 427 (1986) [doi: 10.1016/0550-3213(86)90164-1]. (I–A)

and expectations for lepton-nucleon scattering at very small values x are developed in

356. "Small x physics in deep inelastic lepton hadron scattering," B. Badełek, M. Krawczyk, K. Charchula, and

J. Kwiecinski, *Rev. Mod. Phys.* **64**, 927–960 (1992) [doi: 10.1103/RevModPhys.64.927]. (I–A)

357. "Low Q^2, low x region in electroproduction: an Overview," B. Badełek and J. Kwieciński, *Rev. Mod. Phys.* **68**, 445–471 (1996) [hep-ph/9408318]. (I–A)

358. "Small-x physics in perturbative QCD," L. N. Lipatov, *Phys. Rept.* **286**, 131–198 (1997) [hep-ph/9610276]. (A)

359. "Solution to the evolution equation for high parton density QCD," E. Levin and K. Tuchin, *Nucl. Phys.* **B573**, 833–852 (2000) [hep-ph/9908317]. (A)

Experiments at the $e^{\mp}p$ collider HERA, which operated at c.m. energies up to $\sqrt{s} = 320$ GeV, probed the small-x regime and established a rapid rise in the parton densities as $x \to 0$, as reviewed in Refs. 336,338. However, recombination phenomena have not yet been demonstrated. Implications of the HERA observations for future experiments are explored in

360. "Small-x physics: from HERA to LHC and beyond," L. Frankfurt, M. Strikman, and C. Weiss, *Ann. Rev. Nucl. Part. Sci.* **55**, 403–465 (2005) [hep-ph/0507286]. (A)

Our knowledge of the spin structure of the proton at the constituent level is drawn from polarized deeply inelastic scattering experiments, in which polarized leptons or photons probe the structure of a polarized proton and polarized proton-proton collisions. How current understanding developed, and what puzzles arose, can be traced in

361. "Spin physics and polarized structure functions," B. Lampe and E. Reya, *Phys. Rept.* **332**, 1–163 (2000) [hep-ph/9810270]. (I–A)

362. "Spin structure functions," E. W. Hughes and R. Voss, *Ann. Rev. Nucl. Part. Sci.* **49**, 303–339 (1999) [doi: 10.1146/annurev.nucl.49.1.303]. (I–A)

363. "The spin structure of the nucleon," B. W. Filippone and X.-D. Ji, *Adv. Nucl. Phys.* **26**, 1 (2001) [hep-ph/0101224]. (I–A)

364. "The spin structure of the proton," S. D. Bass, *Rev. Mod. Phys.* **77**, 1257–1302 (2005) [hep-ph/0411005]. (I–A)

365. "The spin structure of the nucleon," W. Vogelsang, *J. Phys.* **G34**, S149–S171 (2007) [doi: 10.1088/0954-3899/34/7/S08]. (I–A)

366. The Spin Structure of the Proton, S. D. Bass (World Scientific, Singapore, 2007). (I–A)

367. "The spin of the proton," A. W. Thomas, *Prog. Part. Nucl. Phys.* **61**, 219 (2008) [arXiv:0805.4437 [hep-ph]]. (I–A)

368. "Spin structure of the nucleon—status and recent results," S. E. Kuhn, J. P. Chen, and E. Leader, *Prog. Part. Nucl. Phys.* **63**, 1–50 (2009) [arXiv:0812.3535 [hep-ph]]. (I–A)

369. "COMPASS and HERMES contributions to our understanding of the nucleon spin," F. Bradamante, *Prog. Part. Nucl. Phys.* **61**, 229–237 (2008) [doi: 10.1016/j.ppnp.2007.12.046]. (I–A)

Progress in making spin-dependent measurements can be traced through the spin physics symposia; the latest in the series is

370. "18th International Symposium on Spin Physics," http://faculty.virginia.edu/spin2008/. (I–A)

For a set of spin-dependent parton distribution functions, with extensive references to the underlying measurements, see

371. "Extraction of spin-dependent parton densities and their uncertainties," D. De Florian, R. Sassot, M. Stratmann, and W. Vogelsang, *Phys. Rev.* **D80**, 034030 (2009) [arXiv:0904.3821 [hep-ph]]. (I–A)

Standard parton distribution functions provide detailed information about how spin and longitudinal momentum and spin are partitioned among the quarks, antiquarks, and gluons in a fast-moving hadron, but the information is integrated over transverse degrees of freedom. The role of orbital angular momentum of the partons in building a spin-$\frac{1}{2}$ proton is obscured. Generalized parton distributions inferred from exclusive scattering processes provide a tool for probing such subtleties of hadron structure.

372. "Generalized parton distributions," M. Diehl, *Phys. Rept.* **388**, 41–277 (2003) [hep-ph/0307382]. (I–A)

373. "Generalized parton distributions," X. Ji, *Ann. Rev. Nucl. Part. Sci.* **54**, 413–450 (2004) [doi: 10.1146/annurev.nucl.54.070103.181302]. (I–A)

374. "Unraveling hadron structure with generalized parton distributions," A. V. Belitsky and A. V. Radyushkin, *Phys. Rept.* **418**, 1–387 (2005) [hep-ph/0504030]. (I–A)

375. "Hadron structure from lattice quantum chromodynamics," P. Hägler, *Phys. Rept.* **490**, 49–175 (2010) [arXiv:0912.5483 [hep-lat]]. (I–A)

An important undertaking of modern hadron physics is to understand how hidden flavors (*e.g.*, virtual $s\bar{s}$ pairs) contribute to the structure of the nucleon. Recent experimental and theoretical progress toward unravelling the role of strange quarks in the nucleon can be traced in

376. "Nucleon electromagnetic form factors," J. Arrington, C. D. Roberts, and J. M. Zanotti, *J. Phys.* **G34**, S23–S52 (2007) [nucl-th/0611050]. (I–A)

377. "Nucleon electromagnetic form factors," C. F. Perdrisat, V. Punjabi, and M. Vanderhaeghen, *Prog. Part. Nucl. Phys.* **59**, 694–764 (2007) [hep-ph/0612014]. (I–A)

In analogy to the hidden flavors of light quarks, hadrons could have an intrinsic component of charm-anticharm pairs:

378. "The intrinsic charm of the proton," S. J. Brodsky, P. Hoyer, C. Peterson, and N. Sakai, *Phys. Lett.* **B93**, 451–455 (1980) [doi: `10.1016/0370-2693(80)90364-0`]. (I–A)

379. "The charm parton content of the nucleon," J. Pumplin, H. L. Lai, and W. K. Tung, *Phys. Rev.* **D75**, 054029 (2007) [`hep-ph/0701220`]. (I–A)

380. "The 3-, 4-, and 5-flavor NNLO parton from deep-inelastic-scattering data and at hadron colliders," S. Alekhin, J. Blümlein, S. Klein, and S. Moch, *Phys. Rev.* **D81**, 014032 (2010) [`arXiv:0908.2766 [hep-ph]`]. (A)

F. Quarkonium

An early opportunity for QCD-inspired models of hadrons came with the discovery of the J/ψ particle and other bound states of charmed quarks and antiquarks,

381. "Experimental observation of a heavy particle J," J. J. Aubert *et al.*, *Phys. Rev. Lett.* **33**, 1404–1406 (1974) [doi: `10.1103/PhysRevLett.33.1404`]. (I)

382. "Discovery of a narrow resonance in e^+e^- annihilation," J. E. Augustin *et al.*, *Phys. Rev. Lett.* **33**, 1406–1408 (1974) [doi: `10.1103/PhysRevLett.33.1406`]. (I)

For an account hard on the heels of the discovery, see

383. "Electron-positron annihilation and the new particles," S. D. Drell, *Sci. Am.* **232**, 50–62 (June, 1975). (E)

Early perspectives on the implications are given in the Nobel Lectures,

384. "The discovery of the J particle: a personal recollection," S. C. C. Ting, *Rev. Mod. Phys.* **49**, 235–249 (1977) [doi: `10.1103/RevModPhys.49.235`]. (I)

385. "From the ψ to charm: the experiments of 1975 and 1976," B. Richter, *Rev. Mod. Phys.* **49**, 251–266 (1977) [doi: `10.1103/RevModPhys.49.251`]. (I)

Quarkonium spectroscopy was enriched by the discovery of the Υ family of $b\bar{b}$ bound states:

386. "Observation of a dimuon resonance at 9.5 GeV in 400-GeV proton-nucleus collisions," S. W. Herb *et al.*, *Phys. Rev. Lett.* **39**, 252–255 (1977) [doi: `10.1103/PhysRevLett.39.252`]. (I)

387. "Observation of structure in the Υ region," W. R. Innes *et al.*, *Phys. Rev. Lett.* **39**, 1240 (1977) [doi: `10.1103/PhysRevLett.39.1240`]. Erratum: *ibid.* 1640 (1977). (I)

An accessible account of these discoveries is

388. "The Upsilon particle," L. M. Lederman, *Sci. Am.* **239**, 60–68 (1978). (E)

For a summary of early comparisons between the $c\bar{c}$ and $b\bar{b}$ families, see

389. "Quarkonium," E. D. Bloom and G. J. Feldman, *Sci. Am.* **246**, 66–77 (May, 1982). (E)

These discoveries spurred the development of potential models (see Sec III B). Reviews of this work from the experimental perspective are in

390. "Upsilon resonances," P. Franzini and J. Lee-Franzini, *Ann. Rev. Nucl. Part. Sci.* **33**, 1–29 (1983) [doi: `10.1146/annurev.ns.33.120183.000245`]. (I)

391. "Upsilon spectroscopy: transitions in the bottomonium system," D. Besson and T. Skwarnicki, *Ann. Rev. Nucl. Part. Sci.* **43**, 333–378 (1993) [doi: `10.1146/annurev.ns.43.120193.002001`]. (I–A)

Calculations of the quarkonium spectrum, once the exclusive province of potential models (*cf.* Sec. III B), are an important theme in lattice QCD. Three of the first papers on calculations with $2 + 1$ flavors of sea quarks are

392. "The Υ spectrum and m_b from full lattice QCD," A. Gray *et al.*, *Phys. Rev.* **D72**, 094507 (2005) [`hep-lat/0507013`]. (A)

393. "Highly improved staggered quarks on the lattice, with applications to charm physics," E. Follana *et al.*, HPQCD Collaboration, *Phys. Rev.* **D75**, 054502 (2007) [`hep-lat/0610092`]. (A)

394. "Quarkonium mass splittings in three-flavor lattice QCD," T. Burch *et al.*, *Phys. Rev.* **D81**, 034508 (2010) [`arXiv:0912.2701 [hep-lat]`]. (A)

The breadth of quarkonium physics—experimental, theoretical, and computational—is surveyed in

395. "Heavy quarkonium physics," N. Brambilla *et al.*, Quarkonium Working Group, *CERN Yellow Report* CERN–2005–005 (2005) [`hep-ph/0412158`]. (I–A)

A novel form of quarkonium arises from binding a bottom quark and a charmed antiquark. The first observation of the pseudoscalar B_c meson is reported in

396. "Observation of the B_c meson in $p\bar{p}$ collisions at $\sqrt{s} = 1.8$ TeV," F. Abe *et al.*, CDF Collaboration, *Phys. Rev. Lett.* **81**, 2432–2437 (1998) [`hep-ex/9805034`]. (I–A)

Precise measurements of the mass did not appear until later:

397. "Evidence for the exclusive decay $B_c^\pm \to J/\psi\pi^\pm$ and measurement of the mass of the B_c meson," A. Abulencia *et al.*, CDF Collaboration, *Phys. Rev. Lett.* **96**, 082002 (2006) [`hep-ex/0505076`]. (I–A)

The mass of the B_c was correctly predicted by PNRQCD:

398. "The B_c mass up to order α_s^4," N. Brambilla and A. Vairo, *Phys. Rev.* **D62**, 094019 (2000) [`hep-ph/0002075`]. (A)

and lattice QCD:

399. "Mass of the B_c meson in three-flavor lattice QCD," I. F. Allison *et al.*, HPQCD and Fermilab Lattice Collaborations, *Phys. Rev. Lett.* **94**, 172001 (2005) [`hep-lat/0411027`]. (I–A)

Recently, a new set of states has appeared in the charmonium spectrum that presents new challenges to hadron dynamics. Some of these may be (mostly) charm-anticharm states above the threshold for decay into charmed-meson pairs. Others cannot readily be identified in the same way. For a recent survey, see

400. "The exotic XYZ charmonium-like mesons," S. Godfrey and S. L. Olsen, *Ann. Rev. Nucl. Part. Sci.* **58**, 51–73 (2008) [`arXiv:0801.3867 [hep-ph]`]. (I–A)

G. Jets in hadron collisions

An account of early evidence for jet structure in the first pp collider, at energies up to $\sqrt{s} = 63$ GeV, is given in

401. "The jet cross section in pp interactions at $\sqrt{s} =$ 45 GeV and its $\sqrt{s}$ dependence," T. Åkesson *et al.*, Axial Field Spectrometer Collaboration, *Phys. Lett.* **B123**, 133 (1983) [doi: `10.1016/0370-2693(83)90973-5`]. (I–A)

Incisive comparisons with QCD were made in experiments at the SPS Collider, at energies up to 630 GeV:

402. "Measurement of the $\sqrt{s}$ dependence of jet production at the CERN $\bar{p}p$ Collider," J. A. Appel *et al.*, UA2 Collaboration, *Phys. Lett.* **160B**, 349 (1985) [doi: `10.1016/0370-2693(85)91341-3`]. (I–A)

403. "Inclusive jet cross-section and a search for quark compositeness at the CERN $\bar{p}p$ Collider," J. Alitti *et al.*, UA2 Collaboration, *Phys. Lett.* **257B**, 232–240 (1991) [doi: `10.1016/0370-2693(91)90887-V`]. (I–A)

404. "Measurement of the inclusive jet production cross section at the CERN $\bar{p}p$ Collider," G. Arnison *et al.*, UA1 Collaboration, *Phys. Lett.* **172B**, 461–466 (1986) [doi: `10.1016/0370-2693(86)90290-X`]. (I–A)

Extensive studies have been carried out at the Tevatron Collider, at energies up to $\sqrt{s} = 1.96$ TeV. We show in Fig. 8 that perturbative QCD, evaluated at next-to-leading order, accounts for the transverse-momentum spectrum of central jets produced in the reaction

$$\bar{p}p \to \text{jet}_1 + \text{jet}_2 + \text{anything} \tag{33}$$

over more than eight orders of magnitude:

405. "Measurement of the inclusive jet cross-section in $p\bar{p}$ collisions at $\sqrt{s} = 1.96$ TeV," V. M. Abazov *et al.*, D0 Collaboration, *Phys. Rev. Lett.* **101**, 062001 (2008) [`arXiv:0802.2400 [hep-ex]`]. (I–A)

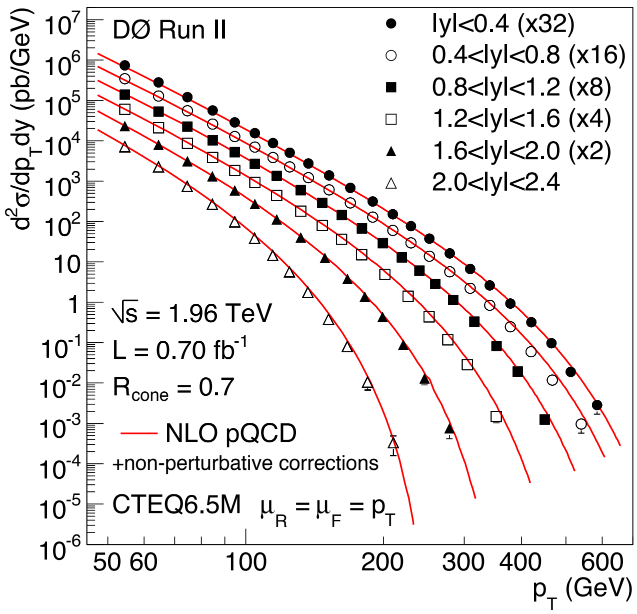

FIG. 8: The inclusive jet cross section measured at $\sqrt{s} =$ 1.96 TeV by the D0 Collaboration (Ref. 405) as a function of transverse momentum in six rapidity bins. The data points are multiplied by 2, 4, 8, 16, and 32 for the bins $1.6 < |y| < 2.0$, $1.2 < |y| < 1.6$, $0.8 < |y| < 1.2$, $0.4 < |y| < 0.8$, and $|y| < 0.4$, respectively. Solid curves show next-to-leading-order (one-loop) perturbative QCD predictions. A 6.1% uncertainty on the integrated luminosity is not included. Theoretical predictions carry an uncertainty of approximately 10%.

Similar results from the CDF experiment are reported in

406. "Measurement of the inclusive jet cross section using the k_T algorithm in $p\bar{p}$ collisions at $\sqrt{s} =$ 1.96 TeV with the CDF II detector," A. Abulencia *et al.*, CDF Collaboration, *Phys. Rev.* **D75**, 092006 (2007) [`hep-ex/0701051`]. (I–A)

For a summary of recent QCD studies at the Tevatron, see

407. "Jet physics at the Tevatron," A. Bhatti and D. Lincoln, `arXiv:1002.1708 [hep-ex]`. (I–A)

The current state of the art is presented in

408. "Hard interactions of quarks and gluons: a primer for LHC physics," J. M. Campbell, J. W. Huston, and W. J. Stirling, *Rept. Prog. Phys.* **70**, 89 (2007) [`hep-ph/0611148`]. (I–A)

Jet phenomena in relativistic heavy-ion collisions are summarized in

409. "Review of hard scattering and jet analysis," M. J. Tannenbaum, *PoS* **CFRNC2006**, 001 (2006) [`nucl-ex/0611008`]. (A)

For a discussion of jet definitions and their interplay with measurements, see Ref. 323.

H. Photon structure function

The proposal to determine the constituent structure of the photon by studying the scattering of a highly virtual photon on a real photon is due to

410. "Deep inelastic scattering of electrons on a photon target," S. J. Brodsky, T. Kinoshita, and H. Terazawa, *Phys. Rev. Lett.* **27**, 280–283 (1971) [doi: `10.1103/PhysRevLett.27.280`]. (I)

411. "Inelastic electron-photon scattering," T. F. Walsh, *Phys. Lett.* **B36**, 121–123 (1971) [doi: `10.1016/0370-2693(71)90124-9`]. (I)

To the extent that a photon behaves as a vector meson, the momentum-fraction (x) and momentum-transfer (Q^2) dependences of its structure function should roughly resemble those of the proton structure function. But a parton-model calculation reveals that a pointlike contribution that arises when the photon fluctuates into a quark-antiquark pair should dominate over the vector-meson component at high Q^2.

412. "Two-photon processes in the parton model," T. F. Walsh and P. M. Zerwas, *Phys. Lett.* **B44**, 195–198 (1973) [doi: `10.1016/0370-2693(73)90520-0`]. (I)

413. "Anomalies in photon-photon scattering reactions," R. L. Kingsley, *Nucl. Phys.* **B60**, 45–51 (1973) [doi: `10.1016/0550-3213(73)90168-5`]. (I)

Remarkably, the x-dependence of the photon structure function is fully calculable at large Q^2, in contrast to the proton structure function, for which the x-dependence at fixed Q^2 results from nonperturbative effects and, in practice, is taken from the data or, in the approach of Ref. 346, from a simple Ansatz.

QCD confirms the calculability of the photon structure function at large Q^2, and differs from the parton-model result, particularly as $x \to 1$. In leading logarithmic approximation, the result is reported in

414. "Anomalous cross-section for photon-photon scattering in gauge theories," E. Witten, *Nucl. Phys.* **B120**, 189–202 (1977) [doi: `10.1016/0550-3213(77)90038-4`]. (I–A)

The next-to-leading-order calculation improves the reliability of the predicted shape of the photon structure function:

415. "Higher-order asymptotic freedom corrections to photon-photon scattering," W. A. Bardeen and A. J. Buras, *Phys. Rev.* **D20**, 166 (1979) [doi: `10.1103/PhysRevD.20.166`]. Erratum: *ibid.* **D21**, 2041 (1980). (A)

also enabling a determination of the strong coupling α_s, now at the 5% level.

For an excellent short review, see

416. "Photon structure functions: 1978 and 2005," A. J. Buras, *Acta Phys. Polon.* **B37**, 609–618 (2006) [`hep-ph/0512238`]. (I)

Extensive experimental summaries appear in

417. "The photon structure from deep inelastic electron photon scattering," R. Nisius, *Phys. Rept.* **332**, 165–317 (2000) [`hep-ex/9912049`]. (I–A)

418. "Survey of present data on photon structure functions and resolved photon processes," M. Krawczyk, A. Zembrzuski, and M. Staszel, *Phys. Rept.* **345**, 265–450 (2001) [`hep-ph/0011083`]. (I–A)

A useful digest appears in Fig. 16.14 of Ref. 8.

I. Diffractive Scattering

The Pomeranchuk singularity, or Pomeron, designates the Regge pole with vacuum quantum numbers that controls the asymptotic behavior of elastic and total cross sections. The Regge intercept of the Pomeron, the location of the pole in the complex angular-momentum plane at zero momentum transfer, would be $\alpha_{\mathbb{P}} = 1$ if total cross sections approached constants at high energies. Comprehensive modern fits to meson-baryon and especially proton-(anti)proton total cross sections initiated in

419. "Total cross-sections," A. Donnachie and P. V. Landshoff, *Phys. Lett.* **B296**, 227–232 (1992) [`hep-ph/9209205`]. (I)

indicate that $\alpha_{\mathbb{P}} \approx 1.08$.

With the advent of quantum chromodynamics, it was natural to begin searching for a dynamical description of the Pomeron's origin. The idea that the Pomeron somehow emerged from the exchange of color-octet gluons between color-singlet hadrons, first articulated in

420. "A model of the bare Pomeron," F. E. Low, *Phys. Rev.* **D12**, 163–173 (1975) [doi: `10.1103/PhysRevD.12.163`]. (I)

has great resonance today. A concrete realization of the Pomeron in QCD as a composite state of two Reggeized gluons was developed (in leading logarithmic approximation) by

421. "The Pomeranchuk singularity in nonabelian gauge theories," E. A. Kuraev, L. N. Lipatov, and V. S. Fadin, *Sov. Phys. JETP* **45**, 199–204 (1977). (A)

422. "The Pomeranchuk singularity in quantum chromodynamics," I. I. Balitsky and L. N. Lipatov, *Sov. J. Nucl. Phys.* **28**, 822–829 (1978). (A)

The dynamics underlying the "BFKL Pomeron" also entail a resummation of leading $1/x$ corrections to structure functions.

In spite of much productive effort, the origin of the Pomeron and the details of its structure are still not entirely clear:

423. Quantum Chromodynamics and the Pomeron, J. R. Forshaw and D. A. Ross (Cambridge University Press, Cambridge & New York, 1997). (I–A)

424. Pomeron Physics and QCD, S. Donnachie, G. Dosch, P. Landshoff, and O. Nachtmann (Cambridge University Press, Cambridge & New York, 2002). (I–A)

Although the Pomeron was conceived to account for "soft" scattering, the QCD interpretation implies that it should have a partonic structure, and thus a "hard" component. For a survey of evidence for a hard component from $e^{\mp}p$ collisions at HERA, see

425. "Review of high-energy diffraction in real and virtual photon-proton scattering at HERA," G. Wolf, `arXiv:0907.1217 [hep-ex]`. (I–A)

The suggestion that Pomeron exchange should result in large rapidity gaps between jets crystallized in

426. "Rapidity gaps and jets as a new physics signature in very high-energy hadron hadron collisions," J. D. Bjorken, *Phys. Rev.* **D47**, 101–113 (1993) [doi: `10.1103/PhysRevD.47.101`]. (I–A)

Evidence for events of the suggested character was presented in

427. "Rapidity gaps between jets in $p\bar{p}$ collisions at $\sqrt{s} = 1.8$ TeV," S. Abachi *et al.*, D0 Collaboration, *Phys. Rev. Lett.* **72**, 2332–2336 (1994) [doi: `10.1103/PhysRevLett.72.2332`]. (I–A)

428. "Observation of rapidity gaps in $\bar{p}p$ collisions at 1.8 TeV," F. Abe *et al.*, CDF Collaboration, *Phys. Rev. Lett.* **74**, 855–859 (1995) [doi: `10.1103/PhysRevLett.74.855`]. (I–A)

If the Pomeron can be exchanged between color-singlets, then, in analogy with two-photon physics or the multiperipheral model, two Pomerons can collide and produce collections of particles with net vacuum quantum numbers, isolated by large rapidity gaps from other particle production in the event, as observed in

429. "Observation of exclusive charmonium production and $\gamma\gamma \to \mu^+\mu^-$ in $p\bar{p}$ collisions at $\sqrt{s} = 1.96$ TeV," T. Aaltonen *et al.*, CDF Collaboration, *Phys. Rev. Lett.* **102**, 242001 (2009) [`arXiv:0902.1271 [hep-ex]`]. (I–A)

A tantalizing possibility is that the Higgs boson might be discovered at the LHC in very quiet events:

430. "Diffractive Higgs production at the LHC," A. D. Martin, V. A. Khoze, and M. G. Ryskin, `hep-ph/0605189`. (I–A)

J. Weak boson production

In hadron colliders, an electroweak vector boson can be produced directly via fusion of a quark and antiquark. The cross section depends on the parton distributions discussed in Sec. IV E. This extension of the parton model was first noted in

431. "Massive lepton pair production in hadron-hadron collisions at high-energies," S. D. Drell and T.-M. Yan, *Phys. Rev. Lett.* **25**, 316–320 (1970) [doi: `10.1103/PhysRevLett.25.316`]. (I–A)

This process provided the basis for the discovery of the W and Z bosons:

432. "Experimental observation of the intermediate vector bosons W^+, W^-, and Z^0," C. Rubbia, *Rev. Mod. Phys.* **57**, 699–722 (1985) [doi: `10.1103/RevModPhys.57.699`]. (E–I–A)

What was then a QCD-guided discovery is now one of the most precise tests of perturbative QCD. The production cross sections and rapidity distributions for the Tevatron and the LHC have been carried out to the next-to-next-to-leading order in α_s in

433. "A complete calculation of the order α_s^2 correction to the Drell-Yan K factor," R. Hamberg, W. L. Van Neerven, and T. Matsuura, *Nucl. Phys.* **B359**, 343–405 (1991) [doi: `10.1016/0550-3213(91)90064-5`]. (A)

434. "High-precision QCD at hadron colliders: electroweak gauge boson rapidity distributions at next-to-next-to-leading order," C. Anastasiou, L. Dixon, K. Melnikov, and F. Petriello, *Phys. Rev.* **D69**, 094008 (2004) [`hep-ph/0312266`]. (A)

These calculations have been validated (except at the largest accessible values of rapidity) in measurements performed at the Tevatron:

435. "Measurement of the shape of the boson rapidity distribution for $p\bar{p} \to Z/\gamma^* \to e^+e^-X$ events produced at $\sqrt{s}$ of 1.96 TeV," V. M. Abazov *et al.*, D0 Collaboration, *Phys. Rev.* **D76**, 012003 (2007) [`hep-ex/0702025`]. (A)

436. "Measurement of $d\sigma/dy$ of Drell-Yan e^+e^- pairs in the Z mass region from $p\bar{p}$ collisions at $\sqrt{s} = 1.96$ TeV," T. Aaltonen *et al.*, CDF Collaboration, `arXiv:0908.3914 [hep-ex]`. (A)

Production of electroweak bosons together with jets is discussed in Sec. IV K.

This history is set to repeat itself in the search for the Higgs boson, which relies on the next-to-next-to-leading order QCD calculation:

437. "Next-to-next-to-leading order Higgs production at hadron colliders," R. V. Harlander and W. B. Kilgore, *Phys. Rev. Lett.* **88**, 201801 (2002) [`hep-ph/0201206`]. (A)

438. "Higgs boson production at hadron colliders: differential cross sections through next-to-next-to-leading order," C. Anastasiou, K. Melnikov, and F. Petriello, *Phys. Rev. Lett.* **93**, 262002 (2004) [`hep-ph/0409088`]. (A)

439. "An NNLO subtraction formalism in hadron collisions and its application to Higgs boson production at the LHC," S. Catani and M. Grazzini, *Phys. Rev. Lett.* **98**, 222002 (2007) [`hep-ph/0703012`]. (A)

The Higgs-boson searches at the Tevatron and the LHC rely on these results, and on comparably precise calculations of background processes, in an essential way.

K. Heavy-quark production

Another probe of the short-distance dynamics of QCD is the production of heavy quark-antiquark pairs in hadron collisions:

440. "The total cross-section for the production of heavy quarks in hadronic collisions," P. Nason, S. Dawson, and R. K. Ellis, *Nucl. Phys.* **B303**, 607 (1988) [doi: 10.1016/0550-3213(88)90422-1]. (A)

441. "QCD corrections to heavy quark production in $p\bar{p}$ collisions," W. Beenakker, H. Kuijf, W. L. Van Neerven, and J. Smith, *Phys. Rev.* **D40**, 54–82 (1989) [doi: 10.1103/PhysRevD.40.54]. (A)

442. "Heavy quark correlations in hadron collisions at next-to-leading order," M. L. Mangano, P. Nason, and G. Ridolfi, *Nucl. Phys.* **B373**, 295–345 (1992) [doi: 10.1016/0550-3213(92)90435-E]. (A)

An important application is the production of the top quark at the Tevatron:

443. "Top quark production cross-section," E. Laenen, J. Smith, and W. L. Van Neerven, *Phys. Lett.* **B321**, 254–258 (1994) [hep-ph/9310233]. (A)

Measurements of the top quark mass have been combined into an average:

444. "Combination of CDF and D0 results on the mass of the top quark," Tevatron Electroweak Working Group, arXiv:0903.2503 [hep-ex]. (A)

yielding the result

$$m_t = 173.1 \pm 1.3 \text{ GeV}, \tag{34}$$

where this mass has a more conventional definition (similar to that of the electron). The top-quark mass is now precise enough that the ambiguities raised in Refs. 144,145 are becoming quantitatively important.

QCD calculations are important not only to gain an understanding of the experimental signal, but also to understand the background, which stems from W production:

445. "On the production of a W and jets at hadron colliders," F. A. Berends, H. Kuijf, B. Tausk, and W. T. Giele, *Nucl. Phys.* **B357**, 32–64 (1991) [doi: 10.1016/0550-3213(91)90458-A]. (A)

446. "Next-to-leading order corrections to $W+2$ jet and $Z+2$ jet production at hadron colliders," J. M. Campbell and R. K. Ellis, *Phys. Rev.* **D65**, 113007 (2002) [hep-ph/0202176]. (A)

447. "$W+3$-jet production at the Tevatron," R. K. Ellis, K. Melnikov, and G. Zanderighi, *Phys. Rev.* **D80**, 094002 (2009) [arXiv:0906.1445 [hep-ph]]. (A)

448. "Next-to-leading order QCD predictions for $W+$ 3-jet distributions at hadron colliders," C. F. Berger *et al.*, *Phys. Rev.* **D80**, 074036 (2009) [arXiv:0907.1984 [hep-ph]]. (A)

L. Inclusive B decays

Another useful application of perturbative QCD is to inclusive decays of hadrons containing a heavy quark. In practice, this approach applies to hadrons with the bottom quark. One again appeals to quark-hadron duality and applies the operator-product expansion to factorize the differential rate into short- and long-distance contributions. This rich subject launched with

449. "Lepton energy distributions in heavy meson decays from QCD," J. Chay, H. Georgi, and B. Grinstein, *Phys. Lett.* **B247**, 399–405 (1990) [doi: 10.1016/0370-2693(90)90916-T]. (I–A)

The arc of this research is explained pedagogically in Ref. 178 and in further detail in

450. "Imprecated, yet impeccable: on the theoretical evaluation of $\Gamma(B \to X_c l\nu)$," D. Benson, I. I. Bigi, T. Mannel, and N. Uraltsev, *Nucl. Phys.* **B665**, 367–401 (2003) [hep-ph/0302262]. (I–A)

This formalism has several applications, using the experimental data to gain insight into long-distance QCD on the one hand, and to determine the bottom quark's flavor-changing weak couplings. Both perspectives are treated in a thorough analysis of the then-current theory and data:

451. "Fits to moment measurements from $B \to X_c l\nu$ and $B \to X_s\gamma$ decays using heavy-quark expansions in the kinetic scheme," O. Buchmüller and H. Flächer, *Phys. Rev.* **D73**, 073008 (2006) [hep-ph/0507253]. (A)

A by-product of these analyses is another determination of the bottom-quark mass. The status is summarized in

452. "Flavor physics in the quark sector," M. Antonelli *et al.*, *Phys. Rept.* in press (2010) [arXiv:0907.5386 [hep-ph]]. (I–A)

This review covers all of flavor physics, including aspects pertaining to this and the next subsection, and well beyond.

M. Exclusive meson decays

Pseudoscalar mesons can decay via the weak interaction to a charged lepton and its neutrino, and the rate can be compared with lattice-QCD calculations of the transition amplitudes. For π and K mesons, the calculations and measurements agree well. For mesons with heavy

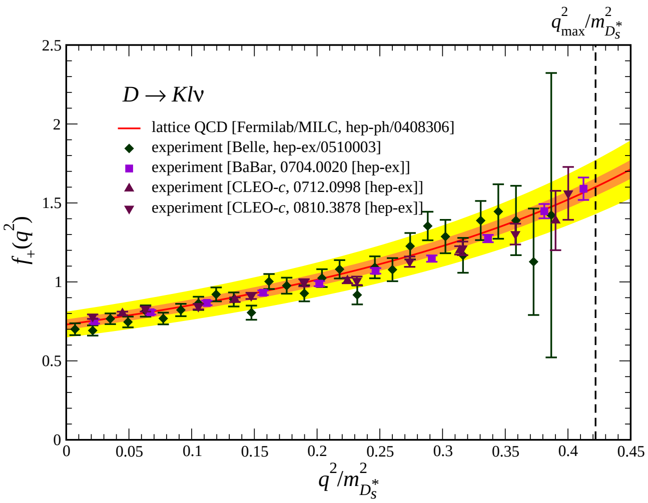

FIG. 9: Comparison lattice-QCD calculations and experimental measurements of the form factor $f_+(q^2)$ for the semileptonic decay $D \to Kl\nu$. The curve and error band show a lattice-QCD calculation; From Ref. 301; adapted from Ref. 453.

quarks, the measurements lag the calculations somewhat, and the agreement is good but not spectacular. The current status is thoroughly discussed in Ref. 301.

Pseudoscalar mesons can also decay via the weak interaction to a lighter hadron in association with the lepton-neutrino pair. These three-body decays are called semileptonic. Lattice-QCD calculations predicted the normalization and kinematic distribution of semileptonic D decays. A good place to start is

453. "Visualization of semileptonic form factors from lattice QCD," C. Bernard *et al.*, Fermilab Lattice and MILC Collaborations, *Phys. Rev.* **D80**, 034026 (2009) [`arXiv:0906.2498 [hep-lat]`]. (I–A)

from which a comparison of a QCD calculation with measurements from several experiments is reproduced in Fig. 9. Similar comparisons can be made for semileptonic kaon and B-meson decays.

Nonleptonic kaon decays are too computationally challenging for lattice QCD and, apart from constraints from chiral perturbation theory, too conceptually challenging via other approaches. Numerous nonleptonic B decays are kinematically allowed, posing conceptual challenges for lattice QCD. The high scale of the bottom-quark mass, however, allows a treatment in perturbative QCD, at least to leading order in $1/m_b$ (Refs. 141,195). Broad studies provide information on flavor-changing couplings of the Standard Model:

454. "QCD factorization in $B \to \pi K$, $\pi\pi$ decays and extraction of Wolfenstein parameters," M. Beneke, G. Buchalla, M. Neubert, and C. T. Sachrajda, *Nucl. Phys.* **B606**, 245–321 (2001) [`hep-ph/0104110`]. (A)

455. "SCET analysis of $B \to K\pi$, $B \to K\bar{K}$, and $B \to \pi\pi$ decays," C. W. Bauer, I. Z. Rothstein, and I. W. Stewart, *Phys. Rev.* **D74**, 034010 (2006) [`hep-ph/0510241`]. (A)

For a comprehensive set of references to the measurements and a comparison with calculations at the third order in α_s, see Table 3 of

456. "NNLO vertex corrections to non-leptonic B decays: tree amplitudes," M. Beneke, T. Huber, and X.-Q. Li, `arXiv:0911.3655 [hep-ph]`. (A)

For an alternative approach, see

457. "Penguin enhancement and $B \to K\pi$ decays in perturbative QCD," Y. Y. Keum, H.-N. Li, and A. I. Sanda, *Phys. Rev.* **D63**, 054008 (2001) [`hep-ph/0004173`]. (A)

N. Heavy-ion collisions and the quark-gluon plasma

One of the goals of relativistic heavy-ion collisions is to investigate the quark-hadron phase transition that presumably occurred in the early universe.

458. "The first few microseconds," M. Riordan and W. A. Zajc, *Sci. Am.* **294N5**, 24–31 (2006). (E)

459. "The hadron to quark/gluon transition," G. E. Brown, *Phys. Rept.* **242**, 261–267 (1994) [doi: `10.1016/0370-1573(94)90162-7`]. (A)

By creating small volumes with high energy density or high particle density, heavy-ion collisions open a window on new phases of matter.

460. "Highly relativistic nucleus-nucleus collisions: the central rapidity region," J. D. Bjorken, *Phys. Rev.* **D27**, 140–151 (1983) [doi: `10.1103/PhysRevD.27.140`]. (I–A)

461. "The condensed matter physics of QCD," K. Rajagopal and F. Wilczek, vol. 3 of Shifman, Ref. 39, ch. 35, p. 2061. [`hep-ph/0011333`]. (I–A)

462. "Color superconducting quark matter," M. G. Alford, *Ann. Rev. Nucl. Part. Sci.* **51**, 131–160 (2001) [`hep-ph/0102047`]. (A)

463. "The color glass condensate and high energy scattering in QCD," E. Iancu and R. Venugopalan, `hep-ph/0303204`. (I–A)

464. "Some features of the glasma," T. Lappi and L. McLerran, *Nucl. Phys.* **A772**, 200–212 (2006) [`hep-ph/0602189`]. (I–A)

465. "Color superconductivity in dense quark matter," M. G. Alford, A. Schmitt, K. Rajagopal, and T. Schäfer, *Rev. Mod. Phys.* **80**, 1455–1515 (2008) [`arXiv:0709.4635 [hep-ph]`]. (I–A)

Experiments at Brookhaven National Laboratory's Relativistic Heavy-Ion Collider (RHIC) imply the existence of a "perfect fluid" of quarks and gluons.

466. "What have we learned from the Relativistic Heavy Ion Collider?," T. Ludlam and L. McLerran, *Phys. Today* **56**, 48–54 (October, 2003) [doi: `10.1063/1.1629004`]. (E–I)

467. "Quark gluon plasma and color glass condensate at RHIC? The perspective from the BRAHMS experiment," I. Arsene *et al.*, *Nucl. Phys.* **A757**, 1–27 (2005) [`nucl-ex/0410020`]. (I–A)

468. "The PHOBOS perspective on discoveries at RHIC," B. B. Back *et al.*, *Nucl. Phys.* **A757**, 28–101 (2005) [`nucl-ex/0410022`]. (I–A)

469. "Experimental and theoretical challenges in the search for the quark gluon plasma: The STAR Collaboration's critical assessment of the evidence from RHIC collisions," J. Adams *et al.*, *Nucl. Phys.* **A757**, 102–183 (2005) [`nucl-ex/0501009`]. (I–A)

470. "Formation of dense partonic matter in relativistic nucleus-nucleus collisions at RHIC: experimental evaluation by the PHENIX Collaboration," K. Adcox *et al.*, *Nucl. Phys.* **A757**, 184–283 (2005) [`nucl-ex/0410003`]. (I–A)

471. "Results from the Relativistic Heavy Ion Collider," B. Müller and J. L. Nagle, *Ann. Rev. Nucl. Part. Sci.* **56**, 93–135 (2006) [`nucl-th/0602029`]. (I–A)

472. "The phase diagram of strongly-interacting matter," P. Braun-Munzinger and J. Wambach, *Rev. Mod. Phys.* **81**, 1031–1050 (2009) [`arXiv:0801.4256 [hep-ph]`]. (I–A)

A series of conference on quark matter may be traced starting at

473. "Quark Matter 2009," `http://www.phy.ornl.gov/QM09/`. (A)

The phase diagram is thought to be much richer beyond the region explored by heavy-ion collisions. Figure 10 shows a current conception of QCD thermodynamics (Ref. 465). The region of Fig. 10 with $\mu \approx 0$ has been demonstrated with lattice QCD:

474. "Status of lattice QCD at finite temperature," E. Laermann and O. Philipsen, *Ann. Rev. Nucl. Part. Sci.* **53**, 163–198 (2003) [`hep-ph/0303042`]. (I–A)

475. "QCD thermodynamics from the lattice," C. DeTar and U. M. Heller, *Eur. Phys. J.* **A41**, 405–437 (2009) [`arXiv:0905.2949 [hep-lat]`]. (I–A)

476. "The phase diagram of quantum chromodynamics," Z. Fodor and S. D. Katz, `arXiv:0908.3341 [hep-ph]`. (I–A)

In addition to providing detailed information that is useful for interpreting heavy-ion collisions, these calculations have shown that QCD contains a phase in which (quasi-particle guises of) quarks and gluons are no longer confined, and the chiral symmetry of the quarks is restored. As shown in Fig. 11, the transition is smooth, but order parameters for deconfinement and for chiral symmetry restoration change qualitatively and quantitatively at essentially the same temperature.

477. "The QCD equation of state with almost physical quark masses," M. Cheng *et al.*, *Phys. Rev.* **D77**, 014511 (2008) [`arXiv:0710.0354 [hep-lat]`]. (A)

478. "Equation of state and QCD transition at finite temperature," A. Bazavov *et al.*, HotQCD Collaboration, *Phys. Rev.* **D80**, 014504 (2009) [`arXiv:0903.4379 [hep-lat]`]. (A)

The transition temperature near 190 MeV corresponds to 2×10^{12} K.

An intriguing result of these lattice-QCD calculations is how the order of the phase transition depends on the light- and strange-quark masses. If they were around half the size needed to explain the nonzero pion and kaon masses, then the transition would be first order, instead of a smooth crossover.

479. "The order of the quantum chromodynamics transition predicted by the standard model of particle physics," Y. Aoki, G. Endrodi, Z. Fodor, S. D. Katz, and K. K. Szabo, *Nature* **443**, 675–678 (2006) [`hep-lat/0611014`]. (I–A)

480. "The chiral critical line of $N_f = 2+1$ QCD at zero and nonzero baryon density," P. de Forcrand and O. Philipsen, *JHEP* **01**, 077 (2007) [`hep-lat/0607017`].

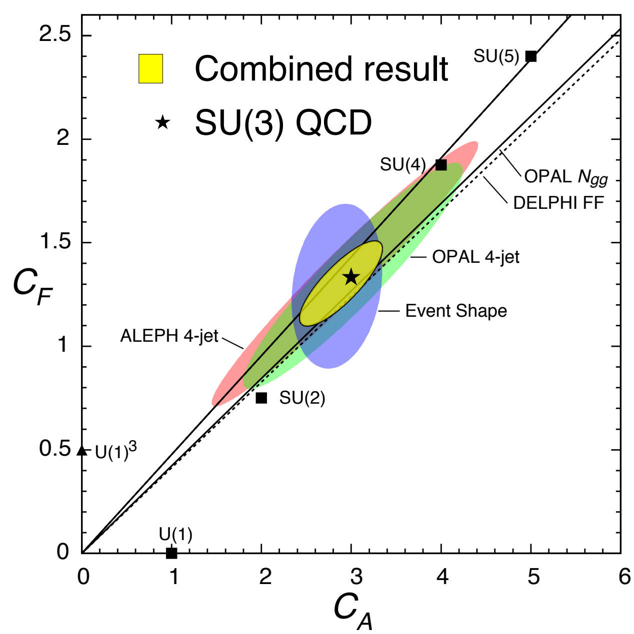

FIG. 10: Phase diagram of QCD in the μ-T plane. Here μ denotes baryon chemical potential and T temperature. At low μ, there is a smooth transition with varying T, probed by heavy-ion collisions and lattice-QCD calculations. At higher μ the phases are informed by models and other theoretical considerations. Hadronic matter denser than neutron stars is thought to exhibit "color superconductivity," first without and eventually with "color-flavor locking." Adapted from Ref. 465.

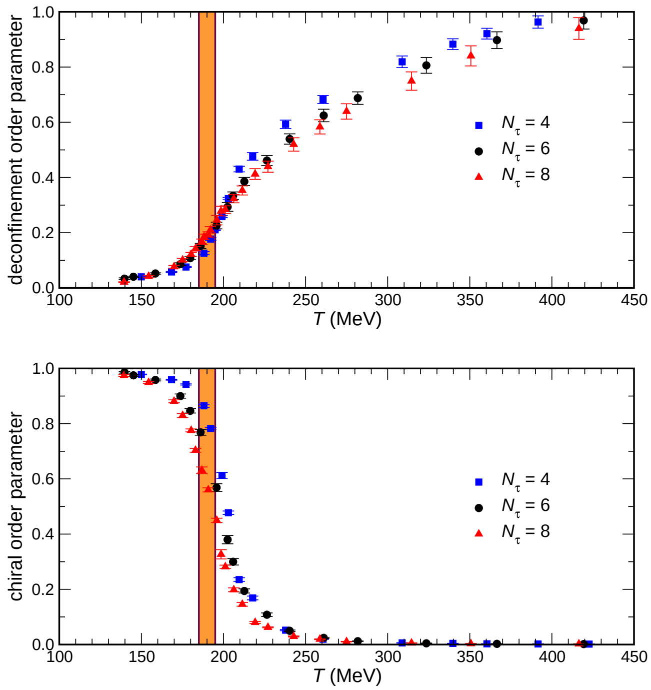

FIG. 11: Order parameters for deconfinement (top) and chiral symmetry restoration (bottom), as a function of temperature. The physical temperature $T = a/N_\tau$, where a is the lattice spacing. Agreement for several values of N_τ thus indicates that discretization effects from the lattice are under control. From Refs. 477,478.

(I–A)

That would expose the early universe to a latent heat as it cools below the critical temperature. Ramifications of the QCD phase transition on the early universe are discussed in

481. "The quark-hadron phase transition in the early universe: isothermal baryon number fluctuations and primordial nucleosynthesis," G. M. Fuller, G. J. Mathews, and C. R. Alcock, *Phys. Rev.* **D37**, 1380 (1988) [doi: `10.1103/PhysRevD.37.1380`]. (I–A)

Numerical lattice QCD is for now limited to baryon chemical potential $\mu \approx 0$, with obstacles to the regime relevant to neutron stars.

Ordinary nuclei consist of protons and neutrons, which are composed of up and down quarks. Because the most stable nuclei have exceedingly long lifetimes—greater than the age of the universe—it is natural to idealize them as absolutely stable, up to the conjectured nucleon decay that arises in unified theories of the strong, weak, and electromagnetic interactions. If the strange-quark mass were comparable to the up- and down-quark masses, then the Pauli principle would be less restrictive, and the ground state of matter would be a mixture of u, d, and s quarks. It has been conjectured that such strange matter is the true ground state in the real world, so that nuclear matter is metastable, as elaborated in

482. "Cosmic separation of phases," E. Witten, *Phys. Rev.* **D30**, 272–285 (1984) [doi: `10.1103/PhysRevD.30.272`]. (I–A)

483. "Strange matter," E. Farhi and R. L. Jaffe, *Phys. Rev.* **D30**, 2379 (1984) [doi: `10.1103/PhysRevD.30.2379`]. (I–A)

Small nuggets of strange matter are called strangelets.

According to the strange-matter hypothesis, compact stars might be strange stars, rather than neutron stars, as reviewed in

484. "Strange quark matter and compact stars," F. Weber, *Prog. Part. Nucl. Phys.* **54**, 193–288 (2005) [`astro-ph/0407155`]. (I–A)

which summarizes strange-matter searches. Conferences on strange quark matter may be traced from

485. "SQM09: International Conference on Strangeness in Quark Matter," `http://omnis.if.ufrj.br/~sqm09/`. (A)

Using techniques of gauge-string duality (Ref. 277), theorists have attempted to infer characteristics of QCD in the strong-coupling regime from analogue theories that possess some degree of supersymmetry. Applications to heavy-ion collisions and confinement are reviewed in

486. "Viscosity, black holes, and quantum field theory," D. T. Son and A. O. Starinets, *Ann. Rev. Nucl. Part. Sci.* **57**, 95–118 (2007) [`arXiv:0704.0240 [hep-th]`]. (I–A)

487. "From gauge-string duality to strong interactions: a pedestrian's guide," S. S. Gubser and A. Karch, *Ann. Rev. Nucl. Part. Sci.* **59**, 145–168 (2009) [`arXiv:0901.0935 [hep-th]`]. (A)

One should bear in mind, however, that the archetype of the analogue theories, supersymmetric Yang-Mills theory with four supercharges, does not share some of the essential features of QCD: the coupling that corresponds to α_s does not run, and the theory does not confine.

O. QCD and nuclear physics

In principle, all of nuclear physics follows from QCD:

488. "Nuclear physics at the end of the century," E. M. Henley and J. P. Schiffer, *Rev. Mod. Phys.* **71**, S205–S219 (1999) [doi: `10.1103/RevModPhys.71.S205`]. (A)

489. Particles and Nuclei: an Introduction to the Physical Concepts, B. Povh, K. Rith, C. Scholz, and F. Zetsche (Springer, New York, 1999), 4th ed. (E)

490. The Structure of the Nucleon, A. W. Thomas and W. Weise (Wiley-VCH, New York, 2001). (I–A)

A recent review of QCD-based nuclear theory, emphasizing symmetries and effective field theories can be found in

491. "Modern theory of nuclear forces," E. Epelbaum, H.-W. Hammer, and U.-G. Meißner, *Rev. Mod. Phys.* **81**, 1773 (2009) [`arXiv:0811.1338 [nucl-th]`]. (I–A)

An assault on nuclear physics using lattice gauge theory and effective field theory is reviewed in

492. "Hadronic interactions from lattice QCD," S. R. Beane, K. Orginos, and M. J. Savage, *Int. J. Mod. Phys.* **E17**, 1157–1218 (2008) [`arXiv:0805.4629 [hep-lat]`]. (I–A)

The nucleon-nucleon potential is governed by pion exchange at distances beyond 2 fm, as recognized by

493. "On the interaction of elementary particles," H. Yukawa, *Proc. Phys. Math. Soc. Jap.* **17**, 48 (1935) [`http://dbserv.ihep.su/~elan/src/yukawa35/eng.pdf`]. (E–I)

At intermediate range, 1 fm $\lesssim r \lesssim$ 2 fm, the nuclear force is determined by the exchange of vector mesons and other multipion states. A repulsive hard core, proposed in

494. "On the nucleon-nucleon interaction," R. Jastrow, *Phys. Rev.* **81**, 165 (1951) [doi: `10.1103/PhysRev.81.165`]. (I–A)

is essential to the understanding of nuclear stability, the maximum mass of neutron stars, and other characteristics of nuclear matter. Microscopic quantum Monte Carlo calculations of the properties of light nuclei demonstrate that nuclear structure, including both single-particle and clustering aspects, can be explained starting from elementary two- and three-nucleon interactions

495. "Quantum Monte Carlo calculations of light nuclei," S. C. Pieper and R. B. Wiringa, *Ann. Rev. Nucl. Part. Sci.* **51**, 53–90 (2001) [`nucl-th/0103005`]. (I–A)

The essential features of the two-nucleon interaction have now been deduced from lattice QCD simulations in an approximation omitting sea quarks, as reported in

496. "The nuclear force from lattice QCD," N. Ishii, S. Aoki, and T. Hatsuda, *Phys. Rev. Lett.* **99**, 022001 (2007) [`nucl-th/0611096`]. (I–A)

See also the commentary in

497. "Hard-core revelations," F. Wilczek, *Nature* **445**, 156–157 (2007) [doi: `10.1038/445156a`]. (E)

Meanwhile, this and many other aspects of low-energy baryon-baryon interactions have been computed in lattice QCD with $2+1$ flavors of sea quarks:

498. "High-statistics analysis using anisotropic clover lattices III: baryon-baryon interactions," S. R. Beane *et al.*, `arXiv:0912.4243 [hep-lat]`. (A)

The European Muon Collaboration (EMC) discovered that the per-nucleon deeply inelastic structure function, $F_2(x)$, was significantly different for iron than for deuterium, with a marked suppression of quarks in the interval $0.3 < x < 0.8$:

499. "The ratio of the nucleon structure functions F_2^N for iron and deuterium," J. J. Aubert *et al.*, European Muon Collaboration, *Phys. Lett.* **B123**, 275 (1983) [doi: `10.1016/0370-2693(83)90437-9`]. (I–A)

Data from many subsequent experiments and candidate interpretations are reviewed in

500. "The nuclear EMC effect," D. F. Geesaman, K. Saito, and A. W. Thomas, *Ann. Rev. Nucl. Part. Sci.* **45**, 337–390 (1995) [doi: `10.1146/annurev.ns.45.120195.002005`]. (I–A)

501. "The EMC effect," P. R. Norton, *Rept. Prog. Phys.* **66**, 1253–1297 (2003) [doi: `10.1088/0034-4885/66/8/201`]. (I–A)

Recently, the experimental information has been extended to light nuclei:

502. "New measurements of the EMC effect in very light nuclei," J. Seely *et al.*, *Phys. Rev. Lett.* **103**, 202301 (2009) [`arXiv:0904.4448 [nucl-ex]`]. (A)

V. QCD IN THE BROADER CONTEXT OF PARTICLE PHYSICS

Quantum chromodynamics is part of the extremely successful "Standard Model of Elementary Particles." Some resources that help put QCD in the broader context of the Standard Model are given here.

A comprehensive source of general knowledge about particle physics, including many aspects of QCD, is the biannual review by the Particle Data Group (Ref. 8).

Many of the themes that came together in quantum chromodynamics may be traced in the contributions to two symposia on the history of particle physics:

503. Pions to Quarks: Particle Physics in the 1950s: Based on a Fermilab Symposium, edited by L. M. Brown, M. Dresden, and L. Hoddeson (Cambridge University Press, Cambridge & New York, 1989). (E–I)

504. The Rise of the Standard Model: Particle Physics in the 1960s and 1970s, edited by L. Hoddeson, L. Brown, M. Riordan, and M. Dresden (Cambridge University Press, New York, 1997). (I–A)

Experimental steps that led to today's standard model of particle physics are surveyed in the well-chosen collection,

505. The Experimental Foundations of Particle Physics, R. N. Cahn and G. Goldhaber (Cambridge University Press, Cambridge & New York, 2009), 2nd ed. (E–I)

Also see

506. "Quarks with color and flavor," S. L. Glashow, *Sci. Am.* **233**, 38–50 (October, 1975). (E)

507. "Gauge theories of the forces between elementary particles," G. 't Hooft, *Sci. Am.* **242**, 104–138 (June,

1980). (E)

508. "Elementary particles and forces," C. Quigg, *Sci. Am.* **252**, 84–95 (April, 1985). (E)

509. The New Cosmic Onion: Quarks and the Nature of the Universe, F. E. Close (Taylor & Francis, London, 2006), second ed. (E)

Like quantum chromodynamics, the electroweak theory is a gauge theory, based on weak-isospin and weak-hypercharge symmetries described by the gauge group $\mathrm{SU}(2)_{\mathrm{L}} \times \mathrm{U}(1)_Y$. For a look back at the evolution of the electroweak theory, see the Nobel Lectures by some of its principal architects:

510. "Conceptual foundations of the unified theory of weak and electromagnetic interactions," S. Weinberg, *Rev. Mod. Phys.* **52**, 515–523 (1980) [doi: `10.1103/RevModPhys.52.515`]. (I)

511. "Gauge unification of fundamental forces," A. Salam, *Rev. Mod. Phys.* **52**, 525–538 (1980) [doi: `10.1103/RevModPhys.52.525`]. (I)

512. "Towards a unified theory: threads in a tapestry," S. L. Glashow, *Rev. Mod. Phys.* **52**, 539–543 (1980) [doi: `10.1103/RevModPhys.52.539`]. (I)

Experiments (and the supporting theoretical calculations) over the past decade have elevated the electroweak theory to a law of nature. The current state of the theory is reviewed in

513. "Unanswered questions in the electroweak theory," C. Quigg, *Ann. Rev. Nucl. Part. Sci.* **59**, 505–555 (2009) [`arXiv:0905.3187 [hep-ph]`]. (I–A)

For general surveys of the standard model of particle physics, and a glimpse beyond, see

514. "The standard model of particle physics," M. K. Gaillard, P. D. Grannis, and F. J. Sciulli, *Rev. Mod. Phys.* **71**, S96–S111 (1999) [`hep-ph/9812285`]. (I)

515. "Resource letter SM-1: the Standard Model and beyond," J. L. Rosner, *Am. J. Phys.* **71**, 302–318 (2003) [`hep-ph/0206176`]. (E–I–A)

The common mathematical structure of QCD and the electroweak theory, combined with asymptotic freedom, encourages the hope that a unified theory of the strong, weak, and electromagnetic interactions may be within reach. The unification strategy, with some consequences, is presented in

516. "A unified theory of elementary particles and forces," H. Georgi, *Sci. Am.* **244**, 40–63 (April, 1981). (E)

VI. FRONTIER PROBLEMS IN QCD

Four decades after the synthesis of quarks, partons, and color into the QCD Lagrangian (Ref. 9)—and the essentially immediate discovery of asymptotic freedom (Ref. 10,11)—QCD has been tested and validated up to energies of 1 TeV. Tests are poised to continue at even higher energies, as operations at the Large Hadron Collider (LHC) commence. It is fair to say, however, that most physicists do not expect big surprises at the LHC in the structure of QCD. Instead, QCD will be treated as basic knowledge, much like electrodynamics, enabling discoveries beyond the realm of the standard model of elementary particles (Ref. 32).

In this arena, future research will focus on techniques for evaluating parton amplitudes with increasingly many real and virtual particles, for both signals and backgrounds. The higher energies of the scattering processes will continue to entail many scales (several TeV compared to the top-quark mass, for example) and, hence, will need tools, such as the soft-collinear effective theory discussed in Sec. III E 3. Future experiments with B decays will also continue to rely on QCD, at moderately high energies, to pin down the weak and any new interactions of quarks (or other particles carrying color).

The strong interactions comprise a richer field than the set of phenomena that we have learned to describe in terms of perturbative QCD or the (near-)static nonperturbative domain of lattice QCD. The technology by which we apply QCD is incomplete, and still evolving. Many aspects of hadron phenomenology and spectroscopy are not yet calculable beginning from the QCD Lagrangian. Much analysis of experimental information relies on highly stylized, truncated pictures of the implications of the theory. While expanding the horizons, it is important to distinguish tests of QCD from tests of auxiliary assumptions.

The rest of the strong interactions, moreover, isn't confined to common processes with large cross sections such as the "soft" particle production, elastic scattering, or diffraction. It may well be that interesting, *unusual* occurrences happen outside the framework of perturbative QCD—happen in some collective, or intrinsically nonperturbative, way. At the highest energies, well into the regime where the pp total cross section grows as $\ln^2 s$, long-range correlations might show themselves in new ways. Quantum chromodynamics suggests new, modestly collective, effects such as multiple-parton interactions. The high density of partons carrying $p_z = 5$–10 GeV may give rise to hot spots in the spacetime evolution of the collision aftermath, and thus to thermalization or other phenomena not easy to anticipate from the QCD Lagrangian.

At lower energies, the basic features of the hadron spectrum have been reproduced in a convincing way. Some of the simplest hadronic transition amplitudes, needed to understand flavor physics, are in similarly good shape. The aspiration here is to compute many simple amplitudes with total errors that are 1% or smaller. Such precision will require nonperturbative matching and the charmed sea. Indeed, a next-generation assault on B decays via $e^+e^- \to \Upsilon(4\mathrm{S})$ will hinge on such lattice QCD calculations (Ref. 452). Calculations of similar difficulty are related to moments of the parton distributions. Reliable lattice-QCD calculations would pin down predictions of signals and backgrounds at the LHC. The most crucial in this regard, and most challenging computationally, are moments of the gluon density inside the proton. See Ref. 375 and

517. "Status and prospects for the calculation of hadron structure from lattice QCD," D. B. Renner, *PoS* **LAT2009**, 018 (2010) [`arXiv:1002.0925 [hep-lat]`]. (A)

Precision perturbative QCD and precision lattice QCD are important and challenging, yet programmatic. Other future avenues for research in QCD will explore its richness in ways that are harder to anticipate. QCD is frequently, and justifiably, hailed as a triumph of reductionist science, distilling the plethora of hadrons and their complicated properties into a simple Lagrangian field theory [Eq. (1)]. Now that QCD is accepted as a law of nature, however, it may be time to characterize QCD research by the phenomena that emerge from this tantalizing simple form. What are hadron masses and chiral symmetry breaking, if not emergent phenomena?

Many avenues offer themselves for quantitative and qualitative study. Although the spectrum of the lowest-lying conventional hadrons is well-computed, it remains a challenge to compute the masses of excited hadrons, and even the lowest-lying glueball, hybrid, and exotic states. While these masses tie into experimental programs, it would simply be intriguing to see towers of bound states emerge from the QCD Lagrangian. Another structure that emerges from QCD is a rich phase structure (see Sec. IV N). A fuller understanding will require experiments with heavy-ion collisions, including the higher-density probes of the Compressed Baryonic Matter experiment.

518. "The Compressed Baryonic Matter Experiment," CBM Collaboration, `http://www.gsi.de/fair/experiments/CBM/`. (I–A)

Complementary theoretical work will require both model studies and lattice QCD calculations, although a breakthrough in finite-density lattice QCD could relegate some model studies to secondary importance. The transition to (effectively) deconfined quarks at nonzero temperature and density may help explain why color cannot be isolated in the (zero-temperature) ground state of QCD. Finally, from the emergent phenomenon of hadrons emerges the whole field of nuclear physics. QCD is just beginning to answer questions about nuclear physics, and some nuclear physicists see the future of their field as QCD (see Sec. IV O).

To elucidate these features of QCD, it will help to study lightweight versions of Yang-Mills theories with quarks. For example, with one flavor there is no chiral symmetry to break—the anomaly represents an explicit breaking of the U(1) chiral symmetry (see Sec. III A 2)—presenting a laboratory to study confinement without spontaneous symmetry breaking:

519. "Hadron masses in QCD with one quark flavour," F. Farchioni *et al.*, *Eur. Phys. J.* **C52**, 305–314 (2007) [`arXiv:0706.1131 [hep-lat]`]. (A)

What properties does this confining theory share with QCD? What does it lose along with the loss of chiral symmetry, spontaneously broken? An irony of nature's version of QCD is that the up- and down-quark masses are so much smaller than $\Lambda_{\rm QCD}$, so isospin is an excellent approximate symmetry. (More properly, isospin follows from $m_d - m_u \ll \Lambda_{\rm QCD}$.) Alternatively, one could imagine a theory with two quarks whose masses, and mass difference, are comparable to or larger than $\Lambda_{\rm QCD}$. Which dynamical features remain, and which are lost?

Whatever results academic investigations bring, QCD will retain a strong and deep connection to particle physics, astrophysics and cosmology, and nuclear physics. Indeed, often QCD binds these fields to each other. As discussed above, QCD will always play a central role, within the standard model and beyond, for collider physics. A dream of particle theorists is to unify the strong, weak, and electromagnetic interactions. Further precision for $\alpha_{\rm s}$ and quark masses will inform and constrain this dream. Now that it is fairly well established that the up-quark mass cannot vanish, the strong CP problem demands other solutions. The most elegant proposal augments QCD with additional symmetry (Ref. 100). The observable consequence is a pseudoscalar particle called the axion, which may comprise part of the "dark" matter of the universe (Ref. 102).

A future challenge is to connect nuclei to QCD. Some aspects befuddle models, because some relevant properties are too hard to measure. For example, the three-nucleon interaction is an important missing piece to the puzzle of nuclear structure. Some questions are almost philosophical: How do $\alpha_{\rm s}$ and the quark masses lead to various happenstances of nuclear physics, some of which seem implausible, yet are necessary for carbon-based life to exist? Looking beyond Earth, details of the quark-gluon plasma influence the evolution of the early universe. Above the transition temperature, hadrons do not "dissolve" quite as fast as sometimes thought. Another interesting QCD calculation, which has not yet been carried out, is to determine the Σ^--nucleon interaction. This is not a prosaic matter of hadronic physics, but a nuclear property that influences whether a supernova evolves to a neutron star or a black hole (Ref. 492).

In summary, QCD is not our "most perfect theory" (Ref. 2) merely because asymptotic freedom ensures its scope on towards the highest energies, temperatures, and densities. It is also a rich and varied physical theory, exhibiting qualitatively different behavior in different regimes, all stemming ultimately on the dynamics of quarks and gluons. For all the explanatory power of QCD, it still provides problems for physicists to work on.

Acknowledgments

We thank Thomas Becher, Johan Bijnens, Lance Dixon, Claudia Glasman, Maxime Gouzevitch, Kenichi Hatekeyama, Frithjof Karsch, Stefan Kluth, Christina Mesropian, and Heath O'Connell for assistance in the preparation of this Resource Letter.

Fermilab is operated by Fermi Research Alliance, LLC under Contract No. DE-AC02-07CH11359 with the United States Department of Energy.

Appendix A: Links to Basic Resources

1. Journals

New research papers on QCD are published in journals of elementary particle physics and of nuclear physics. The principal particle physics journals are

- *European Physics Journal C*, available on-line at `http://epjc.edpsciences.org/`;
- *Journal of High Energy Physics*, available on-line at `http://jhep.sissa.it/jhep/` or `http://www.iop.org/EJ/jhep/`;
- *Journal of Physics G*, available on-line at `http://www.iop.org/EJ/journal/JPhysG/`;
- *Nuclear Physics B*, available on-line at `http://www.elsevier.com/wps/product/cws_home/505716/`;
- *Physical Review D*, available on-line at `http://prd.aps.org/`;
- *Physical Review Letters*, available on-line at `http://prl.aps.org/`;
- *Physics Letters B*, available on-line at `http://www.elsevier.com/wps/product/cws_home/505706/`.

The principal nuclear physics journals are

- *European Physics Journal A*, available on-line at `http://epja.edpsciences.org/`;
- *Journal of Physics G*, available on-line at `http://www.iop.org/EJ/journal/JPhysG/`;
- *Nuclear Physics A*, available on-line at `http://www.elsevier.com/wps/product/cws_home/505715/`;
- *Physical Review C*, available on-line at `http://prc.aps.org/`;
- *Physical Review Letters*, available on-line at `http://prl.aps.org/`;
- *Physics Letters B*, available on-line at `http://www.elsevier.com/wps/product/cws_home/505706/`.

Journals with review articles:

- *Annual Reviews of Nuclear and Particle Science*, available on-line at `http://arjournals.annualreviews.org/loi/nucl`
- *Physics Reports*, available on-line at `http://www.elsevier.com/wps/product/cws_home/505703/`
- *Reports on Progress in Physics*, available on-line at `http://www.iop.org/EJ/journal/RoPP/`
- *Reviews of Modern Physics*, available on-line at `http://rmp.aps.org/`

These websites provide electronic versions (*e.g.*, pdf files) of most—in some cases all—papers published in the corresponding journal. Often a personal or institutional subscription, or the payment of a fee, is necessary.

2. Electronic archives

Most research papers and conference proceedings appear first in the physics e-print archives:

- `http://arxiv.org/archive/hep-ex/` contains e-prints on experimental high-energy (elementary particle) physics, many of which concern QCD;
- `http://arxiv.org/archive/hep-lat/` contains e-prints on lattice gauge theory, most of which address nonperturbative QCD;
- `http://arxiv.org/archive/hep-ph/` contains e-prints on theoretical high-energy physics with focus on observable phenomena, many of which concern QCD;
- `http://arxiv.org/archive/hep-th/` contains e-prints on theoretical aspects of string theory and quantum field theory, some of which concern QCD;
- `http://arxiv.org/archive/nucl-ex/` contains e-prints on experimental nuclear physics, many of which concern QCD explicitly;
- `http://arxiv.org/archive/nucl-th/` contains e-prints on theoretical nuclear physics, many of which concern QCD explicitly.

The arXiv provides free downloads. The arXiv version of this Resource Letter provides hyperlinks to `arXiv.org` where possible, and otherwise provides a hyperlink to the digital object identifier (doi) of other electronically published sources. One should bear in mind, however, that the versions in journals are usually definitive; `arXiv.org` provides doi links.

3. Pedagogical web sites

For a very approachable introduction to the ideas of contemporary particle physics, see the

520. "CPEP Materials about Fundamental Particles and Interactions," Contemporary Physics Education Project, `http://www.cpepweb.org/particles.html`. (E)

and the accompanying

521. The Charm of Strange Quarks: Mysteries and Revolutions of Particle Physics, R. M. Barnett, H. Muehry, and H. R. Quinn (Springer, Heidelberg, 2000). (E)

The ideas of nuclear science are presented in a wall chart and teacher's guide, available at

522. "CPEP Materials about Nuclear Science," Contemporary Physics Education Project, `http://www.cpepweb.org/nuclear.html`. (E)

The Particle Data Group (Ref. 8) maintains a web site as comprehensive as its review, with updates online midway between the biennial editions. Students and the general public should enjoy their Particle Adventure, `http://www.particleadventure.org/`.

Visualizations of some of the main elements of nonperturbative QCD, with helpful explanations, may be found at the URL in Ref. 26.

The laboratories and major experiments in particle and nuclear physics maintain web sites that feature educational materials.

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