FERMILAB–PUB–08–099–T
arXiv: 0804.4581 [hep-ph]

Model-Independent Analysis of Tri-bimaximal Mixing – a Softly-Broken Hidden or an Accidental Symmetry?

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Abstract

To address the issue of whether tri-bimaximal mixing (TBM) is a softly-broken hidden or an accidental symmetry, we adopt a model-independent analysis in which we perturb a neutrino mass matrix leading to TBM in the most general way but leave the three texture zeros of the diagonal charged lepton mass matrix unperturbed. We compare predictions for the perturbed neutrino TBM parameters with those obtained from typical $SO(10)$ grand unified theories with a variety of flavor symmetries. Whereas $SO(10)$ GUTs almost always predict a normal mass hierarchy for the light neutrinos, TBM has a priori no preference for neutrino masses. We find, in particular for the latter, that the value of $|U_{e3}|$ is very sensitive to the neutrino mass scale and ordering. Observation of $|U_{e3}|^2 > 0.001$ to 0.01 within the next few years would be incompatible with softly-broken TBM and a normal mass hierarchy and would suggest that the apparent TBM symmetry is an accidental symmetry instead. No such conclusions can be drawn for the inverted and quasi-degenerate hierarchy spectra.

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Neutrino oscillations seem to point towards a lepton flavor structure completely different from that of the quark sector. In particular, the PMNS lepton mixing matrix has a very different structure from that of the CKM quark mixing matrix. Nevertheless, this seemingly incompatible feature can be reconciled in Grand Unified Theories (GUTs), where quarks and leptons belong to the same multiplets. In particular, GUTs based on $SO(10)$, which allow for a seesaw mechanism [1] without adding singlets by hand, have been frequently studied in the past ten years; see e.g. [2]. The models often specify in addition a particular flavor symmetry with charges assigned to the fermion and Higgs $SO(10)$ multiplets, although so-called "minimal" Higgs models may rely on no flavor symmetry at all.

Another more recent approach for explaining the neutrino mixing scheme has involved the introduction of lepton flavor symmetries. The goal of such models has been to reproduce the approximate tri-bimaximal mixing (TBM) form observed by Harrison, Perkins and Scott among others [3]:

$$U_{\rm PMNS} \simeq U_{\rm TBM} = \begin{pmatrix} \sqrt{\frac{2}{3}} & \sqrt{\frac{1}{3}} & 0 \\ -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & -\sqrt{\frac{1}{2}} \\ -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & \sqrt{\frac{1}{2}} \end{pmatrix} P \ . \tag{1}$$

Here $P = {\rm diag}(1,\, e^{i\alpha},\, e^{i\beta})$ contains the two Majorana phases, α and β, whereas the Dirac phase δ remains unspecified. Lepton flavor symmetries such as A_4, S_4, and S_3 have been applied which lead to the TBM in a straightforward manner [4]. Even more recently symmetries such as T' have been introduced as an extended flavor symmetry in order to treat the quark sector as well in a self-consistent fashion [5].

The question then arises whether the TBM symmetry is a presumably softly-broken hidden symmetry, or whether it is an accidental symmetry in nature. Note that the flavor symmetries originally introduced with $SO(10)$ models were not designed to reproduce the TBM matrix per se, but rather were designed to reproduce quark and lepton mixing schemes in approximate agreement with the then known mixing data. Even with more refined data and fits to the data now available in the literature, many $SO(10)$ models have still survived. For reference, we quote the current best-fit values and 1σ (3σ) ranges for the neutrino oscillation parameters as given in [6]:

$$\begin{aligned} \Delta m^2_{21} &= 7.67\,^{+0.22}_{-0.21}\,\left({}^{+0.67}_{-0.61}\right) \times 10^{-5}\ {\rm eV}^2\,, \\ \Delta m^2_{31} &= \begin{cases} -2.37 \pm 0.15\,\left({}^{+0.43}_{-0.46}\right) \times 10^{-3}\ {\rm eV}^2 & \text{(inverted ordering)}\,, \\ +2.46 \pm 0.15\,\left({}^{+0.47}_{-0.42}\right) \times 10^{-3}\ {\rm eV}^2 & \text{(normal ordering)}\,, \end{cases} \\ \sin^2\theta_{12} &= 0.32 \pm 0.02\,\left({}^{+0.08}_{-0.06}\right)\,, \\ \sin^2\theta_{23} &= 0.45\,^{+0.09}_{-0.06}\,\left({}^{+0.19}_{-0.13}\right)\,, \\ \sin^2\theta_{13} &= 0.0\,^{+0.019}_{-0.000}\,\left({}^{+0.05}_{-0.00}\right)\,. \end{aligned} \tag{2}$$

For exact TBM, the mixing angles correspond to

$$\sin^2\theta_{12} = \frac{1}{3}\,, \quad \sin^2\theta_{23} = \frac{1}{2}\,, \quad \sin^2\theta_{13} = 0\,. \tag{3}$$

In order to address this issue and provide a partial answer to the question raised, we shall study (what we consider) reasonable deviations from TBM and compare those results with the predictions of $SO(10)$ models available in the literature. In doing so, we will not assume a particular model leading to TBM, but instead take the corresponding neutrino mass matrix for exact TBM at face value after adopting a basis in which the charged leptons are real and diagonal, i.e., $U_\ell = \mathbb{1}$. Let us stress here the following: we take the point of view that some unknown flavor symmetry generates TBM and work in the charged lepton basis. The flavor symmetry results obtained in any other basis can readily be cast into this form, so our results will apply in general. With the specified choice of basis the mass matrix, m_ν, uniquely giving rise to TBM is

$$(m_\nu)_{\rm TBM} = U^*_{\rm TBM}\, m_\nu^{\rm diag}\, U^\dagger_{\rm TBM} = \begin{pmatrix} A & B & B \\ \cdot & \frac{1}{2}(A+B+D) & \frac{1}{2}(A+B-D) \\ \cdot & \cdot & \frac{1}{2}(A+B+D) \end{pmatrix}. \tag{4}$$

Here $m_\nu^{\rm diag} = {\rm diag}(m_1, m_2, m_3)$ and the parameters A, B, D are in general complex and functions of the neutrino masses and Majorana phases:

$$A = \frac{1}{3}\left(2\,m_1 + m_2\,e^{-2i\alpha}\right)\,, \quad B = \frac{1}{3}\left(m_2\,e^{-2i\alpha} - m_1\right)\,, \quad D = m_3\,e^{-2i\beta}\,. \tag{5}$$

In this short note we will investigate in a general manner deviations from the tri-bimaximal mixing texture. Our Ansatz is to modify the structure of the mass matrix by multiplying each element of Eq. (4) with an individual complex correction factor, ϵ_i:

$$m_\nu = \begin{pmatrix} A\,(1+\epsilon_1) & B\,(1+\epsilon_2) & B\,(1+\epsilon_3) \\ \cdot & \frac{1}{2}(A+B+D)\,(1+\epsilon_4) & \frac{1}{2}(A+B-D)\,(1+\epsilon_5) \\ \cdot & \cdot & \frac{1}{2}(A+B+D)\,(1+\epsilon_6) \end{pmatrix}. \tag{6}$$

Here the complex perturbation parameters are taken to be $|\epsilon_i| \leq 0.2$ for $i = 1-6$ with their phases ϕ_i allowed to lie between zero and 2π.

Note that had we chosen instead to perturb the original three parameters A, B and D with complex parameters ϵ_A, ϵ_B and ϵ_D, the neutrino masses, m_i and Δm^2_{ij} would be altered but the mixing matrix would remain TBM. Instead we perturb the neutrino mass matrix as above but demand that the three texture zeros in the diagonal charged lepton mass matrix, m_ℓ, remain unperturbed. The same perturbation prescription then applied to m_ℓ simply results in a diagonal phase transformation acting on the matrix U_ℓ, which can be rotated away in $U_{\rm PMNS} = U_\ell^\dagger\, U_{\rm TBM}$.

Still one may insist on applying corrections from the charged lepton sector. For example, with $U_{\rm PMNS} = U_\ell^\dagger \, U_\nu$, one can assume that U_ν corresponds to tri-bimaximal mixing and that the correction is given by

$$U_\ell \simeq \left(\begin{array}{ccc} 1 & \lambda & 0 \\ -\lambda & 1 & 0 \\ 0 & 0 & 1 \end{array} \right) . \tag{7}$$

Then in the basis in which the charged leptons are diagonal, the neutrino mass matrix reads

$$m_\nu' = U_\ell^\dagger \, (m_\nu)_{\rm TBM} \, U_\ell^\ast \; . \tag{8}$$

In the case of a normal hierarchy where $A \simeq B \simeq \sqrt{\Delta m^2_\odot}/3 \ll D \simeq \sqrt{\Delta m^2_{\rm A}}$, one finds for the $e\mu$ entry that $(m_\nu')_{e\mu} \simeq B \, (1 - \frac{D}{B} \frac{\lambda}{2})$. Since $\frac{D}{B} \frac{\lambda}{2} \simeq \frac{3}{2} \sqrt{\Delta m^2_{\rm A}/\Delta m^2_\odot} \, \lambda \lesssim 9 \, \lambda$, we require $\lambda \lesssim 0.02$ in order to have a soft-breaking perturbation to $(m_\nu)_{\rm TBM}$ of less than 20%. A similar number holds in the inverted hierarchy, where for $\alpha = \pi/2$, $A \simeq \sqrt{\Delta m^2_{\rm A}}/3$ and $B \simeq -2\sqrt{\Delta m^2_{\rm A}}/3$, the $\mu\mu$ entry receives the largest corrections, roughly 8λ. Again, demanding a soft-breaking correction less than 20% requires $\lambda \lesssim 0.025$. It is then easy to see from [7] with $U_{\rm PMNS} = U_\ell^\dagger \, U_{\rm TBM}$ and U_ℓ from Eq. (7), the following small deviations from TBM are obtained:

$$|U_{e3}|^2 = \frac{\lambda^2}{2} \text{ and } \sin^2 2\theta_{23} \simeq 1 - \frac{1}{4} \, \lambda^4 \; . \tag{9}$$

The implied small values of λ lead to $|U_{e3}|^2$ well below 10^{-3} and $\sin^2 2\theta_{23}$ very close to 1. In the spirit of this note we do not tolerate a leptonic correction with as large a value as $\lambda = 0.22$ for the soft breaking perturbation. We prefer to work in the charged lepton basis and proceed as indicated in the previous paragraph.

Radiative corrections also lead to perturbations of a tri-bimaximal mass matrix [8]. The relevant small parameters depend on the charged lepton masses, so that the τ contribution is enough to consider. In this limit the $e\tau$ and $\mu\tau$ elements are multiplied with $(1 + \epsilon_\tau)$, while the $\tau\tau$ entry is multiplied with $(1 + 2 \, \epsilon_\tau)$. The small parameter is defined as $\epsilon_\tau = c \, \frac{m_\tau^2}{16\pi^2 \, v^2} \ln \frac{M_X}{m_Z}$, where $v = 174$ GeV and c is given by $3/2$ in the SM and by $-(1 + \tan^2 \beta)$ in the MSSM. Demanding that $|2 \, \epsilon_\tau|$ be less than 0.2 leads for $M_X = 10^{15}$ (10^9) GeV to $\tan \beta \lesssim 71.4$ (97.1). Therefore, our corrections include radiative corrections up to this huge value of $\tan \beta$. A detailed analysis of radiative corrections to TBM can be found in Ref. [9, 10].

Returning to the perturbed matrix in Eq. (6), we will vary the complex ϵ parameters, diagonalize the resulting m_ν's and study the predictions for the neutrino mixing angles. The results obtained for the perturbed mixing matrix will of course depend on the neutrino mass ordering and scale. We shall find that $|U_{e3}|$ depends most sensitively on these observables. Luckily, one expects a sizable improvement on its current upper limit of $|U_{e3}| \lesssim 0.2$ in the near future. We then compare our findings from broken TBM with predictions from successful $SO(10)$ GUTs and study how one may distinguish these two approaches experimentally.

Let us first consider normal mass ordering. The strategy we adopt is as follows: we fix $m_3 = 0.050$ eV, and take as starting values for the other masses, $m_2 = 0.0095$ eV and $m_1 = 0.0037$ eV, corresponding to $\Delta m^2_{21} = 7.66 \times 10^{-5}$ eV^2 and $\Delta m^2_{31} = 2.49 \times 10^{-3}$ eV^2, values well within the center of the currently allowed region. We shall allow for a 20% variation around the initial values of m_2 and m_1 and vary the phases α and β between zero and 2π. Furthermore, the complex perturbation parameters in Eq. (6) are also varied within $|\epsilon_i| \leq 0.2$ for each $i = 1 - 6$, with the full range of phases allowed for each. For each choice of parameters the resulting mass matrix is diagonalized and, if the outcome is within the current 3σ range from Eq. (2), the point is kept. Note that with a maximal perturbation of 20% of the individual mass matrix elements, two entries can have a relative variation of 40%, which is quite generous.

The scatter plots in Figs. 1 and 2 show the results of this analysis. We see that $|U_{e3}|^2$ is predicted to be rather small and lie below 10^{-3}. This number should be compared with the expected sensitivity of the Double Chooz reactor experiment [11], which will start data taking in 2009, and will reach a 90% C.L. limit of 0.018 after one year with one detector, and 0.005 after 3 years of operation with both detectors. The Daya Bay experiment [12], presumably starting after Double Chooz, is expected to improve the limit by another factor of two. Our results show that neither of the two experiments is expected to find a positive signal, if a flavor symmetry predicting tri-bimaximal mixing and a normal mass hierarchy is broken by less than 20%. The same is true of course for the currently running long-baseline MINOS [13] and OPERA [14] experiments. It will take the first generation superbeam experiments, or perhaps even more advanced technologies such as β-beams or neutrino factories, to probe $|U_{e3}|^2$ in the range below $\sin^2\theta_{13} = 10^{-3}$ predicted by the perturbed TBM results. On the other hand, $\sin^2\theta_{12}$ is uniformly populated over its experimentally allowed range and appears to have no bearing on the issue raised.

For comparison, we also give in Table 1 the results for thirteen $SO(10)$ GUT models, all of which involve a conventional type I seesaw mechanism and predict a normal mass hierarchy for the light neutrinos. All of these models predict all three mixing angles in their currently allowed range. According to the names of the authors, the references are A [15], AB [16], BB [17] BM [18], BO [19], CM [20], CY [21], DMM [22], DR [23], GK [24], JLM [25], VR [26] and YW [27]. For details, we refer to the cited works, and also to recent model compilations [28–30]. Note that the $SO(10)$ predictions are well separated from the results of perturbed TBM and are accessible or more nearly accessible to the reactor experiments discussed above. The only exception is the GK model. We note in this respect that this model is very much challenged by its relatively large predictions for lepton flavor violating decays like $\mu \to e\gamma$ [29].

It is of interest to present some approximate analytical results to support the numerical work leading to the scatter plot in Fig. 1. We will not try here to diagonalize the fully perturbed mass matrix, but rather estimate the implied order of magnitude of $|U_{e3}|$ and $\sin^2 2\theta_{23}$. A general statement, independent of the mass ordering, is that if the perturbation occurs only in the ee or $\mu\tau$ entry of the tri-bimaximal m_ν from Eq. (4), then the resulting

Model	Hierarchy	$\sin^2 2\theta_{23}$	$\lvert U_{e3}\rvert^2$	$\sin^2\theta_{12}$
A [15]	NH	0.99	0.0025	0.31
AB [16]	NH	0.99	0.0020	0.28
BB [17]	NH	0.97	0.0021	0.29
BM [18]	NH	0.98	0.013	0.31
BO [19]	NH	0.99	0.0014	0.27
CM [20]	NH	1.00	0.013	0.27
CY [21]	NH	1.00	0.0029	0.29
DMM [22]	NH	1.00	0.0078	–
DR [23]	NH	0.98	0.0024	0.30
GK [24]	NH	1.00	0.00059	0.31
JLM [25]	NH	1.0	0.0189	0.29
VR [26]	NH	0.995	0.024	0.34
YW [27]	NH	0.96	0.04	0.29
S-B TBM	NH	$\gtrsim 0.94$	$\lesssim 10^{-3}$	–
S-B TBM	IH	$\gtrsim 0.91$	$\lesssim 10^{-2}$	–
S-B TBM	QD	–	–	–

Table 1: $SO(10)$ models and their predictions for the lepton mixing angles. If ranges are given we take the central value. Also given are the constraints, if any, on the mixing angles for the three possible mass orderings from the softly-broken tri-bimaximal mixing mass matrices.

mass matrix is still μ–τ symmetric. Consequently the relation $|U_{e3}| = 1 - \sin^2 2\theta_{23} = 0$ still holds in this case. The largest deviation of $|U_{e3}|$ from zero occurs when the $e\mu$ and $e\tau$ entry of the tri-bimaximal m_ν are perturbed [10, 31]. If we set all other perturbations to zero, the extreme case occurs when the $e\mu$ element is multiplied by $(1-\epsilon)$ while the $e\tau$ element is multiplied by $(1+\epsilon)$, where ϵ is here real. We can then diagonalize Eq. (6) and find (ignoring the phases and neglecting m_1 for simplicity) that

$$|U_{e3}|^2 \simeq 4\,A^2\,\epsilon^2 \simeq \frac{4}{9}\,R\,\epsilon^2 \lesssim 7\times 10^{-4}\ . \tag{10}$$

where $R = \Delta m^2_\odot/\Delta m^2_{\rm A}$ is the ratio of the solar and atmospheric mass-squared differences with $\Delta m^2_\odot = m_2^2 - m_1^2$ and $\Delta m^2_{\rm A} = |m_3^2 - m_1^2|$. This is actually quite close to the numerical result, and the small discrepancy can be explained by the effects of non-zero m_1 and the other small terms including ϵ_i. The smallest value for $\sin^2 2\theta_{23}$ is achieved for a perturbation in the μ–τ block of the tri-bimaximal m_ν [10, 31]. The extreme case occurs for a multiplication of the $\mu\mu$ entry by $(1+\epsilon)$ and of the $\tau\tau$ entry by $(1-\epsilon)$, and yields

$$\sin^2 2\theta_{23} \simeq 1 - \epsilon^2 \gtrsim 0.96\ . \tag{11}$$

This is also quite close to the numerical result.

For the inverted hierarchy case, we note that such a stable hierarchy is difficult to obtain in $SO(10)$ models which do not have a type II seesaw structure, i.e., if there is no direct left-handed Majorana contribution arising from a real or effective Higgs triplet. The $SO(10)$ model from Ref. [32], which has a negligible type II (triplet) contribution, is able to fit an inverted hierarchy, but is not very predictive in what regards the mixing angles. The model from Ref. [33] concentrates on embedding a neutrino mass matrix with vanishing diagonal elements, assumes type II (triplet) dominance, and has a best-fit of $|U_{e3}|^2 = 0.0025$ in case of an inverted hierarchy but no other mixing angle predictions.

The results for perturbed tri-bimaximal matrices arising from an inverted hierarchy are shown as scatter plots in Figs. 3 and 4. Here we have started with fixed $m_2 = 0.05076$ eV, $m_1 = 0.050$ eV and $m_3 = 0.0114$ eV, corresponding to $\Delta m^2_{21} = 7.66 \times 10^{-5}$ eV^2 and $\Delta m^2_{31} = -2.37 \times 10^{-3}$ eV^2. Proceeding as in the normal hierarchy case we have varied the phases, α and β, and masses m_1 and m_3 within 20% of their starting values. To guide the eye, we have drawn with a dashed line the value of $|U_{e3}|^2 = 0.001$, which is roughly the upper value found in case of a normal mass hierarchy which separates the perturbed results from $SO(10)$ GUT predictions. It is clear from the plots in Figs. 3 and 4 that $|U_{e3}|^2$ can easily be around, and even above, 0.01 and is therefore testable in up-coming reactor experiments, unlike the normal hierarchy case. Again $\sin^2\theta_{12}$ is unconstrained.

In Figs. 5 and 6 we show results for restricted values of $\alpha = 0$ and $\pi/2$ appearing in the parameters A and B of the TBM neutrino mass matrix. We find that the largest values of $|U_{e3}|^2$ typically occur if the phase α is around $\pi/2$. This value implies that the effective mass governing neutrinoless double beta decay ($0\nu\beta\beta$) takes its minimally allowed value: $\langle m \rangle \simeq \sqrt{\Delta m^2_{\rm A}}\,\cos 2\theta_{12} \simeq \frac{1}{3}\sqrt{\Delta m^2_{\rm A}}$. For the normal hierarchy case, $\langle m \rangle$ is expected to be smaller still. On the other hand, if the phase α is zero or π, $\langle m \rangle \simeq \sqrt{\Delta m^2_{\rm A}}$, and $|U_{e3}|^2$ is tiny. We also note that θ_{23} can deviate more sizably from maximal mixing, if neutrinos are inversely ordered, and that the largest deviation occurs for $\alpha = \pi/2$ as shown in Fig. 6.

Turning to analytic estimates for inverted ordering, for $\alpha = \pi/2$ one has $A \simeq \sqrt{\Delta m^2_{\rm A}}/3$ and $B \simeq -2\sqrt{\Delta m^2_{\rm A}}/3$. Consider first a perturbation of the $e\mu$ entry with $(1+\epsilon)$ and of the $e\tau$ entry with $(1-\epsilon)$ for real ϵ. In this case

$$|U_{e3}|^2 \simeq \epsilon^2 \left(\frac{8}{81} + \frac{16}{27}\,\frac{m_3}{\sqrt{\Delta m^2_{\rm A}}} \right) \lesssim 10^{-2} \text{ and } \sin^2 2\theta_{23} \simeq 1 - \left(\frac{16}{9}\right)^2 \epsilon^2 \gtrsim 0.87\,. \qquad (12)$$

In the case of $\alpha = 0$, we have $B/A \simeq \frac{1}{6}\,\Delta m^2_{\odot}/\Delta m^2_{\rm A}$ and find that $|U_{e3}|^2$ is at most of order $(\epsilon\, B/A)^2 \simeq 10^{-6}$ and therefore completely negligible. In the case of perturbed $\mu\mu$ and $\tau\tau$ entries, $\sin^2 2\theta_{23} \simeq 1 - \epsilon^2 \gtrsim 0.96$. The agreement with the figures is quite reasonable.

For completeness we study also the case of quasi-degenerate neutrinos. For definiteness, we consider the case of normally ordered neutrinos, choosing fixed $m_3 = 0.1$ eV, $m_2 = 0.08778$ eV and $m_1 = 0.08735$ eV, corresponding to $\Delta m^2_{21} = 7.59 \times 10^{-5}$ eV^2 and $\Delta m^2_{31} = 2.37 \times 10^{-3}$ eV^2. The procedure is the same as before and the results are shown in Figs. 7 and 8. Plots for an inverted ordering of quasi-degenerate neutrinos look basically identical. As expected, both $\sin^2 2\theta_{23}$ and $|U_{e3}|^2$ deviate more sizably than before from their initial, tri-bimaximal values. Since this quasi-degenerate case is more similar to the inverted one, it is not surprising that the interplay of the Majorana phase α, the effective mass and the deviation from maximal θ_{23} and zero θ_{13} are similar. Analytically, one finds that an enhancement of roughly $m_1^2/\Delta m_{\rm A}^2 \simeq 4$ occurs for the upper limits of $|U_{e3}|^2$. All mixing angles are populated in their allowed ranges.

In summary, we have raised the question whether the approximately tri-bimaximal mixing observed in the lepton sector results from a hidden symmetry or whether it is accidental. Early proposed mass matrix models based on $SO(10)$ family symmetry were not designed to lead to tri-bimaximal mixing, although a number of them are still successful. To study this issue we have adopted a model-independent approach and perturbed the TBM neutrino mass matrix elements about their central values by 20% in the lepton flavor basis. The charged lepton mass matrix is trivially perturbed by keeping the off-diagonal three texture zeros intact. We found that $|U_{e3}|^2$ and the neutrino mass scale and ordering are of importance in the problem. In general the value of $\sin^2\theta_{12}$ is not constrained, and precision measurements of its value will not help in settling the issue raised. The other mixing parameter, $\sin^2 2\theta_{23}$ has only limited impact. A most striking result obtained is that for a normal neutrino mass hierarchy, the predicted perturbed TBM values of $|U_{e3}|^2$ lie below 10^{-3}, while the type I seesaw $SO(10)$ models typically predict values above this. For an inverted or quasi-degenerate neutrino mass hierarchy, on the other hand, only two of the studied $SO(10)$ models apply, while the allowed perturbed values of $|U_{e3}|^2$ can range noticeably higher, up to the present experimental limit. An interesting correlation with the value of the effective mass governing neutrinoless double beta decay is observed. While the question posed is unanswerable at this time, we can conclude that observation of $|U_{e3}|$ within the next few years would be incompatible with TBM and a normal mass hierarchy. Clearly it will be necessary to determine both $|U_{e3}|$ and the mass hierarchy in order to address the posed question.

Acknowledgments

One of the authors (C.H.A.) thanks Manfred Lindner and members of the Max-Planck-Institut für Kernphysik for their kind hospitality while the research reported here was carried out. The work of W.R. was supported in part by the Deutsche Forschungsgemeinschaft in the Transregio 27 "Neutrinos and beyond – weakly interacting particles in physics, astrophysics and cosmology".

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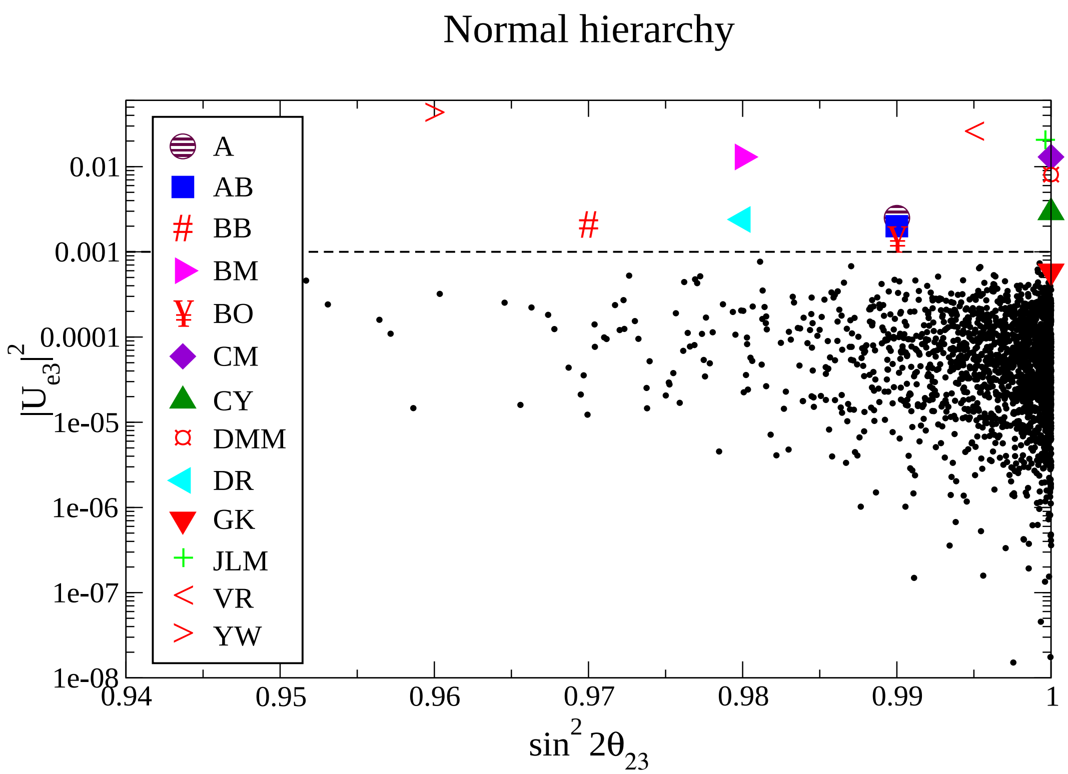

Figure 1: Scatter plot of $\sin^2 2\theta_{23}$ against $|U_{e3}|^2$ for perturbed tri-bimaximal mixing and a normal mass hierarchy. Also given are predictions of thirteen $SO(10)$ GUT models.

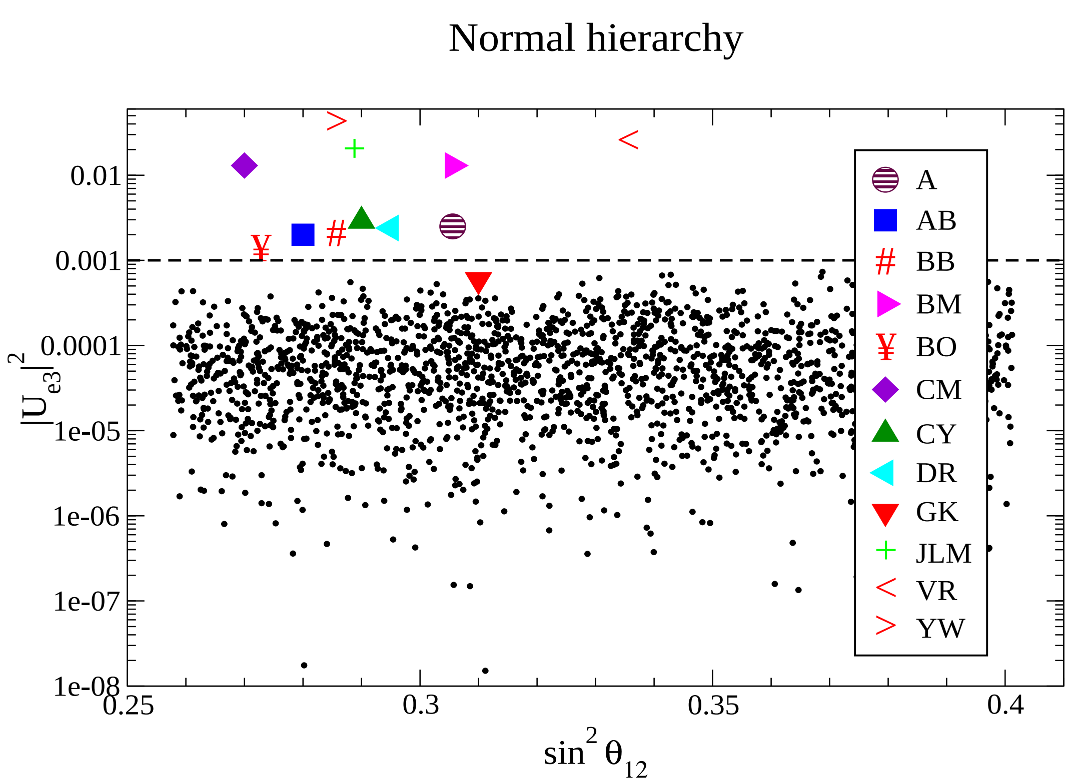

Figure 2: Scatter plot of $\sin^2 \theta_{12}$ against $|U_{e3}|^2$ for perturbed tri-bimaximal mixing and a normal mass hierarchy. Also given are predictions of thirteen $SO(10)$ GUT models.

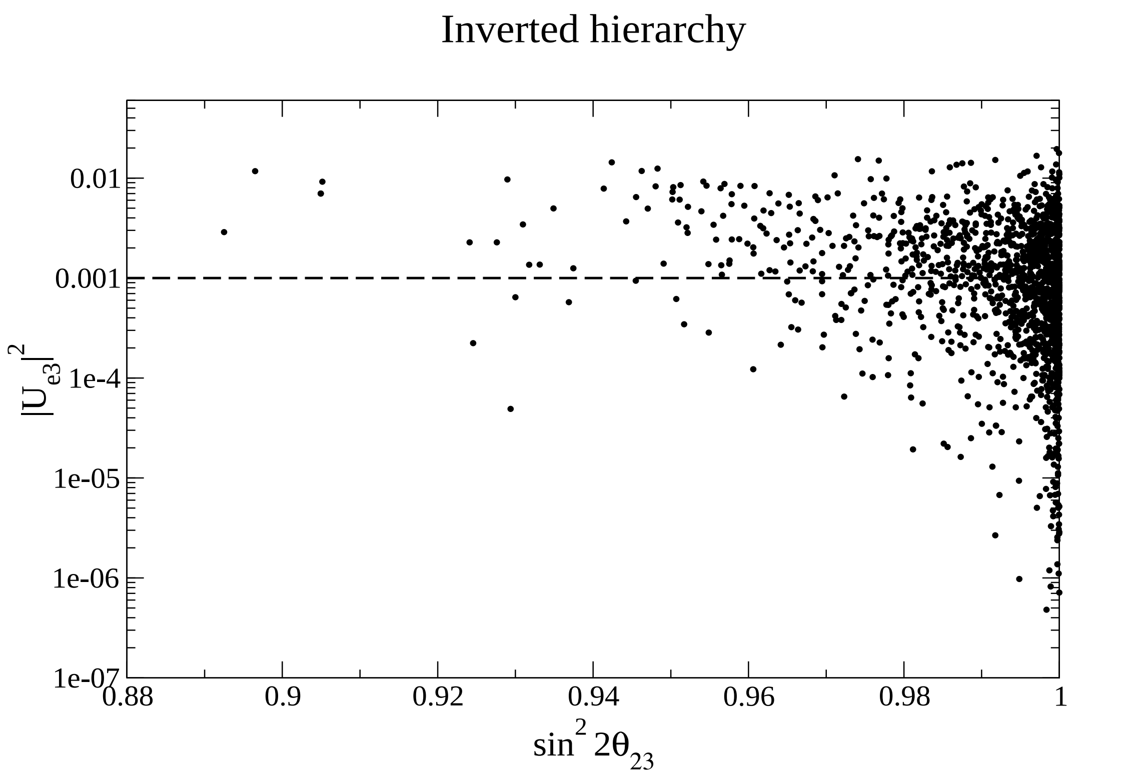

Figure 3: Scatter plot of $\sin^2 2\theta_{23}$ against $|U_{e3}|^2$ for perturbed tri-bimaximal mixing and an inverted mass hierarchy.

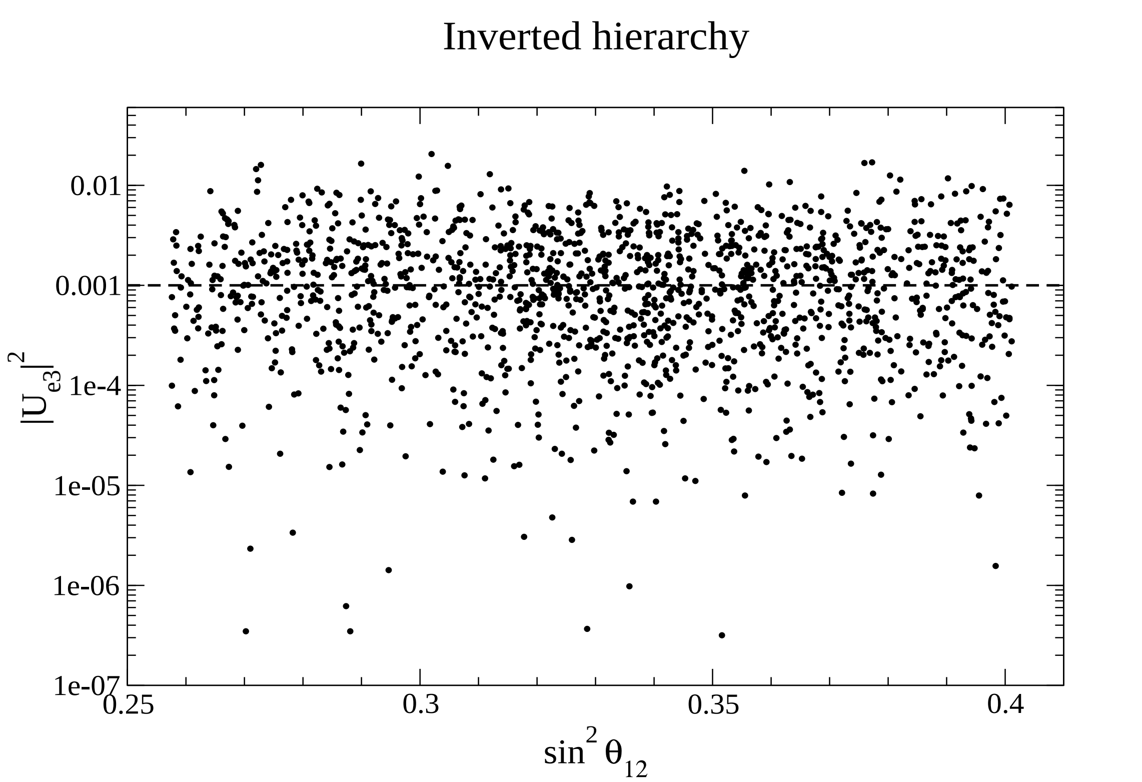

Figure 4: Scatter plot of $\sin^2 \theta_{12}$ against $|U_{e3}|^2$ for perturbed tri-bimaximal mixing and an inverted mass hierarchy.

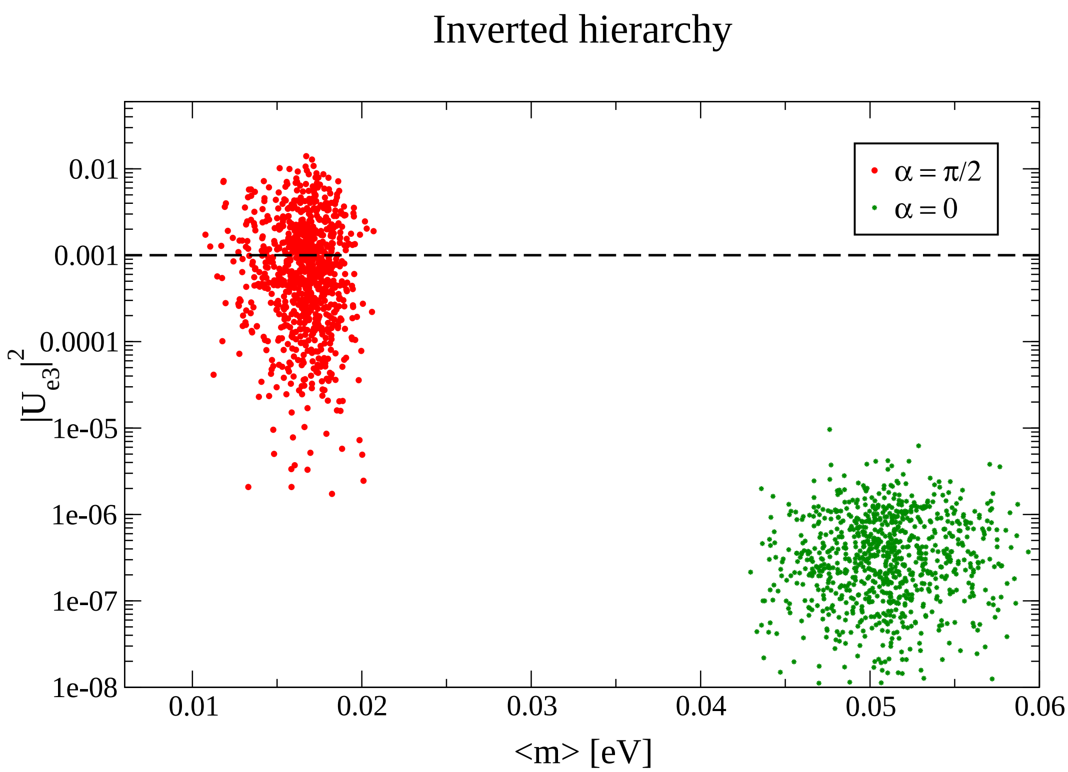

Figure 5: Scatter plot of the effective mass against $|U_{e3}|^2$ for perturbed tri-bimaximal mixing and an inverted mass hierarchy with two different choices of the Majorana phase α, with the $\alpha = \pi/2$ cluster on the left and the $\alpha = 0$ cluster on the right.

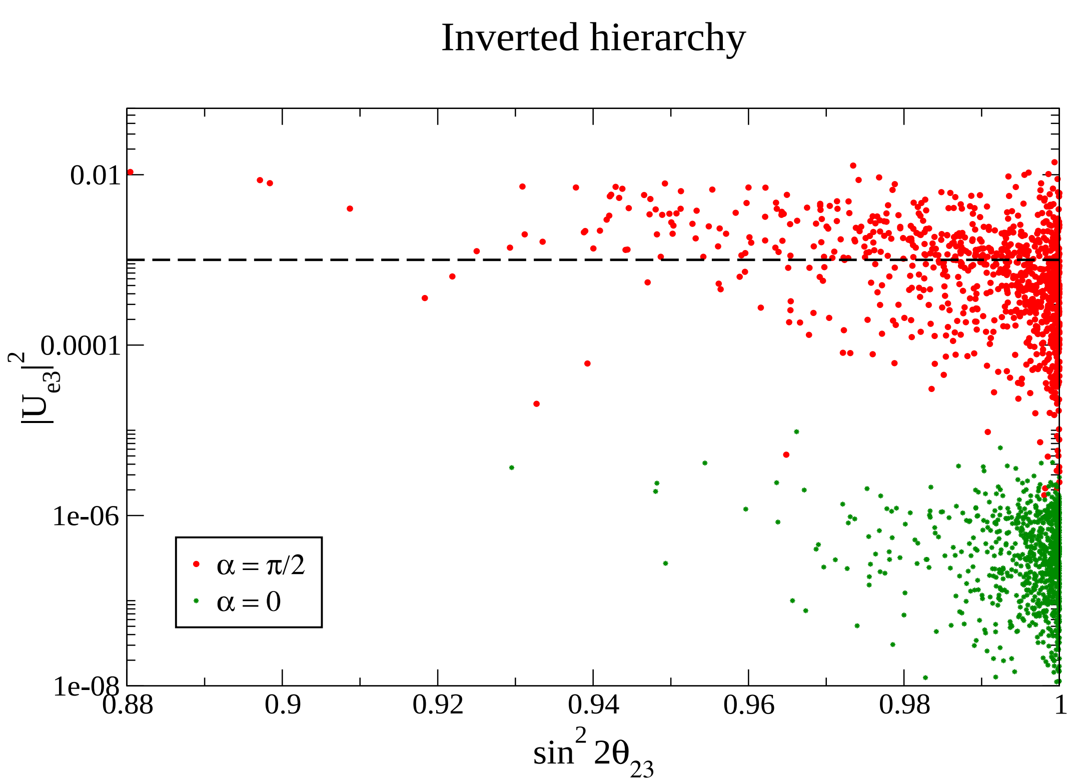

Figure 6: Scatter plot of $\sin^2 2\theta_{23}$ against $|U_{e3}|^2$ for perturbed tri-bimaximal mixing and an inverted mass hierarchy with two different choices of the Majorana phase α, with the upper cluster referring to $\alpha = \pi/2$ and the lower to $\alpha = 0$.

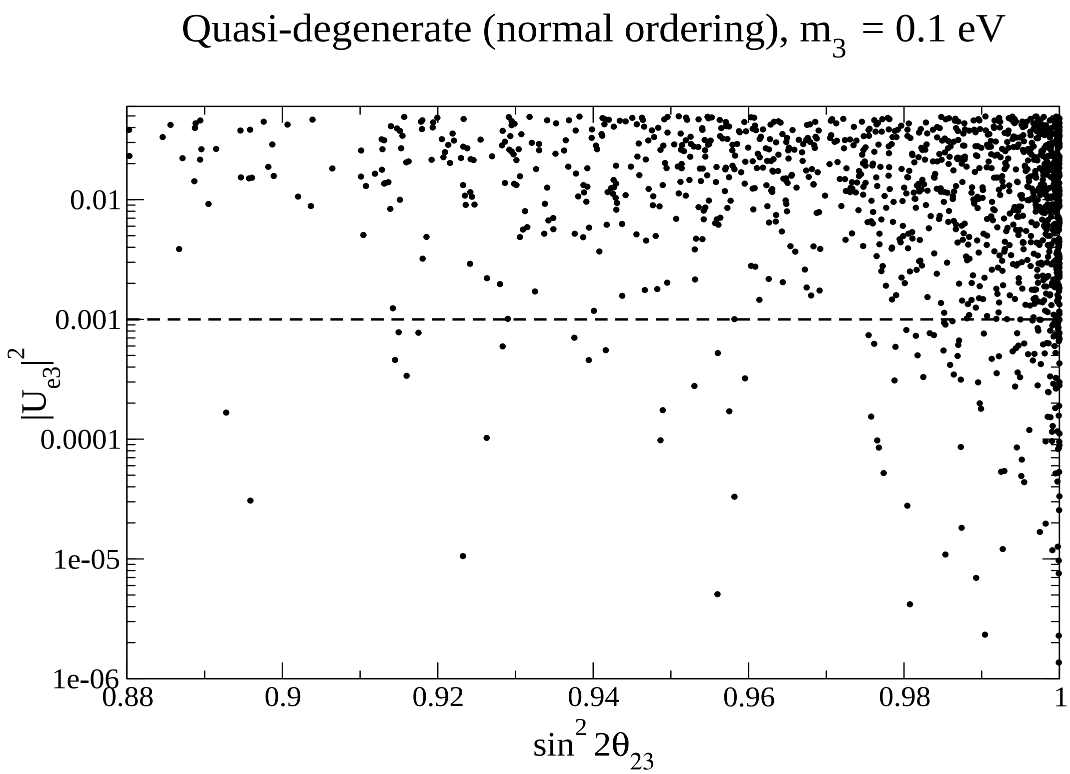

Figure 7: Scatter plot of $\sin^2 2\theta_{23}$ against $|U_{e3}|^2$ for perturbed tri-bimaximal mixing and quasi-degenerate neutrinos.

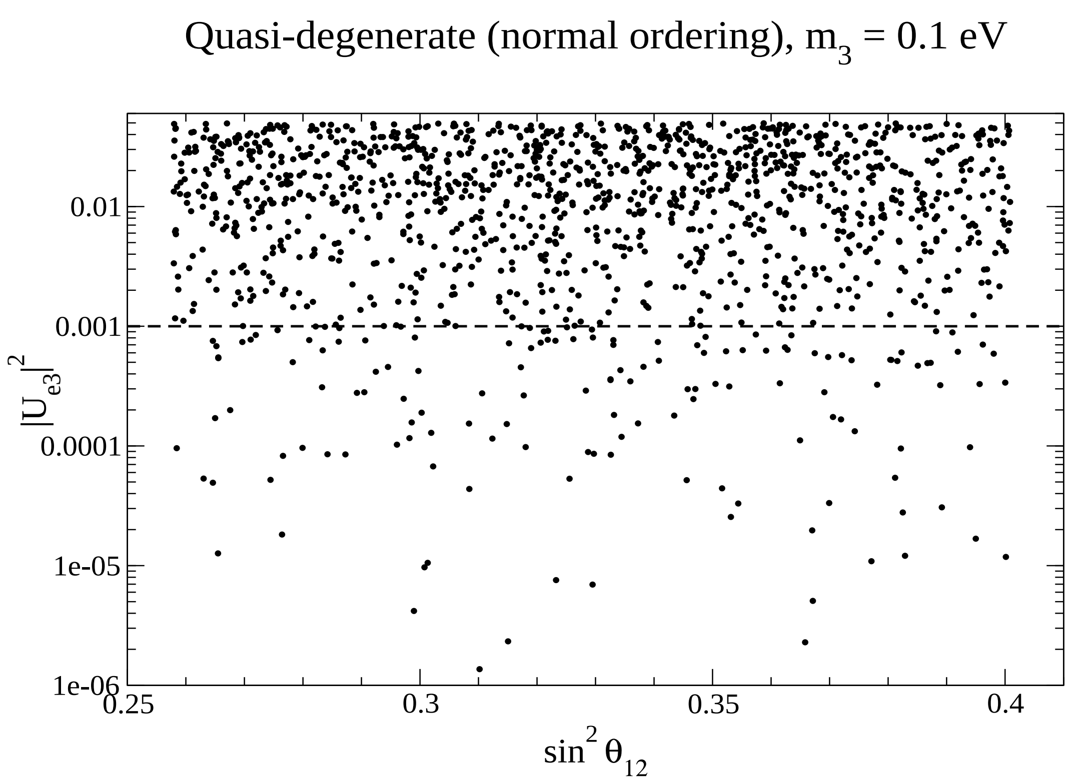

Figure 8: Scatter plot of $\sin^2 \theta_{12}$ against $|U_{e3}|^2$ for perturbed tri-bimaximal mixing and quasi-degenerate neutrinos.