

fastNLO: Fast pQCD Calculations for PDF Fits

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Abstract

We present a method for very fast repeated computations of higher-order cross sections in hadron-induced processes for arbitrary parton density functions. A full implementation of the method for computations of jet cross sections in Deep-Inelastic Scattering and in Hadron-Hadron Collisions is offered by the “fastNLO” project. A web-interface for online calculations and user code can be found at <http://hepforge.cedar.ac.uk/fastnlo/>.

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1 Introduction

The aim of the "fastNLO" project is to make the inclusion of jet data into global fits of parton density functions (PDFs) feasible. Due to the prohibitive computing time required for the jet cross sections using standard calculation techniques, jet data have either been omitted in these fits completely or they were included using a simple approximation. The fastNLO project implements a method that offers exact and very fast pQCD calculations for a large number of jet data sets allowing to take full advantage of their direct sensitivity to the gluon density in the proton in future PDF fits. This includes Tevatron jet data beyond the inclusive jet cross section and also HERA jet data which have been used to determine the proton's gluon density [1], but which are ignored in current PDF fits [2, 3, 4].

2 Concept

2.1 Cross Sections in Perturbative QCD

Perturbative QCD predictions for observables in hadron-induced processes depend on the strong coupling constant α_s and on the PDFs of the hadron(s). Any cross section in hadron-hadron collisions can be written as the convolution of the strong coupling constant α_s in order n , the perturbative coefficient $c_{n,i}$ for the partonic subprocess i , and the corresponding linear combination of PDFs from the two hadrons F_i which is a function of the fractional hadron momenta $x_{a,b}$ carried by the partons

$$\sigma(\mu_r, \mu_f) = \sum_{n,i} c_{n,i}(x_a, x_b, \mu_r, \mu_f) \otimes [\alpha_s^n(\mu_r) \cdot F_i(x_a, x_b, \mu_f)] . \quad (1)$$

The PDFs and α_s also depend on the factorization and the renormalization scales $\mu_{f,r}$, respectively, as does the perturbative prediction for the cross section in finite order n . An iterative PDF fitting procedure using exact NLO calculations for jet data, based on Monte-Carlo integrations of (1), is too time-consuming. Only an approximation of (1) is, therefore, currently being used in global PDF fits.

2.2 A Simple Approach

The "k-factor approximation" as used in [3, 4] parameterizes higher-order corrections for each bin of the observable by a factor $k = \frac{\sigma_{\text{NLO}}}{\sigma_{\text{LO}}} = \frac{\sigma_{(2)} + \sigma_{(3)}}{\sigma_{(2)}}$ computed from the contributions with $n = 2$ ($\sigma_{(2)}$) and $n = 3$ ($\sigma_{(3)}$) for a fixed PDF, averaged over all subprocesses i . In the iterative fitting procedure only the LO cross section is computed and multiplied with k to obtain an estimate of the NLO cross section. This procedure does not take into account that different partonic subprocesses can have largely different higher-order corrections. Fig. 1 shows that the k -factors for quark-only and gluon-only induced subprocesses can differ by more than $\pm 20\%$ from the average. The χ^2 is therefore minimized under an incorrect assumption of the true PDF dependence of the cross section. Further limitations of this approach are:

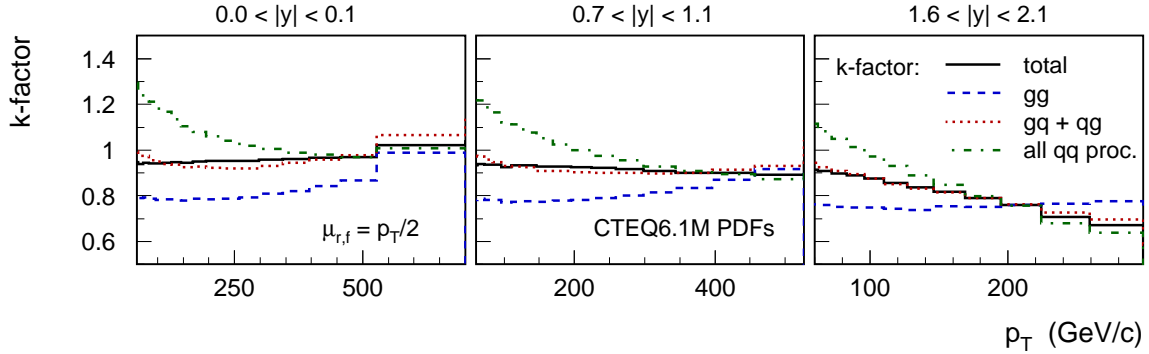


Figure 1: The k -factor for the inclusive $p\bar{p}$ jet cross section at $\sqrt{s} = 1.96$ TeV as a function of p_T at different rapidities y for the total cross section (solid line) and for different partonic subprocesses: gluon-gluon (dashed), gluon-quark (dotted) and the sum of all quark and/or anti-quark induced subprocesses (dashed-dotted).

- Even the LO Monte-Carlo integration of (1) is a trade-off between speed and precision. With finite statistical errors, however, theory predictions are not ideally smooth functions of the fit parameters. This contributes to numerical noise in the χ^2 calculations [5] distorting the χ^2 contour during the PDF error analysis, especially for fit parameters with small errors.
- The procedure can only be used for observables for which LO calculations are fast. Currently, this prevents the global PDF analyses from using Tevatron dijet data and DIS jet data.

In a time when phenomenology is aiming towards NNLO precision [2, 3], the k -factor approximation is clearly not satisfying concerning both its limitation in precision and its restrictions concerning data sets.

2.3 The fastNLO Solution

A better solution is implemented in the fastNLO project. The basic idea is to transform the convolution in (1) into the factorized expression (4). Many proposals for this have been made in the past, originally related to solving the DGLAP parton evolution equations [6] and later to computing jet cross sections [7, 8, 9, 10, 11]. The fastNLO method is an extension of the concepts developed for DIS jet production [7, 10] which have been applied at HERA to determine the gluon density in the proton from DIS jet data [1]. Starting from (1) for the following discussion the renormalization scale is set equal to the factorization scale ($\mu_{r,f} = \mu$). The extension to $\mu_r \neq \mu_f$ is, however, trivial. The x dependence of the PDFs and the scale dependence of α_s^n and the PDFs can be approximated using an interpolation between sets of fixed values $x^{(k)}$ and $\mu^{(m)}$ ($k = 1, \dots, k_{\max}$; $m = 1, \dots, m_{\max}$)

$$\alpha_s^n(\mu) \cdot F_i(x_a, x_b, \mu) \simeq \sum_{k,l,m} \alpha_s^n(\mu^{(m)}) \cdot F_i(x_a^{(k)}, x_b^{(l)}, \mu^{(m)}) \cdot e^{(k)}(x_a) \cdot e^{(l)}(x_b) \cdot b^{(m)}(\mu) \quad [“=” \text{ is true for } k_{\max}, l_{\max}, m_{\max} \rightarrow \infty] \quad (2)$$

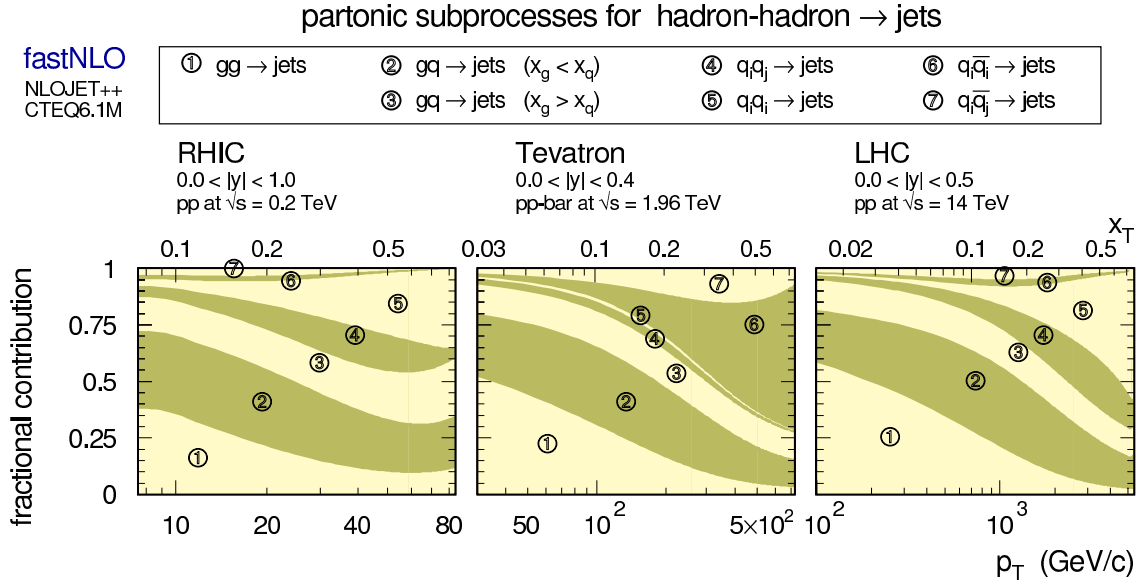


Figure 2: Contributions of different partonic subprocesses to the inclusive jet cross section at RHIC (left), the Tevatron (middle) and the LHC (right) as a function of p_T and $x_T = 2p_T/\sqrt{s}$. The subprocess $gq \rightarrow$ jets has been separated into the contributions (2) and (3) where either the quark- or the gluon momentum fraction is larger.

where $e^{(k,l)}(x)$ and $b^{(m)}(\mu)$ are interpolation functions for the x and the μ dependence, respectively. All information of the perturbatively calculable piece (including phase space restrictions, jet definition, etc. but excluding α_s and the PDFs) is fully contained in the quantity

$$\tilde{\sigma}_{n,i,k,l,m}(\mu) = c_{n,i}(x_a, x_b, \mu) \otimes [e^{(k)}(x_a) \cdot e^{(l)}(x_b) \cdot b^{(m)}(\mu)] . \quad (3)$$

In the final prediction for the cross section the convolution in (1) is then reduced to a simple product

$$\sigma(\mu) \simeq \sum_{n,i,k,l,m} \tilde{\sigma}_{n,i,k,l,m}(\mu) \cdot \alpha_s^n(\mu^{(m)}) \cdot F_i(x_a^{(k)}, x_b^{(l)}, \mu^{(m)}) . \quad (4)$$

The time-consuming step involving the calculation of the universal (PDF and α_s independent) $\tilde{\sigma}$ is therefore factorized and needs to be done only once. Any further calculation of the pQCD prediction for arbitrary PDFs and α_s values can later be done very fast by computing the simple sum of products in (4). While the extension of the method from one initial-state hadron [10] to two hadrons was conceptually trivial, the case of two hadrons requires additional efforts to improve the efficiency and precision of the interpolation. Both are directly related to the choices of the points $x^{(k,l)}$, $\mu^{(m)}$ and the interpolation functions $e(x)$, $b(\mu)$. The implementation in fastNLO achieves a precision of better than 0.1% for $k_{\max}, l_{\max} = 10$ and $m_{\max} \leq 4$. Computation times for cross sections in fastNLO are roughly 40-200 μ s per order α_s (depending on m_{\max}). Further details are given in Ref [12].

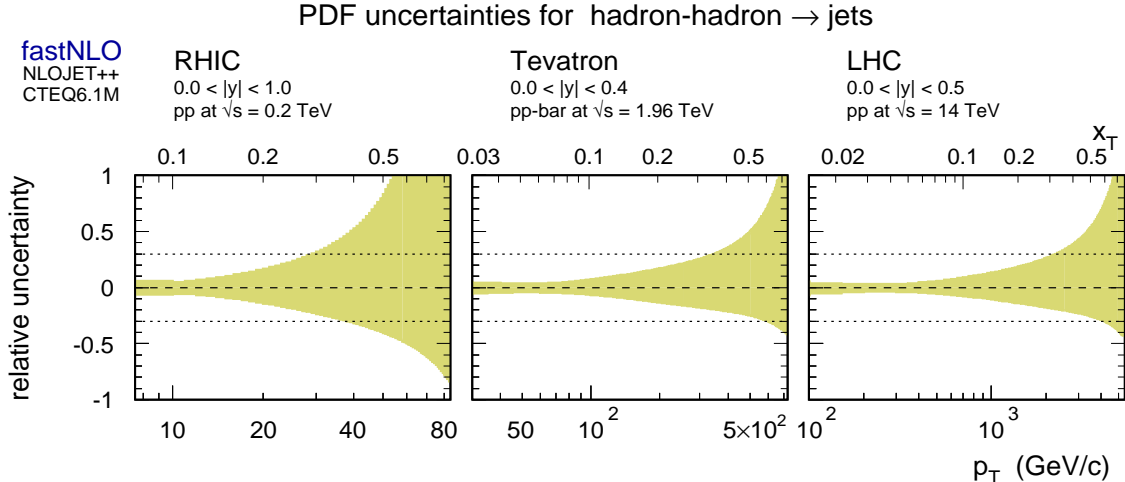


Figure 3: Comparison of PDF uncertainties for the inclusive jet cross section at RHIC (left), the Tevatron (middle) and the LHC (right). The uncertainty band is obtained for the CTEQ6.1M parton density functions and the results are shown as a function of p_T and $x_T = 2p_T/\sqrt{s}$.

The $\tilde{\sigma}$ in (3) are computed using NLOJET++ [13, 14]. A unique feature in fastNLO is the inclusion of the $O(\alpha_s^4)$ threshold correction terms to the inclusive jet cross section [15], a first step towards a full NNLO calculation.

3 Results

fastNLO calculations are available at <http://hepforge.cedar.ac.uk/fastnlo> for a large set of (published, ongoing, or planned) jet cross section measurements at HERA, RHIC, the Tevatron, and the LHC (either online or as computer code for inclusion in PDF fits). Some fastNLO results for the inclusive jet cross section in different reactions are shown in this section. The contributions from different partonic subprocesses to the central inclusive jet cross section are compared in Fig. 2 for different colliders: For pp collisions at RHIC and the LHC, and for $p\bar{p}$ scattering at Tevatron Run II energies. It is seen that the quark-induced subprocesses are dominated by the valence quarks: In proton-proton collisions (RHIC, LHC) the quark-quark subprocesses (4,5) give much larger contributions than the quark-antiquark subprocesses (6,7) while exactly the opposite is true for proton-antiproton collisions at the Tevatron. The contribution from gluon-induced subprocesses is significant at all colliders over the whole p_T ranges. It is interesting to note that at fixed $x_T = 2p_T/\sqrt{s}$ the gluon contributions are largest at RHIC. Here, the jet cross section at $x_T = 0.5$ still receives 55% contributions from gluon-induced subprocesses, as compared to only 35% at the Tevatron or 38% at the LHC. As shown in Fig. 3, this results in much larger PDF uncertainties for the high x_T inclusive jet cross section at RHIC, as compared to the Tevatron and the LHC for which PDF uncertainties are roughly of the same size (at the same x_T). This indicates that the PDF sensitivity at the same x_T is about the same at the Tevatron and at

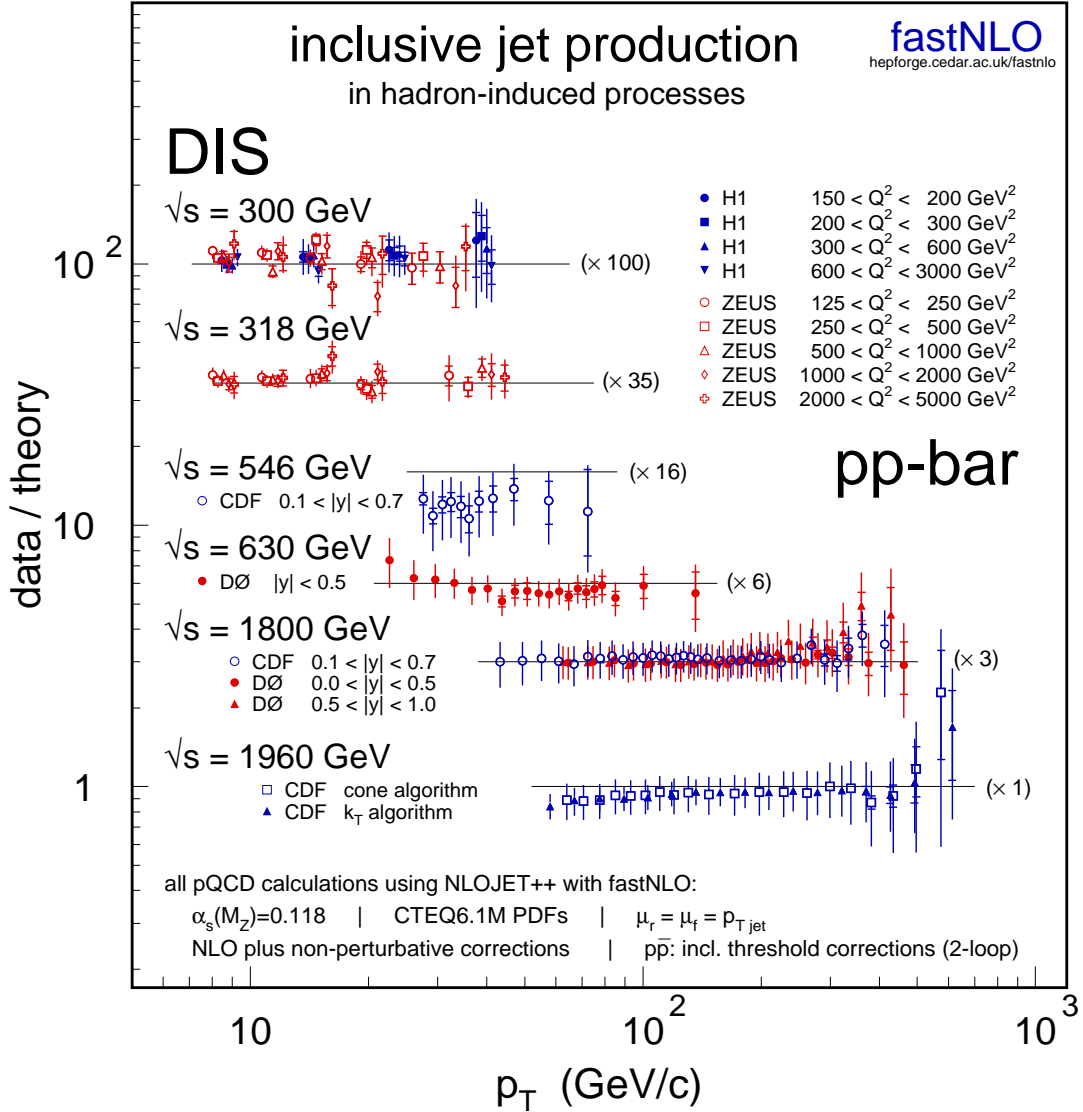


Figure 4: An overview of data over theory ratios for inclusive jet cross sections, measured in different processes at different center-of-mass energies. The data are compared to calculations obtained by fastNLO in NLO precision (for DIS data) and including $\mathcal{O}(\alpha_s^4)$ threshold corrections (for $p\bar{p}$ data). The inner error bars represent the statistical errors and the outer error bars correspond to the quadratic sum of all experimental uncertainties. In all cases the perturbative predictions have been corrected for non-perturbative effects.

the LHC, while it is much higher at RHIC.

An overview over published measurements of the inclusive jet cross section in different reactions and at different center-of-mass energies is given in Fig. 4. The results are shown as ratios of data over theory. The theory calculations include the best available perturbative predictions (NLO for DIS data and NLO + $\mathcal{O}(\alpha_s^4)$ threshold corrections for $p\bar{p}$ data) which have been corrected for non-perturbative effects. Over

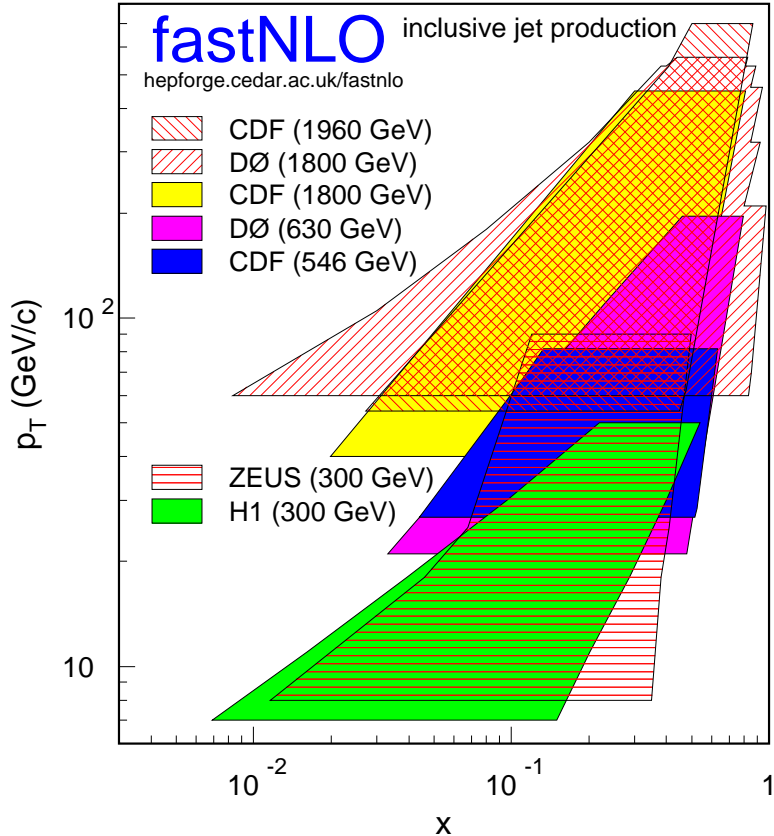


Figure 5: The phase space in x and p_T covered by the data sets shown in the previous figure.

the whole phase space of $8 < p_T < 700$ GeV jet data in DIS and $p\bar{p}$ collisions are well-described by the theory predictions using CTEQ6.1M PDFs [4]. The phase space in x and p_T covered by these measurements is shown in Fig. 5, demonstrating what can be gained by using fastNLO to include these data sets in future PDF fits. A first study using fastNLO on the future potential of LHC jet data has been published in Ref. [16].

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