Mitigation of Plasma Effects in High Pressure RF Cavity for Ionization Cooling

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In this note, we summarize the possible effects of plasma formation during beam propagation through the gas-filled pressurized RF cavity. To mitigate detrimental effects, use of a dopant gas with high electron attachment rate has been discussed.

I. INTRODUCTION

Ionization cooling is the critical building block for the realization of a neutrino factory and a muon collider, because only this cooling scheme can reduce the emittance of the intense muon beam fast enough compared to the short muon lifetime [1–3]. Recently, the idea of using high-pressure hydrogen gas as an energy absorber has been proposed [4]. In particular, when RF cavities are pressurized, there are two unique advantages. First, the momentum loss (both in the transverse and longitudinal directions) and the momentum regain (only in the longitudinal direction) happen simultaneously rather than sequentially. Second, higher accelerating field gradient and better breakdown suppression are possible due to the Paschen effect.

Initial experiments of High-Pressure RF (HPRF) cavity in the absence of the beam have demonstrated that higher field gradients are possible compared to the case of the conventional evacuated cavities [4]. When an intense beam is present, however, several factors associated with the plasma formation may limit the performance of the HPRF cavity [5]. To demonstrate the feasibility of using HPRF cavities in the actual cooling channel, a beam test is planned in the MuCool Test Area (MTA) of Fermilab in early 2009. Since an intense muon beam is not available, a proposed beam test will use proton beam injected from the linac.

In this note, we will discuss several physics issues associated with the formation of the plasma column induced by the beam and possible mitigation methods for them.

II. IONIZATION COOLING

The equation describing the rate of change of the normalized transverse emittance $\epsilon_n$ within the absorber is given by the balance between the energy-loss cooling (first term) and the multiple-scattering heating (second term) effects [1–3]:

$$\frac{d\epsilon_n}{ds} = -\frac{1}{\beta^2} \frac{1}{E} \frac{dE}{ds} \epsilon_n + \frac{\beta \gamma E_s^2 \beta_\perp}{2 \beta^2 E^2 L_R}. \quad (1)$$

Here, $E = \gamma m_e c^2$ is the beam particle energy, $E_s$ is the characteristic scattering energy, $dE/ds = \rho dE/dx$ is the mean rate of energy loss (or stopping power), $L_R$ is the radiation length of the absorber medium, $\beta_\perp (= \beta_x = \beta_y)$ is the transverse beta-function of the magnetic channel, and $\beta = v/c$ and $\gamma = (1 - \beta^2)^{-1/2}$ are the usual relativistic factors. The normalized emittance $\epsilon_n$ is related to the geometric rms emittance $\epsilon_\perp(= \epsilon_x = \epsilon_y)$ by $\epsilon_n = \epsilon_\perp (\beta \gamma)$ and the transverse beam size is given by $\sqrt{\langle x^2 \rangle} = \sqrt{\langle y^2 \rangle} = (\epsilon_\perp \beta_\perp)^{1/2}$.

A. Energy-loss cooling

The energy lost by a beam particle with atomic number $z$ as it propagates through a medium is given by the Bethe-Bloch equation [6],

$$\frac{dE}{ds} = K z^2 Z \rho \frac{I}{A} \frac{1}{\beta^2} \frac{1}{E} \ln \left( \frac{m_e c^2 \beta^2 \gamma^2 T_{\text{max}}}{I^2} \right) - \beta^2 - \frac{\delta}{2}. \quad (2)$$

Here, $K = 0.3071$ MeV g$^{-1}$cm$^2$, $Z/A$ is the ratio of atomic number to weight, $\rho$ is the mass density of the medium in g/cm$^3$, $I$ is the mean excitation energy in eV. $T_{\text{max}}$ is the maximum kinetic energy which can be transferred to a free electron in a single collision, which is approximated as $T_{\text{max}} \approx 2m_e c^2 \beta^2 \gamma^2$ in the low-energy approximation, and $\delta$ is the density effect correction which is small at low energy. For the beam test of a HPRF cavity experiment, the kinetic energy of the proton beam inside the cavity is estimated to be $T \approx 200$ MeV, which yields the stopping power of $dE/dx \approx 10$ MeV cm$^2$/g in the molecular hydrogen (see Fig. 1). Note that when $\partial (dE/dx)/\partial T < 0$, lower energy particle loses more energy than higher energy particles, increasing the longitudinal energy spread.

B. Multiple-scattering heating

The rms scattering angle can be given in terms of the radiation length $L_R$ [7]:

$$\theta_{\text{rms}} = \sqrt{\langle \theta^2 \rangle} \approx \frac{1}{\beta^2 E} \sqrt{\frac{\Delta s}{L_R}}, \quad (3)$$

where $\Delta s$ is the thickness of material. In the limit of small angle, $\frac{1}{2} \theta_{\text{rms}}^2 = \langle \theta^2 \rangle \approx \Delta \langle x^2 \rangle$, where the factor $\frac{1}{2}$ comes from the projection of the deflection angle into the $x-s$ plane. The radiation length $L_R$ is a function of scattering material given by

$$L_R = \frac{X_0}{\rho} = \frac{716.4 \times A}{Z(Z+1) \ln (287/\sqrt{Z})} \frac{1}{\rho}. \quad (4)$$
Since, the rms emittance is \( \epsilon_{\perp} = \sqrt{\langle x'^2 \rangle - \langle x^2 \rangle} \approx \beta_{\perp} \theta_{\text{rms}} \), at a beam waist, the increase in the rms emittance during the transit of \( \Delta s \) around the beam waist is approximated as

\[
\Delta \epsilon_{\perp} \approx \beta_{\perp} \Delta \langle x'^2 \rangle \approx \frac{\beta_{\perp}}{2} \theta_{\text{rms}}^2. \tag{5}
\]

Hence, we have

\[
\frac{\Delta \epsilon_{\perp}}{\Delta s} \approx \frac{\beta_{\perp} \theta_{\text{rms}}^2}{2 \Delta s} \approx \frac{\beta_{\perp}}{2} \frac{1}{\beta_{\perp}} E^2 \frac{1}{L_R}, \tag{6}
\]

which corresponds to the second term of the Eq. (1). Note that emittance increase from multiple-scattering depends on beam size.

### III. PLASMA FORMATION

#### A. Beam-impact ionization

During propagation through a gas, the incident beam undergoes ionizing collisions with neutral gas molecules (as intended for ionization cooling). For the ionization of hydrogen molecules by fast proton impact, single-ionization is dominant \[8\].

\[ p + H_2 \longrightarrow p + H^+_2 + e^- \tag{7} \]

Most of the energy lost by protons through collisions goes into ionization, which leads to the build up of an electron/ion plasma in the volume of the beam path.

The electrons ejected from the primary ionization can have enough energy (larger than the ionization potential \( E_i \) of the gas) to further ionize, producing secondary electron/ion pairs through following processes:

\[ e^- + H_2 \longrightarrow H^+_2 + 2e^- \tag{8} \]

\[ e^- + H_2 \longrightarrow H + H^+ + 2e^- \tag{9} \]

Normally, the dissociative ionization (9) is much smaller than the ionization (8) \[9\], and \( n_e \approx n_{H^+_2} \) to good approximation. The total electron line density after one micropulse (\( \sim 1.5 \times 10^9 \) protons) of the beam passes through the cavity is conveniently expressed by

\[
N_L = n_e \pi r_b^2 = \frac{\rho dE/dx}{W_i} \times (1.5 \times 10^9), \tag{10}
\]

where \( r_b = \sqrt{\frac{2}{T}} \) is the radius of a uniform-density beam, and \( W_i \) is the effective average energy to produce an electron/ion pair. The mass density of the hydrogen gas can be expressed in terms of the pressure as \( \rho [g/cm^3] = 5.7 \times 10^{-6} \rho [\text{psi}] \) for \( T_{\text{room}} = 300 \text{ K} \).

For the analysis of electron evolution at the macropulse time scale, we define the average electron source term as

\[
S \left[ \text{cm}^{-3}s^{-1} \right] \approx \frac{N_L/\pi r_b^2}{\Delta t_p}. \tag{11}
\]

Note that, in fact, micropulse duration (\( \sim 0.13 \text{ ns} \)) is much shorter than the micropulse spacing (\( \Delta t_p = 5 \text{ ns} \)).

#### B. Production of energetic electrons (\( \delta \) rays)

During beam-impact ionization, the ejected electron can have a kinetic energy up to the maximum allowed \( T_{\text{max}} \). The distribution of electrons with kinetic energy \( T \) is approximately given by \[10\]

\[
\frac{dN}{dT} \approx \frac{1}{2} \frac{K_z^2 Z \rho \Delta s}{A \beta^2 T^2}. \tag{12}
\]

Integration of Eq. (12) yields an expression for the number of electrons ejected at a kinetic energy larger than or equal to \( T_0 \) as \[10\]

\[
N(T \geq T_0) = \frac{1}{2} \frac{K_z^2 Z \rho \Delta s}{A \beta^2} \left[ \frac{1}{T_0} - \frac{1}{T_{\text{max}}} \right]. \tag{13}
\]

Before completely slowing down through elastic and inelastic collisions with background gas molecules, energetic electrons will cover a certain distance in the gas. An approximate expression for the practical range (the effective distance including the randomizing effect of the multiple collisions) as a function of electron kinetic energy is \[10\]

\[
R_p \approx [0.71 \times (T[\text{MeV}])^{1.72}/\rho]. \tag{14}
\]

When \( R_p \approx 1 \text{ cm} \), we expect energetic electrons to hit the cavity wall and have some noticeable effects (for example, secondary electron emission). Nonetheless, from Fig. 2, we note that only one out of ten protons ejects a 40 keV electron that has \( R_p \approx 1 \text{ cm} \).
\( \varepsilon = E_0 \sin(\omega t) \)

from the field and transferring it to the gas molecules \( \varepsilon \) of electron thermal energy where illustrated in Fig. 3, together with the micropulse duration \( T_p \) and practical range of electrons \( R_p \) in molecular hydrogen. Here, we set \( p = 500 \text{ psi} \) and \( \Delta s = 3 \text{ cm} \).

C. Electron thermalization

For the nominal operating conditions of the HPRF cavity \( (f = \omega/2\pi \approx 805 \text{ MHz}) \), ions can be assumed to be immobile. On the other hand, electrons not only suffer collisions with background gas molecules, but also interact with the external RF field. The total electron-neutral collision frequency for momentum transfer is approximately given \( \nu_m = 4.4 \times 10^9 [\text{pTorr}] \approx 2.3 \times 10^{11} [\text{psi}] \) for \( T_{\text{room}} = 300 \text{ K} \) \cite{11}, and the response of plasma electrons to the external RF field is described by conductivity:

\[
\sigma = \frac{n_e e^2}{m_e \nu_m} \left[ \frac{\nu_m^2}{\nu_m^2 + \omega^2} - i \frac{\omega \nu_m}{\nu_m^2 + \omega^2} \right]. \tag{15}
\]

A typical axial drift motion of an electron in the RF field in the limit of very high frequency collisions is illustrated in Fig. 3, together with the micropulse duration and spacing. The maximum axial displacement is \( z_0 \approx \varepsilon E_0/(m_e \nu_m \omega) \).

The real component of the plasma conductivity (15) leads to the resistive heating of electrons, and the average power transferred from RF field through collisions can be written as

\[
P_c = \frac{n_e e^2 E_{\text{rms}}^2}{m_e (\nu_m^2 + \omega^2)} \nu_m, \tag{16}
\]

where \( E_{\text{rms}}^2 = E_0^2/2 \) is the rms electric field. The balance of electron thermal energy \( \varepsilon \) is made up of gaining energy from the field and transferring it to the gas molecules (elastic or inelastic):

\[
\frac{d\varepsilon}{dt} = \left[ \frac{P_c}{n_e \nu_m} - \delta_\varepsilon \right] \nu_m, \tag{17}
\]

where \( \delta_\varepsilon \) is the fractional energy loss. During slowing down in the energy range below the ionization potential, electrons mostly dissipate energy by exciting the vibrational and rotational energy levels of molecular hydrogen. In this case, \( \delta_\varepsilon \approx (2m_e/m_{\text{H}_2}) \approx 10 \times (2m_e/m_{\text{H}_2}) \) \cite{12}. The time required for the electrons to reach a new equilibrium is approximately given by \( \tau_\varepsilon \approx 1/\delta_\varepsilon \nu_m \), and the final equilibrium energy is approximated for \( \nu_m \gg \omega \) as

\[
\varepsilon = \frac{1}{2} m_e \langle v_x^2 + v_y^2 + v_z^2 \rangle = \frac{3}{2} T_e \approx \frac{E_{\text{rms}}^2}{m_e \nu_m^2 \delta_\varepsilon}. \tag{18}
\]

If \( \delta_\varepsilon \) and \( \nu_m \) are independent of energy, we obtain \( \varepsilon \propto (E_{\text{rms}}/p)^2 \). More accurately, the equilibrium energy is determined from experiments as a function of a combined parameter, \( E_{\text{rms}}/p \) (see Fig. 4). When RF is off
can be written as

$$\frac{dn_e}{dt} = S + (k_i - k_{DA})n_HZn_e - \beta_e n_e^2 - \frac{D}{\Lambda^2} n_e,$$  \hspace{1cm} (20)

where $S$ is a source term (from beam-impact ionization), $k_{DA}$ is the rate coefficient for dissociative attachment (DA) to background hydrogen molecules, $\beta_e$ is a recombination coefficient, and $D$ and $\Lambda$ are diffusion coefficient and length respectively.

At high pressure, diffusion is often negligible and recombination is slow compared to other processes. Only dissociative attachment,

$$e^- + H_2 \rightarrow H + H^-,$$  \hspace{1cm} (21)

can be a possible mechanism for impeding breakdown in short time scale [15]. Hence, the condition for electron avalanche can be expressed as $k_i > k_{DA}$ or in terms of a threshold value $(E_{rms}/p)_{th} = E_{rms}/p > (E_{rms}/p)_{th}$, where $k_i[(E_{rms}/p)_{th}] = k_{DA}[(E_{rms}/p)_{th}]$. Note that even if $k_i < k_{DA}$, there will be a linear increase of the electron density due to the source term. Since only the lowest vibrational level ($v = 0$) of the ground electronic state of $H_2$ is populated at room temperature ($T_{room} = 300$ K), the rate of this process is typically less than $k_{DA} \sim 10^{-10} \text{ cm}^3\text{s}^{-1}$ [16, 17]. Hence, from Figs. 4 and 5, we expect that only if $E_{rms}/p \geq 10 \text{ V/cm/Torr}$, then $\bar{E} \geq 2 \text{ eV}$, $k_i \geq k_{DA}$, and breakdown can be triggered.

On the other hand, the electron attachment rate is known to increase dramatically with increasing vibrational energy. For example, the rate coefficient approaches to the maximum value of $k_{DA} \sim 10^{-8} \text{ cm}^3\text{s}^{-1}$ for excited vibrational levels around $v = 9$ [16, 17]. Nonetheless, the average energy deposited from one proton micropulse to a hydrogen molecule is only $\Delta E \sim (1.5 \times 10^8) \times (\rho dE/dz)/(n_{H_2} \pi r^2_b) \sim 10^{-8} \text{ eV}$, and the average energy transferred from one RF cycle to a hydrogen molecule is only $\Delta E \sim (e^2E_{rms}/m_e \nu_m^2) \times (v_m/\nu_m) \times (n_e/n_{H_2}) \sim 10^{-7} \text{ eV}$. Therefore, vibrational excitation ($\Delta E > 0.1 \text{ eV}$) is expected to be limited for typical HPRF experiment conditions. To obtain dissociative attachment that is effective in removing electrons, one might consider exciting the higher vibrational states by an infrared laser irradiation [18].

**B. Decrease in quality factor**

In the absence of breakdown, dissociative recombination is the dominant mechanism for bulk removal of electrons in a weakly ionized plasma where molecular ions are abundant [15].

$$e^- + H^+_2 \rightarrow H(n = 1) + H^*(n > 1)$$  \hspace{1cm} (22)

Recombination does not greatly influence the breakdown threshold. The fate of an avalanche (whether it will grow
Recombination rate $n$, $H_{cm}b = Ls > H_{cm}sL$.

or die out) is decided at its early stage, in which the number of electron/ion pairs is so small that recombination has a very low probability (note that the recombination rate is proportional to the electron density squared). On the other hand, recombination can finalize the electron density level reached by beam-impact ionization during the propagation of macropulse (many short micropulses) in the microsecond to millisecond range. The electron density evolution during the macropulse is determined in the microsecond to millisecond range. The electron density level reached by beam-impact ionization during the macropulse without (red) and with (blue) recombination. Here, we pick the parameters to $\beta_r \sim 10^{-8} \text{cm}^3\text{s}^{-1}$, $r_b \sim 1 \text{ cm}$, $p = 500 \text{ psi}$.

Note that for $\beta_r \rightarrow 0$, $n_e = St$ and for $t \rightarrow \infty$, $n_e = \sqrt{Se}/\beta_r$.

Even though the plasma density is expected to be saturated somehow in the absence of breakdown, the locally accumulated plasma will eventually degrade the performance of the RF cavity. When the cavity is filled with a plasma of complex conductivity [Eq. (15)], the imaginary part causes the shift in the resonance frequency $\omega_0$ and the real part the decrease in the $Q$ value. They are given by Slater’s perturbation equations [22]:

$$2\frac{\Delta \omega_0}{\omega_0} = \frac{1}{1 + (\nu_m/\omega_0)^2} \langle n_e \rangle,$$

$$\Delta \left( \frac{1}{Q} \right) = \frac{(\nu_m/\omega_0)^2 \langle n_e \rangle}{1 + (\nu_m/\omega_0)^2} \langle n_e \rangle,$$

where

$$\langle n_e \rangle = \int_V n_e E_0^2(r, z) dV,$$

and $n_e = \epsilon_0 \mu_0 \omega_0^2/e^2$ is the critical density. Here, we assume that the plasma is highly collisional ($\omega_{ce}^2 = n_e e^2/\epsilon_0 m_e \ll \nu_m^2$ and $\omega_0^2 \ll \nu_m^2$) and the electric field is altered only slightly from the initial spatial distribution $E_0(r, z)$. If the plasma consists of a small radius column ($r \approx r_b$), much smaller than the cavity radius $R_c$, we can approximate

$$\langle n_e \rangle \approx \frac{N_L}{\xi R_c^2} \approx n_e(r = 0) \times \frac{r_b^2}{\xi R_c^2},$$

where the geometric factor $\xi$ is calculated to be 0.2695 for the case of a simple pill box cavity. The plasma column may be looked upon as a lumped admittance, shunting the cavity. Since the change in the resonance frequency is of the order of $(\omega_0/\nu_m)^2 \ll 1$, the dominant effect of this plasma column will be the decrease in the loaded $Q$, $Q_L$. Combining Eqs. (26) and (28), we can estimate the change in the $Q_L$ of the cavity according to the propagation of the macropulse. As is apparent in Fig. 8, if the plasma is not removed properly, we expect a significant reduction in the quality factor within a couple of hundreds of micropulses.

FIG. 6: Dissociative recombination rate coefficient as a function of average electron energy [19, 20].

FIG. 7: Examples of electron density evolution over many micropulses without (red) and with (blue) recombination. Here, we pick the parameters to $\beta_r \sim 10^{-8} \text{cm}^3\text{s}^{-1}$, $r_b \sim 1 \text{ cm}$, $p = 500 \text{ psi}$.

FIG. 8: Decrease in the loaded $Q$, $Q_L$, over many micropulses without (red) and with (blue) recombination process. Initially, we assume $Q_L = 6000$. 
C. Enhanced multiple scattering

When a beam ion collides with a background neutral gas, no force is felt until the beam ion is close to the gas molecule, on the scale of atomic dimensions. If the incident beam ion doesn’t approach within atomic dimensions, then the nucleus of the gas molecule is shielded by its bound electrons [23]. When a plasma column is created along the beam path, however, scattering through the plasma column may extend the range of Coulomb interactions. In particular, there is a possibility that small-angle long-range Coulomb scattering between beam ions and background molecular ions could increase the deflection of the beam ion velocity vector [24]. Note that collisions between beam ions and plasma electrons will not contribute to the deflection due to the large mass ratio, $m_b/m_e \gg 1$. For $n_i \approx n_{H_2} \approx n_e$ and $m_i = 2m_b$, the scattering angle can be approximated as [24, 25]

$$\langle \theta^2 \rangle_i \approx \frac{n_i e^2 Z_i^2 e^4}{4\pi\epsilon_0^2 E^2 b^4} \ln \Lambda \times \Delta s. \quad (29)$$

Here, $\ln \Lambda$ is the Coulomb logarithm [23, 25]. By comparing Eqs. (3) and (29), we have the following ordering:

$$\frac{\langle \theta_i^2 \rangle}{\langle \theta^2 \rangle_i} \sim O \left( \frac{n_e}{n_{H_2}} \right) \ll 1. \quad (30)$$

Hence, the contribution of the plasma to the multiple scattering is negligible.

From Eqs. (4) and (29), we note that enhanced multiple scattering can be an issue when high-Z impurities are accumulated inside the cavity (for example, when too much dopant gas is applied).

V. MITIGATION OF PLASMA EFFECTS USING A DOPANT GAS

When plasma electrons are accumulated, the performance of the RF cavity will be degraded considerably. In order to remove plasma electrons, use of a dopant gas with high electron attachment cross section has been proposed. Here, we consider SF$_6$ (sulphur hexafluoride) and $c$-C$_4$F$_8$ (perfluorocyclobutane).

With a dopant gas, the average lifetime of a beam-induced electron is determined by the sum of the time needed for the electron to be thermalized to an energy level at which attachment becomes significant, $\tau_e \approx 1/\langle \delta e/v_m \rangle$, plus the average time required before the electron is then captured, $\tau_a \approx 1/\langle k_a n_g \rangle$ [21]. Here, $k_a$ is the electron attachment rate coefficient (see Fig. 9), $\alpha = \frac{n_{dop}}{n_g} = \frac{n_{dop}}{p}$ is the partial fraction of dopant in the gas mixture, and $n_g[\text{cm}^{-3}] = 1.7 \times 10^{18} \rho[\text{psi}]$ for $T_{room} = 300 \text{ K}$. If $\tau_e + \tau_a < \Delta t_p$, then the equilibrium electron density is approximately given by

$$n_e \approx S(\tau_e + \tau_a). \quad (31)$$

When $\tau_e + \tau_a > \Delta t_p$, the dopant gas cannot fully remove the electrons before next micropulse comes. The minimum effective dopant fraction is, therefore, $\alpha \approx 0.001\%$. Once the dopant fraction is high enough that the attachment time is comparable to the energy equilibration time, little further benefit is obtained by increasing $\alpha$ [21]. Hence, the maximum effective dopant fraction can be estimated by setting $\tau_e \approx \tau_a$, which results in $\alpha \approx \delta_e v_m / k_a n_g \approx 4\%$ for $\delta_e \approx 10 \times (2m_e/m_{H_2})$. Figure 10 shows, indeed, the equilibrium electron density can be reduced by more than two orders of magnitude with a dopant fraction of $< 1\%$. Since $Z = 70$ for SF$_6$ and $Z = 96$ for $c$-C$_4$F$_8$, too much dopant fraction will cause enhanced beam-impact ionization and multiple scattering with the dopant gas.

For most dopant gases, the probability of electron capture drops to zero for high energy electrons. For SF$_6$, the upper energy limit is as low as 0.4 eV, while $c$-C$_4$F$_8$ has finite attachment cross section at up to 1 eV (see Fig. 9).

![FIG. 9: Electron attachment cross section as a function of electron energy (left), and electron attachment rate coefficient as a function of average electron energy of Maxwellian distribution (right) for dopant gases, $c$-C$_4$F$_8$ and SF$_6$.](image)

![FIG. 10: Equilibrium electron density as a function of dopant fraction for $k_a \approx 2 \times 10^{-6}\text{cm}^3\text{s}^{-1}$ and $\delta_e \approx 10 \times (2m_e/m_{H_2})$.](image)
VI. SUMMARY AND CONCLUSION

The beam test planned for the HPRF cavity at Fermilab will be an important milestone for the future direction of ionization cooling channel development. One concern is the possible accumulation of plasma induced by beam-impact ionization. Since recombination and dissociative attachment are quite slow, the plasma is expected to be locally accumulated over the macropulse duration and absorb a considerable amount of RF power. To mitigate this effect, use of a small fraction of dopant gas has been discussed.