A Linear Algorithm for Minimum Phasor Measurement Units Placement

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Abstract-- **A Phasor measurement unit (PMU) measures the voltage and current phasors (angle and magnitude) at a bus in a power system. This paper presents a linear algorithm to minimize the number of PMUs to monitor the entire system by modeling the system with topological observation theory. The algorithm is tested by the computation to standard IEEE 14, 18 and 30 bus systems and New England 39 bus system. Results are compared to those of previously implemented topology based formulated algorithms.**

Index Terms**—** Domination problem, PMU, Topological observation theory

I. INTRODUCTION

A Phasor measurement unit (PMU) measures the voltage and current phasor (angle and magnitude) at a bus in power system. In Smart Grids, PMUs are widely used for state estimate, transient analysis, and protection purposes. Due to the high cost of a PMU, it is desirable to have minimum number of PMUs to monitor the entire system.

Studies have been done towards optimal PMU placement in past years. Generally these efforts can be classified into two categories: topology based algorithms and numerical methods. Topology methods are developed from graph theories, compared to numerical methods that are mainly based on numerical factorization of measurement Jacobi matrices. Numerical methods are less suitable for large systems because they are involved with large dimension matrices that increase the computational complexity.

If we model the buses in a power system by vertices, and model the transmission and/or distribution lines connecting buses by edge, this problem is converted to be a domination problem and requires the extension of the topological observation theory.

The observation rules can be described as following:

Any bus (or alternatively called, nodes, vertices) with a PMU is observed. So is (are) the directly adjacent bus(s).

Any line (or alternatively called, edges) between two observed buses is observed.

Any bus that is incident to an observed line is observed

If a bus is incident to a total of k>1 lines and if k-1 of those lines are observed, then all k of these lines are observed. This rule is based on Kirchhoff's current law.

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In all the topological approaches the goal is to find out the minimum number of PMUs and the dominating set S (the set of buses where PMUs are mounted on) for a given power system [1-4]. In Tarjan's research [5], in order to get the fast solution, a good initial guess of PMU placement was found through a procedure which built a spanning measurement subgraph according to a depth-first search strategy, taking advantage of graph theory. Baldwin et al advanced this method and proposed a dual search algorithm which used both a modified bisecting search and a simulated annealing-based method to find out the minimum PMU set [6]. This algorithm was tested for a list of power systems and proven efficient. However, "there is no guarantee that the resulting placement set is optimal [6]", i.e., there is no guarantee that the PMU number has been minimized.

In 2002, Haynes et al [7] mathematically proved that, for a tree having k vertices of degree at least 3, the "power dominating number"

$$\gamma p(T) \geq \frac{k+2}{3} \tag{1}$$

And

$$\gamma p(T) \leq \frac{n}{3} \tag{2}$$

where n is the total number of vertices. Equation (1) and (2) give the upper and lower bounds for the power dominating number. Although a power system does not have to have a tree topology, these theorems corresponded to the computation results from [6] very well. Haynes et al in their paper also gave an algorithm to find out the dominating set S and a partition of the whole set G (the power system) into $|S|$ so that each subset induces a "spider". This algorithm was strictly proven in this paper. However, this algorithm is designed specifically for tree topologies; the logic in the algorithm is complicated and the effectiveness of this algorithm is to be tested.

In 2009, Mohammadi-Ivatloo summarized most available topological based formulated algorithms, including generic algorithm (2009), Tabu search (2006), Integer linear programming (2008) and etc. However, the measurement, or the complexity of each algorithm, was still left to be discussed.

In engineering practice, we are most interested in the dominating set, i.e., where to mount the PMUs. The partition is not the primary concern. Taking advantage of the upper and lower bounds, a linear algorithm is proposed in this study. Backed up by graph theory, this algorithm is proven especially effective for small systems.

II. The Linear Algorithm

A. Node-Incidence Matrix for Power System

The topological information of a power system can be stored in the so-called node-incidence matrix for computer analysis.

The rule is simple:

If node i is adjacent to node j, then $A_{ij} = 1$, $i \neq j$

$A_{ii} = 0$

Normally A is a large sparse matrix. For example, for the IEEE 14-bus system,

```
A= [0 1 0 0 1 0 0 0 0 0 0 0 0 0
    1 0 1 1 1 0 0 0 0 0 0 0 0 0
    0 1 0 1 0 0 0 0 0 0 0 0 0 0
    0 1 1 0 1 0 1 0 1 0 0 0 0 0
    1 1 0 1 0 1 0 0 0 0 0 0 0 0
    0 0 0 0 1 0 0 0 0 0 1 1 1 0
    0 0 0 1 0 0 0 1 1 0 0 0 0 0
    0 0 0 0 0 0 1 0 0 0 0 0 0 0
    0 0 0 1 0 0 1 0 0 1 0 0 0 1
    0 0 0 0 0 0 0 0 1 0 1 0 0 0
    0 0 0 0 0 1 0 0 0 1 0 0 0 0
    0 0 0 0 0 1 0 0 0 0 0 0 1 0
    0 0 0 0 0 1 0 0 0 0 0 1 0 1
    0 0 0 0 0 0 0 0 1 0 0 0 1 0]
```

It is easy to find out that there are 7 nodes with degree 3 or more. So, the bounds of number of PMUs needed are

$$\gamma p(T) \geq \frac{7+2}{3} = 3, \qquad (3)$$

$$\gamma p(T) \leq \frac{14}{3} = 4.67 \rightarrow 4 \quad . \qquad (4)$$

That is, the only possible values for $|S|$ are 3 and 4.

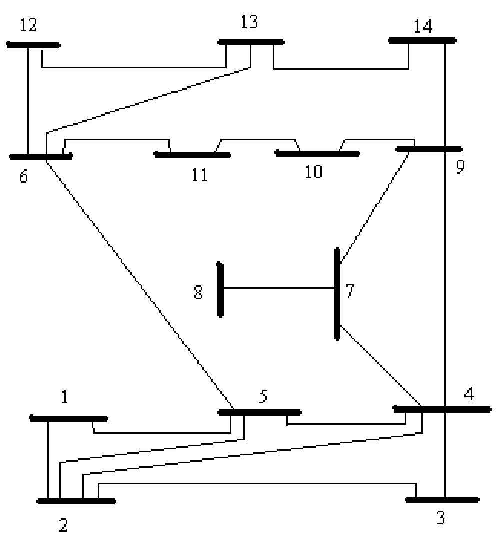

Fig. 1. IEEE 14-bus Power System

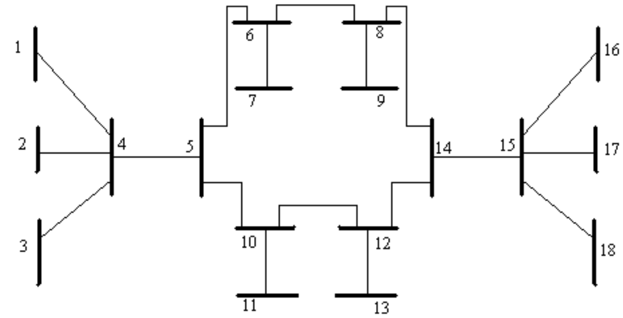

Fig. 2. IEEE 18-bus Power System

B. The Measurement of the Algorithm

Having known the bounds, we want to find out the minimum number of PMUs, and the dominating set S. The basic idea of this algorithm is to test all possible node combinations by the observation rules, until one combination is found to be able to "observe" all the system. We call a test for a combination (with bound constrains) a measurement. We wish the number of measurements as few as possible, to make the algorithm more effective.

Obviously, for the IEEE 14-bus system, the maximum number of measurements is

$$\begin{pmatrix} 3 \\ 14 \end{pmatrix} + \begin{pmatrix} 4 \\ 14 \end{pmatrix} , \qquad (5)$$

because the S-set may contain either 3 or 4 nodes.

Taking advantage of the observation 4 in [7] that every node in S-set has degree at least 3 (buses with at least 3 lines), we can tell the maximum number of measurements for the 14-bus system is

$$\begin{pmatrix} 3 \\ 7 \end{pmatrix} + \begin{pmatrix} 4 \\ 7 \end{pmatrix} = 70 \quad . \qquad (6)$$

We need to keep in mind that, in the implementation of the algorithm, we may not have to run all the 70 measurements to find out the S-set. The number of measurement before we get an S-set (which is usually not unique) can be any number between 1 and 70.

By the nature of this algorithm, the minimum number of PMUs needed can be guaranteed.

C. Flowchart of the Algorithm

Begin

1. Read in node-incidence matrix A. All buses (nodes) are in G.
2. Determine the bounds of $|S|$.
3. Loop starting from the lower bound, to the upper bound:
    a. Generate a node combination, e.g., {2, 4, 5}. These nodes are mounted with PMUs, thus observed. Save them in array C.
    b. Find out all nodes adjacent to these 3 or 4 nodes.

Save them in array C

c. Find out all nodes are not in C.

   a) Pick up such a node j, use rule 4 to judge if it is observed.

   b) If yes, put j in C, and pick up another node and check.

   c) If all "not-in-C" nodes have been checked, compare C to the whole set G.

      - If C=G, output the node combination in a). That is the S-set. Quit.
      - If C is not equal to G, go to a), to generate another node combination.

4. Output how many number of combination has been tried. That is the number of measurement.

End

III. Case Study Results

The linear algorithm is implemented by Matlab (version 6.5) and is tested for IEEE 14, 18, 30-bus systems and the New England 39 bus system. The results are given and compared to [6] and [8]. Table 1 gives the comparison of computation results by different algorithms.

TABLE I
COMPARISON OF COMPUTATION RESULTS BY DIFFERENT ALGORITHMS

Power systems (bus number N)	**Method**	**v**	**v/N**	**m**	**m/N**
IEEE 14-bus	***Proposed linear algorithm***	***3***	***0.214***	***2***	***0.143***
	Dual search [6]	**3**	**0.214**	**32**	**2.286**
	Graphic theoretic procedure [8]	**5**	**0.357**		
IEEE 18-bus	***Proposed linear algorithm***	***4***	***0.222***	***15***	***0.833***
IEEE 30-bus	***Proposed linear algorithm***	***7***	***0.233***	***72***	***2.400***
	Genetic algorithm [8]	**7**	**0.233**		
	Graphic theoretic procedure [6,8]	**11**	**0.367**		
New England 39-bus	***Proposed linear algorithm***	***8***	***0.205***	***72***	***1.846***
	Dual search [6]	**8**	**0.205**	**84**	**2.154**
	Genetic algorithm [8]	**8**	**0.205**		
	Branch and bound [8]	**9**	**0.231**		

Normal font: results from linear algorithm proposed
Italic: results from proposed linear algorithm
N: number of buses
v: minimum number of PMUs needed
m: number of measurements

IV. Discussion

The proposed linear algorithm takes advantage of the upper and lower bounds and the graph theorems that were mathematically proven in [7], which greatly reduced the computation in seeking a dominating set in a power system. It is applied to power systems up to 39 buses. Compared to the algorithms in [6] and [8], this proposed algorithm can theoretically guarantee the minimum number of PMUs, and results in relatively smaller measurement (see results for the 14-bus and 39-bus systems). Compared to the algorithm in [7], it is simpler and easier to be implemented.

It has not been tested for larger systems. However, the computation complexity reflected by measurement study indicates that the linear algorithm should be very competitive to other topology based algorithm and other numerical methods.

The v/N ratios are all between 0.2 and 0.3 for all tested systems.

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VI. Biographies

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