

**LONG BASELINE NEUTRINO OSCILLATIONS:
PARAMETER DEGENERACIES
AND
JHF/NUMI COMPLEMENTARITY***

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A summary of the parameter degeneracy issue for long baseline neutrino oscillations is presented and how a sequence of measurements can be used to resolve all degeneracies. Next, a comparison of the JHF and NuMI Off-Axis proposals is made with emphasis on how both experiments running neutrinos can distinguish between the normal and inverted hierarchies provided the E/L of NuMI is less than or equal to the E/L of JHF. Due to the space limitations of this proceedings only an executive style summary can be presented here, but the references and transparencies of the talk contain the detailed arguments.

1. Parameter Degeneracies: Overview

The probability of $\nu_\mu \rightarrow \nu_e$ depends on θ_{13} , δ_{CP} , θ_{23} and sign of δm_{31}^2 . Untangling the degeneracies associated with these parameters is the subject of this section, see Ref. [1]-[4].

*<http://www-sk.icrr.u-tokyo.ac.jp/noon2003/transparencies/11/parke.pdf>

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1.1. θ_{13} and δ_{CP} Degeneracy

If the probabilities $P(\nu_\mu \rightarrow \nu_e)$ and $P(\bar{\nu}_\mu \rightarrow \bar{\nu}_e)$ are precisely determined by long baseline experiments then in general there are four solutions of parameters $(\theta_{13}, \delta_{CP})$ for a fixed value^a of θ_{23} . This is shown on the right bi-probability diagram (see Ref. [5]) of Fig. 1 where the four ellipses intersect at a single point. Two of these ellipse are assuming normal hierarchy and the other two are the inverted hierarchy. Note, that the values of $\sin^2 2\theta_{13}$ varies significantly between the ellipses of the same hierarchy.

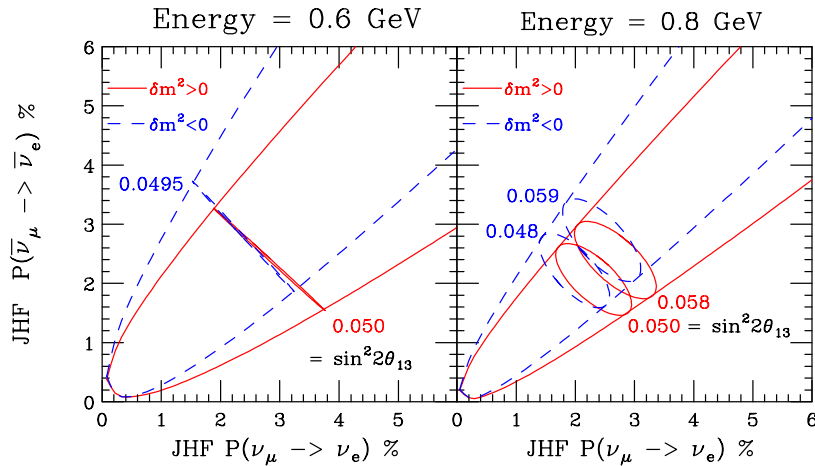


Figure 1. The allowed region in the bi-probability plot at oscillation maximum (left panel) and at 30% above oscillation maximum (right panel) for $\nu_\mu \rightarrow \nu_e$ versus $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$ for JHF at 295 km. The ellipses are for fixed $\sin^2 2\theta_{13}$ but allowing δ_{CP} to vary from 0 to 2π . Except where noted the other mixing parameters are fixed to be $|\Delta m_{13}^2| = 2.5 \times 10^{-3} \text{ eV}^2$, $\sin^2 2\theta_{23} = 1.0$, $\Delta m_{12}^2 = +7 \times 10^{-5} \text{ eV}^2$ and $\sin^2 2\theta_{12} = 0.85$. The electron density for JHF is fixed to be $Y_e \rho = 1.15 \text{ g cm}^{-3}$ (for NuMI we will use 1.4 g cm^{-3}).

The left hand panel of Fig. 1 shows that these four ellipses collapse to a line at the energy that corresponds to oscillation maximum and that the value of $\sin^2 2\theta_{13}$ can be determined precisely at this special energy, see Ref. [3]. For a given hierarchy, the complicated $(\theta_{13}, \delta_{CP})$ degeneracies factorizes into a fixed value for θ_{13} and a $(\delta_{CP}, \pi - \delta_{CP})$ degeneracy. In general, the hierarchy degeneracy still exists unless nature has chosen one of the edges of the allowed region in bi-probability space represented in Fig. 1.

^aThe ambiguity in θ_{23} will be discussed in the next section.

In Fig. 2 we have plotted the fractional difference in the allowed values of θ_{13} for the same hierarchy in the left panel and different hierarchies in the right panel. This fractional uncertainty in the allowed value grows as θ_{13} gets smaller but for values of θ_{13} not too far below the current Chooz bound of $\sin^2 2\theta_{13} < 0.1$ the fractional uncertainty is less than say 20%.

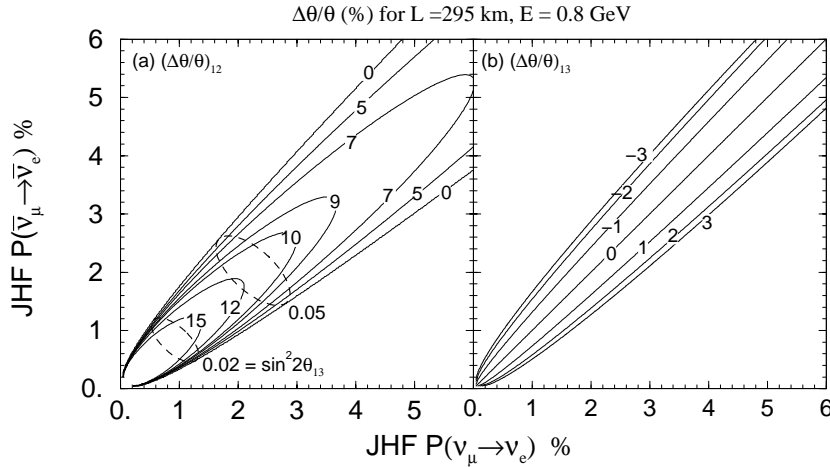


Figure 2. The fractional difference in the values of θ_{13} for the same hierarchy (left panel) and different hierarchy (right panel).

1.2. θ_{23} Degeneracy

In general, θ_{23} is determined from $\nu_\mu \rightarrow \nu_\mu$ disappearance experiments. Unfortunately in the disappearance probability θ_{23} appears as $\sin^2 2\theta_{23}$. If $\sin^2 2\theta_{23}$ differs from 1 by ϵ^2 then the two solutions for $\sin^2 \theta_{23}$ are $(1 \mp \epsilon)/2$. Since the appearance probability for $\nu_\mu \rightarrow \nu_e$ depends on $\sin^2 \theta_{23}$ this ambiguity leads an ambiguity in the determination of θ_{13} . However, the quantity $(\sin \theta_{23} \sin \theta_{13})$ can be determined accurately from the appearance experiments. See Fig. 3.

In Fig. 3 we have assumed that $\sin^2 2\theta_{23} = 0.96 = 1 - (0.2)^2$ and drawn the bi-probability ellipses for $(2 \sin^2 \theta_{23} \sin^2 2\theta_{13}) = 0.02, 0.05$ and 0.09 . In the right panel the energy is chosen at 30% above oscillation maximum and the four ellipses (two different θ_{23} times the two hierarchies) are approximately, but not exactly, degenerate. The left panel is at oscillation maximum and the degeneracy of the four ellipse is nearly exact.

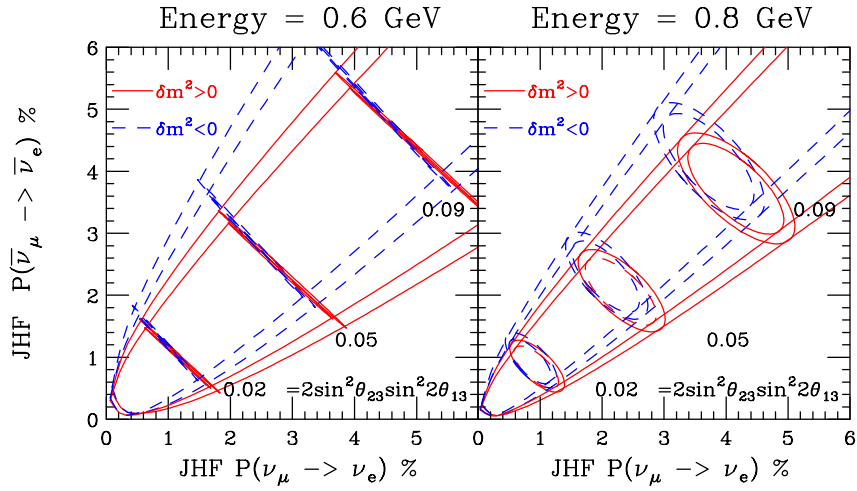


Figure 3. The bi-probability plots at oscillation maximum (left) and at an energy 30% above oscillation maximum (right) assuming that θ_{23} differs from $\pi/4$ for constant values of $(\sin \theta_{23} \sin \theta_{13})$. Here, $\sin^2 2\theta_{23} = 0.96$ giving two solutions for $\sin^2 \theta_{23} = 0.4$ and 0.6 .

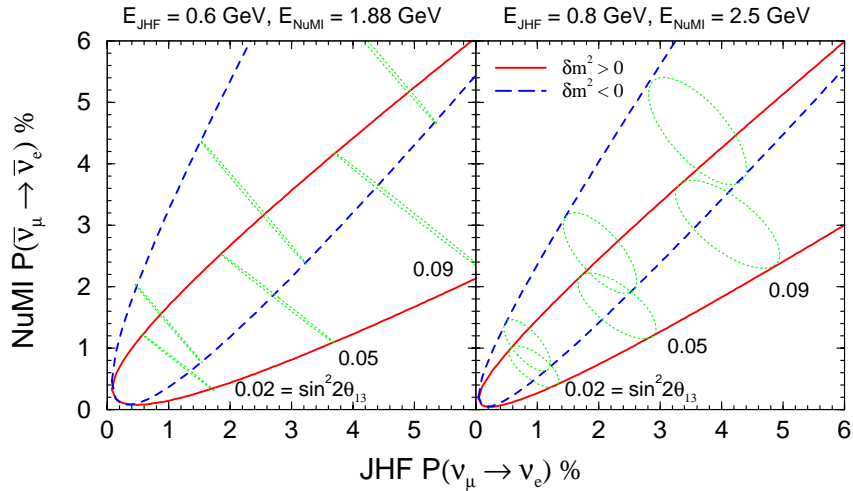


Figure 4. Allowed region in the bi-probability plane for JHF $\nu_\mu \rightarrow \nu_e$ versus NuMI $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$ at 915 km with represented ellipses for fixed values of $\sin^2 2\theta_{13}$. The left panel is both experiments at oscillation maximum while the right panel is both experiments 30% above oscillation maximum.

Thus the ambiguity in the determination of θ_{23} leads to an ambiguity in the determination of θ_{13} in long baseline oscillation experiments. However, the quantity $(\sin\theta_{23}\sin\theta_{13})$ can be precisely determined especially at oscillation maximum. To break the degeneracy in θ_{23} ; $(\sin\theta_{23}\sin\theta_{13})$ and $(\cos\theta_{23}\sin\delta_{CP})$ can be measured at oscillation maximum then $(\cos\theta_{23}\cos\delta_{CP})$ can be determined above oscillation maximum. The combination of these measurements leads to a determination of $\cos\theta_{23}$, breaking this θ_{23} degeneracy.

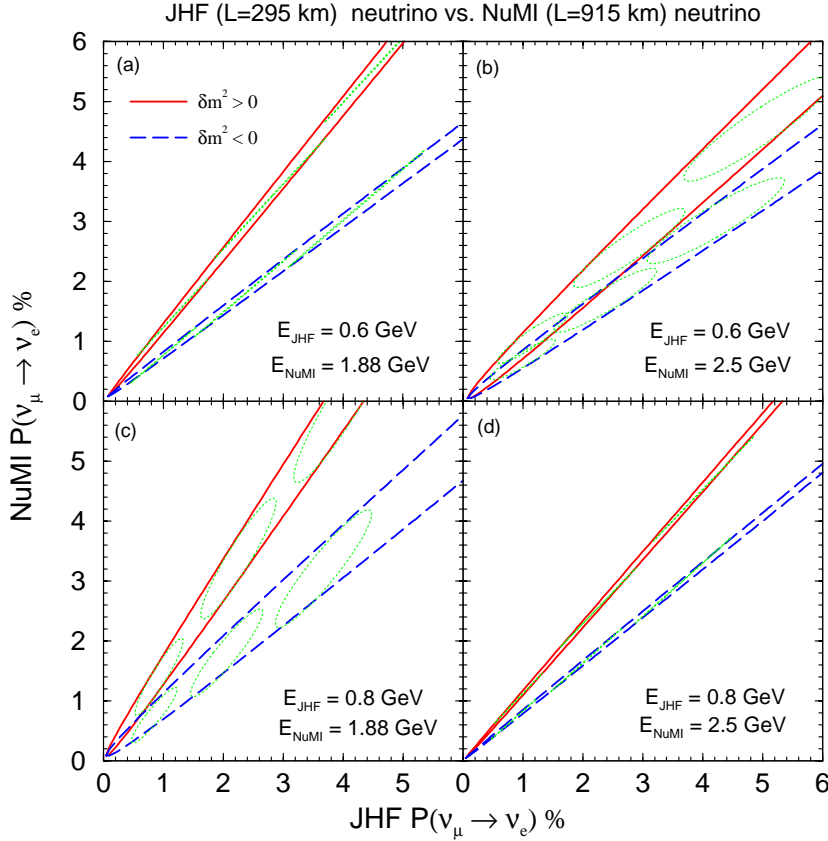


Figure 5. Allowed region in the bi-probability plane for JHF $\nu_\mu \rightarrow \nu_e$ versus NuMI $\nu_\mu \rightarrow \nu_e$ with represented ellipses for fixed values of $\sin^2 2\theta_{13}$. The top (left) two panel are JHF (NuMI) at oscillation maximum while the bottom (right) panel are JHF (NuMI) at 30% above oscillation maximum. The ellipse are for $\sin^2 2\theta_{13} = 0.02, 0.05$ and 0.09 .

2. JHF/NuMI Complementarity

Detailed discussions can be found in references [6], [7] and [8].

2.1. *Neutrino-Antineutrino*

Fig. 4 represents the allowed region in bi-probability space for one experiment neutrinos and the other anti-neutrinos. Note the similarity between this plot and the plots where both the neutrino and anti-neutrino probability are from the same experiment (Fig. 1 for example).

2.2. *Neutrino-Neutrino*

Fig. 5 represents the allowed region in bi-probability space for both experiments neutrinos. The allowed regions are narrow “pencils” which grow in width as the energy of either or both experiments differ from oscillation maximum. The ratio of slopes of these pencils increases (decreases) as the energy of the experiment with smaller (larger) matter effect increases. For JHF/NuMI this means that the best separation occurs when

$$\frac{E}{L}|_{NuMI} \leq \frac{E}{L}|_{JHF}. \quad (1)$$

The top right panel of Fig. 5 violates this condition and there is substantial overlap between the two allowed regions, see Ref. [8].

3. Conclusions

The eight fold parameter degeneracy in $\nu_\mu \rightarrow \nu_e$ can be resolved with multiple measurements in the neutrino and anti-neutrino channel. A neutrino as well as an anti-neutrino measurement at oscillation maximum plus a neutrino measurement above oscillation maximum is sufficient if chosen carefully. Exploitation of the difference in the matter effect between JHF and NuMI can be used to determine the mass hierarchy provided that the E/L of NuMI is smaller than or equal to the E/L of JHF.

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