



# Natural Theories of Ultra-Low Mass PNGB's: Axions and Quintessence

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## Abstract

We consider the Wilson Line PNGB which arises in a  $U(1)^N$  gauge theory, abstracted from a latticized, periodically compactified extra dimension  $U(1)$ . Planck scale breaking of the PNGB's global symmetry is suppressed, providing natural candidates for the axion and quintessence. We construct an explicit model in which the axion may be viewed as the 5th component of the  $U(1)_Y$  gauge field in a  $1+4$  latticized periodically compactified extra dimension. We also construct a quintessence PNGB model where the ultra-low mass arises from Planck-scale suppressed physics itself.

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# 1 Introduction

Ultra-low mass spin-0 fields are desired in particle physics, and often carry cosmological implications. As usual, “low-mass” means a mass scale hierarchically small compared to much larger mass scales in the problem. The Planck mass,  $M_P$ , is the largest mass scale present in any field theory problem. Achieving a “low-mass” spin-0 field is generally challenging, owing at least to putative Planck scale quantum effects, and always requires the judicious use of symmetry. Key examples of these are the Higgs boson and the axion. We will not recite the issues associated with electroweak symmetry breaking and the Higgs boson naturalness problem presently, though it has been the main driver of speculation about physics beyond the weak scale (e.g., see the recent reviews, [1, 2]). Rather, we focus presently on the axion, and ultra-low-mass pseudo-Nambu-Goldstone bosons (PNGB’s).

The axion was proposed as a massless particle, in the absence of QCD effects, which arises as a Nambu-Goldstone-boson (NGB) when the  $U(1)$  global Peccei-Quinn symmetry (PQ-symmetry) is spontaneously broken at a very high mass scale  $f$  [3, 4]. We also have small explicit symmetry breaking effects which arise from QCD, via the axial vector current anomaly, which are of order  $\sim \Lambda_{QCD}$ . The axion develops a residual mass of order  $m_{axion} \sim \Lambda_{QCD}^2/f$ , becoming a pseudo-Nambu-Goldstone-boson (PNGB). With  $f \sim 10^{12}$  GeV this is a tiny mass scale of order  $10^{-5}$  eV. At the minimum of its potential, the VEV of the axion identically cancels the unwanted  $\theta$  angle of QCD to a precision of  $\leq 10^{-9}$ .

This picture can be criticized as follows. The PQ-symmetry is postulated as a global symmetry at all scales above  $\Lambda_{QCD}$ . However, at the scale  $M_P$  we expect quantum gravity (string theory) to become important. The vacuum at this scale will contain large fluctuations involving the formation and rapid evaporation of tiny black-holes. Black holes can eat global symmetries, i.e., fields carrying global charges will experience global charge nonconserving interactions in the presence of mini-black-holes. Hence we expect large explicit PQ-symmetry breaking effects at  $M_P$ . The size of these effects depends upon what fields carry the PQ-symmetry, but in general these effects will give symmetry breaking corrections to the axion (mass)<sup>2</sup> of order  $f^{p+2}/M_P^p$ . In order to avoid upsetting the delicate cancellation of  $\theta$  to the precision of  $10^{-9}$  we require  $f^{p+4}/M_P^p \lesssim 10^{-9}\Lambda_{QCD}^4$  or  $p \gtrsim 9$  using the parameters described above. We would naively expect  $p \approx 2$  (or smaller!) in a scalar field effective theory. Hence, the theory of the axion has a major hurdle to overcome in being reconciled with the large “ $\theta$ -pull” induced by Planck scale effects.

As we will show in the present paper, the effective Lagrangian of a latticized extra

dimension offers a simple solution to this problem. It also provides a natural origin for the axion: the axion can be viewed as the 5th component of the  $U(1)_Y$  weak hypercharge gauge group in a world with a periodically compactified extra dimension. It can alternatively be viewed as arising in a natural way, immune from Planck scale physics, within a particular 1 + 3 dimensional generalization of the Standard Model, where  $U(1)_Y$  is embedded into a chain of  $N$  gauge groups,  $U(1) \times U(1) \times \cdots \times U(1)$  [5, 6]. A viable minimal scheme exists for  $N$  as small as  $N \sim 14$ .

Ultra-low mass (pseudo)scalars are also desired to solve a number of cosmological problems, going under the rubric of “quintessence” (for some recent cosmo-phenomenological discussion, see e.g., [7]). Peebles and Ratra [8] proposed a fundamental scalar theory, where no symmetries are present, which is one of the first “quintessence” models. Though a useful phenomenological construction, from a fundamental point of view this model suffers serious unnaturalness: *(i)* the cosmological constant is tuned to zero at the minimum of the potential; *(ii)* the ultra-low mass scale is put in by hand, and is otherwise arbitrary and unmotivated; *(iii)* Planck scale effects in such a theory would be expected to pull the scalar mass term up to  $\sim M_P$ . The cosmological constant problem *(i)* is common to all approaches to issues in cosmology and particle physics, and we offer no insight into its resolution at present.

Independently, the other earliest “quintessence” model, and a theory of late-time phase transitions, was based upon established concepts of spontaneous symmetry breaking in particle physics. It was proposed by Hill, Schramm and Fry, and subsequently developed in detail with Frieman, Gupta, Holman, Kolb, Stebbins, Waga, and Watkins, [9, 10, 11, 12].<sup>1</sup> These models contain an ultra-low mass PNGB, with a characteristic potential of the form  $\sim m_\nu^4 \cos(\chi/f)$ . Here  $f$  ranges from  $f \sim M_{GUT} \sim 10^{15}$  GeV to  $f \sim M_P$ , the scale of *spontaneous symmetry breaking*, and  $m_\nu$  is a “neutrino mass scale,” or some other comparable low mass scale, the scale of *explicit symmetry breaking*. Expanding the potential to quadratic order we see that  $\chi$  has the cosmologically interesting mass of  $m_\nu^2/M_P$ , and a feeble coupling  $\lambda \sim m_\nu^4/M_P^4$ . These small parameters were installed by hand in the Peebles-Ratra model, but arise automatically here, under control of symmetries, and thus, the arbitrariness problem *(ii)* described above does not arise in a PNGB scheme.

With regard to problem *(iii)* above, the proposal of ultra-low mass PNGB’s has also been criticized (e.g., [13]), again based upon Planck-scale explicit symmetry breaking

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<sup>1</sup>In fact, the idea of coupling these fields to  $F_{\mu\nu}F^{\mu\nu}$  for electrodynamics and QCD was elaborated in [9] and applied in [12] to constrain time dependent fundamental constants.

effects in complete analogy to the axion. Here the problem is more severe than in the case of the axion because  $f \sim M_P$  and there is evidently no power law suppression to terms of the form  $f^p/M_P^p$ , and the PNGB can apparently never have low mass.

In the present paper we will show that the Planck scale effects for quintessence PNGB's are likewise suppressed when they are reinterpreted as  $U(1)$  gauge fields periodically compactified in an extra dimension. We will see that the bound on Planck scale effects typically restricts  $f_\chi$  to be less than  $\sim$  a few  $\times 10^{17}$  GeV. Indeed, in the minimal quintessence theory we present below, we actually use the Planck scale effects to generate the ultra-low mass of the PNGB. This latter result appears to us to be very natural within the framework of the parameters of the deconstructed theory.<sup>2</sup>

The key idea to solving the naturalness problem of ultra-low-mass PNGB's is to replace the PNGB by a gauge field in a periodically compactified extra dimension. Consider a  $U(1)$  gauge theory in  $1 + 4$  dimensions. This theory contains the normal vector potential,  $A_\mu$ , where  $\mu = 0, 1, 2, 3$ , plus a fifth component,  $A_4$ . We take  $x^4$  to be periodically compactified. The  $A_4$  gauge field, at low energies (i.e., no momentum in the  $x^4$  dimension), is then effectively a “Wilson line” which wraps around a compact extra dimension,  $\int_{loop} dx^4 A_4$ . This object has the effective Lagrangian of the PNGB of a global  $U(1)$  symmetry. We will refer to it as the Wilson Line PNGB (WLPNGB).

We can derive the effective Lagrangian for the WLPNGB in  $1 + 3$  dimensions using the technique of latticizing (“deconstructing”) the extra dimension [15, 16] (see also [17]). The case of  $U(1)$  electrodynamics with a heavy fermion was studied in detail in a previous companion paper ref. [18], and will play a major role in the present discussion. The lattice provides a useful tool for regulating the enhanced quantum loop divergences of the extra dimensional theory, and generates a gauge invariant low energy effective Lagrangian with a finite subset of Kaluza-Klein modes. A lattice description of a  $1 + 4$  theory involves chopping the fifth dimension,  $x^4$ , into  $N$  slices, or “branes,” each brane describing the  $1 + 3$  spacetime with its own copy of the gauge group.  $N$  plays the role of a UV cutoff, and cannot exceed  $\sim 4\pi/\tilde{\alpha}$ , where  $\tilde{\alpha} = \tilde{g}^2/4\pi$  is the low energy effective coupling constant of the  $U(1)$  theory in  $1 + 3$  dimensions (the coupling “runs” to large values,  $N\tilde{\alpha} = \alpha_{max}$  at short distances, and  $\alpha_{max}$  is typically the unitarity bound of the coupling constant [15, 19]).

The effective Lagrangian in  $1+3$  appears as  $N$  copies of a  $U(1)$  gauge theory, of the form  $U(1) \times U(1) \times \dots \times U(1) \equiv U(1)^N$ . The  $U(1)$ 's are linked together by  $N$  Higgs fields,  $\Phi_n$ ,

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<sup>2</sup>For another recent proposal for quintessence fields arising from extra dimensions see, e.g., [14].

each having a common VEV. When each  $U(1)$  has the same coupling constant, and each  $\Phi_n$  the same VEV, there is a  $Z_N$  symmetry which replaces the continuous translational invariance in the extra dimension. In fact, one can simply forget about the existence of an extra dimension and view this as a procedure for constructing a new kind of theory in 1+3 dimensions, with a new symmetry group,  $Z_N$ . This procedure is conceptually powerful, and has suggested new directions and dynamics in building models of physics beyond the Standard Model, and provides a new rationale for dynamical electroweak symmetry breaking (see, e.g. [20]).

Planck scale effects are suppressed because the WLPNGB is contained in the  $Z_N$  invariant product of the linking Higgs fields,  $\tilde{\Phi} = \Phi_1 \Phi_2 \cdots \Phi_N$ . Only the  $Z_N$  invariant field  $\tilde{\Phi}$  is gauge invariant. Powers of any individual factor field, i.e.,  $\Phi_n^p$ , are gauge dependent, owing to the local  $U(1)_n$ , and cannot be generated in the Lagrangian by quantum gravity. Symmetry breaking effects from Planck scale physics must be gauge invariant and can involve only  $\tilde{\Phi}^p$ , hence terms like:

$$\sim \frac{(\Phi_1 \Phi_2 \cdots \Phi_N)^p}{M_P^{pN-4}} + h.c. \quad (1.1)$$

These terms are seen to be very highly suppressed owing to the dimensionality of  $\tilde{\Phi}$ . Such terms do produce symmetry breaking effects (e.g., corrections to the PNGB mass) that are of order  $(f/M_P)^{pN}$ . For large  $N$  this provides the required suppression of Planck scale breaking.

A conventional mass of the WLPNGB can also come through its couplings to matter in the bulk. We have computed the Coleman-Weinberg potential for the WLPNGB with a heavy Dirac fermion propagating in the 1+4 bulk, and studied the anomalous couplings of the WLPNGB to other gauge fields [18]. The Coleman-Weinberg effective potential for the WLPNGB is finite when  $N \geq 3$ . Finite potentials for PNGB's similar to these were discussed long ago by Hill and Ross [9]. The finiteness is a consequence of the  $Z_N$  invariance of the full theory. The ‘‘schizon’’ models of Hill and Ross [9], exploited  $Z_{2L} \times Z_{2R}$  to reduce the degree of divergence from quadratic to logarithmic and implement ultra-low-mass PNGB's to provide natural ‘‘5th’’ forces in the Standard Model, and remedy certain limits on the axion. With  $Z_3$  symmetry, finite neutrino-schizon models have been used to engineer the first ‘‘quintessence’’ models, late-time cosmological phase transitions, and place limits upon time dependent fundamental constants [10]. The finite temperature behavior of such models is also striking [11]. These models are structurally equivalent to the present extra-dimensional scheme with latticized fermions when written in the

momentum space expansion in the fifth dimension [18].

The present approach in principle solves a second outstanding issues for the axion: Why should the axion exist at all? In our present view the axion is the  $U(1)_Y$  gauge field propagating in a compact extra dimension. Indeed, there does exist the  $U(1)_Y$  in nature, and the axion can occur if there also exists the periodic extra dimension. Probing the physics of the axion, e.g., measuring its decay constant  $f_a$ , is a direct probe of the physics of the extra dimension, e.g.,  $f_a \propto 1/R$  where  $R$  is the circumference of the extra dimension.

We take up the problem of constructing a bona-fide theory of the axion as a consequence of the existence of Standard Model  $U(1)_Y$  and an extra compact dimension in Section 3. We begin in Section 2 with the simpler problem of a minimal PNLGB theory for quintessence or late-time phase transitions.

## 2 The Wilson Line Pseudo-Nambu-Goldstone Boson (WLPNLGB)

### 2.1 Gauge Field Lattice

Consider a free  $U(1)$  gauge theory in  $1 + 4$  dimensions that is *periodically compactified* to  $1 + 3$ :

$$\mathcal{S} = \int d^4x \int_0^R dy \left[ -\frac{1}{4} F_{AB} F^{AB} + \mathcal{L}_P \right], \quad (2.2)$$

where  $(A, B)$  run from 0 through 4, and  $y = x^4$ .  $\mathcal{L}_P$  describes the Planck scale symmetry breaking effects, which we consider below.

The Wilson latticization of the gauge theory with a periodic fifth dimension is straightforward. We slice the extra dimension into  $N$  slices, or “branes,” each labeled by  $n$ . The effective Lagrangian becomes the gauged chiral Lagrangian in  $1 + 3$  dimensions for  $N$  copies of the  $U(1)$  gauge group:

$$\mathcal{S} = \int d^4x \left[ -\sum_{n=1}^N \frac{1}{4} F_{n\mu\nu} F_n^{\mu\nu} + \sum_{n=1}^N D_\mu \Phi_n^\dagger D^\mu \Phi_n + \mathcal{L}_P \right]. \quad (2.3)$$

In eq. (2.3) we have  $N$  gauge groups,  $U(1)_n$ , with a common gauge coupling  $g$ , and  $N$  “link-Higgs” fields,  $\Phi_n$ , having nonzero charges in the  $U(1)_n$  and  $U(1)_{n+1}$  gauge groups

only, as  $(0, 0, \dots, 1_n, -1_{n+1}, 0, 0, \dots, 0)$  (Note:  $\Phi_N$  then has charges  $(-1, 0, \dots, 1)$ , and we identify  $n = N + 1 = 1$ , i.e.  $n$  is an integer mod $_N$ ). Hence,  $\Phi_n$  “links” the  $U(1)_n$  to the  $U(1)_{n+1}$  gauge group. The covariant derivative therefore acts upon  $\Phi_n$  as:

$$D_\mu \Phi_n = \partial_\mu \Phi_n - ig(A_{n+1\mu} - A_{n\mu})\Phi_n. \quad (2.4)$$

$g$  is the “high energy” value of the coupling constant of the theory, while the low energy coupling of the zero-mode photon is  $\tilde{g} = g/\sqrt{N}$  [15, 16], or  $\tilde{\alpha} = \tilde{g}^2/4\pi = \alpha/N$ .

Each  $\Phi_n$  has a vacuum expectation value, in which the Higgs mode is very heavy, i.e., it is effectively a nonlinear- $\sigma$  model field, depending only upon its phase  $\chi_n(x^\mu)$ :

$$\Phi_n \rightarrow (v/\sqrt{2}g) \exp(ig\chi_n/v). \quad (2.5)$$

The normalization is convenient and defines  $v$  to be the inverse lattice spacing,  $v = 1/a$ . The VEV’s can easily be arranged with a judicious choice of potential, common to each  $\Phi_n$  [15, 16, 18].

Through the equivalence of eq. (2.2) and eq. (2.3) we have mapped the  $1 + 4$  theory into a  $1 + 3$  dimensional description. These two theories are equivalent in their low energy physics (modulo certain lattice subtleties as addressed in [18]). Since we are only interested in the extreme low energy limit, this procedure is particularly useful, and it buys a bonus: anything we say about the low energy physics of the  $1 + 4$  dimensional theory of eq. (2.2), also applies to the equivalent  $1 + 3$  dimensional theory, eq. (2.3). We can therefore use geometrical intuition from  $1 + 4$  to understand the behavior of the  $1 + 3$  theory and *vice versa*.

The  $\Phi_n$  kinetic terms then go over to a mass-matrix for the gauge fields:

$$\sum_{n=1}^N D_\mu \Phi_n^\dagger D^\mu \Phi_n \longrightarrow \frac{1}{2}v^2 \sum_{n=1}^N \left| \left( (A_{n+1\mu} - A_{n\mu}) - \frac{1}{v} \partial_\mu \chi_n \right) \right|^2 \quad (2.6)$$

Eq. (2.6) is easily diagonalized [18]. Eq. (2.3) in the conjugate momentum space basis becomes:

$$\int d^4x \left[ \frac{1}{2}(\partial_\mu \chi_0)^2 - \sum_{p=-J}^J \frac{1}{4} F_{p\mu\nu} F^{p\mu\nu} + \sum_{p=-J}^J 2v^2 A_{p\mu}^2 \sin^2(\pi p/N) + \mathcal{L}_P \right], \quad (2.7)$$

(where without loss of generality consider  $N$  odd, and then  $J = (N - 1)/2$ ; see [18] for more details). The spectrum contains the zero-mass-mode  $U(1)$  gauge field,  $A_\mu$ , and the zero-mass-mode  $\chi_0$  (which is the zero- $p_4$ -momentum  $A_4$  or WLPNGB),

$$\chi_0 \equiv -\frac{iv}{g\sqrt{N}} \ln \left[ \prod_{n=1}^N (\sqrt{2}g\Phi_n/v) \right] = \frac{1}{\sqrt{N}} \sum_{n=1}^N \chi_n. \quad (2.8)$$

This mode is contained in the nonlocal product of links. It corresponds to the continuum Wilson line around the compact 5th dimensions.

The spectrum has a tower of spin-1 Kaluza-Klein-modes, appearing as massive gauge fields, each labeled by  $p$ , of mass:

$$M_p^2 = 4v^2 \sin^2(\pi p/N). \quad (2.9)$$

The tower of KK modes is linear in the large- $N$  limit. All the  $\chi_n$ , except the zero-mode linear combination  $\chi_0$ , are “eaten” to become longitudinal modes of the massive “KK-photons” [16, 18].

## 2.2 Suppressing Planck Scale Effects

We now include in the Lagrangian of eq. (2.3,2.4) the Planck scale breaking effects. The leading allowed term is:

$$\mathcal{L}_P = \frac{1}{2} \kappa \frac{e^{i\theta_P}}{M_P^{N-4}} \prod_{n=1}^N \Phi_n + h.c. + \dots \quad (2.10)$$

where  $\theta_P$  is an arbitrary “gravitational CP-angle,” an effective  $\theta$ -angle of the Planck scale vacuum.  $\kappa$  is a coefficient expected to be of order unity associated with the details of Planckian physics. This implies that the WLPNGB zero mode acquires a potential:

$$\kappa M_P^4 \left( \frac{v}{\sqrt{2}gM_P} \right)^N \cos(g\sqrt{N}\chi_0/v + \theta_P) \quad (2.11)$$

$g$  is the *high energy* value of the coupling constant in this theory.

It is useful to recast eq. (2.11) in terms of the low energy quantities in the theory. We refer the reader to the companion paper [18]. The low energy value of the coupling, as would be determined by the exchange of low energy  $U(1)$  gauge fields between currents carrying any one of the  $U(1)_n$  charges, is [16]:

$$\tilde{g} = g/\sqrt{N}; \quad \tilde{\alpha} = \frac{\tilde{g}^2}{4\pi} = \frac{\alpha}{N}. \quad (2.12)$$

From a continuum perspective, this is just the power-law “running” of the gauge coupling in an extra dimension. It arises in the lattice description as a consequence of the mixing of the chain of gauge fields in the  $1 + 3$  effective Lagrangian. The physical compactified circumference of the extra dimension is given by:

$$R = N/v \quad (2.13)$$



and the *decay constant* is [18]:

$$f_\chi = \frac{v}{g\sqrt{N}} = \frac{1}{\tilde{g}R} = \frac{1}{\sqrt{4\pi\tilde{\alpha}}R}. \quad (2.14)$$

The maximum value that  $N$  can achieve is dictated by  $\tilde{\alpha}$  and the upper bound of  $\alpha \lesssim \alpha_{max}$ , to which our theory applies. This implies:

$$N_{max} = \frac{\alpha_{max}}{\tilde{\alpha}}. \quad (2.15)$$

The free theory has no upper bound, in principle, but any couplings to matter will lead a unitarity bound on  $\alpha_{max}$ . If we take  $\alpha_{max}$  close to its unitarity upper bound, we have chosen a fine grained lattice with brane interspacing  $1/v_{max}$ , where the hopping parameter  $v_{max} = N_{max}/R$ , corresponding to the shortest distance scales at which we can apply the present field-theoretic description of the system. We must, at shorter distances, go over to the overarching theory, e.g., string theory. The unitarity bound is dependent upon the matter content and details of the theory, but a reasonable limit corresponds to the breakdown of perturbation theory, expected when  $\alpha \sim 4\pi$ . Hence, we would expect:

$$N_{max} \lesssim \frac{4\pi}{\tilde{\alpha}}. \quad (2.16)$$

Using these definitions we can rewrite the gravitationally induced potential eq. (2.11):

$$V = \kappa M_P^4 \exp \left[ - \left( \frac{\alpha_{max}}{\tilde{\alpha}} \right) \ln \left( \sqrt{\frac{2\tilde{\alpha}}{\alpha_{max}}} \frac{M_P}{f_\chi} \right) \right] \cos(\chi_0/f_\chi + \theta_P). \quad (2.17)$$

We see that the induced potential is exponentially suppressed for large  $N$ , provided we satisfy the requirement:

$$f_\chi^2 < \left( \frac{2}{N} \right) M_P^2 = \left( \frac{2\tilde{\alpha}}{\alpha_{max}} \right) M_P^2. \quad (2.18)$$

If we use the largest possible value for  $\alpha_{max} \approx 4\pi$ , we have the bound  $f_\chi^2 \lesssim (\tilde{\alpha}/2\pi)M_P^2$ . Hence, as we would expect, there is an upper bound on the decay constant of a soft-PNGB such that Planck scale effects are suitable suppressed. In fact the constraints are nontrivial and arbitrary suppression is not possible. This result is dependent upon the cut-off  $N$  because the leading term in  $\mathcal{L}_P$  is  $N$  dependent, and  $\tilde{\Phi}$  is a nonlocal object.

A weakly coupled theory,  $\tilde{\alpha} \ll 1$ , can have large  $N_{max}$  and negligible Planck scale corrections, provided  $f_\chi$  is also taken small. As we will see below, this is the case for the axion with its normal parameter range,  $f_a \sim 10^{12}$  GeV and with  $\tilde{\alpha} \sim \alpha_Y$ , the coupling of the  $U(1)_Y$  weak hypercharge. Taking larger values of  $f_\chi$  requires larger  $N_{max}$  to enhance the  $Z_N$  symmetry.

The mass of the WLPNGB, owing solely to Planck scale effects is obtained by expanding the potential to quadratic order about its minimum:

$$m_\chi^2 = \kappa(M_P^4/f_\chi^2) \exp \left[ - \left( \frac{\alpha_{max}}{\tilde{\alpha}} \right) \ln \left( \sqrt{\frac{2\tilde{\alpha}}{\alpha_{max}}} \frac{M_P}{f_\chi} \right) \right]. \quad (2.19)$$

While the result is expected in a true extra-dimensional theory of the kind we have specified, it also applies in 3 + 1 theories having the structure of our effective Lagrangian. We can abandon the notion of the parent 1 + 4 theory and view this construction as a purely 1 + 3 theory. This is the philosophy of “deconstruction” advocated in [17]. In either case,  $N$  is a parameter which we are free to vary up to  $N_{max}$ .

### 2.3 Minimal Theory of Quintessence

A minimal model of quintessence can be constructed by using a 1 + 4 dimensional  $U(1)$  pure gauge theory, as in eq. (2.2), in a compact extra dimension, or its equivalent 1 + 3 dimensional latticized description eq. (2.3), as a source for the WLPNGB. We couple the  $U(1)$  theory to a single Dirac fermion, which also propagates in the bulk extra dimensions, suitably latticized, below. For the moment, we neglect the effects of the fermion. We tune the coupling constant so that the ultra-low mass of the WLPNGB is given by the Planck scale effects. As we see below, the matter fields can produce additional effects, but these are not problematic for the present scheme.

The potential is therefore given by eq. (2.17). Let us take  $\kappa = 1$ . If we saturate the unitarity bound of  $\alpha_{max} \lesssim 4\pi$  we see that the potential becomes:

$$V = M_P^4 \exp \left[ - \left( \frac{4\pi}{\tilde{\alpha}} \right) \ln \left( \sqrt{\frac{\tilde{\alpha}}{2\pi}} \frac{M_P}{f_\chi} \right) \right] \cos(\chi_0/f_\chi + \theta_P). \quad (2.20)$$

If this potential represents quintessence, it must have an overall scale of order the closure density,  $V \sim \rho_c \sim 3H_0^2/8\pi G_N$ , and we choose  $H_0 = 100h$  (km s<sup>-1</sup> Mpc<sup>-1</sup>) with  $h = 0.7$ , hence  $\rho_c = 4 \times 10^{-47}$  GeV<sup>4</sup>. Let  $y = \ln(M_P/f_\chi)$  and  $x = 4\pi/\tilde{\alpha}$ . Then:

$$y = \frac{1}{x} \ln(M_P^4/\rho_c) + \frac{1}{2} \ln(x) - \frac{1}{2} \ln(2). \quad (2.21)$$

Thus the minimum value of  $y$  is:

$$x = \frac{4\pi}{\tilde{\alpha}} = 2 \ln(M_P^4/\rho_c), \quad y = \ln(M_P/f_\chi) = \frac{1}{2} \left( 1 + \ln[\ln(M_P^4/\rho_c)] \right). \quad (2.22)$$

Note that  $\ln(M_P^4/\rho_c) \approx 2.83 \times 10^2$ , and we thus see that the optimal value of  $N_{max} \sim \alpha_{max}/\tilde{\alpha} \sim 4\pi/\tilde{\alpha} \sim 5.65 \times 10^2$ , or  $\tilde{\alpha} \approx 0.022$ . We can compare  $\tilde{\alpha}$  to the  $U(1)_Y$  coupling

constant of the Standard Model,  $\alpha_Y \sim 0.01$ , and we see that  $\tilde{\alpha}$  is physically reasonable. Moreover, we have  $f_\chi \sim 0.036 \times M_P \sim$  or  $f_\chi \sim 4 \times 10^{17}$  GeV. This causes the quintessence field to have a mass,  $m_\chi \sim \sqrt{\rho_c}/f_\chi \sim 1.6 \times 10^{-41}$  GeV, or, expressed as a Compton wavelength, of order  $\sim (400 \text{ Mpc})^{-1}$ . Larger values of the overall scale of the potential can be chosen, and correspondingly smaller  $f_\chi$  and larger  $\chi$  masses can be generated, since the field can relax while rolling in its potential.

We have provided only a crude estimate, and we will not presently give a detailed “cosmic evolution” of the  $\chi$  field. We remark, however, that the scale of the potential is so sensitive to the values of these parameters that it is conservative to say that  $f_\chi \gtrsim \text{few} \times 10^{17}$  GeV would yield an unacceptably large vacuum energy in the  $\chi$  field today. For these parameters we have a field that may begin rolling in its potential at redshifts  $z \sim 10$ . This is roughly in accord with the Late Time Phase Transition of Hill, Schramm and Fry, [10]. A network of soft domain walls may form. The extent to which  $\chi$  can be trapped in its potential, e.g., dwell near the maximum until redshifting like matter, thus providing an effective cosmological constant, remains to be explored. The cosmic evolution of a quintessence field  $\chi$  with these parameters is subtle and beyond the scope of the present paper.

One thing we learn immediately from this exercise is that it is evidently not possible to have a PNGB field that *naively* provides a cosmological constant today by being trapped in its potential due to Hubble damping. To achieve this condition we would require a potential of the form:

$$\sim \Lambda_c \cos(\chi/f) \tag{2.23}$$

and the mass would have to satisfy  $m_\chi^2 = \Lambda_c/f^2 \lesssim H^2 \sim \Lambda_c/M_p^2$ , hence we would require  $f \sim M_P$ . However, this requires  $\tilde{\alpha} \rightarrow \alpha_{max}$ . and we cannot adequately suppress the Planck effects in this case. We suspect that this objection may be more general, and apply to any natural quintessence model. It also implies difficulties for PNGB inflaton models [21], which typically require  $f \sim M_P$  and Hubble damping to provide a cosmological constant phase.

The resulting overall scale of the potential for this “pure quintessence” model is extremely sensitive to the parameters we have used, and there is thus considerable fine-tuning of such a model. It would evidently be better to raise  $\alpha_{max}/\tilde{\alpha}$ , or  $M_P/f_\chi$  slightly and thus turn the effects of Planck scale physics off completely. In this case we require some other mechanism to generate the small WLPNGB mass. This can come from fermions to which the  $U(1)$  theory couples. Examples are the neutrino schizon models de-

scribed in refs. [10, 11], though some additional  $U(1)^N$  and  $Z_N$  apparatus must be added to satisfy the constraints described here. We examine presently an alternative scheme, a fermion which “propagates in the bulk.”

## 2.4 Effect of Fermions in the Bulk

Let us now incorporate a Dirac fermion which can be viewed as carrying the  $U(1)$  charge and propagating in the bulk. The continuum action is:

$$\mathcal{L} = \int d^4x \int_0^R dy \bar{\Psi}[(i\cancel{D} - g\cancel{A}) - (\partial_4 + igA_4)\gamma^5 - M]\Psi. \quad (2.24)$$

Latticizing and passing to a chiral basis of  $L$ -handed and  $R$ -handed fields on each brane, the theory can be written (*c.f.* eq. (2.37) of [18]):

$$\begin{aligned} & \sum_{n=1}^N \int d^4x \left[ \bar{\Psi}_{nL}(i\cancel{D})\Psi_{nL} + \bar{\Psi}_{nR}(i\cancel{D})\Psi_{nR} - \widetilde{M}(\bar{\Psi}_{nL}\Psi_{nR} + h.c.) \right] \\ & - \sqrt{2} \sum_{n=1}^N \int d^4x g \left[ \eta_1 \bar{\Psi}_{nL}\Phi_n\Psi_{(n+1)R} - \eta_2 \bar{\Psi}_{nR}\Phi_n\Psi_{(n+1)L} + h.c. \right] \end{aligned} \quad (2.25)$$

This lattice action for the Dirac fermion is discussed in detail in [18]. Here  $\Psi_{R,L} = \frac{1}{2}(1 \pm \gamma^5)\Psi$  and  $\widetilde{M} = M - \eta v$ , where  $M$  is the continuum theory’s Dirac mass of eq. (2.24). eq. (2.25) represents a faithful lattice description of the continuum theory. We have included the “Wilson term” which eliminates an unwanted spurious fermion flavor doubling in the spectrum. When the latticized model is constrained to match the low energy spectrum of the continuum theory, we find a nontrivial constraints on  $\eta_i$  [18],

$$\eta_2 \ll \eta_1 \equiv \eta. \quad (2.26)$$

Note that we can swap  $\eta_2 \leftrightarrow \eta_1$  under a parity transformation. The matching of the spectrum further requires:

$$v^2 = -\eta v \widetilde{M} = \eta v(\eta v - M), \quad \eta = \frac{M \pm \sqrt{M^2 + 4v^2}}{2v}. \quad (2.27)$$

If we implement these constraints, and substitute  $\Phi_n = (v/\sqrt{2}g) \exp(ig\chi_n/v)$ , the fermionic lattice action can be rewritten in terms of the parameters  $v$  and  $\eta$ , as:

$$\sum_{n=1}^N \int d^4x \left[ \bar{\Psi}_n(i\cancel{D})\Psi_n + (v/\eta) \bar{\Psi}_n\Psi_n - (\eta v \bar{\Psi}_{nL}\Psi_{(n+1)R} e^{ig\chi_n/v} + h.c.) \right] \quad (2.28)$$

We can regard eq. (2.28) as a 1+3 dimensional model with effectively two mass parameters,  $m_1 = v/\eta$  and  $m_2 = \eta v$ . A third allowed set of terms,  $\eta_2 v \Psi_{nR} \Psi_{(n+1)L} e^{ig\chi_n/v}$ , can in principle occur, but we assume  $\eta_2 \ll \eta$  by at least an order of magnitude, and does not affect the present estimates.

The fermion couplings to the Higgs-link fields of eq. (2.25) cause the WLPNGB to develop a Coleman-Weinberg effective potential [18]. For  $\eta \approx \pm 1$  the  $\chi_0$  acquires a large mass of order  $\tilde{\alpha} v^2/N^2 \sim \tilde{\alpha}^2 f_\chi^2 \sim \tilde{\alpha}/R^2$ . In this limit the WLPNGB cannot be an ultra-low-mass spin-0 field if the compactification scales,  $R^{-1} = v/N$ , are at least as large as  $\sim$  many (TeV) $^{-1}$ .

For large or small  $\eta$  we see that either the chiral breaking  $m_1$  terms disappear, or the chiral coupling  $m_2$  terms disappear, and the Coleman Weinberg effective potential becomes small. It actually does not matter which limit to use, since the Coleman-Weinberg potential depends symmetrically upon  $\eta^2 + \eta^{-2}$  and  $v$  in this limit. Define:

$$\omega = \sqrt{\eta^2 + \eta^{-2}}. \quad (2.29)$$

In the large  $\omega$  limit we obtain an exponentially suppressed Coleman-Weinberg potential [18]:

$$V = -\frac{v^4}{4\pi^2} \frac{\omega^{-2(N-2)}}{N^2} \cos\left(g\sqrt{N}\chi_0/v\right) \quad (2.30)$$

which can be written in terms of low energy parameters:

$$V = -\frac{\omega^4 \alpha_{max}^2 e^{-(\alpha_{max}/\tilde{\alpha}) \ln(\omega^2)}}{4\pi^2 R^4 \tilde{\alpha}^2} \cos(\chi_0/f_\chi) \sim -4\pi^2 (\omega^4 f_\chi^4) e^{-(4\pi/\tilde{\alpha}) \ln(\omega^2)} \cos(\chi_0/f_\chi). \quad (2.31)$$

In the latter expression we used the ‘‘unitarity bound,’’  $\alpha_{max} = 4\pi$ .

We can thus choose the Planck-scale effects to be arbitrarily small, and generate the WLPNGB mass for quintessence from eq. (2.31). For example, with the choice of parameters  $x = 4\pi/\tilde{\alpha} = 10^2$  ( $\tilde{\alpha} \approx 0.12$ ) and  $f_\chi \lesssim 6.3 \times 10^{16}$  GeV, we see that the Planck scale effects become miniscule. We can then have the fermion contribution to the potential dominate and provide vacuum energy of order  $\rho_c$  with  $\omega \sim 3.9$ . This field has a mass of  $m_\chi \sim 5.0 \times 10^{-41}$  GeV, hence a Compton wavelength of order  $\sim 120$  Mpc.

### 3 Theory of the Axion

The previous discussion shows that it is possible to reduce Planck scale effects well below the order of the closure density with  $f \lesssim 10^{-2} \times M_P$  and  $\tilde{\alpha}$  of order the  $U(1)_Y$  electroweak

coupling constant. This is a far better control than is required to adapt the scheme to a theory of the axion.

Consider a scheme in which we incorporate the Standard Model gauge groups  $SU(3) \times SU(2)_L$  and all matter fields and Higgs fields into the latticized effective theory. We place all of these Standard Model fields on a particular lattice brane, e.g.,  $n = 1$  without loss of generality. We allow, however, the  $U(1)_Y$  to fill the bulk, i.e., we replace  $U(1)_Y \rightarrow [U(1)_Y]^N$ , hence we take the  $U(1)_Y$  Lagrangian to be of the form of eq. (2.3), describing a latticized, periodically compactified, extra dimension. One can simply view this as a chain model of  $U(1)$ 's in  $1 + 3$  dimensions.

Hence, the model we are considering from the  $1 + 3$  perspective is  $SU(3) \times SU(2)_L \times U(1)_Y^N$ . The weak hypercharges of the  $U(1)_{Y_n}$  will be denoted  $Y_n$ . All Standard Model fermions and the Higgs field couple only to  $U(1)_{Y_1}$  with the usual weak hypercharges,  $Y_q = Y_{q_1}$ . The  $U(1)_{Y_1}$  coupling constant is  $g$ , the common high energy value of all  $U(1)_{Y_n}$ 's, but the zero mode gauge field will couple with strength  $g_1 = g/\sqrt{N}$ , hence:

$$\alpha(M_Z)_{QED}^{-1} \sim 128; \quad \tilde{\alpha}(M_Z) = \frac{\alpha(M_Z)_{QED}}{\cos^2 \theta_W} \approx 0.01. \quad (3.32)$$

This model reproduces as a low energy effective Lagrangian, the Standard Model, with the zero-mode gauge field of the  $U(1)_{Y_n}$  playing the role of the usual  $U(1)_Y$  gauge field (for details of this kind of structure see, [16]).

One might object to a periodic compactification of the  $U(1)_Y$  given the chiral fermionic zero-modes on brane-1. Chiral fermions are usually engineered in extra-dimensional theories with domain walls, and/or require ‘‘orbifold’’ compactification. Orbifold compactification, unlike periodic compactification, does not produce a zero-mode  $\chi_0$ . The orbifold can, however, be considered as a dynamical system. It can be viewed as a ‘‘parallel brane capacitor’’ bounding an extra dimension, with the branes at, e.g.,  $x^4 = 0$  and  $x^4 = R$ , or a single brane with  $x^4 = 0$  and  $x^4 = R$  identified. The capacitor branes are a *Type II magnetic superconductor*, which is equivalent to a *confining phase* (magnetic superconductors admit electric flux tubes which confine electric charge). The gauge field boundary conditions are accordingly  $F_{\mu 4} = 0$  at  $x^4 = 0$  and  $x^4 = R$ . This implies no electric (magnetic) field perpendicular (transverse) to the brane at the boundary. It also implies (i)  $\partial_4 A_\mu = 0$  and (ii)  $\partial_\mu A_4 = 0$  at the boundary. (i) implies that there can exist a zero  $p^4$  momentum mode of  $A_\mu$  while (ii) implies that there cannot exist a zero- $p^4$  momentum mode of  $A_4$ . Hence,  $\chi_0 = 0$  with orbifold compactification. From such a ‘‘dynamical orbifold’’ point of view, certain fields, e.g.,  $SU(3)$ , might propagate in the bulk as if an orbifold, and ‘‘feel

the capacitor branes,” while other fields such as the  $U(1)_Y$  are decoupled from the parallel branes and are periodic. Hence, a geometrical extra-dimension interpretation may exist for our present model with dynamical orbifolding, but we will not elaborate this presently.

Moreover, from a 1 + 3, or “theory space,” point of view there is no problem with such a construction, since it may be viewed strictly as a 1 + 3 theory with  $N$  copies of the  $U(1)_Y$ , and we summarize a viable minimal scheme with  $N = 14$  below. We thus use, for the gauge part of our  $U(1)_Y^N$  theory, the Lagrangian of eq. (2.3). The zero-mode of the  $A_4$  gauge field in the continuum version of the theory is again the chiral phase in the product of linking Higgs fields,  $\prod_{n=1}^N \Phi_n$  of the  $[U(1)_Y]^N$  theory. This phase,  $\chi_0$ , will be identified with the axion. If no fermions propagate in the bulk, the  $\chi_0$  is massless, up to Planck scale effects. The Planck scale effects are especially small, much smaller than those considered for quintessence, given that we typically want  $f_\chi \sim 10^{12}$  GeV, and the  $\ln(M_P/f_\chi)$  is becoming  $\sim \mathcal{O}(10^1)$ , large. The problem, however, is that the axion, without fermions, is decoupled from the QCD field strength combination  $G^*G$ , where  $*G_{\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu\rho\sigma}G^{\rho\sigma}$ . This is required to provide the mechanism for canceling the  $\theta$  term.

Setting aside the problem of controlling Planck scale effects momentarily, we note that a particularly simple “bare-bones” theory of the axion could always be engineered by postulating a heavy, e.g.,  $M \gg \text{TeV}$ , vector-like color triplet whose mass is chiral,  $\sim M\bar{Q}_L Q_R e^{i\chi_0/f_\chi} + h.c..$  This theory is classically  $U(1)_L \times U(1)_R$  invariant, but the  $U_{R-L}(1)$  PQ-symmetry is broken by the QCD axial vector anomaly. Rephasing  $Q$  as  $Q_L \rightarrow e^{i\chi/2f}Q_L$  and  $Q_R \rightarrow e^{-i\chi/2f}Q_R$  removes the axion phase from the mass term, and gives it a derivative coupling to the axial current, but induces the Wess-Zumino term:

$$\frac{\alpha_{QCD}}{4\pi} \frac{\chi_0}{f_\chi} G_{\mu\nu}^a *G^{a\mu\nu}. \quad (3.33)$$

The point is that it is unnecessary to directly couple the axion into the light quark mass matrix. Through the effects of the anomaly in QCD, the W-Z term of eq. (3.33) will lead to mixing of the  $\chi$  field with the  $\eta', \eta, \pi^0$  and a mass for  $\chi$  will be induced of order  $\Lambda_{QCD}^2/f$  (this mixing also induces the  $(\chi_0/f_\chi)F^*F$  anomalous electromagnetic coupling). The VEV of  $\chi_0$  at the potential minimum will identically cancel the unwanted QCD  $\theta$  angle.

It is actually not necessary that the PQ-symmetry be classically exact. One can add (before rephasing) a “small chiral breaking” term of the form  $\delta m \bar{Q}_L Q_R + h.c..$  This would induce in loops a quadratically divergent axion mass,  $\sim \Lambda^2 \delta m M \cos(\chi/f)$ . However, we know how to soften this by invoking a  $Z_N$  symmetry, with more  $Q$  fields, so we can ignore

it presently. This, however, affects the anomaly, since it redefines  $\chi$  as:

$$\overline{M}e^{i\chi'/f} = Me^{i\chi/f} + \delta m, \quad \overline{M}^2 = M^2 + (\delta m)^2 + 2\delta m M \cos(\chi/f), \quad (3.34)$$

and it is  $\chi'$  that is ultimately rephased and couples as in eq. (3.33). This is a correction to the anomaly coming from the explicit chiral breaking embodied in  $\delta m$ . Note that as  $\delta m/M \rightarrow 0$ , we have  $\chi' \rightarrow \chi$ . As long as  $\delta m$  is not too large, and the induced potential for the axion is sufficiently negligible, this has no effect on the low energy dynamics.

For us, the simplest method to engineer the necessary coupling of the axion to  $G^*G$  is to imitate this bare-bones model and introduce  $N$  copies of a vector-like (heavy) quark color triplet  $\Psi_n$  with sequentially different weak hypercharges,  $(0, 0, \dots, 0, -1_n, 0, 0, 0)$  for  $(U(1)_{Y1}, \dots, U(1)_{YN})$ . One can view this as a fermion that propagates in the bulk. We could also allow  $SU(3)$  to propagate in the bulk, however, it is convenient to simplify the model, and simply assign each of these fields to be a color triplet relative to single  $SU(3)_{QCD}$  gauge group. We thus have an anomaly free representation of  $\Psi_n$ 's under the full  $SU(3)_{QCD} \times SU(2)_L \times U(1)_Y^N$ .

This common color triplet assignment indeed appears to have nothing to do with a geometrical theory. To make contact with geometry, we require QCD itself propagate in the bulk, and to have the lattice description  $[SU(3)]^N$ . Indeed, we have an impetus to allow QCD in the bulk: Topcolor, and in particular, the Top Seesaw model [20], is a successful dynamical scheme for breaking the weak interactions in analogy to BCS theory, and can be viewed as an extra dimensional theory with  $SU(3)$  and the top quark propagating in the bulk. One can view the present common triplet model as a technical simplification of a more complete QCD-in-the-bulk theory.

The common triplet scheme is sufficient to generate the required anomalous coupling of the axion. Again we have the Lagrangian of eq. (2.25,2.28). The couplings to the axion, which is the  $\chi_0$  zero mode, are:

$$\begin{aligned} & \sum_{n=1}^N \int d^4x \left[ \overline{\Psi}_n (i\not{D}) \Psi_n + (v/\eta) \overline{\Psi}_n \Psi_n \right] \\ & + \sum_{n=1}^N \int d^4x \left[ (-\eta v \overline{\Psi}_{nL} \Psi_{(n+1)R} + \eta_2 v \overline{\Psi}_{nR} \Psi_{(n+1)L}) e^{i\chi_0/f} + h.c. \right] \end{aligned} \quad (3.35)$$

Again we assume  $\eta_2 \ll \eta_1 \equiv \eta$ , the form apropos the Wilson term, but we emphasize that  $\eta_2$  need not be hierarchically smaller than  $\eta_1$ .

The  $(v/\eta)$  diagonal terms which explicitly break the  $U(1)_{PQ}$  chiral symmetry are the



analogue of  $\delta m$  described above. We would require fixed  $\eta v$  and  $v/\eta \rightarrow 0$  to recover exact chiral symmetry, whence the Coleman-Weinberg potential would vanish.

The Coleman-Weinberg potential is given as before in terms of  $\omega = \sqrt{\eta^2 + \eta^{-2}}$ , for  $\omega$  large, by the exponentially suppressed potential:

$$V \sim -4\pi^2 \left( \omega^4 f_\chi^4 \right) e^{-(4\pi/\tilde{\alpha}) \ln(\omega^2)} \cos(\chi_0/f_\chi). \quad (3.36)$$

where we have used  $\alpha_{max} = 4\pi$ .

The dominant contribution to the axion potential will come from QCD, due to the usual instanton effects. The key to our present theory is that we must insure the axion is not pulled away from its QCD potential minimum, where  $\theta_{QCD}$  is canceled, by more than  $1 : 10^{-9}$ . We thus require:

$$4\pi^2 (f_\chi^4 \omega^4) e^{-(4\pi/\tilde{\alpha}) \ln(\omega^2)} \lesssim 10^{-9} \Lambda_{QCD}^4. \quad (3.37)$$

Using  $f_\chi \sim 10^{12}$  GeV, and  $\tilde{\alpha} \sim \alpha_Y \sim 0.01$ , we see that  $\omega \gtrsim 1.1$  causes the *lhs* of eq. (3.37) to become negligible. Since  $\omega \geq \sqrt{2}$ , any value of  $\eta$  may be used, i.e.,  $\eta$  need not be tuned excessively large or small. More relevant is the validity of our large- $\omega$  approximation; we expect that it requires  $\eta \sim \omega \gtrsim 10$ . Moreover, the Planck scale effects must also be suppressed to a comparable level:

$$M_P^4 e^{-(4\pi/\tilde{\alpha}) \ln[\sqrt{\tilde{\alpha}} M_P / \sqrt{2\pi} f_\chi]} \lesssim 10^{-9} \Lambda_{QCD}^4. \quad (3.38)$$

With the parameters  $f_\chi \sim 10^{12}$  GeV, and  $\tilde{\alpha} \sim \alpha_Y \sim 0.01$  we see that the *lhs* is negligible.

Since the Planck scale effects are miniscule with  $f_\chi = 10^{12}$  GeV, we might ask what is the smallest value of  $N = \alpha_{max}/\tilde{\alpha}$  we can choose such that both the Planck scale and  $\Psi_n$   $\theta$ -pull effects are still negligible? Our underlying theory will contain a fermion mass term  $v\eta$  or  $v\eta^{-1}$  in excess of  $M_P$  unless we also enforce  $\omega \lesssim M_P/v \sim M_P/\tilde{g} N f_\chi \sim 3.4 \times 10^7/N$ . The Planck scale  $\theta$ -pull effects are:

$$M_P^4 e^{-N \ln[\sqrt{2} M_P / \sqrt{N} f_\chi]} \lesssim 10^{-9} \Lambda_{QCD}^4 \quad (3.39)$$

We see that  $N > 13$  suffices to reduce these to a negligible status with  $f_\chi \sim 10^{12}$  GeV. Then the fermion  $\theta$ -pull effects require:

$$4\pi^2 (f_\chi^4 \omega^4) e^{-2N \ln(\omega)} \lesssim 10^{-9} \Lambda_{QCD}^4 \quad (3.40)$$

Hence, with  $N = 14$  the  $\theta$ -pull effects are satisfied with the range  $4.1 \times 10^2 \lesssim \omega \lesssim 2.5 \times 10^6$ . Thus, the minimal model has  $N = 14$  copies of  $U(1)_Y$ , with the low energy coupling

$\tilde{\alpha} = \alpha_Y = 0.01$ ,  $f_\chi = 10^{12}$  GeV. Using  $\omega = 4.1 \times 10^2$ , the Lagrangian of eq. (3.35) has the chirally invariant mass term  $v\eta \approx 2 \times 10^{15}$  GeV and the chiral breaking term  $v/\eta \approx 10^{10}$  GeV.

Computing the loops involving the  $\Psi_n$  generates an anomalous coupling of  $\chi_0$  to two gluons, and to pairs of the  $U(1)_Y$  field and its KK-modes [18]. The anomalous coupling of  $\tilde{\chi}_0$  to the  $[U(1)_Y]^N$  was computed in the companion paper [18], and the QCD coupling is similarly easy to derive. We obtain:

$$\frac{\eta\chi_0}{16\pi^2\omega f_\chi} \left[ 4\pi N\tilde{\alpha}_3 G_{\mu\nu}^a \star G^{a\mu\nu} + 4\pi\tilde{\alpha}_1 \sum_{n=1}^N F_{n\mu\nu} \star F^{a\mu\nu} \right]. \quad (3.41)$$

Notice the large coefficient of  $N$  in the QCD piece. This coefficient would be unity if we allowed QCD to propagate in the bulk, and  $G_{\mu\nu}^a \star G^{a\mu\nu} \rightarrow \sum_{n=1}^N G_{n\mu\nu}^a \star G_n^{a\mu\nu}$ . The  $\eta/\omega$  coefficient reflects the explicit chiral symmetry breaking for small  $\eta$ , and the decoupling of  $\chi_0$  from the fermions. For large  $\eta$ , we have  $\eta/\omega \rightarrow 1$  and the coupling is given purely by the anomaly.

In summary, from a  $1 + 3$  dimensional (“theory space”) point of view we have built a natural model of the axion based upon the gauge group  $SU(3) \times SU(2)_L \times U(1)_Y^N$  where, e.g., we can minimally choose  $N = 14$ .  $U(1)_Y^N$  is broken by  $N$  linking Higgs-fields  $\Phi_n$  as in eq. (2.3). The model includes  $N$  vector-like color triplets that carry sequential  $U(1)_n$  charges with the Lagrangian of eq. (3.35). The axion is then the chiral phase (zero-mode) contained in the product  $\tilde{\Phi} \propto \prod_{n=1}^N \Phi_n$ . This suppresses the Planck scale corrections to the axion mass adequately to permit the cancellation of the QCD  $\theta$ -angle to well within  $1 : 10^9$ , and to yield the usual QCD-induced axion mass  $\sim \Lambda_{QCD}^2/f_\chi$ . From an extra-dimensional point of view the axion is the Wilson line wrapping around a fifth compact dimension. More generally,  $N$  can be taken as large as  $4\pi/\alpha_Y \sim 1.3 \times 10^3$  in the latticized extra dimension scheme, or as small as  $N \approx 14$ .

## 4 Conclusions

We have generated naturally low-mass pseudo-Nambu-Goldstone bosons, relatively immune to the effects of Planck scale breaking of global symmetries, and which can be used for a number of phenomenological purposes. These low mass PNGB's are the Wilson lines wrapping around extra compact dimensions in the presence of heavy fermions that propagate in the bulk. Or, they are products of fields occurring in an equivalent  $1 + 3$  theory. Because they are nonlocal objects, Planck scale effects that normally destroy global symmetries are suppressed.

It is, in fact, somewhat remarkable that we can make reasonable estimates of Planck scale effects in these models and obtain nontrivial constraints. This stems from using the lattice approach and latticizing the physics of a compact extra dimension, to rewrite it in terms of a  $1 + 3$  dimensional theory. In a sense, latticization (or deconstruction) which is normally viewed as a configuration space cut-off, is an operator product expansion which isolates the relevant components of the theory necessary to describe its low energy physics. The operator product expansion, which reliably accommodates the scaling behavior of various operators, is what allows us to estimate the magnitude of the Planck scale effect in the  $1 + 3$  effective theory.

These fields can be quintessence fields. We have not given a detailed analysis of the cosmic evolution of these fields, but they can yield interesting effects at late times in the early Universe. Such fields have been anticipated in earlier works [10, 11] but until now the deleterious Planck scale effects have not been addressed [13].

We have also been able to solve the nagging problem of Planck scale effects and the axion. With  $f_a \sim 10^{12}$  GeV, we can construct models in “theory space” with  $14 \lesssim N \lesssim 10^3$  heavy fermions and an approximate  $U(1)$  PQ-symmetry, sufficient to yield an axion with no significant Planck effects, with no significant  $\theta$ -pull, thus allowing the axion to cancel the unwanted  $\theta$ -term of QCD.

A point worth emphasizing is that the Coleman-Weinberg potential arises naturally in these schemes as a finite quantity [9]. There is no renormalization of the potential that is implemented “by hand,” as in the case of massless scalar electrodynamics. This suggests new directions in thinking about improved models of the inflaton.

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