



Dark Energy as a Modification of the Friedmann Equation

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ABSTRACT

Dark energy could actually be the manifestation of a modification to the Friedmann equation arising from new physics (e.g., extra dimensions). Writing the correction as $(1 - \Omega_M)H^\alpha/H_0^{\alpha-2}$, we explore the phenomenology and detectability of such. We show that: (i) α must be $\lesssim 1$; (ii) such a correction behaves like dark energy with equation-of-state $w_{\text{eff}} = -1 + \frac{\alpha}{2}$ in the recent past ($10^4 > z \gg 1$) and $w = -1$ in the distant future and can mimic $w < -1$ without violating the weak-energy condition; (iii) w_{eff} changes, $dz/dw|_{z \sim 0.5} \sim \mathcal{O}(0.2)$, which is likely detectable; and (iv) a future supernova experiment like SNAP that can determine w with precision σ_w , could determine α to precision $\sigma_\alpha \approx 2\sigma_w$.

1 Introduction

The discovery that the expansion of the Universe is speeding up and not slowing down [1, 2] has presented cosmologists and particle physicists with a profound (and wonderful) puzzle. In the context of general relativity this surprising result can be accounted for by the existence of a smooth component of energy with large negative pressure ($w \equiv p/\rho \lesssim -1/2$), dubbed dark energy, which accounts for about 2/3 of the critical density [3].

A number of suggestions for the dark energy have been discussed including quantum vacuum energy (cosmological constant), a very light and slowly evolving scalar field, and a frustrated network of topological defects. None is compelling and all have serious conceptual problems [3].

Another logical possibility is that the phenomenon of accelerated expansion is actually a sign of a breakdown of the standard Friedmann equation which governs the expansion rate, which is the idea we explore here.

The high degree of isotropy and large-scale homogeneity observed in the Universe implies that the metric of our 4-d spacetime can be written in the Robertson – Walker form with a single function – the cosmic scale factor $R(t)$ – describing the large-scale dynamics of the Universe. The issue then is the equation(s) that govern the evolution of the cosmic scale factor.

It should be noted that the kinematics of the expansion – acceleration or deceleration – can be discussed without regard to dynamics, and further, that the current type Ia supernova (SNIa) data indicate a recent period of acceleration ($\ddot{R}/HR^2 > 0$ for $z \lesssim 0.5$) preceded by an earlier period of deceleration ($\ddot{R}/H^2R < 0$ for $z \gtrsim 0.5$) [4]. Thus, simply allowing for modified dynamics cannot eliminate the puzzling phenomenon of accelerated expansion.

In this paper, we investigate the addition to the Friedmann equation of a term, $(1 - \Omega_M)H^\alpha/H_0^{\alpha-2}$, which can arise with theories with extra dimensions [5]. We know that at early times the Friedmann equation is a good approximation and this fact constrains α to be $\lesssim 1$. We further show that such a modification has an equivalent description as dark energy with time varying equation-of-state $w_{\text{eff}}(z)$. Finally, we show that future supernova measurements envisioned with SNAP [6] can constrain α to a precision of about half that of w , i.e., $\sigma_\alpha \approx 2\sigma_w$, or about $\sigma_\alpha \sim 0.1$.

In the next Section we discuss some theoretical motivations for a modification to the Friedmann equation, and in the following Section we discuss the cosmological phenomenology of “ α dark energy.” We end with a brief summary.

2 Motivations

Both the hierarchy [7, 8] and cosmological constant [10] problems motivate theories with large extra dimensions. Extra dimensions that are either compact or have finite volume manifest themselves exclusively at high energies, above the compactification scale. Such theories modify laws of gravity only at short distances, below the size of the extra dimensions.

Consequently, all long-distance physics, including the late-time cosmological evolution of the Universe, is very close to the standard picture [9].

In contrast, theories with *infinite volume* extra dimensions [5, 10] modify the laws of gravity in the far infrared, and at short distances (or early times) the gravitational dynamics is very close to that of the 4-dimensional Einstein gravity. Consequently the cosmological evolution is very close to the standard FRW picture at early times, but gets modified at late times. This modification can account for the observed accelerated expansion of the Universe without dark energy; that is, the modified gravitational dynamics leads to a “self-accelerated” Universe [11, 12].

For simplicity, we describe a model with a single extra dimension. The effective, low-energy action takes the following form [5]

$$S = \frac{M_{\text{Pl}}^2}{r_c} \int d^4x dy \sqrt{g^{(5)}} \mathcal{R} + \int d^4x \sqrt{g} \left(M_{\text{Pl}}^2 R + \mathcal{L}_{\text{SM}} \right). \quad (1)$$

where $M_{\text{Pl}}^2 = 1/8\pi G$, $g_{AB}^{(5)}$ is 5-dimensional metric ($A, B = 0, 1, 2, \dots, 4$), and y is the extra spatial coordinate. The first term in Eq. (1) is the bulk 5-dimensional Einstein action, and the second term is the 4-dimensional Einstein localized on the brane (at $y = 0$). For simplicity we do not consider brane fluctuations. The induced metric on the brane is given by

$$g_{\mu\nu}(x) \equiv g_{\mu\nu}^{(5)}(x, y = 0). \quad (2)$$

The quantity r_c is the crossover scale, the single new parameter. It sets the scale beyond which the laws of 4-d gravity breakdown and become 5-dimensional.

The existence of late-time, self-accelerated solutions can be seen from Einstein’s equations, obtained by the variation of Eq. (1) with an arbitrary 4-d matter source $T_{\mu\nu}$:

$$\frac{1}{r_c} \mathcal{G}_{AB} + \delta(y) \delta_A^\mu \delta_B^\nu (G_{\mu\nu} - 8\pi G T_{\mu\nu}) = 0. \quad (3)$$

Here \mathcal{G}_{AB} and $G_{\mu\nu}$ are the 5-d and 4-d Einstein tensors respectively. Without the first term, Eq. (3) would be the standard Einstein equations for the induced 4-d metric $g_{\mu\nu}(x)$. If we exclude the matter source ($T_{\mu\nu} = 0$), then the only maximally-symmetric solutions to Eq. (3) are flat. However, the presence of the first term modifies this equation, though the modification becomes significant only for very small values of the 4-d curvature. Consider the maximally symmetric FRW ansatz,

$$ds_5^2 = f(y, H) ds_4^2 - dy^2, \quad (4)$$

where ds_4^2 is the 4-dimensional maximally-symmetric metric, and H is the 4-d Hubble parameter, the second term in Eq. (3) takes the standard form $G_{\mu\nu} \propto g_{\mu\nu} H^2$.

In the first term the y -derivatives of the warp-factor $f(y, H)$ must take care of the $\delta(y)$ -function, which leads to an effective 4-d equation for H . From simple dimensional arguments

it follows that the first term must scale as $\sqrt{H^2}$. A calculation leads to Friedmann equation for arbitrary 4-d brane-localized matter source $\rho_M(t)$ [11, 12]:

$$H^2 \pm \frac{H}{r_c} = \frac{8\pi G \rho_M}{3} \quad (5)$$

These general features persist for the arbitrary number of dimensions.

The basic point is that the higher-dimensional action is suppressed relative to 4-dimensional one by an inverse power of the crossover scale. As a result, the scaling arguments suggest that for the maximally symmetric ansatz the higher-dimensional contribution should scale as a lower power of H^2 , and thus be important at late times (small H). This argument suggests that infinite-volume extra-dimensional theories have the potential to explain cosmic acceleration with dark energy.

Before proceeding, let us make a few important points. Naively, it appears that as far as dark energy is concerned the self-accelerated solutions of higher-dimensional theory have the same number of new parameters as the simplest model of dark energy, a cosmological constant. The cosmological constant is replaced by the crossover scale r_c . However, from the quantum field theory point of view there is a crucial difference, the crossover scale r_c is stable under quantum corrections.

There is an interesting coincidence, which also motivates the Hubble-scale value of r_c and is unrelated to dark energy. The scale r_c also sets the distance at which corrections (coming from higher-dimensional gravity) to the usual metric for a gravitating source become important. In particular, there are corrections to the Schwarzschild metric [13], which effect planetary motions. The existing phenomenological bounds on such deviations demand that the crossover scale be large. For instance, for Eq. (1) the most stringent bound comes from lunar laser ranging experiments that monitor the moon's perihelion precession with a great accuracy and imply a lower bound for r_c which is close to the present cosmological horizon H_0^{-1} [14].

In the present paper we shall take a more radical and generic attitude. We shall use the notion of infinite extra dimensions to motivate the modification of Friedmann equation at late times. In that spirit, we shall assume that the physics that modifies Friedmann equation satisfies the following simple requirements:

- There is a single crossover scale r_c
- To leading order, the corrections to Friedmann equation can be parameterized a single term, H^α

These two assumptions fix the form of the modified Friedmann equation:

$$H^2 - \frac{H^\alpha}{r_c^{2-\alpha}} = \frac{8\pi G \rho_M}{3} \quad (6)$$

In order to eliminate the need for dark energy, this term must specifically be: $(1 - \Omega_M)H^\alpha / H_0^{\alpha-2}$, which implies that $r_c = (1 - \Omega_M)^{\frac{1}{\alpha-2}} H_0^{-1}$.

In the examples discussed above this stability of $r_c \sim H_0^{-1}$ is manifest (in contrast to the choice of a small vacuum energy). It should remain true in our generalized analysis: Since corrections to standard Einstein gravity are non-analytic in H^2 , they cannot be generated perturbatively in the limit $1/r_c^{2-\alpha} \rightarrow 0$. Thus, the value of $1/r_c$ can only be renormalized through *itself* and cannot be UV sensitive. In other words, $1/r_c \sim H_0^{-1}$ plays the role of an infrared cut-off for standard gravity, below which it is replaced by a modified theory.

Finally, we would like to stress that we do not attempt to solve the cosmological constant problem, i.e., the smallness of vacuum energy. We simply postulate that it is zero due for whatever reason, and study how the observed accelerated expansion could result from a modification of the Friedmann equation in far infrared. In this respect our approach is different (and complementary) to the idea of solving cosmological constant problem by long-distance modification of gravity [10, 15, 16]. In Refs. [15, 16] such modification ensures that the vacuum energy (no matter how large) gravitates extremely weakly, so that its over-all effect is reduced to the one of a small cosmological constant *at all times*. Thus, the resulting cosmology is indistinguishable from the standard FRW picture with a small Λ -term, as opposed to the present case.

3 Phenomenology of α Dark Energy

Suppose that the effects of extra dimensions manifest themselves as a modification to the Friedmann equation of the form

$$H^2 - (1 - \Omega_M) \frac{H^\alpha}{H_0^{\alpha-2}} = \frac{8\pi G\rho}{3}, \quad (7)$$

where a flat universe has been assumed, ρ is the total energy density (today dominated by matter with $\Omega_M \approx 0.33$) and H_0 is the present value of the Hubble constant ($\approx 72 \pm 7 \text{ km sec}^{-1} \text{ Mpc}^{-1}$). The coefficient of the new term is fixed by requiring that it eliminates the need for dark energy.

In the distant future, cosmic scale factor $R \gg 1$,

$$H \longrightarrow (1 - \Omega_M)^{\frac{1}{2-\alpha}} H_0,$$

corresponding to asymptotic deSitter expansion. For $\alpha = 0$ the new term always behaves just like a cosmological constant, while for $\alpha = 2$, the additional term corresponds to a “renormalization” of the Friedmann equation.

Equation (7) can be recast in a more suggestive form

$$\left(\frac{H}{H_0}\right)^2 = (1 - \Omega_M) \left(\frac{H}{H_0}\right)^\alpha + \Omega_M(1+z)^3. \quad (8)$$

Provided $\alpha < 2$ (see below), the matter term will dominate the r.h.s. for $10^4 \gtrsim z \gg 1$, in which case $H \propto (1+z)^{3/2}$. This implies that during the matter-dominated era the α term

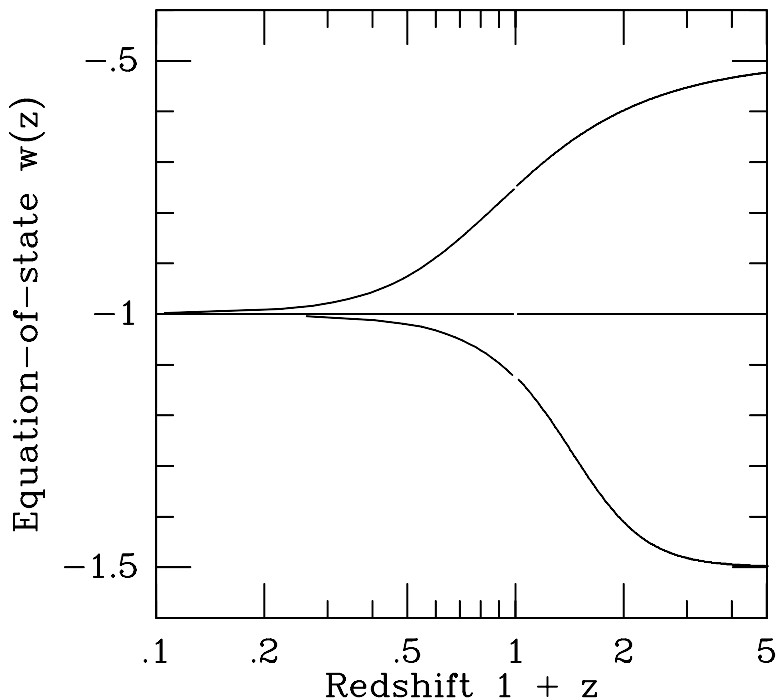


Figure 1: Effective equation-of-state, $w_{\text{eff}} \equiv -1 - \frac{1}{3} \frac{d \ln H^\alpha}{d \ln R}$, vs. $1+z$ for $\alpha = -1, 0, 1$ (top to bottom). Note, $1+z < 1$ corresponds to the future, i.e., scale factor $R = 1/(1+z) > 1$, where $R = 1$ today.

varies as $(1+z)^{3\alpha/2}$, which corresponds to an effective equation-of-state

$$w_{\text{eff}} = -1 + \frac{\alpha}{2} \quad \text{for } 10^4 \gtrsim z \gg 1.$$

[Recall, for constant equation-of-state $w \equiv p/\rho$, the energy density varies as $(1+z)^{3(1+w)}$.] During the earlier radiation-dominated epoch ($z \gg 10^4$), where $\rho \propto (1+z)^4$ and $H \propto (1+z)^2$, α dark energy has an effective equation of state $w_{\text{eff}} = -1 + \frac{2\alpha}{3}$ for $z \gg 10^4$.

To summarize, the effective equation-of-state of α dark energy varies from $-1 + \frac{2\alpha}{3}$ during the radiation-dominated era to $-1 + \frac{\alpha}{2}$ during the matter-dominated era to -1 in the distant future (see Fig. 1). The change in the effective equation-of-state is largest around $z \sim 0-1$, with $dw/dz \sim 0.2$. This happens to be where SNe measurements of the expansion rate are most sensitive to a variation in w and a variation of this size may well be detectable [17]. It may be of some interest that for $\alpha < 0$, the effective equation-of-state can be more negative than $w = -1$, without violating the weak-energy condition.

To eliminate the need for dark energy, today the “ α -term” must be about twice the matter term; however, to avoid interfering with the successful predictions of the standard Friedmann equation, the α term must have been smaller in the past. In particular, the successful predictions of big-bang nucleosynthesis set a limit to any new forms of energy density in the Universe at $z \sim 10^{10}$ of less than a few percent of the standard value (usually

stated as limit to the number of neutrino species) [18]. This in turn imposes an upper bound to α :

$$\alpha \lesssim 1.95.$$

A more stringent bound follows from requiring that the α term not interfere with the formation of large-scale structure: To achieve the structure seen today from the primeval fluctuations whose imprint was left on the cosmic microwave background requires a long matter-dominated epoch [19]. In terms of w this bound is: $w \lesssim -\frac{1}{2}$. Requiring $w_{\text{eff}} \lesssim -\frac{1}{2}$ during the matter dominated era leads to the bound:

$$\alpha \lesssim 1.$$

We remind the reader that these bounds apply only to modifications of the Friedmann equation that aspire to explain dark energy, i.e., where the corrections are significant today. In finite-volume, extra-dimensional models it is expected that α is larger than 2 so that the corrections are most important at early times. For example, in many brane-world scenarios the modifications effectively correspond to $\alpha = 4$ [20].

An expansion history (i.e., $H(z)$ vs. z) may be obtained by picking a value for H/H_0 and solving for z . From this, the comoving distance as function of redshift z , $r(z) = \int_0^z dz/H(z)$, can be computed. Distance vs. redshift diagrams [$r(z)$ vs. z] are shown in Fig. 2 (and in more detail in Fig. 3) for $\alpha = 1, 0, -1, -2, -3$. Also shown are distance vs. redshift diagrams for constant- w dark-energy models. From redshift $z = 0$ to $z = 2$ α dark energy can be described approximately by a constant equation-of-state model with

$$w_{\text{eff}} \approx -1 + 0.3\alpha.$$

An α dark energy model is completely described by two parameters – α and Ω_M – and so it is straightforward to address precisely the power of supernovae observations to test it. To this end, we have constructed the Fisher matrix for a set of 2500 supernovae observations spanning $z = 0.2 - 1.7$ (like those that might be made by SNAP [6]):

$$F_{ij} = \sum_k \frac{1}{\sigma_k^2} w_i(z_k) w_j(z_k),$$

$$w_i(z) \equiv \frac{\partial m(z)}{\partial p_i}, \quad (9)$$

where $m(z) \propto 5 \log[r(z)]$ is the expected apparent magnitude of a SNIa at redshift z , σ_k is the expected measurement accuracy (here assumed to be 0.15 mag), and the parameters $p_i = \alpha$ and Ω_M . From the Fisher matrix we “forecast” the 1σ error ellipses in the usual way.

Figure 4 compares the error ellipse in the α - Ω_M plane with that in the w - Ω_M plane for constant- w dark-energy models with $w = -1.3, -1, -0.7$. What is most relevant is the relative sizes of the error ellipses since the exact size will depend upon details of the supernovae survey. While the $\alpha = 1$ model ellipse aligns approximately with $w = -0.7$

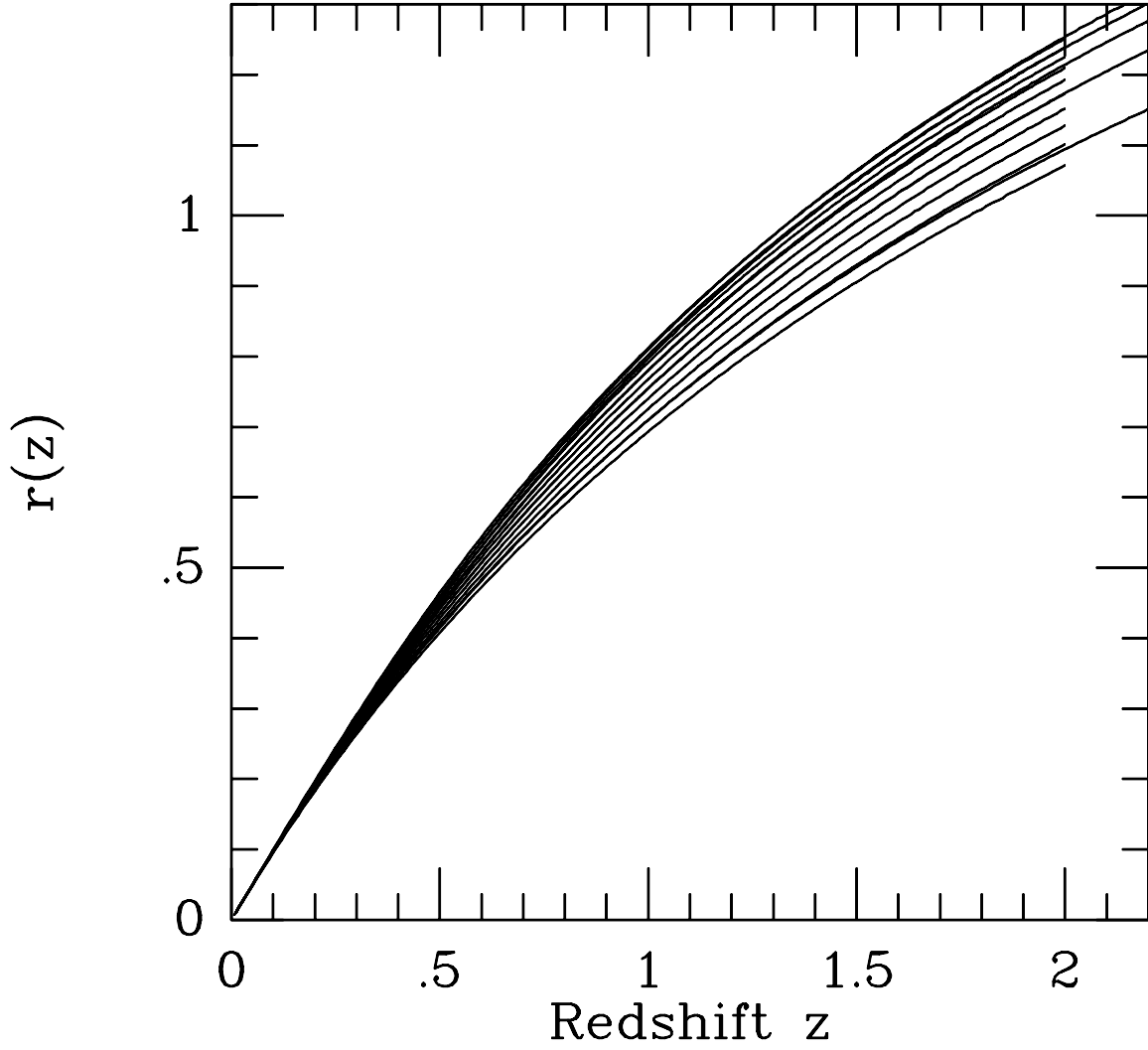


Figure 2: Distance vs. redshift ($r(z)$ vs. z) for $\alpha = 1, 0, -1, -2, -3$ (full curves, from bottom to top) and for $w = -0.6, -0.7, -0.8, -0.9, -1.0, -1.1, -1.2, -1.3, -1.4, -1.5$ (short curves, from bottom to top).

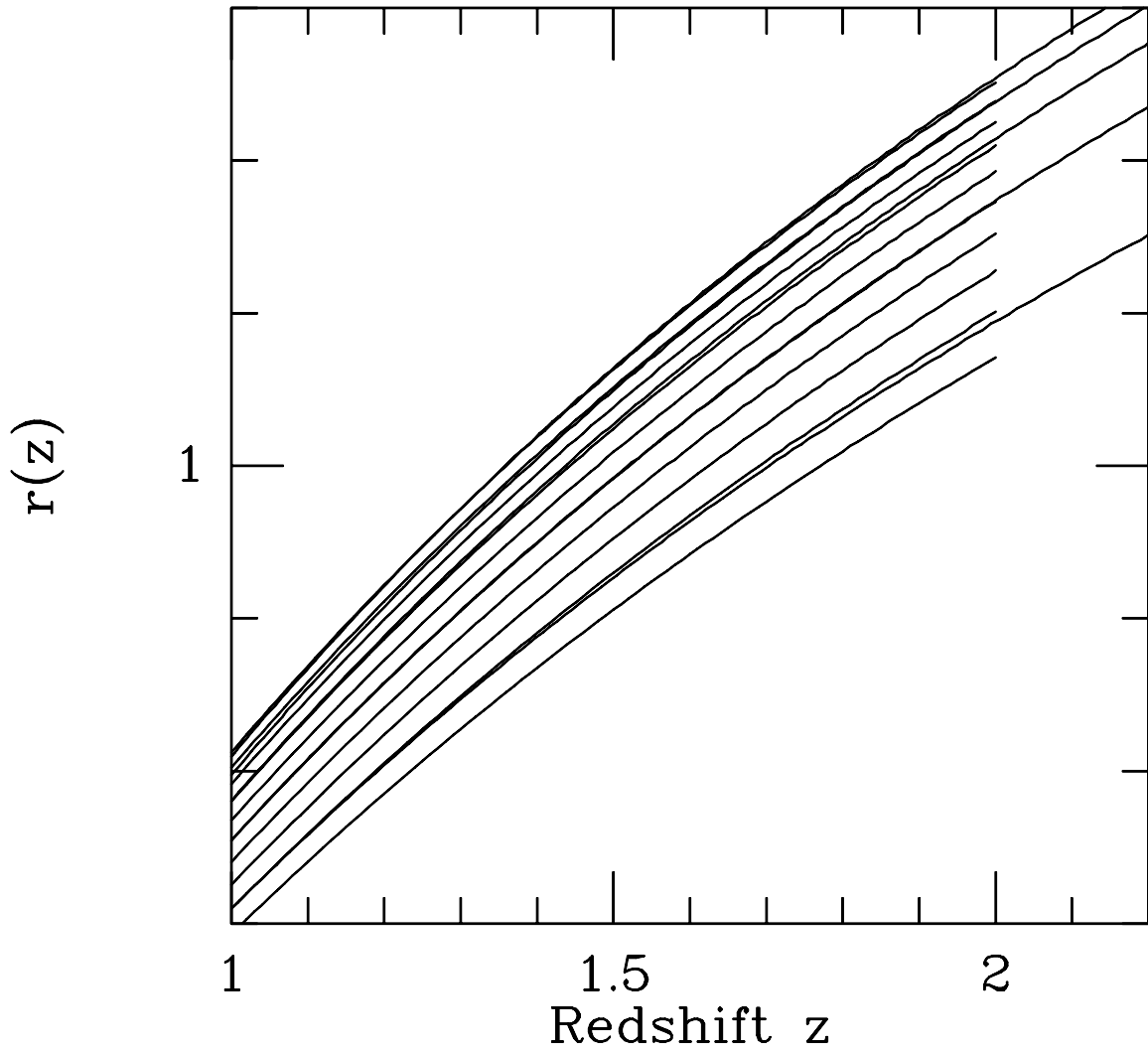


Figure 3: Blow up of Fig. 2.

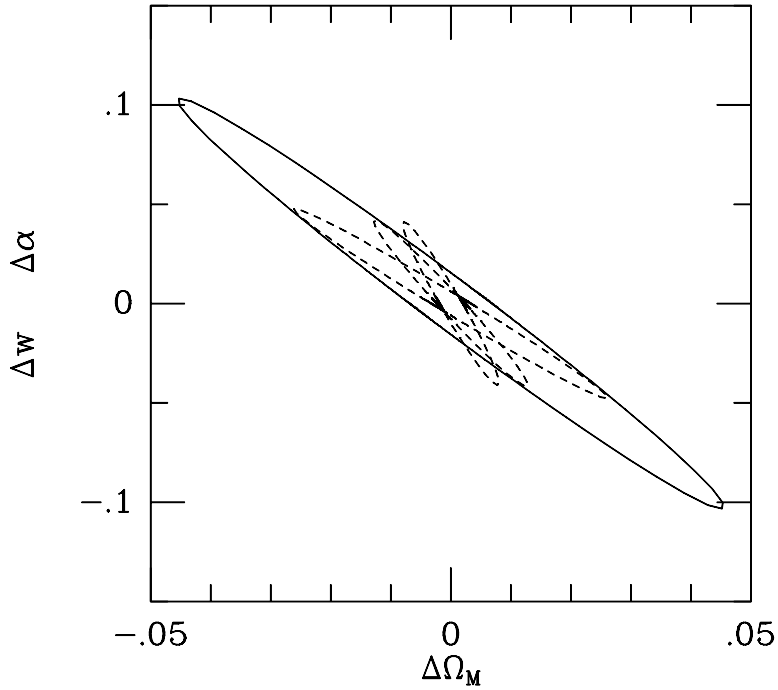


Figure 4: Predicted error ellipses in the $\alpha - \Omega_M$ and $w - \Omega_M$ planes for $w = -1.3, -1.0, -0.7$ (moving counterclockwise) and for $\alpha = 1$ for a SNAP-like supernova experiment, assuming $\Omega_M = 0.33$.

as expected, it is only about twice as large (not quite as large as expected from the rough relationship between w and α). Assuming that SNAP can determine w to around ± 0.05 , the same observations could constrain α to around ± 0.1 , sufficiently well to distinguish integer values of α .

4 Concluding Remarks

We have argued that dark energy may actually be a modification to the standard Friedmann equation arising from new physics (e.g., infinite-volume extra dimensions). Writing the modification as $(1 - \Omega_M)H^\alpha/H_0^{\alpha-2}$, the cosmological phenomenology can be summarized by:

- The effective equation-of-state of α dark energy evolves from $-1 + \frac{2\alpha}{3}$ ($z \gg 10^4$) to $-1 + \frac{\alpha}{2}$ ($10^4 \gtrsim z \gg 1$) and asymptotically to -1 in the distant future ($R \rightarrow \infty$ or $z \rightarrow -1$).
- The maximal change in w_{eff} occurs recently, $z \sim 0 - 1$, with $(dw/dz)_{z \sim 0.5} \approx 0.2$. This is where measurements by a future experiment like SNAP are most sensitive and could probably detect a variation of this magnitude.

- For $\alpha < 0$, the effective equation-of-state w_{eff} can be less than -1 without violating the weak-energy condition.
- Around $z \sim 0-2$, an α dark-energy model can be described approximately by constant w with $w \approx -1 + 0.3\alpha$.
- If SNAP can measure w to a precision $\sigma_w \approx 0.05$, it could determine α to about twice that, $\sigma_\alpha \approx 2\sigma_w \approx 0.1$.

If the phenomenon of dark energy is associated with corrections to the Friedmann equations from new physics and involves corrections like H^α , it seems likely that future supernova measurements could test this hypothesis and determine α to a precision of order 10%. Thus, α dark energy is not only intriguing, but it is also testable.

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