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THE THERMAL MEMORY OF REIONIZATION HISTORY

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ABSTRACT

The recent measurement by *WMAP* of a large electron scattering optical depth $\tau_e = 0.17 \pm 0.04$ is consistent with a simple model of reionization in which the intergalactic medium (IGM) is ionized at redshift $z \sim 15$, and remains highly ionized thereafter. Here, we show that existing measurements of the IGM temperature from the Lyman-alpha (Lyα) forest at $z \sim 2-4$ rule out this "vanilla" model. Under reasonable assumptions about the ionizing spectrum, as long as the universe is reionized before $z = 10$ *and remains highly ionized thereafter*, the IGM reaches an asymptotic thermal state which is too cold compared to observations. To simultaneously satisfy the CMB and Lyα forest constraints, the reionization history must be complex: reionization begins early at $z \gtrsim 15$, but there must have been significant (order unity) changes in fractions of neutral hydrogen and/or helium at $6 < z < 10$, and/or singly ionized helium at $4 < z < 10$. We describe a physically motivated reionization model that satisfies all current observations. We also explore the impact of a stochastic reionization history and show that a late epoch of (HeII$\rightarrow$HeIII) reionization induces a significant scatter in the IGM temperature, but the scatter diminishes with time quickly. Finally, we provide an analytic formula for the thermal asymptote, and discuss possible additional heating mechanisms that might evade our constraints.

Subject headings: cosmology: theory — intergalactic medium — quasars: absorption lines — cosmic microwave background

1. INTRODUCTION

The detection by the Wilkinson Microwave Anisotropy Probe (*WMAP*) of a large optical depth τ_e to electron scattering has opened a new window in studies of the ultra–high redshift ($z \sim 15$) universe (Kogut et al. 2003, Spergel et al. 2003). Taking the value $\tau_e = 0.17 \pm 0.04$, inferred from the polarization and temperature anisotropies of the cosmic microwave background (CMB), at face value, implies the reionization of the universe must begin very early.

The optical depth to electron scattering is given by (e.g. Dodelson 2003)

$$\tau_e = \int_0^\infty \frac{dz}{(1+z)H(z)} \sigma_T n_e(z) \tag{1}$$

where $H(z)$ is the Hubble parameter at redshift z, σ_T is the Thompson cross-section, and $n_e(z)$ is the proper free electron density. This can be rewritten as

$$\tau_e = 0.0691 \times \Omega_b h \int_0^\infty \frac{(1+z)^2 dz}{\sqrt{\Omega_m (1+z)^3 + \Omega_\Lambda}} \tag{2}$$
$$[(1-Y_P)X_{\rm HII} + \frac{1}{4} Y_P (X_{\rm HeII} + 2X_{\rm HeIII})]$$

where Ω_b, and Ω_m are the baryon and matter densities in fraction of the critical, h is the Hubble constant in units of 100 km/s/Mpc, and $Y_P = 0.244 \pm 0.002$ is the helium mass fraction (Burles, Nollett & Turner 2001). The fractions of ionized hydrogen $X_{\rm HII}$, singly ionized helium $X_{\rm HeII}$, and doubly ionized helium $X_{\rm HeIII}$, are functions of z.

Ignoring helium, the observed $\tau_e = 0.17 \pm 0.04$ is consistent with a universe in which $X_{\rm HII}$ changes from essentially zero to close to unity at $z = 17 \pm 3$, and $X_{\rm HII} \sim 1$ since (Kogut et al. 2003). If helium is fully ionized together with hydrogen, the reionization redshift changes slightly to $z = 15.3 \pm 2.7$. [1]

Two features are noteworthy. First, helium reionization has a sub-dominant effect on τ_e compared to hydrogen. Second, since the electron scattering optical depth is controlled by the free electron density, it is insensitive to the neutral fractions of hydrogen ($X_{\rm HI} = 1 - X_{\rm HII}$) and helium ($X_{\rm HeI} = 1 - X_{\rm HeII} - X_{\rm HeIII}$), as long as they are small.

In contrast, the (hydrogen) Lyman-alpha (Lyα) optical depth, inferred from the spectra of distant quasars, is extremely sensitive to small amounts of neutral hydrogen. The Lyα optical depth at mean density is (Gunn & Peterson 1965)

$$\tau_\alpha = 41.8 \left[\frac{X_{\rm HI}}{10^{-4}}\right] \left[\frac{1+z}{7}\right]^3 \left[\frac{H(z=6)}{H(z)}\right], \tag{3}$$

Whether $X_{\rm HI}$ is 10^{-4} or 10^{-5}, for instance, makes a big difference to τ_α. Using a model for large scale structure in the intergalactic medium (IGM), the observed mean Lyα transmission at $z \sim 6$ (or the lack thereof i.e. the Gunn-Peterson trough; Becker et al. 2001) implies a (1σ) lower limit on the hydrogen neutral fraction for a fluid element at mean density: $X_{\rm HI} > 10^{-4}$. The analog for Lyβ provides a stronger constraint, due to the larger absorption cross-section: $X_{\rm HI} > 5 \times 10^{-4}$ (taken from Lidz et al. 2002; see also Cen & McDonald 2002, Fan et al. 2002).[2]

[1] Throughout this paper, we adopt $\Omega_b h^2 = 0.024$, $\Omega_m h^2 = 0.14$, and $h = 0.72$ the central best–fit values measured by WMAP (Spergel et al. 2003).

[2] Note that some authors quote volume– or mass–weighted neutral fractions. We here use the neutral fraction for a fluid element that happens to be at the cosmic mean density, motivated by the fact that we will discuss the temperature evolution of fluid elements with the same property in this paper.

Taken at face value, these two separate observations are therefore consistent with the simplest "vanilla" model in which the universe is reionized in a single step: the neutral fraction experiences a drop of order unity at at $z \gtrsim 15$, and it remains $\ll 1$ thereafter. Our goal in this paper is to confront this model with other existing observations, and to see if additional constraints can be put on the reionization history of our universe.

Several authors have pointed out that the *evolution* of the Lyα optical depth suggests reionization might take place not much earlier than $z \sim 6$ (Becker et al. 2001, Djorgovski 2001, Cen & McDonald 2002, Gnedin 2001, Razoumov et al. 2002; but see also Barkana 2001, Songaila & Cowie 2002). This is based on an extrapolation of the mean transmission measurements from lower redshifts. The small number of lines of sight used (a single quasar was employed for the measurement at $z \sim 6$; but see Fan et al. 2003 for three new $z > 6$ sources with Gunn-Peterson troughs), the challenge of sky subtraction and continuum extrapolation in Gunn-Peterson trough measurements (see discussion in Becker et al. 2001; see also Hui et al. 2001), as well as our still maturing understanding of radiative transfer during reionization, motivates us to look for other clues for a late period of reionization.

The main idea is quite simple. Reionization typically heats up the IGM to tens of thousands of degrees, and the gas subsequently cools due to the expansion of the universe as well as due to other processes. If the universe was reionized early, and has stayed highly ionized thereafter, photo–ionization heating of the gas cannot overcome the overall cooling, and the IGM might reach too low a temperature at low redshifts compared to observations. This idea is not new (e.g. Miralda-Escude & Rees 1994, Hui & Gnedin 1997, Haehnelt & Steinmetz 1998, Hui 2000, Theuns et al. 2002). Our objective here is to seek a formulation of this argument that is as clean and robust as possible, that reveals clearly the underlying assumptions, and to check the consistency with the IGM temperatures of specific models that produce the high value of τ_e measured by *WMAP*.

The Lyα forest temperature measurements we will use are taken from Zaldarriaga, Hui & Tegmark (2001; ZHT01 thereafter): $T_0 = 2.1 \pm 0.9 \times 10^4$ K at $z = 2.4$, $2.3 \pm 0.7 \times 10^4$ K at $z = 3$, and $2.2 \pm 0.4 \times 10^4$ K at $z = 3.9$. Note that 2 σ errorbars are quoted, and the temperature T_0 was derived for fluid elements at the mean density, consistent with our modeling of the IGM temperature in the rest of this paper. There have been several other measurements in the past (Ricotti, Gnedin & Shull 2000, Schaye et al. 2000 [ST00], Bryan & Machacek 2000, McDonald et al. [MM01], Meiksin, Bryan & Machacek 2001). The ones that are easiest to compare, because they are based on very similar datasets, are ST00, MM01 and ZHT01. The former two are based on line width measurements, while the last one makes use of the small scale transmission power spectrum. A virtue of the last method is that the temperature constraints come from marginalizing over a wide array of parameters, including the slope and amplitude of the primordial power spectrum (e.g. Hui & Rutledge 1999), and the equation of state index. It is reassuring that MM01 and ZHT01, using very different methods and employing different simulations (MM01 using hydrodynamic simulations, and ZHT01 using N-body simulations with a marginalized smoothing to mimic Jeans smoothing), agree well with each other. The results of ST00 are somewhat discrepant from these two works – the reader is referred to ZHT01 for further discussions.

An important issue in determining the temperature of the IGM is the second ionization of helium (HeII→HeIII). As we will see, this can be an important source of heating at low redshifts $z \sim 3-4$. There are several lines of evidence that suggest HeII might be reionized at $z \sim 3$, including observations of HeII patches that do not seem to correlate with HI absorption (Reimers et al. 1997, Anderson et al. 1999, Heap et al. 2000, Kriss et al. 2001, Jakobsen et al. 2003), increase in IGM temperature (Schaye et al. 2000; Theuns et al. 2002), the evolution of the hardness of the ionizing background spectrum (Songaila 1998), and evolution of the mean transmission (Bernardi et al. 2003). On the other hand, it is unclear if the fluctuations in HeII absorption observed in a few lines of sight might not be due to a naturally fluctuating IGM (Miralda-Escude, Haehnelt & Rees 2000); the IGM temperature measurements by MM00 and ZHT01 are consistent with no feature at $z \sim 3$; power spectrum evolution also seems to argue against HeII reionization at $z \sim 3$ (McDonald & Seljak, private communication). In this paper, when we consider constraints from the temperature measurement, primarily at $z = 3.9$, we therefore leave two options open: HeII can be ionized or not ionized by $z = 3.9$. With this explained, we can now state our vanilla model in more concrete terms. It has two variants: 1. both hydrogen and helium are fully reionized at $z \gtrsim 15$, and they remain highly ionized thereafter; 2. the same as 1. except that helium is only singly ionized, and remains so until at least past $z = 3.9$.

The paper is organized as follows. In §2, we explain the idea of a thermal asymptote for the IGM, and use it to derive a constraint on the hardness of the ionizing spectrum if the universe were to reionize before $z = 10$, and remains highly ionized thereafter. In §3, we discuss limits on the hardness of the ionizing spectra, and show that these spectra fall short of making the thermal asymptote sufficiently hot to match observations. We then work out illustrative examples in §4 of how order unity changes in the neutral fractions at $z \lesssim 10$ can reproduce the temperature measurements, while being consistent with the *WMAP* data. We go on to offer a physically motivated model in §5, and discuss the implications of the fact that different fluid elements are reionized at different times (stochastic reionization). We conclude in §6.

2. THERMAL ASYMPTOTICS

Several thermal processes are at work in a photoionized IGM. They are described in detail in Hui & Gnedin (1997). Here is a brief qualitative summary:

- Adiabatic heating/cooling. Gas elements can heat up or cool simply due to adiabatic contraction or expansion. The overall expansion of the universe drives a temperature fall-off as $(1+z)^2$ as z decreases.
- Photoionization heating. Photons inject energy into the gas by ionizing hydrogen or helium.
- Recombination cooling. Protons and electrons (or

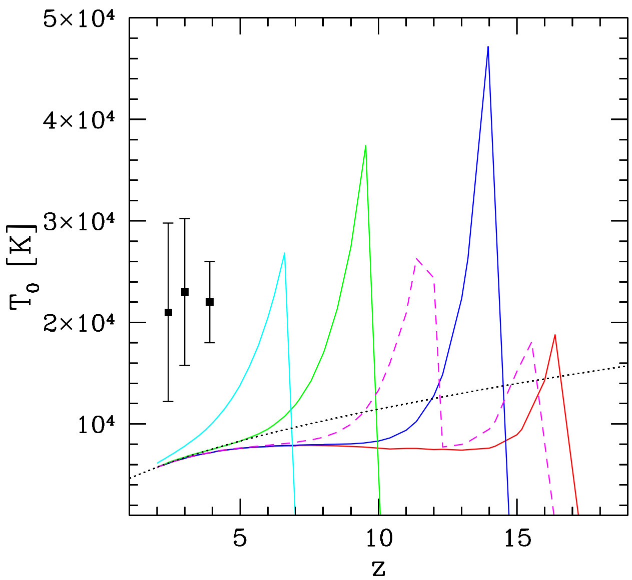

FIG. 1.— The figure shows the thermal asymptote (black dotted line; eq. [4]), and illustrates the fact that a wide range of ionization histories result in the same IGM temperature by redshift $z = 4$, unless reionization occurs late. Each colored solid line describes the evolution of the temperature (T_0) for a fluid element at mean density, according to a different reionization history (and a different initial reheat-temperature). The (magenta) dashed line illustrates the thermal evolution for a complex reionization history – such complexities do not stop the temperature from reaching the late time asymptote, as long as they take place early, before $z \sim 10$. The points with ($2\ \sigma$) error-bars on the left are measurements of T_0 from the Lyα forest (Zaldarriaga et al. 2001).

ionized helium and electrons) can cool by recombining and radiating energy away.

- Compton cooling. At sufficiently high redshifts, $z \gtrsim 10$, Compton scattering of free electrons with the lower temperature CMB can be an important source of cooling.

Typically, photoionization heating provides the dominant source of heating for the tenuous IGM. This occurs in primarily two forms. One is during what we call reionization 'events' – these are (possibly extended) periods in which a given gas element experiences order unity changes in the fractions of HI/HeI/HeII. The other is photo-heating for an already highly ionized plasma – this occurs through photoionization of small amounts of HI/HeI/HeII, whose abundances are determined by photoionization equilibrium. The former provides a big boost to the temperature, while the latter determines the asymptotic thermal state of the IGM. Typical temperature evolutions are illustrated in several toy models in Fig. 1.

Fig 1 shows the evolution in temperature T_0 for a fluid element at mean density, with a variety of reionization histories. These histories have different reionization redshifts and initial temperature jumps, selected here only for illustration. The code for computing the thermal and chemical evolution is described in Hui & Gnedin (1997). For each thermal curve, one can see a rise to high temperatures after a reionization event. Thereafter, the varieties of reionization histories and initial reheat temperatures all result in a rather similar late time thermal asymptote (black dotted line). In fact, if reionization occurs before $z \sim 10$ (and the IGM stays highly ionized since, or to use our earlier terminology: no reionization events take place thereafter), the temperature is always very close to the asymptote for $z < 4$, which is where temperature measurements exist (points with error-bars). This is true even if reionization is not a single step process, as illustrated by the dashed (magenta) line, as long as complexities in the reionization process occur before $z \sim 10$. The term 'complexities' here has a rigorous meaning: it refers to two or more episodes during which the neutral fractions of hydrogen and/or helium undergo changes of order unity. In all thermal curves shown in Fig. 1, with one exception, such changes take place before about $z \sim 10$. The exception, i.e. the left-most curve where reionization takes place at $z \sim 7$, is also the only one where the asymptote is *not* reached by $z = 4$ – this is because there is not sufficient time for the IGM to cool after a recent episode of reionization.

Fig. 1 therefore illustrates two very important general facts:

1. *The thermal state at $z \leq 4$ does not remember the part of reionization history prior to $z \sim 10$.*

2. *The IGM does, however, retain short-term memory of reionization events in its recent past.*

What determines the late time thermal asymptote? It is the combination of photoionization heating, recombination cooling, and adiabatic cooling. An approximate analytic expression can be derived for the thermal evolution of the IGM (Hui & Gnedin 1997). We find that the following simple formula provides a more accurate ($\sim 5\%$) fit to the numerically computed thermal asymptote (for T_0 at $z = 2-4$), for a variety of spectral shapes we have tested:

$$T_0 = [B(1+z)^{0.9}]^{\frac{1}{1.7}} \tag{4}$$

where

$$\begin{aligned} B \equiv\ & 18.8\,\mathrm{K}^{0.7}\left[\sqrt{\frac{0.14}{\Omega_m h^2}}\frac{\Omega_b h^2}{0.024}\right] \\ & \times\left(T_{\mathrm{HI}} + \frac{X_{\mathrm{HeII}}}{16}T_{\mathrm{HeI}} + \frac{5.6 X_{\mathrm{HeIII}}}{16}T_{\mathrm{HeII}}\right) \\ T_i \equiv\ & k_B^{-1}\frac{\int_{\nu_i}^{\infty} J_\nu \sigma_i (\mathrm{h}\nu - \mathrm{h}\nu_i) d\nu/(\mathrm{h}\nu)}{\int_{\nu_i}^{\infty} J_\nu \sigma_i d\nu/(\mathrm{h}\nu)} \end{aligned} \tag{5}$$

Here, k_B is the Boltzmann constant, the symbol J_ν denotes the ionizing intensity as a function of frequency (it has unit of energy per frequency per time per area per ster-radian), σ_i denotes the ionization cross-section for the respective species (i = HI, HeI or HeII), ν_i is the ionizing threshold frequency, and h is the Planck constant (in distinction from the Hubble constant h). It is useful to remember $\mathrm{h}\nu_{\mathrm{HI}}/k_B = 1.57807 \times 10^5$ K, $\nu_{\mathrm{HeI}} = 1.808\,\nu_{\mathrm{HI}}$, and $\nu_{\mathrm{HeII}} = 4\,\nu_{\mathrm{HI}}$.

The asymptote given in eq. (4) assumes that at late times, hydrogen is highly ionized. That is why the term due to photo-heating of HI, $T_{\rm HI}$, has the form shown in eq. (5). The denominator of $T_{\rm HI}$, when multiplied by 4π, is the photoionization rate of neutral hydrogen – its inverse is hence proportional to the (small) hydrogen neutral fraction under ionization equilibrium. Its numerator gives the photo-heating rate per neutral atom. The combination of factors in $T_{\rm HI}$ therefore gives a temperature scale whose amplitude is proportional to the net amount of HI photo-heating. Similarly, the terms $X_{\rm HeII} T_{\rm HeI}/16$, and $5.6 X_{\rm HeIII} T_{\rm HeII}/16$ in eq. (5) quantify the importance of photo-heating of HeI and HeII respectively. The factors of $X_{\rm HeII}$ and $X_{\rm HeIII}$ arise due to photoionization equilibrium. If, asymptotically, helium is doubly (singly) ionized, then $X_{\rm HeII}$ ($X_{\rm HeIII}$) can be set to zero.

It is important to emphasize that the thermal asymptote given in eq. (4) is determined completely by the shape (or 'hardness') of the ionizing spectrum J_ν, but not its amplitude. For a power-law $J_\nu \propto \nu^{-\alpha}$, the thermal asymptote can be recast simply as:

$$T_0 = 2.49 \times 10^4 \, {\rm K} \, \times (2+\alpha)^{-\frac{1}{1.7}} \left(\frac{1+z}{4.9} \right)^{0.53} \qquad (6)$$

assuming, asymptotically, helium is doubly ionized.

Given the above, we can ask the following question: how hard does the ionizing spectrum have to be for the thermal asymptote to match the observed temperatures at low redshifts? It suffices to use the measurement at the highest redshift, $z = 3.9$: $T_0 = 2.2 \pm 0.4 \times 10^4$ K (2 σ error-bar; ZHT01). More concretely, what constraint can be placed on α if the thermal asymptote were to reach $T_0 > 1.6 \times 10^4$ K (3 σ lower limit), by $z = 3.9$? It is straightforward to show that this requires $\alpha < 0.12$, from eq. (6).

One can also derive a similar limit if it is assumed helium is only singly ionized by $z = 3.9$ (i.e. $X_{\rm HeI}, X_{\rm HeIII} \ll 1$, $X_{\rm HeII} \sim 1$). Assuming again a power-law $J_\nu \propto \nu^{-\alpha}$, but with a cut-off for $\nu > \nu_{\rm HeII}$, the requirement is $\alpha < -2.2$. In other words, the spectrum needs to be even harder with no HeII photo-heating, and a spectrum as hard as $\alpha < -2.2$ is unrealistic in comparison with stellar and quasar spectra.

To summarize: *if reionization takes place at $z > 10$, and the fractions of HI/HeI/HeII experience no significant (order unity) change after $z = 10$, a sufficiently hard ionizing spectrum is necessary to keep the IGM temperature high enough to match observations at $z = 3.9$.* Parameterizing the ionizing background by a power-law $J_\nu \propto \nu^{-\alpha}$, this requires $\alpha < 0.12$ if HeII is reionized by $z = 3.9$, or $\alpha < -2.2$ if HeII is not reionized by then (the latter assumes J_ν is cut off for $\nu > \nu_{\rm HeII}$). One should keep in mind that the only relevant slope of the spectrum is the slope just above each ionization threshold (unless the spectrum is very hard), because the ionization cross section $\sigma_i \sim \nu^{-3}$. In other words, α can deviate greatly from the values given above as long as the deviation takes place at frequencies far away from the ionization thresholds. Note also that the spectrum described here refers to the asymptotic spectrum at $z = 3.9$. If the spectrum changes significantly after $z \sim 10$, one can view the above limits as applicable to the hardest spectrum between $z = 3.9 - 10$.

The power index we find, $\alpha < 0.12$, or $\alpha < -2.2$, represents a very hard spectrum. We next turn to the question: how hard can a realistic ionizing spectrum be?

3. THE IONIZING SPECTRUM

Two kinds of ionizing spectrum are generally discussed in the literature. One is quasar-like and the other is star-like.

Zheng et al. (1998) finds a quasar spectral shape of $\sim \nu^{-1.8}$ for the relevant ionizing frequencies in high resolution *HST* spectra. In this paper, we will follow Haardt & Madau (1996) and consider, conservatively, a quasar spectrum that goes as $\nu^{-1.5}$. One should keep in mind that, at the relevant redshifts $\gtrsim 4$, the known populations of quasars probably cannot contribute significantly to the (hydrogen) ionizing background. Here, we take the conservative view that there might be a population of dim quasars that still contribute significantly to a hard spectrum (Haiman & Loeb 1997).

Stellar spectra are generally softer than a typical quasar spectrum. An exception is the spectrum of metal-free stars (Tumlinson & Shull 2000, Bromm, Kudritzki & Loeb 2001; Schaerer 2002). In their theoretical models, Bromm et al. (2001) find a spectrum that has the following form: $J_\nu \sim \nu$ for ν just above $\nu_{\rm HI}$, $J_\nu \sim \nu^0$ at $\nu \gtrsim \nu_{\rm HeI}$, and $J_\nu \sim \nu^{-4.5}$ for $\nu \gtrsim \nu_{\rm HeII}$. Such a spectrum is harder than the quasar spectrum at frequencies below the HeII threshold. Note that metal free stars probably cause reionization early on, but it is unlikely they contribute significantly to the asymptotic ionizing spectrum at low redshifts (Haiman & Holder 2003). We consider a metal free stellar spectrum here for the sake of being conservative i.e. assume a spectrum that is as hard as possible.

The actual ionizing spectrum seen by a fluid element is different from the above, due to processing by the IGM. Haardt & Madau (1996) have done a careful calculation of such effects. Absorption generally hardens the spectrum[3] However, diffuse recombination radiation from the absorbing medium tends to compensate for this hardening. Haardt & Madau (1996) found that the spectrum above $\nu_{\rm HeII}$ hardens by 1.5 (to be precise, $\alpha \rightarrow \alpha - 1.5$) (see also Zuo & Phinney 1993). The spectrum just above $\nu_{\rm HI}$ and $\nu_{\rm HeI}$ maintains roughly the same slope as the source.

We are therefore led to consider the following IGM modified spectra. Let us use the symbols $\alpha_{\rm HI}$, $\alpha_{\rm HeI}$ and $\alpha_{\rm HeII}$ to denote the spectral slopes (more precisely, its negative), above the three relevant ionizing thresholds. A quasar-like processed spectrum has $\alpha_{\rm HI} = \alpha_{\rm HeI} = 1.5$, and $\alpha_{\rm HeII} = 0.0$. A metal-free-stellar spectrum gives rise to $\alpha_{\rm HI} = -1$, $\alpha_{\rm HeI} = 0$, and $\alpha_{\rm HeII} = 3$. At the moment(s) of reionization, the relevant spectrum can, however, be even harder (Abel & Haehnelt 1999). In principle, the spectrum can be hardened relative to the source by as much as a power-law index of 3, the 3 coming from the scaling $\sigma_i \propto \nu^{-3}$ (see arguments in Abel & Haehnelt (1999), and also Zuo & Phinney 1993). We therefore allow the quasar-spectrum to have $\alpha_{\rm HI} = \alpha_{\rm HeI} = \alpha_{\rm HeII} = -1.5$, and the stellar-spectrum to have $\alpha_{\rm HI} = -4, \alpha_{\rm HeI} = -3, \alpha_{\rm HeII} = 1.5$, at the initial

[3]Note the apparent paradox: absorption, by taking away ionizing photons, *increases* the photo-heating rate. This is a result of photoionization equilibrium, which makes the amplitude of the ionizing background irrelevant (eq. [5]). Rather it is the spectral shape that is important.

moment(s) of reionization. This is not relevant for the thermal asymptote, but is relevant for the magnitude of temperature boosts during reionization events.

Comparing the above spectral slopes against the limits obtained in the last section suggests neither quasars, nor metal–free stars can match the observed temperature at $z = 3.9$, if reionization occurs at $z > 10$, and no reionization event takes place afterwards. However, those limits were based on a strict power-law spectrum. The thermal asymptotes (eq. [4]) for our more realistic spectra for quasars and metal–free stars are shown in Fig. 2 and Fig. 3 (black dotted lines), respectively. Indeed, in both cases, the thermal asymptotes fall short of the observed (3 σ) lower limit of $T_0 > 1.6 \times 10^4$ K at $z = 3.9$, regardless of whether or not helium is doubly ionized.

Hence, if the universe is reionized before $z \sim 10$, and remains highly ionized thereafter, neither reasonably hard spectra can reproduce the IGM temperatures inferred from the Lyα forest.

4. ILLUSTRATIVE EXAMPLES

The conclusion from the last section, together with the large electron scattering optical depth measured by *WMAP*, therefore implies that one or more of the fractions $X_{\rm HI}, X_{\rm HeI}, X_{\rm HeII}$ must change by order unity at $z < 10$, to give the IGM temperature boosts above the thermal asymptote. *We therefore rule out the vanilla reionization models laid out in §1.* One should keep in mind, however, this is predicated upon (reasonable) assumptions about the hardness of the ionizing spectrum. In this section, we work out some illustrative examples of what it takes to match the IGM temperature constraints, using the spectra from the last section.

Fig. 2 shows the thermal evolution for a fluid element at mean density subject to a quasar-like ionizing spectrum, and experiencing a variety of ionization histories, for both the case of having helium doubly ionized (upper panel), and the case of only having singly ionized helium (lower panel; except for model E, see below). Models A and C both describe early reionization before $z = 10$, and no significant (order unity) changes in $X_{\rm HI}$, $X_{\rm HeI}$, $X_{\rm HeII}$ thereafter. As explained before, both converge to their respective thermal asymptotes by $z \sim 4$, which fall short of the observed temperatures, especially for model C, which has no HeII reionization, and therefore lower temperatures. Model B demonstrates how early reionization can be made consistent with the forest temperature measurements. It has a late second episode of (both hydrogen and helium I and II) reionization at $z \sim 7$, which boosts the temperature significantly above the thermal asymptote. Model E is similar, except that in the second episode, only HeII is reionized (HI and HeI were already highly ionized before then). It shows that *HeII reionization alone can provide a significant boost to the IGM temperature.*

Model D is intriguing, as it shows a case where a second episode of hydrogen (and HeI, but not HeII) reionization occurs at the smallest redshift ($z = 6$) allowed by SDSS observations (Fan et al. 2002). Even with such a late reionization epoch, the temperature can barely match the observations (the temperature at $z = 3.9$ is 1.5×10^{-4}, which is just below the 3 σ limit). This is due to the lack of HeII reionization in this model. *Therefore, with a quasar spectrum like the one assumed here, some amount of HeII reionization is necessary prior to $z \sim 4$, to heat the IGM sufficiently.* If the evidence that suggests that HeII reionization is occurring at $z \sim 3$ holds up (see §2), then this implies either that a spectrum harder than assumed here for quasars exists prior to $z \sim 4$, or else HeII reionization must last for a an extended period, from $z \sim 3$ back to at least $z > 4$.

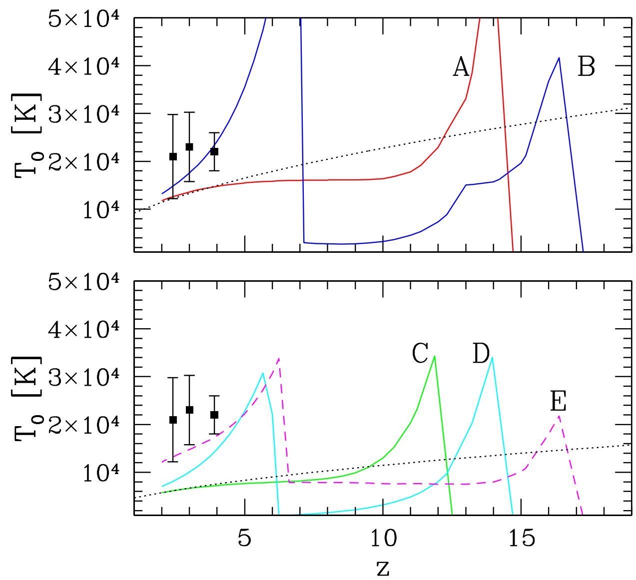

Fig. 2.— Thermal evolution for a quasar-like ionizing spectrum. *Upper panel: helium is doubly ionized together with hydrogen.* The (black) dotted line shows the thermal asymptote (eq. [4]). The (colored) solid lines show the thermal evolution for two different reionization histories. Model A has a single-episode reionization at $z = 14$, and stays highly ionized thereafter. Model B also has early reionization, but experiences a drop in ionizing flux thereafter, and starts recombining until $z \sim 7$ at which time it undergoes a second period of hydrogen and helium (I and II) reionization. Model B, but not A, reaches a high enough T_0 to match observations (points with error-bars; ZHT01). *Lower panel: helium is singly ionized together with hydrogen; the double ionization of helium is never reached (except in model E).* The (black) dotted line shows the asymptote in such a case. Models C, D (colored solid lines) are analogs of A and B above – the only difference is that here, helium remains only singly ionized. In model E (magenta dashed line), hydrogen and helium is (singly) ionized at $z \sim 17$, and then HeII is ionized at $z \sim 6.5$.

Fig. 3 shows a similar exercise for a metal free stellar spectrum (as defined in §3). We emphasize again, however, that it is unlikely that metal free stellar spectra remain a dominant contribution to the ionizing background down to low redshifts (Haiman & Holder 2003). This figure should only be viewed as an illustration of possibilities. The curves are exact analogs of those in Fig. 2. Two features are different from the previous figure. Model E, where a late period of HeII reionization occurs around $z \sim 6$, can no longer heat up the IGM sufficiently to be

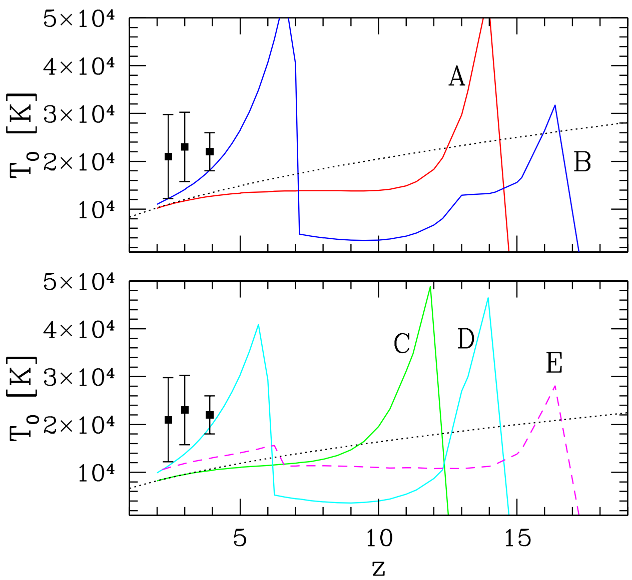

FIG. 3.— Thermal evolution using a spectrum motivated by metal-free stars. All reionization histories are exact analogs of those in Fig. 2. All symbols inherit the same meanings.

consistent with observations. This is because of the softness of a stellar spectrum above the HeII threshold. On the other hand, model D, which has a late hydrogen (and HeI, but no HeII) reionization at $z \sim 6$, has no problem matching the observed temperatures, unlike its quasar analog shown in Fig. 2. This is because the stellar spectrum we adopted is, in fact, harder than the quasar spectrum for frequencies just above $\nu_{\rm HI}$ and $\nu_{\rm HeI}$.

5. STOCHASTIC REIONIZATION HISTORY – RESULTS FROM A PHYSICALLY MOTIVATED MODEL

In the above sections we have used toy models of reionization to illuminate the key issues that determine the temperature evolution of the IGM. It is interesting to consider this evolution in a physically motivated model of the reionization history that appears to fit all the relevant observations (including the electron scattering optical depth $\tau_e = 0.17$ measured by *WMAP*). Based on assumptions about the nature and efficiency of the ionizing sources, the reionization history can be predicted from "first principles" in numerical simulations (e.g .Gnedin & Ostriker 1997; Nakamoto, Umemura, & Susa 2001; Gnedin 2001; Razoumov et al. 2002), and in semi–analytical models (e.g. Shapiro, Giroux & Babul 1994; Tegmark et al. 1994; Haiman & Loeb 1997, 1998; Valageas & Silk 1999; Wyithe & Loeb 2003; Cen 2003). Here we consider a semi–analytical model adopted from Haiman & Holder (2003), but modified to include the ionization of HeII→HeIII. For technical details, the reader is referred to that paper. Inclusion of HeIII is conceptually straightforward.

In this model, we follow the volume filling fractions $x_{\rm HII}$ and $x_{\rm HeIII}$ of HII and HeIII, assuming that discrete ionized Strömgren spheres are being driven into the IGM by ionizing sources located in dark matter halos. As can be seen from eq. (5), photoionization of HeI (to HeII) plays a sub-dominant role in heating the IGM. For simplicity, we assume that HeI is reionized at the same time as HI, so there is no need to separately keep track of the filling fraction of HeI. Note that before the Strömgren spheres percolate, the radiation background is extremely inhomogeneous: fluid elements inside ionized regions see the flux of a single (or a cluster of a few) sources, whereas fluid elements in the still neutral regions see zero flux.[4] In this picture, each fluid element is engulfed by an ionization front at a different time. In effect, each fluid element therefore has a different reionization history. Rather than considering a single temperature for a fluid element at the mean density, it is more appropriate to consider an *ensemble* of fluid elements at the mean density, each with a different reionization history and temperature evolution. Note that this stochasticity is *in addition* to the IGM having a distribution of temperatures due to density variations (see §6 for a discussion of the latter).

The evolutions of $x_{\rm HII}$ and $x_{\rm HeIII}$ in our model are shown by the solid and dashed curves in the top panel of Figure 4 (ignore the QSO curve to the left for the moment). Reionization has an interesting history that reflects contributions from three distinct types of ionizing sources, and two different feedback effects (all of which have physical motivations as described in detail in Haiman & Holder 2003). In short, ionizing sources (assumed to be massive metal–free stars) first appear inside gas that cools via H_2 lines, and collects in the earliest non–linear halos with virial temperature of $100\,{\rm K} \lesssim T \lesssim 10^4{\rm K}$. These sources ionize $\sim$ 50% of the volume in hydrogen, and they have sufficiently hard spectra (see discussion above) that they reionize $\sim 10\%$ of the helium. However, at this stage (redshift $z \sim 17$) the entire population of these first generation sources effectively shuts off due to global H_2–photodissociation by the cosmic soft UV background they had built up. Soon more massive halos, with virial temperatures of $10^4\,{\rm K} \lesssim T \lesssim 2\times10^5{\rm K}$ form, which do not rely on H_2 to cool their gas (they cool via neutral H excitations), and new ionizing sources turn on in these halos. These are assumed to be "normal" stars, since the gas had already been enriched by heavy elements from the first generation. These sources continue ionizing hydrogen, but since they produce little flux above the HeII edge, helium starts recombining. This second generation population is also self–limiting: gas infall to these relative shallow potential wells is prohibited inside regions that had already been ionized and photo–heated to 10^4K. As a result, hydrogen reionization starts slowing down around $z \sim 10$ (see the solid curve in the bottom panel of Fig. 4). However, at this stage, still larger halos with virial temperatures of $T \gtrsim 2\times10^5$K start forming. These relatively massive, third generation halos are impervious to photoionization feedback, and complete the reionization of hydrogen.

As far as the late time thermal state is concerned, the relevant ionizing spectrum is that of the sources that turn on after $z \sim 17$. These are Population II stars – a rea-

[4]These statements would no longer hold if the early ionizing sources had a hard spectrum extending to $\gtrsim$1 keV energies; an interesting possibility (e.g. Oh 2001, Venkatesan, Giroux & Shull 2001) that we do not consider in this paper.

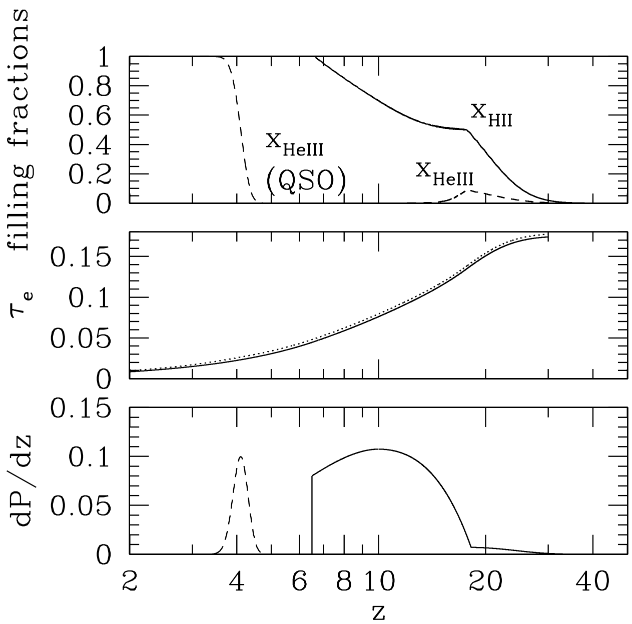

FIG. 4.— *Top panel.* The evolution of the volume filling fractions $x_{\rm HII}$ and $x_{\rm HeIII}$ of ionized hydrogen and helium in a physically motivated reionization model. The solid curve corresponds to HII, and the dashed curve to HeIII regions. The extra dashed curve to the left (QSO) corresponds to the evolution of volume filling fraction of HeIII regions due to a late period of HeII reionization, driven by the population of known quasars at $z > 3$. *Middle panel.* The electron scattering optical depth, integrated from 0 to z. Two barely distinguishable curves are shown here: excluding/including free electrons from the quasar-induced HeII ionization. *Bottom panel.* The solid curve shows the probability distribution of hydrogen reionization redshifts. The dashed curve shows the distribution of (late) HeII reionization redshifts. The amplitude of the latter distribution has been divided by a factor 20, for clarity of presentation.

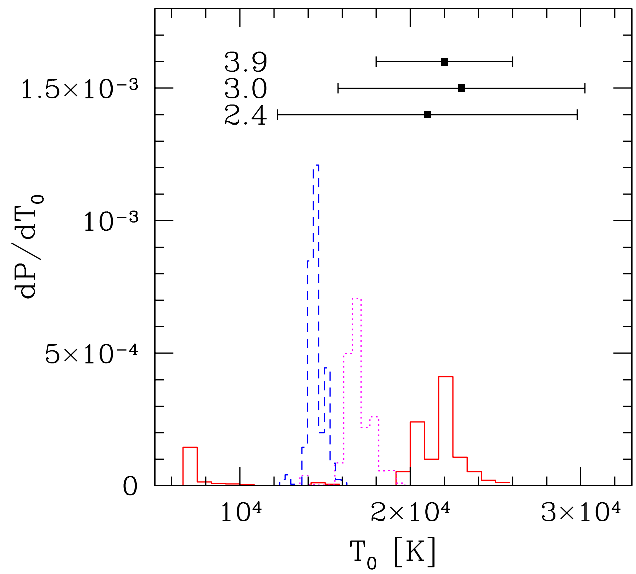

FIG. 5.— Probability distribution of temperature T_0 for fluid elements at mean density, for three different redshifts $z = 2.4$ (blue dashed histogram), 3.0 (magenta dotted histogram), and 3.9 (red solid histogram). The observed temperatures with their 2 σ errorbars are shown at the top.

sonable spectrum is $\alpha_{\rm HI} = 1$, and $\alpha_{\rm HeI} = 4$, and heavily truncated beyond $\nu_{\rm HeII}$ (Leitherer et al. 1999). As we have illustrated with examples in the last section, a spectrum as soft as this, even with some amount of late (HI, HeI but not HeII) reionization after $z \sim 10$, has difficulty heating up the IGM to high enough temperatures (even after accounting for processing of spectrum by the IGM, see §3).

We are therefore led to consider the effect of quasars. Sokasian, Abel & Hernquist (2002) showed, based on a 3D radiative transfer simulation, that HeII reionization should occur around $z \sim 4$, driven by the population of known quasars. [5] According to this work, HeII→HeIII reionization completes within a relatively short redshift range (see the left dashed curve in the top panel of Fig. 4). We approximate their probability distribution of (late) HeII reionization redshifts as a Gaussian centered at 4.1 with a full-width-at-half-maximum of 0.5 (dashed curve in the bottom panel of Fig. 4). The quasar spectrum is as described in §3.

We generate reionization histories for an ensemble of fluid elements (at mean density) in a stochastic way: determining the redshifts of HI (and HeI) reionization, and HeII reionization by drawing from the probability distributions described in the bottom panel of Fig. 4. The resulting temperatures at $z = 2.4$, 3.0, and 3.9 have a scatter because of the stochastic history, and their probability distributions are shown in Fig. 5.

The results shown in this figure are intriguing. At $z = 3.9$, the distribution (red solid histogram) peaks around $T_0 = 2.2 \times 10^4$ K, but it has a significant bump below 10^4 K as well. This bimodal distribution is a result of the stochastic reionization history: at $z = 3.9$, there is a minority of fluid elements that have not undergone HeII reionization, and so they are significantly colder, by more than a factor of 2. There have been attempts in the past to look for temperature fluctuations (beyond what one expects from variations with density) in the Lyα forest (Zaldarriaga 2002, Theuns et al. 2002b). So far there has been no detection. Our results indicate that the stochasticity of reionization can detectably increase the width of the temperature distribution. It will be very interesting to confirm our results by a more detailed analysis that folds in a realistic density distribution, and to apply the observational search techniques to larger datasets and to higher redshifts ($z \sim 3$ so far).

The formal mean and rms scatter of T_0 at $z = 3.9$ is $1.8 \times 10^4 \pm 7 \times 10^3$ K. It can be seen from Fig. 5 that

[5] Wyithe & Loeb (2002) considered HeII reionization by the known quasar population in a semi-analytical model and obtained a similar result: HeII reionization extends beyond $z \sim 4$, but is complete by around then.

the scatter gets progressively smaller as one goes to lower redshifts: at $z = 3.0$, $T_0 = 1.7 \times 10^4 \pm 8 \times 10^2$ K; at $z = 2.4$, $T_0 = 1.5 \times 10^4 \pm 5 \times 10^2$ K. The scatter at these lower redshifts are no larger than the expected scatter from shock-heating as well as dynamics (e.g. Croft et al. 1997, Hui & Gnedin 1997, Dave & Tripp 2001).

6. DISCUSSION

We find that the temperature of the IGM, as inferred from Lyα forest spectra at redshifts $z \approx 2-4$, leads to several interesting conclusions. Especially interesting are the conclusions we obtain when the Lyα forest temperature is considered together with the *WMAP* results. Our main conclusions are summarized as follows:

- *A vanilla reionization model, where $X_{\rm HI}, X_{\rm HeI}$, and/or $X_{\rm HeII}$ undergo order unity changes at $z > 10$, but suffer no such changes at $z < 10$, is ruled out by temperature measurements of ZHT01, especially at $z = 3.9$.* This relies on assumptions about the ionizing spectrum, which are discussed in §3. For a power-law spectrum, rigorous requirements can be put on the hardness of the ionizing background to evade the the above argument: $\alpha < 0.12$ if helium is doubly ionized, or $\alpha < -2.2$ if helium is singly ionized, for $J \propto \nu^{-\alpha}$ (with a cutoff at $\nu_{\rm HeII}$ if helium is only singly ionized). The idea of an asymptotic evolution for the thermal state of the IGM is quite useful in formalizing the above argument. The asymptote (at $2 \leq z \leq 4$) is described quite accurately ($\sim 5\%$) by eq. (4). It is reached as long as any order unity changes in $X_{\rm HI}, X_{\rm HeI}$, and/or $X_{\rm HeII}$ occur prior to $z = 10$.

- *Conversely, the requirement by WMAP that the universe reionizes early, at $z > 10$, implies the reionization history is complex. To fulfill the temperature constraints, there must be additional periods of order unity changes in one or more of the fractions $X_{\rm HI}, X_{\rm HeI}$, $X_{\rm HeII}$ at redshift below 10.* This is an argument separate from the argument based on the evolution of mean transmission (Haiman & Holder 2003; based on the Gunn-Peterson troughs observed by Becker et al. 2001 and Fan et al. 2003 at $z \sim 6$, and comparison against an extrapolation from lower redshifts).

- *Exactly what kind of changes at $z < 10$ are necessary to match the temperature constraint at $z = 3.9$ depends critically on the ionizing spectrum.* If the spectrum is harder than $\nu^{-1.5}$ above the HI and HeI ionization thresholds (i.e. harder than the 'quasar' spectrum discussed in §3), then order unity changes in $X_{\rm HI}$ and $X_{\rm HeI}$ in the period $6 \lesssim z \lesssim 10$ (the lower limit of 6 being set by the SDSS mean transmission measurements; Fan et al. 2002), even without accompanying changes in $X_{\rm HeII}$ (i.e. no HeII reionization), are sufficient to heat up the IGM to within the constraint at $z = 3.9$. Conversely, if the spectrum is not hard enough (for instance, a population II stellar spectrum described in §5), some amount of HeII reionization-heating is *necessary* prior to $z = 3.9$.

- *The IGM temperature can have a broad distribution (beyond what one expects based on variation with density) close to reionization events, but the scatter diminishes with time.* The reionization history of the universe is almost certainly stochastic, in the sense that different fluid elements reionize at different times, depending on when a given element becomes engulfed in expanding HII (or HeIII) regions around ionizing sources. Based on simple semi-analytic models, we predict the probability distribution of ionization redshifts. The resulting temperature scatter is particularly large close to reionization events, but diminishes quickly with time. This is illustrated in Fig. 5. It would be very interesting to search for the broad initial temperature scatter that is predicted by physically motivated reionization models.

- *The known quasar population at $z < 5$ may heat the IGM to sufficiently high temperatures by reionizing HeII.* In §5, we describe a physically motivated reionization model with a stochastic history. Hydrogen (and HeI) reionization is accomplished early on by metal-free stars ($17 \lesssim z \lesssim 30$), and later on by population II stars ($z \lesssim 17$), and completes by $z \sim 6.5$. This fulfills the dual requirements of a large CMB optical depth, and the Gunn-Peterson trough seen by SDSS. Because the population II stars have a rather soft spectrum, photo-heating of HI and HeI alone is not enough to bring the temperature up to the observed level, and this model *fails* to predict sufficiently high IGM temperatures – despite the percolation occurring at the relatively low redshift of $z \sim 7$. We therefore make use of the predictions of Sokasian et al. (2002) for a brief period of HeII reionization around $z \sim 4$, based on the *known* population of quasars. We find that such a model can satisfy the temperature constraints. It is important to note, however, that this is not the only way to achieve the observed temperatures. For instance, a population of dim quasars can turn on at $z > 6$ (Haiman & Loeb 1997), which has a sufficiently hard spectrum to either heat the IGM via photo-heating of HI and HeI alone (but their spectrum has to be harder than $\nu^{-1.5}$ if HeII is not reionized), or boost the temperature via HeII reionization as well. If HeII reionization takes place via these mini-quasars, the turn-on of the known population of quasars at $z \lesssim 4$ then has a much weaker influence on the thermal state at low redshifts.

- *HeII reionization makes little difference to the electron scattering optical depth, particularly if it occurs late.* For instance, in the model shown in Fig. 4, extra electrons from HeII reionization changes τ_e at a 2% level fractionally.

An important caveat in our discussion so far is the possible existence of additional heating mechanisms. Two such possibilities are galactic outflows and Compton heating by a hard X-ray background. Adelberger et al. (2002) recently observed signatures of galactic winds into the IGM. Such outflows can in principle heat up the IGM. However, observations by Rauch et al. (2001) of close pairs of lines of sight in lens systems suggest that the IGM is not turbulent on small-scales, arguing against significant stir-up of the IGM by winds. Moreover, galactic outflows can heat up the IGM to a variety of temperatures – the fact that the observed temperatures are in the range of expectations for a photo-ionized, and photo-heated, gas suggests photo-heating is the simplest explanation.

Another important question is whether Compton heating by a hard X-ray background (XRB) could be more important than photoelectric heating. This question was considered by Madau & Efstathiou (1999), who assumed that the redshift evolution of the sources of the hard XRB parallels the flat distribution that had been determined for the soft X–ray AGN luminosity function beyond $z \sim 2$. Under this assumption, they found the hard XRB to be an important source of heating relative to the UV background at redshifts $z > 2$, raising the IGM temperature to 1.5×10^4K at $z \sim 4$. Two new developments make it unlikely for the hard XRB to be an important source of heating. First, as pointed out by Abel & Haehnelt (1999), the photoelectric heating rate can be significantly increased in optically thick gas (when hydrogen and helium first gets ionized); we here adopt these increased rates. Second, the sources of the hard XRB have been resolved by the *Chandra* satellite, and, rather than paralleling the soft X–ray luminosity function, the hard X–ray sources exhibit a steep decline towards high redshift beyond $z \sim 2$ (Cowie et al. 2003).

Our investigation raises a number of interesting issues. Is HeII reionization prior to redshift 4 necessary to heat up the IGM to the right level? This depends critically on what kind of sources and spectra are available. The prediction for a large scatter in temperature (factor of about 2) close to the epoch of HeII reionization is something one could look for, if indeed HeII reionization occurs late (Zaldarriaga 2002, Theuns et al. 2002b). In this paper, we have focused entirely on the thermal state of fluid elements at the mean density. The formalism of Hui & Gnedin (1997) can be used to compute the same for elements at $\delta\rho/\rho \lesssim 10$. It is interesting to explore how the variation of temperature with density ($T \propto \rho^{\gamma-1}$, an effective equation of state) might place additional constraints on the reionization history. At present, measurements of γ are quite noisy (e.g. ZHT01). New approaches to constrain it better will therefore be very useful (Dijkstra, Lidz & Hui 2003).

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