Fermilab FERMILAB-Pub-02/109-T June 2002

FERMILAB-Pub-02/109-T
hep-ph/0206058

Remark on the Theoretical Uncertainty in B^0-$\bar{B}^0$ Mixing

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June 11, 2002

Abstract

We re-examine the theoretical uncertainty in the Standard Model expression for B^0-$\bar{B}^0$ mixing. We focus on lattice calculations of the ratio ξ, needed to relate the oscillation frequency of B^0_s-$\bar{B}^0_s$ mixing to the poorly known CKM element V_{td}. We replace the usual linear chiral extrapolation with one that includes the logarithm that appears in chiral perturbation theory. We find a significant shift in the ratio ξ, from the conventional 1.15 ± 0.05 to $\xi = 1.32 \pm 0.10$.

It is anticipated that the oscillation frequency of B^0_s-$\bar{B}^0_s$ mixing will be measured during Run 2 of the Tevatron [1]. It is thus timely to assess the measurement's impact on tests of the Cabibbo-Kobayashi-Maskawa (CKM) picture of flavor and CP violation. The CKM interpretation is limited by the poorly known hadronic matrix elements for $B^0_s \leftrightarrow \bar{B}^0_s$ and $B^0_d \leftrightarrow \bar{B}^0_d$ transitions. In this paper we re-examine lattice calculations of these matrix elements, focusing on the chiral extrapolation. We find that the range usually quoted is probably incorrect.

In the Standard Model, the theoretical expression for the oscillation frequency is

$$\Delta m_q = \left(\frac{G_F^2 m_W^2 S_0}{16\pi^2 m_{B_q}} \right) |V^*_{tq} V_{tb}|^2 \eta_B \mathcal{M}_q, \tag{1}$$

where $q \in \{d,\, s\}$, S_0 is an Inami-Lim function, η_B is a short-distance QCD correction, and $\mathcal{M}_q$ is the hadronic matrix element for $B^0_q \leftrightarrow \bar{B}^0_q$ transitions. In Eq. (1), the parentheses consists of accurately known quantities, and $|V^*_{tq} V_{tb}|$ is the CKM factor. The hadronic matrix element

$$\mathcal{M}_q = \langle \bar{B}^0_q | [\bar{b}\gamma^\mu (1-\gamma^5) q][\bar{b}\gamma_\mu (1-\gamma^5) q] | B^0_q \rangle \tag{2}$$

and η_B depend on the renormalization scheme, but the product $\eta_B \mathcal{M}_q$ does not. The renormalization-group invariant value of the short-distance factor is $\hat{\eta}_B = 0.55$.

One should keep in mind that non-Standard physics at short distances can modify Eq. (1). For convenience we shall couch the discussion as using Δm_q and the hadronic matrix element to determine $|V_{tq}|$. The resulting value of $|V_{tq}|$ can then be compared to other CKM determinations to test for deviations from the Standard Model.

$\mathcal{M}_q$ must be computed with a non-perturbative method, such as lattice gauge theory. For historical reasons one usually writes

$$\mathcal{M}_q = \frac{8}{3} m_{B_q}^2 f_{B_q}^2 B_{B_q} \tag{3}$$

and focuses on the decay constants f_{B_q} and the bag parameters B_{B_q}. But lattice QCD gives $\mathcal{M}_q$ directly (and f_{B_q} separately from $\langle 0|\bar{b}\gamma_\mu\gamma^5 q|B_q^0\rangle$). The separation does, however, turn out to be useful, as we shall see below, when considering the dependence of f_{B_q} and B_{B_q} on the masses of the light quarks.

At present the uncertainty in the matrix elements is large. A recent review [2] of lattice calculations quotes

$$f_{B_s} = 230 \pm 30 \text{ MeV}, \qquad \hat{B}_{B_s} = 1.34 \pm 0.10, \tag{4}$$

$$f_{B_d} = 198 \pm 30 \text{ MeV}, \qquad \hat{B}_{B_d} = 1.30 \pm 0.12. \tag{5}$$

These estimates take into account the first (partially) unquenched calculations of f_{B_q} [3–7], several quenched calculations of B_{B_q} and preliminary results suggesting that B_{B_q} changes little when the quenched approximation is removed [6]. The raw Monte Carlo data in lattice calculations are generated with the light quark mass m_q in the range 0.2–$0.5 < m_q/m_s < 1$, and the physical matrix elements are obtained by extrapolating m_q to the down quark's mass m_d. This method of reaching physically light quarks is called the chiral extrapolation, and it plays an important role in our analysis below.

The frequency for B_d^0-$\bar{B}_d^0$ mixing has been measured precisely, $\Delta m_d = 0.494 \pm 0.007 \text{ ps}^{-1}$ [8]. With Eqs. (1) and (5) the uncertainty on $|V_{td}|$ is limited to 15% by $f_{B_d}\sqrt{B_{B_d}}$. The precision on $|V_{td}|$ will not improve until better (unquenched) lattice calculations have been carried out. The frequency for B_s^0-$\bar{B}_s^0$ mixing is known to be high, $\Delta m_s > 15 \text{ ps}^{-1}$ [8]. But details of the way Δm_s is extracted from the data mean that the first measurement will immediately have a precision at the percent level [1]. Thus, it is interesting to form the ratio

$$\frac{\Delta m_s}{\Delta m_d} = \left|\frac{V_{ts}}{V_{td}}\right|^2 \frac{m_{B_s}}{m_{B_d}} \xi^2, \tag{6}$$

where

$$\xi^2 = \frac{f_{B_s}^2 B_{B_s}}{f_{B_d}^2 B_{B_d}}, \tag{7}$$

and use Eq. (6) to determine $|V_{td}|$. The measurement uncertainties are (or soon will be) negligible. By CKM unitarity $|V_{ts}| = |V_{cb}|$ to good approximation. Thus, the error in $|V_{td}|$ is

$$\delta|V_{td}| = \sqrt{(\delta|V_{cb}|)^2 + (\delta\xi)^2}. \tag{8}$$

The uncertainty in $|V_{cb}|$, determined from semileptonic B decay, is also dominated by QCD, but it is only 2–4% and relatively well understood [9–11].

The conventional wisdom, coming from several reviews of lattice B physics, is that $\delta\xi$ is small. Based on such endorsement, recent efforts to fit a wide range of precisely measured flavor observables have used $\xi = 1.14 \pm 0.03 \pm 0.05$ [12] or $\xi = 1.16 \pm 0.03 \pm 0.05$ [13]. The second error bar is meant to reflect the uncertainty from the quenched approximation; the first covers all other sources of uncertainty in lattice calculations. Central values in this range are reproduced by many quenched, and some unquenched, calculations.

Such a small error is, however, not universally accepted in the lattice community. Booth [14], noting that chiral logarithms in the quenched approximation differ strikingly from those of QCD, predicted that ξ in QCD would be 0.15–0.28 larger than in the quenched approximation. Sharpe and Zhang [15], with a similar point of view, reckoned that $\delta(\xi-1)/(\xi-1)$ could be 100%. Bernard, Blum and Soni [16] studied two different ways of carrying out the analysis. Treating f_{B_s}/f_{B_d} and B_{B_s}/B_{B_d} separately (as usual), they found $\xi = 1.17 \pm 0.02^{+0.12}_{-0.06}$; treating instead $\mathcal{M}_s/\mathcal{M}_d$ directly, they found $\xi = 1.30 \pm 0.04^{+0.21}_{-0.15}$. (In Ref. [16] the second error comes from studying the lattice spacing dependence; the difference is significant source of concern [17].) Finally, the JLQCD collaboration studied the effect of the chiral log in lattice calculations with two light flavors, finding that the extrapolated value of ξ could change significantly [6].

At first glance, $\delta\xi/\xi$ could well be smaller than $\delta f_{B_q}/f_{B_q}$. ξ is a ratio of similar quantities, so, in numerical lattice calculations, most of the Monte Carlo statistical fluctuations do cancel. Similarly, the short-distance normalization factor of the lattice operator also cancels. But one is still left with a multi-scale problem, with the heavy quark mass m_b, the QCD scale $\Lambda_{\rm QCD}$ and the range of light quark masses from m_s down to m_d. Because the numerator and denominator of ξ are the same, except for the light quark, one may expect ξ to be insensitive to the heavy-quark and QCD scales, but not to scales between m_s and m_d.

Let us examine the uncertainties associated with each scale in more detail. Heavy-quark corrections to ξ are suppressed by $(m_s - m_d)/m_b \sim 2\%$. In lattice calculations, one should also worry about discretization effects of the heavy quark, because $m_b a \sim 1$. There are several ways to handle this problem and some debate over the best method [18]. But the various discretizations yield consistent results for f_{B_s}/f_{B_d} and B_{B_s}/B_{B_d}. Thus, we conclude that errors from the short distance scales are under control.

Next let us consider $\Lambda_{\rm QCD}$. Implicit in the quenched approximation (also called the valence approximation) is that the omitted sea quarks are compensated by a shift in the bare gauge coupling [19]. This treats light-quark vacuum polarization in a dielectric approximation. Such approximations can be accurate when looking at a narrow range of scales. In the case at hand, that means that ratios of decay constants or bag parameters could be accurate as long as all quark masses are not too different. Thus, it is plausible that the quenched approximation accurately determines the slope of ξ, viewed as a function of $r = m_q/m_s$, when $r \sim 1$. Unquenched calculations [3–7] do not contradict this expectation. These calculations, and the justification of the quenched approximation [19], suggest that the scale $\Lambda_{\rm QCD}$ is also under control.

That leaves us with contributions to ξ from the long distances between $1/m_s$ and $1/m_d$. Here the quenched approximation is known to break down [14,15], and it is not obvious that the quenching error could be as small as 5%. One must take a careful look at how the chiral extrapolation is done, and consider what methods of extrapolation are reliable.

The correct framework to discuss the long-distance behavior of QCD, and the chiral extrapolation in particular, is chiral perturbation theory. We neglect $1/m$ corrections and write

$$\sqrt{m_{B_q}}\, f_{B_q} = \Phi\left[1 + \Delta f_q\right], \tag{9}$$
$$B_{B_q} = B\left[1 + \Delta B_q\right], \tag{10}$$

where Φ and B are independent of both heavy and light quark masses, and Δf_q and ΔB_q denote the (one-loop) contribution of the light meson cloud. The "chiral logarithms" reside in Δf_q and ΔB_q.

Neglecting isospin breaking, the one-loop corrections to the decay constants are [20,21,14,15]

$$\Delta f_s = -\frac{1+3g^2}{(4\pi f)^2}\left[m_K^2 \ln\left(m_K^2/\mu^2\right) + \tfrac{1}{3}m_\eta^2 \ln\left(m_\eta^2/\mu^2\right)\right] + \left(m_K^2 + \tfrac{1}{2}m_\pi^2\right) f_1(\mu) + \left(m_K^2 - \tfrac{1}{2}m_\pi^2\right) f_2(\mu), \quad (11)$$

$$\Delta f_d = -\frac{1+3g^2}{(4\pi f)^2}\left[\tfrac{3}{4}m_\pi^2 \ln\left(m_\pi^2/\mu^2\right) + \tfrac{1}{2}m_K^2 \ln\left(m_K^2/\mu^2\right) + \tfrac{1}{12}m_\eta^2 \ln\left(m_\eta^2/\mu^2\right)\right] + \left(m_K^2 + \tfrac{1}{2}m_\pi^2\right) f_1(\mu) + \tfrac{1}{2}m_\pi^2 f_2(\mu), \quad (12)$$

and to the bag parameters

$$\Delta B_s = -\frac{1-3g^2}{(4\pi f)^2}\tfrac{2}{3}m_\eta^2 \ln\left(m_\eta^2/\mu^2\right) + \left(m_K^2 + \tfrac{1}{2}m_\pi^2\right) B_1(\mu) + \left(m_K^2 - \tfrac{1}{2}m_\pi^2\right) B_2(\mu), \quad (13)$$

$$\Delta B_d = -\frac{1-3g^2}{(4\pi f)^2}\left[\tfrac{1}{2}m_\pi^2 \ln\left(m_\pi^2/\mu^2\right) + \tfrac{1}{6}m_\eta^2 \ln\left(m_\eta^2/\mu^2\right)\right] + \left(m_K^2 + \tfrac{1}{2}m_\pi^2\right) B_1(\mu) + \tfrac{1}{2}m_\pi^2 B_2(\mu), \quad (14)$$

where f and g are (the chiral limit of) the light pseudoscalar decay constant and B-B^*-π coupling. The "low-energy" constants $f_i(\mu)$ and $B_i(\mu)$ encode QCD dynamics from distances shorter than μ^{-1}, whereas the logarithms are long-distance properties of QCD, constrained by chiral symmetry. The dependence on μ cancels in the total.

It is convenient to look separately at the f_B and $\sqrt{B_B}$ factors in ξ. The chiral logarithm in the $\sqrt{B_B}$ factor could be small because it is multiplied by $1-3g^2$. On the other hand, the chiral logarithm in the f_B factor could be significant, because it is multiplied by $1+3g^2$. Consequently, we focus on

$$\xi_f = f_{B_s}/f_{B_d} \quad (15)$$

and study its chiral extrapolation. Our strategy is to use lattice calculations as an (indirect) way of determining the low-energy constants, and then we reconstitute ξ_f. Repeating our analysis for the chiral extrapolation of $\xi_B = \sqrt{B_{B_s}/B_{B_d}}$ verifies that ξ_B has a small effect.

Combining Eqs. (11) and (12), the first non-trivial order in the chiral expansion is

$$\xi_f - 1 = (m_K^2 - m_\pi^2) f_2(\mu) - \frac{1+3g^2}{(4\pi f)^2}\left[\tfrac{1}{2}m_K^2 \ln(m_K^2/\mu^2) + \tfrac{1}{4}m_\eta^2 \ln(m_\eta^2/\mu^2) - \tfrac{3}{4}m_\pi^2 \ln(m_\pi^2/\mu^2)\right]. \quad (16)$$

All lattice estimates of ξ are obtained not at physical light meson masses, but by chiral extrapolation. Therefore, we use Gell-Mann–Okubo formulae to replace the meson masses with

$$m_\pi^2 = m_{qq}^2, \quad (17)$$

$$m_K^2 = (m_{ss}^2 + m_{qq}^2)/2, \quad (18)$$

$$m_\eta^2 = (2m_{ss}^2 + m_{qq}^2)/3. \quad (19)$$

Varying the light quark mass changes $m_{qq}^2 \propto m_q$. Lattice calculations typically take m_{qq}^2 not too different from m_{ss}^2, so we write $m_{qq}^2 = r m_{ss}^2$. Then

$$\xi_f(r) - 1 = m_{ss}^2(1-r)\left\{\tfrac{1}{2}f_2(\mu) - \frac{1+3g^2}{(4\pi f)^2}\left[\frac{5}{12}\ln(m_{ss}^2/\mu^2) + l(r)\right]\right\}, \quad (20)$$

where

$$l(r) = \frac{1}{1-r}\left[\frac{1+r}{4}\ln\left(\frac{1+r}{2}\right) + \frac{2+r}{12}\ln\left(\frac{2+r}{3}\right) - \frac{3r}{4}\ln(r)\right]. \quad (21)$$

The function $\chi(r) = (1-r)l(r)$ contains the chiral logarithms. It is plotted in Fig. 1. The curvature over $0.5 \leq r \leq 1.0$ is too small to be resolved when there are percent-level statistical uncertainties on ξ_f. But once $r \ll 1$, which is appropriate for the down quark with $r_d \approx 1/25$, the curvature required by the chiral log has a significant effect. Fig. 2 shows this effect, comparing the conventional linear chiral extrapolation with Eq. (20), for $f_2(\mu)$ in the range coming from Eq. (23), below.

When ξ is calculated in lattice gauge theory, the range of r is restricted to $r \lesssim 1$ but $r \not\ll 1$. Usually, it is fit to a straight line

$$\xi_f(r) - 1 = (1-r)S_f \tag{22}$$

and similarly $\xi_B^2(r) - 1 = (1-r)S_B$. Usually one assumes this linear extrapolation holds down to the chiral limit, quoting $\xi = [1 + (1-r_d)S_f][1 + \frac{1}{2}(1-r_d)S_B]$. The chiral log says, however, that this procedure is not trustworthy. It has been employed because there was, until recently, no independent reliable estimate of the B-B^*-π coupling g^2 in the coefficient of the chiral log.

The CLEO collaboration has recently measured the width of the D^* meson, which yields a value for the D-D^*-π coupling [22]. Heavy-quark symmetry suggests that the B-B^*-π coupling is nearly the same. On this basis, we shall set $g^2 = 0.35$, although below we allow for 20% deviations. With $g^2 = 0.35$, the chiral log in ξ_B is truly small, because $1 - 3g^2 = -0.05$, but the chiral log in ξ_f is multiplied with $1 + 3g^2 = +2.05$.

With this handle on g^2, we can interpret the lattice results for S_f as a calculation of $f_2(\mu)$. We assume the linear fit given by Eq. (22) makes sense around $r = r_0 \sim 1$, even though we do not trust it when $r \ll 1$. So, at r_0 we set the right-hand side of Eq. (20) equal to the right-hand side of Eq. (22) and find

$$m_{ss}^2 \tfrac{1}{2} f_2(\mu) = S_f + m_{ss}^2 \frac{1+3g^2}{(4\pi f)^2} \left[\frac{5}{12}\ln(m_{ss}^2/\mu^2) + l(r_0)\right]. \tag{23}$$

Then, inserting this result into Eq. (20)

$$\xi_f(r) - 1 = (1-r)\left\{S_f + m_{ss}^2 \frac{1+3g^2}{(4\pi f)^2}\left[l(r_0) - l(r)\right]\right\}. \tag{24}$$

To evaluate the right-hand side, one needs estimates of f, g^2 and S_f. We use $f = 130$ MeV and $g^2 = 0.35$. In addition, we take [2]

$$(1-r_d)S_f = 0.15 \pm 0.05 \tag{25}$$

which brackets many quenched calculations (for which there is a lot of experience and reproducibility) as well as less well-developed unquenched calculations.[1]

Once we have made the Ansatz to use the slope from lattice QCD to determine the low-energy constant via Eq. (23), another source of uncertainty is the choice of r_0. Fig. 3 shows the result from Eq. (24) for the physical value $\xi_f(r_d)$, as a function of r_0 from 0 to 1.5. (The lower end 0 is not natural, but recovers the conventional result; the upper end 1.5 is where this order of chiral perturbation theory is less trustworthy.) Since the typical range of fits leading to Eq. (25) is $0.5 < r < 1.0$, we choose r_0 in this range and use Fig. 3 to obtain

$$\xi_f = 1.32 \pm 0.08. \tag{26}$$

[1] In fact, some "unquenched" calculations only have $n_f = 2$.

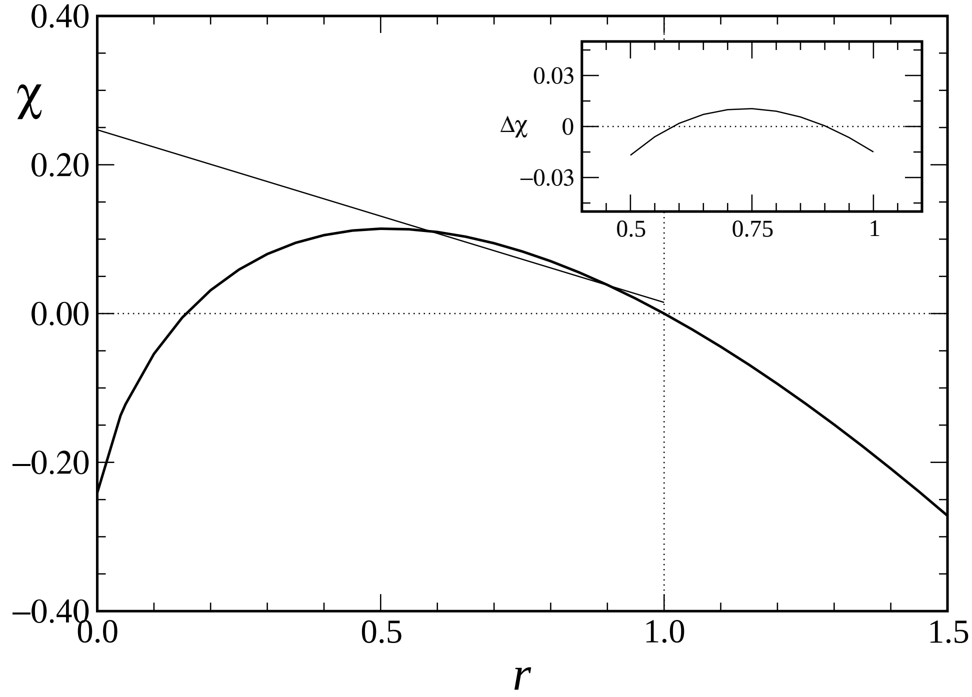

Figure 1: Plot of the chiral logarithm $\chi(r)$ as the mass ratio $r = m_{qq}^2/m_{ss}^2 = m_q/m_s$ is varied, compared with a straight line fit for $0.5 \le r \le 1.0$. The difference between the curve and the fit is shown in the inset.

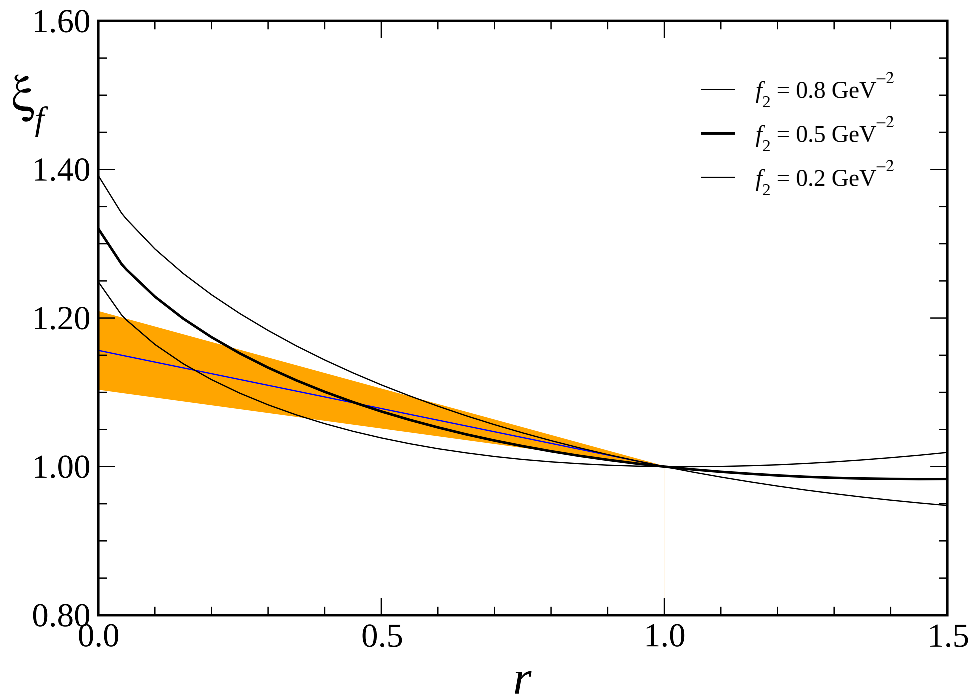

Figure 2: Plot of ξ_f against r for several values of the low-energy constant: $f_2(1\ \mathrm{GeV}) =$ 0.2, 0.5, 0.8 GeV^{-2}. Also shown is the linear extrapolation with $\xi_f(r_d) = 1.15 \pm 0.05$.

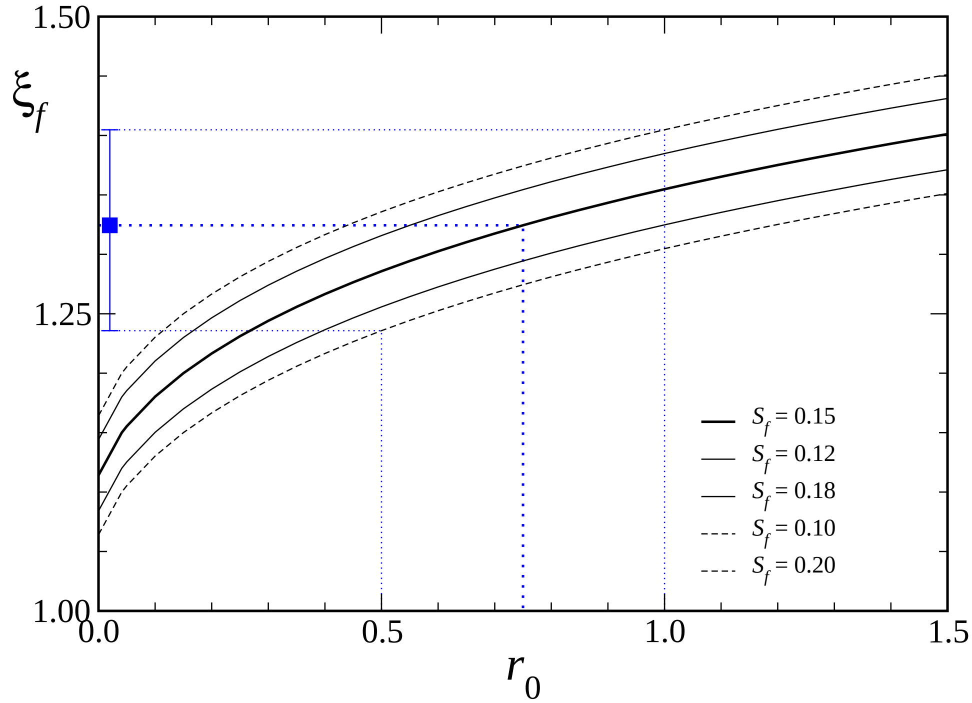

Figure 3: Plot of ξ_f from Eq. (24), with $(1 - r_d)S_f = 0.15 \pm 0.05$, $m_{ss}^2 = 2(m_K^2 - m_\pi^2)$, and $r = r_d = m_\pi^2/m_{ss}^2$, as a function of r_0.

With separation scale $\mu = 1$ GeV, $\xi_f - 1$ receives nearly equal contributions from the low-energy constant (0.159) and the chiral log (0.165).[2]

We have carried out a similar analysis for ξ_B and also allowed for a $\pm 20\%$ range on g^2. (See the appendix for details.) The chiral logs in ξ_f and ξ_B pull in opposite directions, so the resulting $\xi = \xi_f \xi_B$ is insensitive to g^2:

$$\xi = 1.32 \pm 0.10, \tag{27}$$

which is quite different from the range usually used in CKM fits, although it agrees with qualitative discussions of chiral logs [14,15], a direct analysis of $\mathcal{M}_s/\mathcal{M}_d$ [16], and chiral log fits to preliminary unquenched calculations [6]. The shift in central value from 1.15 to 1.32 can be thought of as a correction to the quenched approximation: mature unquenched calculations will certainly see the curvature required by the chiral log.

Because our result is so different than the conventional one, let us stress the differences in methodology. Usually ξ is obtained via a linear chiral extrapolation, although chiral log fits have been tried in Ref. [6]. We have relied completely on the functional form predicted by chiral perturbation theory. It is difficult to determine the coefficient of the chiral logs directly from the lattice calculation. We have circumvented this obstacle by using the D^* width [22], which, with heavy-quark symmetry, implies $g^2 = 0.35$. The uncertainty in Eq. (27) is larger than in many other papers, mostly because we have assigned ± 0.05 instead of ± 0.03 uncertainty to the lattice calculations. On the other hand, we have also not done a complete error analysis: for example, we have neglected uncertainties from higher orders in the chiral expansion.

[2] Loops with excited B_q^{**} mesons are expected to contribute significantly to ξ_f [23,24], but the ensuing r dependence is well described by linear extrapolation, so it is accurate to lump them into $(1 - r)f_2(\mu)$.

One could easily reduce the theoretical uncertainty in B^0-$\bar{B}^0$ mixing by carrying out lattice calculations designed to determine the low-energy constants in Eqs. (11)–(14). If one takes closely-spaced values of the light quark mass, even if close to the strange mass, one can compute the derivative $d\xi/dr$. If one is willing to take g^2 from experiment, these derivatives give $f_2(\mu)$ and $B_2(\mu)$, and one can proceed to determine ξ for physically light quark masses. The same procedure could be applied to f_{B_q} and B_{B_q} although now one must also compute $f_1(\mu)$ and $B_1(\mu)$, and also cope with further low-energy constants in the $1/m_b$ corrections [25,26]. Chiral extrapolations with chiral logs may well change f_{B_q} from the estimates in Eqs. (4) and (5) in the same way they changed ξ_f.

From a (lattice) purist's point of view it may be unsatisfactory to take g^2 from experiment. In the long run it will, however, be possible to solidify our knowledge of g^2 (in the B system) through lattice calculations and other applications of chiral perturbation theory to B physics. To relate the very precise measurements to the CKM matrix, the combination of phenomenology for g^2 and lattice calculation for the low-energy constant is very satisfactory, especially since we find that ξ varies by less than 2% when g^2 is varied by 20%. Fig. 4 shows how the combination of $\sin 2\beta$ and $\Delta m_s/\Delta m_d$ work together to constrain the apex of the unitarity triangle. We take $\sin 2\beta = 0.79 \pm 0.10$ from averaging CDF [27], BaBar [28] and Belle [29] measurements. For illustration we take $\Delta m_s = 20$ ps^{-1}, and compare $\xi = 1.15 \pm 0.05$ (conventional wisdom) with $\xi = 1.32 \pm 0.05$ [Eq. (27) with error halved]. With a larger value of ξ the mixing side is longer, scaling like $\xi/\sqrt{\Delta m_s}$. By the same token, our result suggests that the Standard-Model prediction for Δm_s (16–19 ps^{-1} [12]) should be increased, perhaps by 25–35%.

Acknowledgments: We would like to thank Claude Bernard, Gustavo Burdman, Shoji Hashimoto, Aida El-Khadra, Zoltan Ligeti, Vittorio Lubicz, Ulrich Nierste, and Norikazu Yamada for discussions related to this work.

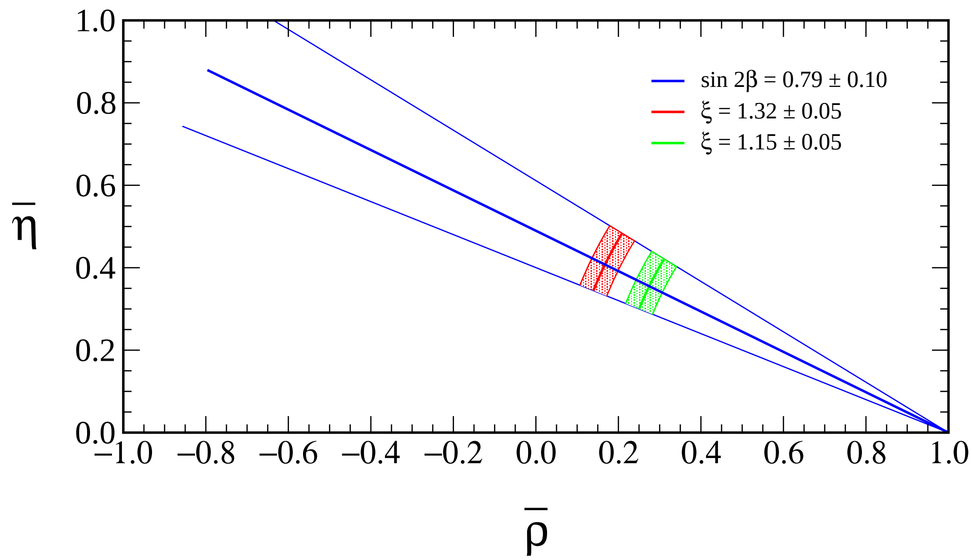

Figure 4: Sketch of constraints on the apex of the unitarity triangle with $\sin 2\beta = 0.79 \pm 0.10$, $\Delta m_s = 20$ ps^{-1} and $\xi = 1.32 \pm 0.05$ or 1.15 ± 0.05.

Appendix: Analysis including ξ_B and varying g^2

Let $\xi_B^2 = B_{B_s}/B_{B_d}$, with linear chiral extrapolation

$$\xi_B^2(r) - 1 = (1-r)S_B. \tag{28}$$

Then, eliminating $B_2(\mu)$ in the same way as $f_2(\mu)$ in ξ_f,

$$\xi_B^2(r) - 1 = (1-r)\left\{S_B + m_{ss}^2 \frac{1-3g^2}{(4\pi f)^2}\left[l_B(r_0) - l_B(r)\right]\right\}, \tag{29}$$

where

$$l_B(r) = \frac{1}{1-r}\left[\frac{2+r}{6}\ln\left(\frac{2+r}{3}\right) - \frac{r}{2}\ln(r)\right]. \tag{30}$$

To evaluate the right-hand side, we take [2]

$$S_B = 0.00 \pm 0.05. \tag{31}$$

Then we find $\xi_B = 0.998 \pm 0.025$.

In the main analysis, we have used $g^2 = 0.35$, which assumes that the B-B^*-π and D-D^*-π are the same. Repeating the analysis with $g^2 = 0.20$ and 0.50 we find the results in Table 1. Although the chiral extrapolation of ξ_B is no longer completely insignificant, and ξ_f changes a little, the result for ξ is very stable.

Table 1: Comparison of chiral extrapolations for ξ_f, ξ_B and ξ for three values of the B-B^*-π coupling $g^2 = 0.20$, 0.35, 0.50.

g^2	ξ_f	ξ_B	ξ
0.20	1.29±0.08	1.01±0.03	1.30±0.09
0.35	1.32±0.08	1.00±0.02	1.32±0.09
0.50	1.36±0.09	0.98±0.02	1.34±0.09

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