Weakly Interacting Stable Pions

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An unbroken discrete symmetry, analogous to $G$-parity in QCD, exists in standard model extensions with vector-like coupling of electroweak $SU(2)$ to “hidden sector” fermions that are confined by a strong gauge force. For an irreducible $SU(2)$ representation of the hidden sector fermions, the lightest hidden sector states form an isotriplet of “pions” with calculable mass splittings and couplings to standard model fields. The parity can be extended to fermions in real representations of color $SU(3)$, and can provide dark matter candidates with distinct collider signatures.

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Introduction. Cosmological and astrophysical observations indicate the existence of dark matter. This motivates the investigation of standard model (SM) extensions with new stable particles. For example, in theories with an exact parity symmetry, the lightest parity-odd particle is stable against decay, and can act as dark matter. Here we describe the implications of a parity selection rule that naturally arises in a “hidden sector” involving strongly-coupled fermions, e.g. a QCD-like theory with large confinement scale.

At the renormalizable level, such a hidden sector can communicate with the standard model only through gauge interactions: $SU(3)_c$, $SU(2)_W$ or $U(1)_Y$. To avoid spontaneous breaking of these symmetries at a high scale we focus on vector-like gauging, i.e., identical coupling of the gauge fields to left- and right-handed fermions.

We will see that the class of models with gauge-confined fermions coupled vectorially to $SU(2)_W$ has some remarkably simple universal features. A parity symmetry acting only on the hidden sector fields is left unbroken. The new parity symmetry is an analog of $G$-parity in QCD \cite{1}, which is broken in the SM by the presence of $U(1)_Y$ and non-vectorial coupling of $SU(2)_W$. Below the hidden sector confinement scale, the lightest physical states are Nambu-Goldstone bosons (NGBs) of the spontaneously broken chiral symmetries. In the limit of vanishing electroweak coupling ($g_2 \to 0$) the pions are degenerate in mass, as implied by exact global symmetries. At finite coupling, and after electroweak symmetry breaking, radiative corrections induce mass splittings, and the electric-neutral component of an $SU(2)_W$ triplet is the “lightest $G$-odd particle” (LGP). If such a hidden sector exists, this particle can be a dark matter candidate. Its mass and standard model couplings are calculable in terms of a free parameter representing the confinement scale of the hidden sector fermions.

Models with vector-like coupling of standard model gauge fields to hidden sector matter have previously been considered from the point of view of dark matter \cite{2} and collider \cite{3} phenomenology. In a large class of models, our conclusions differ from previous work. We demonstrate how stable particles naturally arise in such models without ad hoc assumptions, and provide a chiral lagrangian framework for systematically computing properties of the new particles.

Fermion description. Let us extend the standard model by introducing Dirac fermions in a complex representation of a hidden sector gauge group, e.g. the fundamental of $SU(N)_h$ with $N \geq 3$, i.e., a QCD-like theory. Suppose there are $n_f = 2j + 1$ flavors transforming in an irreducible spin $j$ representation of $SU(2)_W$ \cite{14}. The basic lagrangian is

$$\mathcal{L} = \mathcal{L}_{\text{SM}} - \frac{1}{4} (\hat{F}_{\mu\nu}^a)^2 + \bar{\psi} \left( i \partial^\mu \hat{A}^\mu_b + g_2 W^a \sigma^\mu \right) \psi, \quad (1)$$

where hatted quantities refer to the hidden sector. The Hermitian matrices $\hat{b}^a$, $b = 1, \ldots, N^2 - 1$, denote generators in the fundamental $SU(N)$ representation and $J^a$, $a = 1, \ldots, 3$, are $n_f \times n_f$ isospin matrices.

All representations of $SU(2)$ are real. The matrix $S$ that relates the original and conjugated generators, $S^+ J^a S = - J^a$, can be expressed in terms of a group element \cite{4}: $S = \exp[i \pi J^2]$. On the basis of spin $j$ states, $|jm\rangle$ with $m = j, j-1, \ldots, -j$, we have

$$S_{m_1 m_2} = (-1)^{j+m_2} \delta_{m_1, -m_2} = (\exp \left[ i \pi J^2 \right])_{m_1 m_2}. \quad (2)$$

For example, for $j = 1/2$ we have $J^+ = \gamma^2 J^2 / 2$ and $S = i \gamma^2$ where $\gamma^a$ are the usual Pauli matrices. We use the reality of $SU(2)$ representations to define a modified charge conjugation operation, “$G$-parity”:

$$\hat{\psi} \hat{C} \to S \psi \hat{C} = S i \gamma^2 \hat{\psi}^c, \quad \hat{A}^b \hat{b}^c \to (\hat{A}^b)^c \hat{b}^c = \hat{A}^b (-\hat{\psi}^c), \quad (3)$$

with $C$ denoting “ordinary” charge conjugation and $S$ acting on the fermion flavor index. All SM fields, in particular $W^a_{\mu}$, are left invariant. It is readily verified that the lagrangian \cite{14} is invariant under the transformation \cite{5} \cite{13}. $G$-parity is a good quantum number of the theory, and all SM particles are $G$-even.

Pion description. The $SU(N)_h$ gauge theory is assumed to have a behavior similar to QCD. The gauge
coupling becomes strong in the infrared, triggering confinement and chiral symmetry breaking at a scale \( \Lambda_h \). Below \( \Lambda_h \), the effective theory is described by "pions" which are NGBs associated with the spontaneously broken global flavor symmetry of the hidden sector: \( SU(2)_L \times SU(2)_R \times U(1)_B \to SU(2)_L + U(1)_Y \). Here \( U(1)_B \) is the unbroken dark baryon number symmetry.

The pions are collected into the \( SU(2)_L \) matrix field \( U \equiv e^{2i\theta/f_\Pi} \), where \( f_\Pi \) is the analog of \( f_\pi \approx 93 \text{ MeV} \) in QCD. The \((2j+1)^2 - 1 = \sum_{j=1}^{\infty} (2j+1) \) pions can be classified in irreducible representations, \( J \), of \( SU(2)_L \).

The generators \( t^{(JM)} \) associated with pions of definite total isospin \( J \) and electric charge \( M \) can be expressed using Clebsch-Gordan coefficients for \( j \times j \) as

\[
 t^{(JM)} = (−1)^{j−m_j} (jjJMjjm_1,−m_2). \tag{4}
\]

The generators \( \Pi \) are normalized as \( \text{Tr}[t^{(JM)}t^{(J'M')}] = (−1)^{MJ'}\delta_{M,M'} \), and satisfy \( [t^{(J'M')}t^{(JM)}] = (−M)\delta_{J,J'}\delta_{M,M'} \). The canonically normalized isospin generators appearing in \( \Pi \) are \( J^3 = \sqrt{\frac{2}{J(J+1)}} \), \( J^\pm = \pm \sqrt{\frac{J(2J+1)}{2}} \), and \( C^a(J) = \frac{1}{\sqrt{3}}(1+1/2j+1/3) \) is the normalization constant for the spin-\( j \) representation. \( J^aJ^b = C(j)\delta^{ab} \).

The pion field can be expanded as

\[
\Pi = \sum_{j=1}^{\infty} \Pi^{(J)}t^j = \sum_{j=1}^{\infty} \sum_{M=−J}^{J} \Pi^{(JM)}t^{(JM)}. \tag{5}
\]

Under the transformation \( \Pi^{(JM)} \to G \Pi^{(JM)} \),

\[
(−1)^J\Pi^{(JM)}, \tag{6}
\]

so that pions with odd (even) \( J \) are odd (even) under \( G \) parity. Note that \( G \)-parity generalizes the notion of pion number parity in QCD to arbitrary number of flavors.

**Spectrum.** The leading, two-derivative, term in the symmetric chiral Lagrangian is

\[
\mathcal{L}_2 = \frac{f_\Pi^2}{4} \text{Tr}[D_\mu U^\dagger D^\mu U^\dagger], \tag{7}
\]

where \( D_\mu U = \partial_\mu U - i g_2 W_\mu^a \left[ J^a, U \right] \). This term describes massless, self-interacting scalar fields. Note that the neutral pions \( \Pi^{(0)} \) correspond to generators commuting with \( J^3 \), and hence have no direct interactions with the physical photon or \( Z \) boson. The gauge coupling \( g_2 \) explicitly breaks the global chiral symmetry. At one loop, a quadratically divergent counterterm is required,

\[
\mathcal{L}_\text{c} \sim g_2^2 \Lambda_h^4 f_\Pi^2 \text{Tr}(J^aU^\dagger J^a U^\dagger). \tag{8}
\]

This loop effect lifts the degeneracy among multiplets,

\[
m_\Pi^{(J=0)} \approx J(J+1)\alpha_2 \Lambda_h^4, \tag{9}
\]

where \( g_2^2 \equiv 4\pi\alpha_2 \). The hidden sector fermions transform to the SM Higgs field via the \( SU(2)_W \) gauge field intermediary. After electroweak symmetry breaking, such interactions induce a finite splitting between components of each multiplet. In the limit \( m_\Pi \gg m_W \), \( m_{\Pi^{(J=0)}} \approx \alpha_2 Q^2 m_W \sin^2 \frac{\theta}{2} \). In particular, \( m_{\Pi^{(J=1)}} - m_{\Pi^{(10)}} \approx 170 \text{ MeV} \), leaving \( \Pi^{(10)} \) as the LGP.

**Interactions.** The interactions of the pions amongst themselves, and with SM fields, are constrained by chiral symmetries of the new strong interaction. Taking over results familiar from QCD, the symmetric lagrangian can be expanded in powers of \( 1/f_\Pi^2 \): \( \mathcal{L} = \mathcal{L}_2 + \mathcal{L}_4 + \ldots \). The leading term is displayed in \( \Pi \). Of particular interest at four-derivative order is the Wess-Zumino-Witten (WZW) interaction, containing terms with odd pion number,

\[
\mathcal{L}_4 = \frac{N g_2^2}{16\pi^2 f_\Pi^2} \text{Tr} \left[ \Pi W_{\mu\nu} W_{\rho\sigma} + O(\Pi^5) \right] + \ldots \tag{10}
\]

Let us consider the decays of the lightest states, focusing on the limit \( m_\Pi \gg m_W \). The charged members of the ground state multiplet decay with weak strength into the LGP via QCD pion emission:

\[
\Gamma(\Pi^{(1\pm 1)} \to \Pi^{(10)} + \pi^\pm) = \frac{4G_\text{F}^2}{\pi} \Delta^2 \sqrt{\Delta^2 - m_\Pi^2 f_\Pi^2}; \tag{11}
\]

where \( \Delta = m_{\Pi^{(1\pm 1)}} - m_{\Pi^{(10)}} \) and we have assumed \( m_\Pi \Delta \gg m_\Pi^2 \). In this limit the decay length \( \lambda \approx 5 \text{ cm} \) is independent of \( m_\Pi \). The lightest G-even pions, \( \Pi^{(2M)} \), decay into electroweak bosons via the WZW term \( \Pi \). The relevant trace is

\[
C(j) \text{Tr} \left[ t^{(2M)} \left[ \Pi^{(1L)} \right] \right] \approx (−1)^{M} \left( 112M \right)[11−L′−L] × \frac{4}{\sqrt{15}} \left( j−3/2 \right)(j+1/2)(j+1)(j+3/2). \tag{12}
\]

The amplitude is sensitive both to the number of colors \( N \) and to the spin \( j \) of the underlying fermions. The rate scales as \( \Gamma(\Pi^{(2M)} \to W(W) \to W(W) ) \sim \alpha_2^2 m_\Pi^2/(4\pi)^3 f_\Pi^2 \). Note that the process \( \Pi^{(2M)} \to W(W) \) (the analog of \( \phi^0 \to \gamma \gamma \) in the SM) is forbidden by the vanishing of the relevant isospin trace ("d symbol").

The next-to-lightest G-odd pions, \( \Pi^{(3M)} \), decay into the ground-state multiplet via loops containing G-even pions, \( \Pi^{(2M)} \). The interaction vertices are obtained by expanding \( \mathcal{L}_2 \) and \( \mathcal{L}_4 \). The decay amplitude scales as \( \Gamma(\Pi^{(3M)} \to \Pi^{(2M)} + 2W) \sim \alpha_2^2 m_\Pi^2/(4\pi)^3 f_\Pi^2 \). For \( J \geq 3 \), loops containing G-even pions mediate the decays \( \Pi^{(JM)} \to J(−2,M') + 2W \). The end result is a multi-W final-state. For odd \( J \), the stable \( \Pi^{(10)} \) remains, possibly after decay of the long-lived \( \Pi^{(1\pm 1)} \) as above.

**Couppling to SU(3)c.** The parity operation \( \Pi \) can be extended to real representations of \( SU(3)c \). (Extension to \( U(1)c \) would require embedding the gauge fields inside a larger, real representation.)

Consider, e.g., hidden sector fermions transforming in the adjoint representation of \( SU(3)c \), and the fundamental representation of \( SU(2)c \). In the usual basis where adjoint generators are purely imaginary, the
generalized $G$-parity for this model is defined as in (3) where now $S = \mathbb{1} \otimes \text{i} r^2$ acting on $SU(3)$ and $SU(2)$ indices. Using the product decomposition $8 \times 8 = (8 + 10 + \overline{10}) \Lambda + (1 + 8 + 27) s$ (“A” and “S” denoting antisymmetric and symmetric tensors) and $2 \times 2 = 1 + 3$, we can express the $16^2 - 1 = 255$ pion generators as products of $SU(3)$ and $SU(2)$ generators. The resulting symmetric (anti-symmetric) pure $SU(3)$ generators are $G$-even ($G$-odd) and the decomposition into representations of $SU(3)_c \times SU(2)_W$ with the corresponding $G$-parity is $(8, 2) \times (8, 2) - (1, 1) = (3, 3)^- + (8_A, 1)^- + (8_S, 1)^+ + (8_A, 3)^+ + (8_S, 3)^- + \ldots$. The pion masses are proportional to the sum of second-order Casimir invariants as in (2), $m_{\Pi}^2 \approx \Lambda^2 \left[ \alpha_2 C_2(d_2) + \alpha_2 C_2(d_2) \right]$, with $C_2(3) = 2$ for $SU(2)$, and $C_2(8) = 3$ for $SU(3)$. The lightest states, $(1, 3)^-$, are again an $SU(2)_W$ triplet of $G$-odd pions.

Operators such as (1), (3) and (10) again mediate transitions between pion states, after appropriate substitution of $C$ with $\Lambda$ serves as in (9), $\Lambda$ being symmetric (anti-symmetric) pure $SU(3)$ singlet dark matter (interacting via higher-dimension operators), or as nearly-degenerate iso-triplets. Another possibility for the hidden sector includes reducible representations of $SU(3)$, e.g., multiple “gen-

Operators such as (1), (3) and (10) again mediate transitions between pion states, after appropriate substitution to account for $SU(3)_c \times SU(2)_W$ gauging. The decays of the two lightest $G$-even multiplets $(8_A, 1)^+$ and $(8_A, 3)^+$, proceed primarily through WZW interactions. The two next-to-lightest $G$-odd pions, $(8_A, 1)^-$ and $(8_S, 3)^+$, decay to the LSP plus two SM gauge bosons through loop diagrams with $G$-even pions. The spectrum and interactions in this model can produce interesting collider signatures. For example, the pair production of the next to lightest $G$-odd particle $(8_A, 1)^-$ leads to a final state with 2 photons + 2 jets + missing transverse energy.

Other possibilities for the hidden sector include reducible representations of $SU(2)_W$, i.e., multiple “generations” in the hidden sector. These cases might be interesting to investigate from the standpoint of SM gauge-singlet dark matter (interacting via higher-dimension operators), or as nearly-degenerate iso-triplets. Another possibility (not relevant to weakly coupled dark matter) is to consider $SU(2)_W$ singlet fermions in an irreducible real representation of $SU(3)_c$. Various models can be embedded in five dimensional gauge theory constructions. 

**Pececi-Quinn symmetry and axion.** We have so far neglected two gauge invariant terms in the lagrangian (1) that can appear at the renormalizable level. The first is a theta term for the hidden gauge fields, and the second a bare mass term for the fermions (16).

$$L_\theta = \theta \epsilon_{\mu \nu \rho \sigma} F_{\mu \nu}^a \tilde{F}_{\rho \sigma}^a, \quad L_m = - m_\phi \psi^\dagger \psi. \quad (13)$$

For nonzero $m_\phi$ and $\theta$, these terms give rise to $P$ and $T$ violation in the hidden sector but do not affect $G$-parity conservation [12]. For simplicity we focus on the case $m_\phi < \Lambda_h$. For $m_\phi \gg \Lambda_h$, the lightest $G$-odd states become iso-triplet nonrelativistic “quarkonium”.

It is natural to consider whether some mechanism suppresses $P$ and $T$ violation in the hidden sector, as happens in QCD. Note that at $m_\phi = 0$, there is a Peccei-Quinn (PQ) symmetry present at the classical level, $\psi \to e^{i \alpha} \psi$. The PQ symmetry can be preserved (and $P$ and $T$ violation suppressed) if the mass is generated by spontaneous symmetry breaking. Consider a scalar field $\sigma$ that transforms under the PQ symmetry as $\sigma \to e^{-2i \alpha} \sigma$, and with interactions:

$$L_{\sigma} = |\partial_\mu \sigma|^2 - V(\sigma) - \lambda \sigma \psi_L \psi_R + \text{h.c.}, \quad (14)$$

where $V(\sigma)$ is such that $\sigma$ acquires a VEV, $\langle \sigma \rangle = f_\sigma$. The mass parameter is then $m_\phi = \lambda f_\sigma$, and with $\sigma(x) \sim f_\sigma e^{i a(x)/\sqrt{2} f_\sigma}$, the low-energy spectrum includes an axion $a(x)$ which is massless at the classical level.

As with the QCD axion, we assume that the $SU(N)_h$ anomaly generates an effective potential for $a(x)$ with minimum such that the effective $\theta$ term vanishes. Gauge invariance prevents the hidden axion from direct interaction with SM fermions. The physical axion acquires mass and interactions through mixing with the NGB of the $U(1)_A$ global flavor symmetry in the hidden sector, $\Lambda_A \approx f_\sigma$.

$\Lambda_A = m_\phi^2 f_\phi^2 2 f_\phi^2 - \frac{N C_L \lambda_3^2}{32 \pi^2} a \epsilon_{\mu \nu \rho \sigma} W^a \mu W^a \nu W^a \rho W^a \sigma + \ldots, \quad (15)$

where $m_\phi^2 \sim m_\phi \Lambda_h f_\phi^2 / f_\sigma^2$. Note that the $SU(2)_W$ couplings to the hidden sector do not break the $U(1)_A$ symmetry, and so do not contribute to the axion mass: only in the limit $m_\phi / \Lambda_h \gg g_2 \Lambda_h$ does the QCD-like relation $m_\phi f_\phi \sim m_\phi f_\phi H_{\mu \nu}$ hold.

**Discussion.** Hidden sector fermions coupled vectorially to SM gauge fields are an ingredient in typical axion models addressing the strong CP problem [6]. It is natural to inquire whether other such hidden sectors exist. We have considered approximately massless Dirac fermions in a complex representation of a QCD-like gauge group, and an arbitrary vector-like coupling to $SU(2)_W$. The lightest state of such a hidden sector is the neutral component of an approximately degenerate iso-triplet “pion”, and is stable against decay from any interactions appearing at the renormalizable level.

Could weakly interacting stable pions (WISPs) be a component of cosmological dark matter, produced thermally in the early universe? A lifetime of order $\tau_{\text{inverse}} \approx 10^{10}$ years imposes tight constraints and we must consider whether $G$ parity could be broken either explicitly (by unknown UV physics) or spontaneously (from the choice of vacuum). Suppose that the SM extension (1) is viewed as an effective theory valid up to some scale $\Lambda_U$. Corrections to the renormalizable lagrangian can appear at dimension five. Two such operators, $B_{\mu \nu} \tilde{\psi} \sigma^{\mu \nu} \psi /\Lambda_{\text{UV}}$ and $H^{\mu \nu} \tilde{\psi} J^\mu \psi /\Lambda_{\text{UV}}$, violate $G$-parity. Even for $\Lambda_{\text{UV}}$ of order the Planck scale, an additional suppression would be necessary to ensure cosmological stability. Note however that these operators violate the PQ symmetry present at the renormalizable level. Enforcing this symmetry implies the appearance of the $\sigma$ field, leading to an additional suppression $s(\sigma(x)/\Lambda_{\text{UV}} \sim f_\sigma /\Lambda_{\text{UV}}$. Then $\tau_{\text{LGP}} \sim$
that determines $\Sigma_0$ gauged vector-like symmetries are unbroken [6]. Interaction between fermion mass (13) is included in the hidden sector: the $\Sigma_0$ fixed at order $g_2^2$. As indicated by the positive pion masses [3], the potential has a minimum at $\Sigma_0 \propto 1$. This conclusion is unaffected if a bare fermion mass [13] is included in the hidden sector: the gauged vector-like symmetries are unbroken [6]. Interactions with the SM lead to $O(g_1^4)$ corrections to the effective potential. Since $G$ parity is not explicitly broken, the potential is invariant under $G$ parity, and by a simple continuity theorem [7], the shift (if any) of $\Sigma_0$ is similarly invariant. Barring a conspiracy of higher-order terms, $G$ parity should not be spontaneously broken.

What about hidden sector baryons? These are also stable particles and potentially contribute to the dark matter relic abundance. In the absence of hidden baryon nonconservation, baryon-antibaryon pairs are created in thermal equilibrium. At freezeout, the energy density of baryons to pions scales as $\Omega_\Pi \sim \sigma v / \langle \sigma v \rangle$ where $\sigma v$ is not too large, influencing the thermal history and branching fraction, influencing the thermal history and potential dark matter annihilation signals.

In conclusion, we have argued that an unbroken and nontrivial discrete symmetry emerges in a class of models with vector-like SM gauging of fermions with QCD-like strong dynamics. The mechanism leads to testable and distinct dark matter and LHC phenomenologies.

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[14] We assume that $N$ and $n_f$ are such that spontaneous chiral symmetry breaking occurs. We are not here concerned with issues such as perturbativity and unification at high scales.
[15] The path integral measure is similarly invariant, so that the parity is well defined at the quantum level.
[16] We have used the freedom to perform a chiral rotation of the fermions and redefine $\theta$ to eliminate any $\bar{\psi} \gamma_5 \psi$ term.
[17] A Majorana mass term violating $G$ parity is forbidden by gauge invariance for complex representations of SU($N$)$_h$.
[18] The PQ symmetry could be extended to accommodate a right-handed neutrino coupling by assigning a common vector PQ charge to all SM leptons.