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The dipion mass spectrum in e^+e^- annihilation and τ decay : Isospin Symmetry breaking effects from the (ρ, ω, ϕ) mixing

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A way to explain the puzzling difference between the pion form factor as measured in e^+e^- annihilations and in τ decays is discussed. We show that isospin symmetry breaking, beside the already identified effects, produces also a full mixing between the ρ^0, ω and ϕ mesons which generates an isospin 0 component inside the ρ^0 meson. This effect, not accounted for in current treatments of the problem, seems able to account for the apparent mismatch between e^+e^- and τ data below the ϕ mass.

1. INTRODUCTION

In order to get a theoretical estimate of the muon anomalous magnetic moment $g-2$, one needs to estimate precisely the photon vacuum polarization (see Jegerlehner [1] for a comprehensive review). Its leptonic part can be computed theoretically to a high precision from QED, but the dominance of non–pertubative effects in the low energy region prevents to perform likewise starting from QCD in order to estimate the hadronic part. This is instead done by means of a dispersion integral involving the measured cross section $\sigma(e^+e^- \rightarrow \text{hadrons})$; however, the integration kernel is such that the low energy region contribution is enhanced by a $\sim 1/s^2$ factor. Because of this, the non–pertubative region provides, by far, the largest contribution to the hadronic vacuum polarization (VP). Additionally, the annihilation process $e^+e^- \rightarrow \pi^+\pi^-$ alone happens to provide more than 60 % of the total hadronic VP. Detailed accounts of this matter have been provided by Teubner and Jegerlehner at this Conference[2].

Several data sets are now available which allow a precise estimate of $\sigma(e^+e^- \rightarrow \pi^+\pi^-)$. This covers the former data sets collected by the OLYA, CMD and DM1 Collaborations – all gathered in the review by Barkov *et al* [3] –, and the data sets more recently collected by the CMD2 [4–6] and SND [7] Collaborations at VEPP2M. Additional data sets taking advantage of the initial state radiation mechanism have also been collected by the KLOE (see F. Nguyen transparencies[2]), BaBar and Belle Collaborations and are expected to become available soon.

Moreover, high statistics data on the decay $\tau^\pm \rightarrow \nu_\tau \pi^\pm \pi^0$ are also available from the ALEPH [8] and CLEO [9] Collaborations ; an additional data set from the Belle Collaboration, presented at this Conference (see H.Hayashii [2]) is expected to become public shortly. As the pion form factor in τ decays and in e^+e^- annilhilations are related by the Conserved Vector Current (CVC) assumption, these data are expected to be useful in order to improve the estimate of the photon hadronic VP. Indeed, they can only differ by isospin symmetry breaking (ISB) effects, in principle, subject to identified small corrections.

ISB effects have been especially studied in order to include the τ data in the estimation of the photon hadronic VP. This covers non trivial effects specific of the τ decay like the short range [10] and long range [11,12] ISB factors, but also more standard effects easier to account for : mass differences between charged and neutral pions and charged and neutral ρ mesons, the $\rho^{\pm}-\rho^0$ width difference, the ω and ϕ contributions to the e^+e^- annihilation amplitude (see, for instance, Ref. [13,14]).

As a preliminary step in the process of including τ data in estimating the photon hadronic VP, the comparision has been done of the pion form factor as measured in e^+e^- annihilations and as derived from τ decays while accounting for all known isospin breaking effects appropriately [13,1,14]. This comparison, however, clearly exhibits an unexpected s–dependence of the difference between the e^+e^- data and the pion form factor function reconstructed from (ALEPH) τ data, as reported still recently by Davier [15].

This mismatch is an important issue as the photon hadronic VP derived from e^+e^- data leads to a theoretical prediction for the muon $g-2$ at $\simeq 3.3\ \sigma$ from its measured value [16]; in contrast, the corresponding prediction derived from τ data is in close agreement [15] with the measured $g-2$ value ; this issue has been revisited at this Conference by Teubner and Jegerlehner [2] without significant changes.

Therefore, the question is whether the $(e^+e^- - \tau)$ mismatch can be explained by physics only connected with isospin symmetry breaking, or if it calls for another kind of physics effect. Responding this question by leaving e^+e^- data beyond any doubt may point towards a new physics effect exhibited by the muon anomalous magnetic moment (see D. Stöckinger talk at this Conference [2]).

2. A MISSING PIECE OF ISOSPIN SYMMETRY BREAKING ?

As it is clear that all the identified (and already listed) effects produced by ISB should be considered when comparing e^+e^- and τ data, the issue becomes the possible existence of a missing piece in the standard ISB procedure described above.

Actually, a clue has been given by Maltman [17] : Using sum rules derived from an OPE input, he concluded that the ρ part of the e^+e^- form factor data was inconsistent with being isospin 1, in contrast with the corresponding information provided by τ data. This statement implies that either the quality of the available e^+e^- data can be questioned – which seems by now unlikely – or that the ρ^0 meson is not a (pure) isospin 1 object.

Up to now, the model amplitudes used to describe the neutral and charged ρ mesons may differ by feeding their propagators with different masses and widths to be fit with data; of course, the pion mass difference is also fed in, together with the ω (and ϕ) meson(s) propagators, generally Breit–Wigner formulae. However, as the e^+e^- data clearly exhibit [3–6] the narrow (isospin 1 part of the) ω interfering with the broad ρ^0, one may ask oneself about the existence of a (broad) isospin 0 part of the ρ^0 meson which might make it differing from its charged partner beyond genuine mass effects. Such a component should be small and might well be broad enough that it might barely be visible. Stated otherwise, the question is whether mass and width differences for the ρ mesons exhaust ISB in the pion form factor.

3. THE PION FORM FACTOR AT ONE LOOP

In order to make our statements explicit, we have found it appropriate to work in the framework of the Hidden Local Symmetry (HLS) model widely described in Refs.[18,19] and outlined in [20] with special emphasis on the present issue. In the HLS model, the pion form factor for both e^+e^- annihilation and τ decay writes :

$$\left\{ \begin{array}{l} F_\pi(s) = (1 - \dfrac{a}{2}) - \dfrac{f_\rho g_{\rho\pi\pi}}{D_V(s)} \ , \\ \\ f_\rho = a g f_\pi^2 \ \ \text{and} \ \ g_{\rho\pi\pi} = \dfrac{ag}{2} \ . \end{array} \right. \tag{1}$$

a is a parameter specific of the HLS model – numerically close to 2 – and g is the universal vector coupling – close to 5.5 (from QCD sum rules). $D_V(s) = s - m_\rho^2$ is the inverse ρ bare propagator ($m_\rho^2 = ag^2 f_\pi^2$). While including one loop effects, $D_V(s)$ acquires a pion (and kaon) loop term $\Pi_\rho(s)$ which shifts the ρ pole off the real s–axis. The transition amplitude from γ/W to (neutral/charged) ρ is also dressed by loop effects;

this turns out to perform the change :

$$f_\rho \longrightarrow F_\rho = f_\rho - \Pi_{W/\gamma}(s) \tag{2}$$

in the expression for $F_\pi(s)$. $\Pi_\gamma(s)$ refers to the pion form factor in e^+e^-, which will be named $F_\pi^e(s)$, while $\Pi_W(s)$ refers to the pion form factor in τ decay denoted by $F_\pi^\tau(s)$.

The 3 loop functions $\Pi_\rho(s)$ and $\Pi_{W/\gamma}(s)$ just defined fulfill each a dispersion relation [20] and their imaginary parts are influenced by SU(3) flavor symmetry breaking. Each of these carries a subtraction polynomial, which has been chosen of degree 2 and vanishing at the origin. Additionally, it has been possible to relate the subtraction polynomials for $\Pi_W(s)$ and $\Pi_\gamma(s)$. These conditions allow to fulfill $F_\pi^{e/\tau}(0) = 1$ on the one hand, and permit a significant reduction of the parameter freedom while performing fits on the other hand.

ISB, as usually done, turns out to multiply $|F_\pi^\tau(s)|^2$ by some specific factors [10–12] not discussed here (see [20]) and add the ω and ϕ contributions to $F_\pi^e(s)$. Additionally, the mass difference between charged and neutral pions is considered and the charged and neutral ρ mesons are allowed to carry different (fit) values for their masses and widths.

This procedure, however, has been shown insufficient in order to restore consistency between e^+e^- and τ data [13–15].

4. ISOSPIN SYMMETRY BREAKING AND THE ρ^0, ω, ϕ MIXING

As it is clear that the isospin symmetry breaking effects listed above have each to be taken into account, the question is rather about a missing piece in the scheme outlined in Section 3. While working at one loop order, the HLS model provides self–masses already referred to for the ρ meson propagators. However, it also contains the piece :

$$\frac{iag}{4z_A}\Big[(\rho_I^0 + \omega_I - \sqrt{2}z_V\phi_I)\ K^- \overleftrightarrow{\partial} K^+ + (\rho_I^0 - \omega_I + \sqrt{2}z_V\phi_I)\ K^0 \overleftrightarrow{\partial}\, \overline{K}^0\Big] \tag{3}$$

which – through kaon loops – generates transitions among the so–called ideal (bare) fields ρ_I^0, ω_I and ϕ_I with no counter part affecting the $\rho^\pm$ field. z_A and z_V are flavor SU(3) breaking parameters determined by fit. We have :

$$\left\{\begin{array}{l} \Pi_{\omega\phi}(s) = -g_{\omega KK}g_{\phi KK}[\Pi_\pm(s) + \Pi_0(s)] \\ \Pi_{\rho\omega}(s) = g_{\rho KK}g_{\omega KK}[\Pi_\pm(s) - \Pi_0(s)] \\ \Pi_{\rho\phi}(s) = -g_{\rho KK}g_{\phi KK}[\Pi_\pm(s) - \Pi_0(s)] \end{array}\right. \tag{4}$$

as transition amplitudes between the (ideal) ρ_I^0, ω_I and ϕ_I. $\Pi_\pm(s)$ and $\Pi_0(s)$ denote, resp. the charged and neutral kaon loops amputated from coupling constants factored out for sake of clarity. These loops are defined by dispersion integrals over their imaginary parts and contain subtraction polynomials ($P_\pm(s)$ and $P_0(s)$) real for real s, the invariant mass squared flowing through the vector meson lines. These polynomials are chosen of degree 2 and vanishing at $s = 0$; their coefficients have to be fixed by external conditions. If isospin symmetry is conserved one may assume that $P_\pm(s) = P_0(s)$ and, then, $\Pi_{\rho\omega}(s)$ and $\Pi_{\rho\phi}(s)$ identically vanish; when isospin symmetry is broken this condition is certainly no longer fulfilled. Therefore, the HLS model which always predicts $\omega_I - \phi_I$ transitions (as $\Pi_{\omega\phi}(s)$ never vanishes identically), predicts additionally $\rho_I^0 - \omega_I$ and $\rho_I^0 - \phi_I$ transitions when isospin symmetry is broken. One should also note that the anomalous and the Yang–Mills pieces of the full HLS Lagrangian provide resp. K^*K and $K^*\overline{K}^*$ loops which comes supplementing the kaon loop mechanism and follow the pattern[20] exhibited in Eqs 4. They only modify the logarithmic part therein and can be considered effectively accounted for by the subtraction polynomials.

Therefore, in the general case of isospin symmetry breaking, there are transitions among the ideal vector fields. If one defines the *physical* vector fields as eigenstates of the vector mass matrix, as the amplitudes in Eqs. 4 provide non–vanishing entries in the vector meson squared mass matrix, these cannot coincide with their ideal partners at one loop order. Let us define the vector V and V_I as the vectors constructed with (resp.) the ρ^0, ω and ϕ fields on the one hand, and ρ_I^0, ω_I and ϕ_I fields the other hand. Then the mass eigenstates of the vector meson squared mass matrix and their ideal partners are related by $V = R(s)\ V_I$ and $V_I = \widetilde{R(s)}\ V$ with [20] :

$$R(s) = \begin{pmatrix} 1 & \dfrac{\epsilon_1}{\Pi_{\pi\pi}(s) - \epsilon_2} & -\dfrac{\mu\epsilon_1}{(1-z_V)m^2 + \Pi_{\pi\pi}(s) - \mu^2\epsilon_2} \\ -\dfrac{\epsilon_1}{\Pi_{\pi\pi}(s) - \epsilon_2} & 1 & -\dfrac{\mu\epsilon_2}{(1-z_V)m^2 + (1-\mu^2)\epsilon_2} \\ \dfrac{\mu\epsilon_1}{(1-z_V)m^2 + \Pi_{\pi\pi}(s) - \mu^2\epsilon_2} & \dfrac{\mu\epsilon_2}{(1-z_V)m^2 + (1-\mu^2)\epsilon_2} & 1 \end{pmatrix} \tag{5}$$

where $\epsilon_1 = \Pi_{\rho\omega}(s)$ and $\epsilon_2 = \Pi_{\rho\phi}(s)$ are functions of s, real below $s \sim 1$ GeV^2. Indeed, the loop imaginary parts start at the corresponding two–kaon thresholds (For the additional loops mentioned above, the threshold is much higher). One neglects terms of second order in ϵ_1 and/or ϵ_2. $\Pi_{\pi\pi}(s)$ is the pion loop representing the bulk of the ρ self–energy and $m^2 = ag^2 f_\pi^2$ is the unperturbed ρ meson mass squared.

Performing the change to physical fields into the HLS Lagrangian generates [20] isospin symmetry violating couplings of the ω and ϕ fields to $\pi^+\pi^-$, while leaving the ρ^0 coupling to $\pi^+\pi^-$ identical to that of its ideal partner at leading (first) order in the ϵ_i.

In contrast, the $\gamma - \rho^0$ transition amplitude (named f_ρ in Eq. 1) is modified to [20] $f_\rho + \delta f_\rho(s)$ where $\delta f_\rho(s)/f_\rho$ is equal to a weighted sum of the first column entries in $R(s)$ – *i.e.* it is of first order in the ϵ_i's – and can be found explicitly in [20]. In contrast, the $W - \rho^\pm$ amplitude amplitude still coincides with f_ρ.

Therefore, because of one–loop effects, isospin symmetry breaking introduces a s–dependent difference between the $\gamma - \rho^0$ and $W - \rho^\pm$ transitions; this is entirely due to the fact that ideal neutral vector fields cease to coincide with physical neutral vector fields, when defined as mass matrix eigenstates. Loop effects always affect the $(\omega,\ \phi)$ sector, but the whole $(\rho^0,\ \omega,\ \phi)$ is affected when, additionally, isospin symmetry is broken. Clearly, this effect has not been accounted for in previous analyses of the pion form factor in e^+e^- and τ data.

5. HOW TO WORK OUT THE MODEL ?

The issue now is whether the $(\rho^0,\ \omega,\ \phi)$ mixing we just sketched is able to account *numerically* for the long standing mismatch between e^+e^- and τ data.

From the point of view of data analysis, the number of parameters (coupling constants, U(3)/SU(3) breaking parameters, subtraction parameters from dispersion integrals. . .) in our HLS based model is too large to hope fixing them reasonably well using only the e^+e^- and τ data. Fortunately, there is a way out.

It indeed happens that the radiative decays ($PV\gamma$ and $P\gamma\gamma$), which are accounted for by the anomalous sector [21] of the HLS Lagrangian, depend on a large part of the parameters involved in our model and can serve to fix them quite reliably, even by fitting them in isolation [22,23]. If one adds to this data set the leptonic decay information for the ω and ϕ mesons on the one hand, and two–pion decay information of the ϕ meson on the other hand, the minimization program becomes numerically well defined. This additional data set will be referred to as "decay data".

Therefore, the resolution method we propose is to consider the e^+e^- and τ data together with the decay data. One should stress that [1] the form factor $F_\pi^\tau(s)$ is entirely determined, from a numerical point of view, by the e^+e^- and decay data in isolation, since actually all parameters it depends on are already involved in the decay widths considered or in $F_\pi^e(s)$. Stated otherwise, $F_\pi^\tau(s)$ can be *predicted* from our model using only the e^+e^- and decay data. We actually consider this last property as the main test of validity of our approach. Actually, a possible mass difference between ρ^0 and $\rho^\pm$ is the single information which cannot be predicted from e^+e^- and decay data and should be externally fixed [20].

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[1] Except for a parameter δm_V^2 which may accout for a (possible) mass difference between ρ^0 and $\rho^\pm$.

6. BRIEF ANALYSIS OF FIT RESULTS

Detailed fit information can be found in Ref. [20] where they are lengthily presented and discussed. Here, we limit ourselves to the most relevant. One should also mention that data on the pion form factor in the close spacelike region [24,25] are included in our fits.

Table 1
Pion form factor information from global fits.

	Full Fit	Excluding τ data
χ^2/dof	313.83/331	257.73/274
Probability	74.4%	75.2%
Timelike Data		
(χ^2/points)	187.15/(209)	176.70/(209)
τ ALEPH		
(χ^2/points)	23.86/(33)	**42.27**/(33)
τ CLEO		
(χ^2/points)	26.06/(25)	**26.16**/(25)

Table 1 clearly shows that the description of the global data set is quite satisfactory. The second data column gives mostly the χ^2 distance of the model to the τ data points *left out from the fit procedure*; this clearly illustrates that $F_\pi^\tau(s)$ is indeed numerically derived from the HLS model together with data independent of the τ form factor. One can remark that CLEO data [9] are as well accounted for as when including them in the fit procedure.

Looking at the comparison presented at this Conference by H. Hayashii [2] between ALEPH, CLEO and Belle data, one may guess that the Belle data set will behave rather like the CLEO data set (including the behaviour at top of the invariant mass distribution) than like the ALEPH spectrum. This, for instance, is important concerning the possible $\rho^\pm - \rho^0$ mass difference.

Therefore, one may conclude that introducing the effects of isospin symmetry breaking (a non–zero $\epsilon_1(s)$) on vector meson mixing, together with the already reported effects, is enough to reconcile the e^+e^- and τ data. A missing piece in the current isospin symmetry breaking procedure is then identified as the effects of the isospin 0 component of the ρ^0 meson which has no counter part inside the $\rho^\pm$ meson.

Other information is provided by Figs. 1,2 and 3 which exhibit the fit residuals. One can clearly consider them as structureless in the region below 0.9 GeV. One also clearly sees the effects of higher mass vector meson resonances starting as early as around the GeV region.

As final conclusion, one may indeed consider that e^+e^- and τ data do not exhibit any mismatch once all consequences of isospin symmetry breaking are indeed considered, including the isospin 0 component generated inside the ρ^0 meson. Then, it follows from this work that the predicted value of the muon anomalous moment derived using e^+e^- data is indeed reliable and that the actual mismatch is between the prediction of the muon $g-2$ and its direct (BNL) measurement [16], rather than between e^+e^- and τ data. Therefore, getting an improved measurement (see Ref. [26] and D. Hertzog[2]) of the muon anomalous magnetic moment becomes a key issue, possibly a window on New Physics.

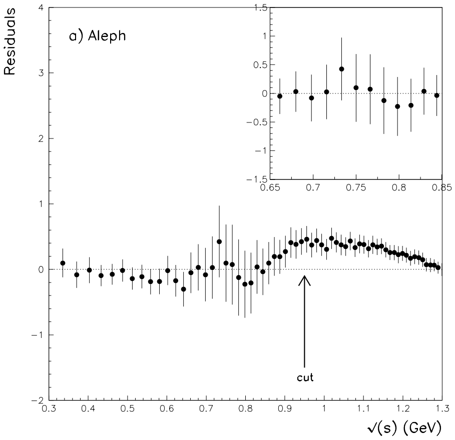

Figure 1. ALEPH data fit residuals.

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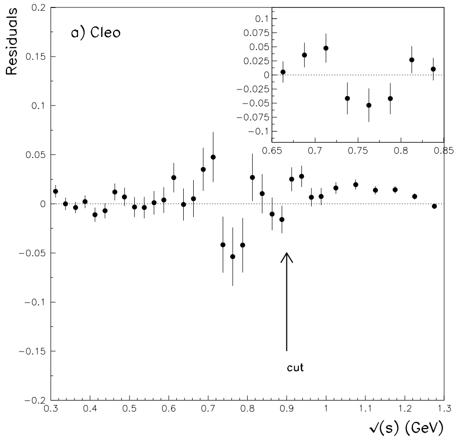

Figure 2. CLEO data fit residuals.

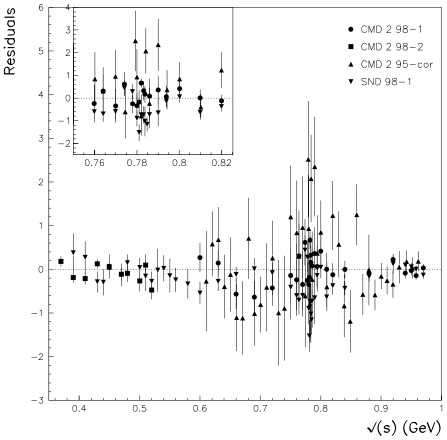

Figure 3. Fit residuals of some timelike data sets.

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