A TUTORIAL ON NON-INTERCEPTING ELECTROMAGNETIC MONITORS
FOR CHARGED PARTICLE BEAMS

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Parameters of fundamental interest for charged particle beams are the beam current, temporal distribution, and transverse position. These quantities are amenable to measurement with non-intercepting devices sensitive to the electromagnetic fields of the beam. A conceptual introduction to such devices is presented. The basic interactions with the electromagnetic fields of the beam and methods for estimating signal levels, sensitivities, and frequency response are described. An overview of typical devices appropriate for different beam energies, intensities, and time structures is given.

I. INTRODUCTION

Electromagnetic beam monitors produce signals by sampling the electromagnetic fields of the charged particle beam. They offer a non-disruptive means to observe and quantify numerous important properties of that beam itself or of the accelerator or transport line environment in which the beam travels.

Fundamental parameters that can be measured directly with these monitors include the beam current, the temporal distribution of particles in the beam, and the transverse position of the beam in the chamber. Signal frequencies ranging from DC to several GHz are accessible with suitably designed monitors and electronics.

A fundamental understanding of the electromagnetic fields carried by a charged particle beam and the interaction of these fields with the beam environment is important in order to interpret beam monitor signals and to appreciate the parameters that drive the design and limit the performance of various monitors.

Beam monitors can be designed to sense a beam’s electric field, magnetic field, or combination of each. Devices that rely primarily on interaction with the beam's electric field are often called capacitive pickups. The amplitude of a capacitive pickup signal is independent of the direction the beam is traveling; signal polarity depends on the sign of the beam particles’ charge. Pickups designed to interact with the magnetic field are called magnetic pickups or more commonly current monitors. Magnetic pickup signal amplitude is also independent of beam travel direction; signal polarity is determined by the product of particle charge and direction of travel, i.e. the sign of electric current. Pickups that couple to both the electric and magnetic fields can produce signals of either polarity depending on the relative electric and magnetic coupling. These monitors offer the possibility of an amplitude response that is dependent on the direction of beam travel without regard to the particle charge, a directional coupler.

II. THE FIELDS

The beam, as an assembly of electrically charged particles, carries an electric field with strength proportional to the total charge. Beams are normally enclosed inside an evacuated chamber bounded by an electrically conducting metallic wall. The electric field will induce an image charge on the inner surface of the chamber wall and under static conditions no electric field exists outside the conducting chamber.

A beam, however, does not normally stand still. As an ensemble of charged particles in motion, the beam represents an electric current and carries a corresponding magnetic field. The beam current may be continuous as in a storage ring or pulsed as in an induction linac; within that macroscopic time scale, the beam is typically modulated with an RF structure and formed into bunches.

For an isolated assembly of charge at rest, the electric field is radial in three dimensions from the ‘center-of-mass’ of the distribution according to Gauss’s Law; the magnetic field is absent. When the distribution is enclosed in a conducting tube, the typical beam chamber, the density of induced charge in the tube wall is greatest where the electric field is largest, that is, nearest to the beam charge distribution. The electric field configuration of the combined beam/induced charge distribution is concentrated radially between the beam and the nearest tube walls; the component along the axis of the tube is

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greatly reduced. If the tube radius is small relative to the axial beam dimensions of interest (e.g., bunch length), the static situation is well approximated by a two-dimensional electric field that is radial and orthogonal to the axis of the tube.

When the assembly of charge moves, a magnetic field appears in accordance with Ampere’s Law. The direction of the magnetic field is perpendicular to the direction of motion and encircles the beam current. As the beam velocity becomes relativistic, the electric and magnetic fields become completely orthogonal to each other and to the direction of motion as described by the Lorentz transformation. These fields exhibit the characteristics of TEM waves. The TEM condition is a useful and accurate approximation for most beam monitor design purposes, except in the case of extremely non-relativistic beams in geometries with unfavorable beam-to-environment aspect ratios.

It is convenient to consider the beam and associated fields using Fourier analysis methods and to describe the current in the frequency domain. This naturally leads to discussion of a beam monitor’s performance in terms of frequency response. The beam current typically presents a broad frequency spectrum. There is always a non-zero component of the current at zero frequency, DC, due to the fact that a beam represents a net transport of particles with (normally) like charges from one point to another. At the other end of the spectrum, there is often a RF frequency component to the beam, and if not, there remains an intrinsic high frequency component, the shot noise, due to the quantized nature of discrete charged particles.

Figure 1 shows a simplified beam structure described by just one non-zero frequency term. This is a biased cosine-like line charge distribution with unit wavelength and amplitude. More generally, such a distribution with amplitude A and wavelength (bunch length) $L_b$ moving at velocity $\beta c$ is described by:

$$\frac{d}{dx} \Delta q(x,t) = A \left[ 1 + \cos \left( 2\pi \frac{(x - \beta \cdot c \cdot t)}{L_b} \right) \right] = A \left[ 1 + \cos \left( 2\pi \frac{\omega}{\beta \cdot c} \right) \right]$$

To an observer at a fixed position in x, the distribution presents a bunch frequency of $f = \beta \cdot c / L_b$. The beam current of this charge distribution, observed at $x = 0$ is:

$$i_b = \beta \cdot c \frac{d \Delta q_b}{dx} = \beta \cdot c \cdot A \left[ 1 + \cos \left( 2\pi \frac{\beta \cdot c \cdot t}{L_b} \right) \right] = \beta \cdot c \cdot A \left( 1 + \cos \left( \omega \cdot t \right) \right)$$

where $\omega = 2\pi f$. The zero-frequency term of magnitude $\beta \cdot c \cdot A$ represents the DC or average beam current.

It is important to realize that $f$, $\beta$, and $L_b$ are not independent; when two are chosen, the third is determined. In a Linac, the frequency is generally a fixed parameter. In this case, $L_b$, the bunch length, is the dependent variable and it is proportional to the beam velocity. Therefore, all other conditions being equal, the bunch length will increase as a function of energy, i.e. position along the Linac, until the beam becomes relativistic. In a synchrotron the harmonic number, the ratio of beam bunch frequency to revolution frequency, is fixed. This establishes a fundamental wavelength, bunch spacing, of the charge distribution. In this case, the frequency is the independent variable and it must change in proportion to beam velocity.

Any physical beam structure can be represented by a linear superposition of frequencies according to Fourier analysis methods; therefore this simple single frequency example paves the way to interpretation of any beam.

### III. ACCESS TO THE FIELDS OF THE BEAM

An electromagnetic beam monitor depends on interaction with the fields of the beam to produce a signal. Therefore, it must include a sensor located in a region where the fields are present.

In the case of static fields, the typical conducting, non-magnetic, beam chamber effectively shields regions outside the chamber. This implies that the sensing electrode of a capacitive monitor must reside inside the chamber. That same chamber, however, is completely transparent to the static magnetic field associated with the DC component of the beam current.* This offers the possibility of a magnetic monitor to measure DC and low frequency beam current signals from outside the typical metallic beam chamber without any special modification!

The static signals are normally of secondary interest.

* Try it! Experiment with a permanent magnet and magnetic paper clips. You will find no difference in magnetic strength at a given distance whether the magnet is bare or contained within a non-magnetic (copper or stainless steel) metal enclosure.
To access the information-rich parts of the spectrum at moderate and high frequency, the time-varying components of the electromagnetic fields must be detected. At even relatively low frequencies, the typical conducting chamber walls begin to attenuate the magnetic field. According to Faraday’s Law, a time-varying magnetic field will induce an emf in the chamber walls forcing currents to flow so as to counteract changes in the intercepted magnetic flux. The ‘wall currents’ will be opposite polarity from the beam current. As the induced wall current magnitude approaches equality with the beam current, the field outside the chamber is cancelled.

The attenuation of an electromagnetic wave in a conductor is captured in the concept of skin depth. The skin depth is the characteristic length in which the wave amplitude is reduced by a factor of e or -8.69 dB. In a non-magnetic, good conductor the skin depth is given by:

\[
\delta = \frac{10^{10}}{2\pi f} \cdot \sqrt{\frac{\rho}{f}}
\]

where \(\rho\) is the resistivity of the conductor and \(f\) is the wave frequency.

The attenuation through a material thickness, \(t\), is 8.69*t/\(\delta\) dB. For example, at 10 MHz, a typical 1/32” (0.794mm) stainless beam tube wall presents a thickness of 6.1 skin depths and will attenuate magnetic fields propagating through to the exterior by 53dB. This is sufficient to clobber the sensitivity of most practical beam monitors. At any given frequency and thickness, the shielding effectiveness of a copper chamber is 55dB better than that of stainless steel.

Thus, nearly all practical beam monitor designs, capacitive or magnetic require a "window to the beam". The monitor, capacitive or magnetic, must be located within the vacuum chamber or the conducting path in the chamber wall must be broken.

**IV. A CAPACITIVE BEAM PICKUP**

Figure 2 shows an example capacitive beam pickup in a beam tube with a representative positively charged beam structure. It is a thin-walled, isolated annular electrode of length \(L_e\) concentric with and located inside a conducting beam tube with a single tap point connection to transmit the signal. This pickup is sensitive only to the electric field of the beam; the geometry presents no loop area to intercept any azimuthal magnetic flux lines.

Figure 3 a plot showing a cross-section of the electrode and a simple beam charge distribution. Assuming a purely transverse electric field, the charge induced on the inside of the capacitive electrode at any time is equal to the total beam charge contained within the linear extent of the electrode and opposite in sign. This is given by:

\[
q_e(t) = \frac{L_e}{2} \int_{-L_e/2}^{L_e/2} dx \left[ A + \cos \left( \frac{x}{\beta c} - t \right) \right] dx
\]

which evaluates to

\[
q_e(t) = A \cdot L_e \left( 1 + \cos \left( \frac{L_e}{2\beta c} \right) \right)
\]

where \(L_e\) is the electrode length and \(L_b = 2\pi \beta c/\omega\) is the spatial period, the wavelength, of the sinusoidal modulation of the line charge distribution.

The signal tap on the electrode allows observation of the monitor signal. The electric potential between the unloaded electrode and the beam tube, that is the electrode voltage, is simply \(V = q_e/C\), the electrode charge as observed at the tap point divided by the electrode capacitance.
The capacitance is purely a geometrical factor and for this geometry approaches a value of

\[ C = \frac{2 \pi \varepsilon_r \varepsilon_0}{\ln \left( \frac{b}{a} \right)} L_e \]

for \( L_e \gg (b-a) \) where the electrode length is large relative to electrode-to-tube wall spacing.

At this point it must be noted that the electrode charge observed at the tap point, \( q_e \), is not identical to the total electrode charge, \( q_e \), as computed above. Missing from that analysis is consideration of the time delay for a charge induced at any longitudinal position along the electrode to be recognized at the tap point. This information can only travel at the speed of an electromagnetic wave through the medium. In the case of a monitor with vacuum as the only dielectric, this is the speed of light. At low frequencies the effect is negligible, but at frequencies where the transit times become a significant fraction of the period and the bunch length is comparable to the electrode length, the impact is large.

Accounting properly for signal propagation times (ignoring azimuthal effects on the assumption of cylindrical symmetry), the electrode charge observed at the tap point at any time is equivalent to the sum of the charges induced at all longitudinal segments of the electrode at a time earlier by an amount \( x/v \), where \( x \) is the position of the segment relative to the tap point and \( v = \beta c \) for vacuum. Mathematically the time at each position along the electrode is weighted by the transit time to the tap point and the expression for the charge observed at the tap point becomes

\[ q_{\text{tap}}(t) = \frac{L_e}{2} \int_{-L_e/2}^{L_e/2} q_b(x,t - \frac{x}{\beta c}) \, dx \]

\[ q_{\text{tap}}(t) = A \left[ 1 + \cos \left( \frac{2 \pi x}{\beta c} \right) \right] \]

The result of the integral is

\[ q_{\text{tap}}(t) = A L_e \left( 1 + \frac{2 \pi L_e}{L_b} \cos(\omega t) \right) \]

Comparing this to the earlier expression for \( q_e(t) \), there is a factor of two difference in the argument of the \( \sin(x)/x \) term. This correctly places the first null in the frequency response, \( \sin(2 \pi L_e/L_b) = 0 \), at the frequency where the electrode length is one half the bunch (wave) length. The first null of the \( q_e \) expression appears when \( L_e = L_b \). At low frequencies, long wavelengths, where \( L_e \ll L_b \) and \( \sin(x)/x \approx 1 \), the transit time effect has little impact as expected and the two expressions converge.

From a circuit perspective, it is useful and correct to model the pickup as a current source driving the electrode capacitance in parallel with any external monitoring circuit load. The source current is just the time derivative of the beam-induced charge as observed at the tap point:

\[ i_{\text{tap}}(t) = \frac{d}{dt} q_{\text{tap}}(t) = -A L_e c \omega - \frac{\sin(\omega L_e)}{\beta c} \sin(\omega t) \]

For a given charge distribution amplitude, the signal current is directly proportional to frequency and is modulated by the \( \sin(x)/x \) term. The current is zero at zero frequency and at frequencies where \( \omega L_e/\beta c = N \pi \), that is when the electrode length is an integer number of half wavelengths. In the common case, where the electrode is short compared to the wavelength, the \( \sin(x)/x \) term is unity and the signal current magnitude becomes simply \( A L_e \omega \).

With a simple resistor as a monitoring load and in the ‘short electrode’ regime, the circuit model is the electrode current source driving the parallel combination of the electrode capacitance and the load resistor. The frequency response of the resulting signal voltage will be:

\[ V(\omega) = \left( \frac{R}{1 + j \omega \tau} \right) \]

where \( \tau = R \cdot C \) is the time constant of the system. At low frequencies, \( \omega \ll 1/\tau \), the signal voltage is \( A L_e \omega R \), directly proportional to the frequency. Above the corner frequency, \( \omega > 1/\tau \), the signal voltage becomes constant at magnitude \( A L_e \omega C \) until the short electrode limit is violated and the \( \sin(x)/x \) modulation becomes manifest.

Finally, signal power is considered. This is the power available to compete with noise and is what gets burned in the load resistor. In the low frequency regime, \( \omega \ll 1/\tau \), the signal power is \( V^2/R = (1/R)(A L_e \omega R)^2 \). In the mid-band region, \( \omega > 1/\tau \) and \( \omega < 2 \pi \beta c/L_e \), where the signal amplitude is constant with frequency, the power is \( V^2/R = (1/R)(A L_e C)^2 \).

The effect on signal power of operating the capacitive monitor at signal frequencies above or below
The corner frequency is underscored by taking the ratio of the power in the two regimes. That ratio is:

\[
\frac{\text{low frequency}}{\text{midband}} = \left[ \frac{A \cdot L_e \cdot \omega \cdot R}{A \cdot L_e} \cdot \frac{\omega}{C} \right]^2 = \frac{2 \cdot R^2 \cdot C^2}{2 \cdot \omega^2 \cdot R^2 \cdot C^2}
\]

The defining low frequency condition, \( \omega << (1/RC) \), implies that \( \omega^2 \cdot R^2 \cdot C^2 << 1 \). Therefore the available power from a given monitor at a selected signal frequency will always be less if that frequency is below the corner frequency of the system than if it is in the mid-band region.

The mid-band power can be written as

\[
V^2/R = (1/R) \cdot (A \cdot L_e/C)^2 = (A \cdot L_e)^2 / (C \cdot \tau)
\]

Substituting for the capacitance find:

\[
P_{\text{wr}} = \frac{A^2 \cdot L_e \cdot \ln \left( \frac{b}{a} \right)}{\tau \cdot 2 \cdot \pi \cdot \varepsilon_r \cdot \varepsilon_o}
\]

The power is proportional to geometrical parameters including the length of the electrode, proportional to the square of the amplitude of the charge distribution, and inversely proportional to the system time constant.

Maximum signal power at a particular frequency, \( \omega \), is obtained if the time constant is set approximately equal to 1/\( \omega \). A larger time constant sacrifices signal power as 1/\( \tau \) and, as shown above, operating below cutoff is a losing proposition.

This discussion was based on cylindrical electrode geometry with 360° angular extent around the beam. Useful pickups are regularly made with electrodes that subtend a much smaller angle. Button electrodes are an example. The previous discussion applies directly to electrodes occupying only a fraction of the circumference with only a reduction in the signal amplitude by the fraction of the circumference covered. An important difference, however, is that the smaller electrode is sensitive to the position of the beam relative to the axis of the beam tube. At low to moderate frequencies, where the wavelength is long compared to all dimensions of the pickup and azimuthal wave propagation can be ignored, the cylindrical electrode is completely insensitive to the beam position. All electric field lines within the longitudinal extent of the electrode terminate on the electrode regardless of the transverse beam position.

V. A MAGNETIC BEAM PICKUP

Magnetic pickups operate according to Faraday’s law of induction: a loop experiences an induced electromotive force equal to the negative of the time rate of change of magnetic flux through the loop. Detect the emf in a loop that intercepts magnetic flux from the beam current and you have a magnetic beam monitor.

The most common magnetic beam monitor is the current transformer shown in schematic form in Figure 4. The beam serves as a single turn primary winding inducing magnetic flux into the annular cross-section of the toroidal magnetic core. An N-turn sense winding on the core intercepts the flux in the core and generates a signal on the load resistor. The response of the monitor is like that of any transformer, except that the beam current is unaffected by induced currents flowing in the secondary winding (provided that the secondary voltage is not comparable to the beam energy).

In a core with moderate to high permeability, the flux due to any current \( i_k \) in an N-turn winding \( k \) is:

\[
\Phi_k = \frac{\mu_r \mu_o \cdot N_k \cdot h \cdot \ln \left( \frac{b}{a} \right)}{2 \cdot \pi}
\]

where \( h \) is the thickness of the core along the beam axis, \( a \) is the inner radius of the core and \( b \) is the outer radius of the core. The total flux in the core is the sum of that due to the single-turn beam current and that due to current in the N-turn sense winding. Note that the flux due to the sense current will be directed opposite to that of the beam current according to Faraday’s Law.

\[
\Phi_{\text{total}} = \frac{\mu_r \mu_o \cdot h}{2 \cdot \pi} \cdot \ln \left( \frac{b}{a} \right) \cdot (i_b - N_s \cdot i_s)
\]

Faraday’s Law also requires that the sense winding voltage obey:

\[
\frac{d}{dt} \Phi_{\text{total}} = \frac{-V_s}{N_s} = \frac{-i_s \cdot R_s}{N_s}
\]
The self-inductance of the secondary winding is a geometrical property of the system and is given by

\[ L_S = \mu \frac{N_S^2 \cdot h}{2\pi} \ln \left( \frac{b}{a} \right) \]

Simultaneous solution of the two flux equations and substitution of the expression for the inductance yields the familiar transformer response:

\[ V_s(\omega) = \frac{j \cdot \omega \cdot \tau}{1 + j \cdot \omega \cdot \tau} \cdot R_s \cdot \left( \frac{\mu_r}{N_s} \right) \]

where \( \tau = L_s/R_s \) is the time constant of the sense circuit. Below the corner frequency, \( \omega \ll 1/\tau \), the signal voltage magnitude is \( \omega \cdot \tau \cdot R_s \cdot i_b/N_s \). In mid-band, the magnitude is \( R_s \cdot i_b/N_s \) and the sense winding current equals the beam current divided by the number of sense winding turns.

Signal power available from this monitor in the mid-band frequency range is

\[ \frac{V^2}{R} = (1/R_s) \cdot (R_s \cdot i_b/N_s)^2 \]

Using the equation for \( L_s \) to eliminate \( N_s \), the power expression for the magnetic pickup becomes

\[ P_{wr} = \frac{i_b^2}{\tau} \cdot \frac{\mu_r \cdot \mu_0 \cdot h \cdot \ln \left( \frac{b}{a} \right)}{2\pi} \]

This result can be compared to that of the capacitive pickup operating in mid-band recalling that the amplitude of the beam current, \( i_b \), for the charge distribution used in the capacitive example is \( (as \; simple \; as) \; A \cdot \beta \cdot c \).

The ratio of the signal power available from a magnetic and capacitive monitor of the same length, \( h = L_s \), the same radial dimensions \( a \) and \( b \), and the same time constant \( \tau \) is:

\[ \frac{P_{wr\_mag}}{P_{wr\_cap}} = \beta^2 \cdot \mu_r \cdot \varepsilon \]

Several observations are noteworthy.

- The addition of magnetic or dielectric material enhances the relative performance of the magnetic monitor, an advantage that can be dramatic as commonly available magnetic materials can offer permeability > 10,000. Hence, the predominance of magnetic type beam current monitors.
- Capacitive monitors, in the relativistic beam regime, can offer benefits in instances where the signal power is adequate and at high frequencies where the advantage of magnetic materials can be lost.

As with capacitive pickups, a magnetic pickup need not be cylindrically symmetric about the beam. Figure 5 shows an example of a simple loop type magnetic pickup. It consists of a section of semi-rigid coaxial cable forming a loop inside the beam tube. The center conductor, slightly longer than the outer conductor, is connected to the inside wall of the beam tube. The outer conductor is connected to the beam tube where the cable or feedthrough connector passes through the wall and is left open at the free end inside. The center conductor in conjunction with the beam tube wall defines a loop area will intercept a fraction of magnetic flux lines from the beam in the tube. Flux changes generate an emf that appears at the output terminals. The outer conductor of the coaxial line inside the beam tube serves to shield the center conductor from electric fields enabling the coupling to be purely magnetic.

The simple construction of this monitor with a uniform coaxial transmission line facilitates preservation of good performance at high frequencies. The coaxial line is usually terminated in its characteristic impedance to maintain flat frequency response. The inductance of a pickup of this style with centimeter scale loop dimensions is of the order of 100 nH. This, combined with a typical 50 ohm load resistance, results in a time constant on the nanosecond scale. This puts the mid-band response of such a pickup into the hundreds of MHz range. Nevertheless, due to the simplicity of design and ease of implementation, these monitors often find application.

Figure 5. Magnetic loop pickup.
with signals below the corner frequency. This loop monitor offers significant sensitivity to the transverse beam position since the magnetic fields of the beam are strongest in near proximity to the beam axis. Frequency response of the monitor exhibits the \( \sin(x)/x \) modulation, alike the capacitive monitor, with zeroes where the length of the loop parallel to the beam axis is an integer multiple of half wavelengths.

VIII. CONCLUSION

An attempt has been made to provide a conceptual understanding of the operation of simple beam monitors based on principles of fundamental physics.

Basic concepts of electric (capacitive) and magnetic beam monitors have been presented. Equations for estimating the quantitative performance of monitors of each type with simple geometries were derived. Comparison of the signal power available from each of the two types of monitors has been made.

It is difficult to give justice to many important aspects of beam monitors in a short paper. It is hoped that the present discussion provides a sound base for further exploration of the subject.

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REFERENCES

For numerous tutorials on beam instrumentation and a wealth of information on the topic the reader is referred to the American Institute of Physics conference proceedings publications for the series of Beam Instrumentation Workshops from 1989 to the present. Tutorials by this author are included in: