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A New Very Cool White Dwarf Discovered by the Sloan Digital Sky Survey

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ABSTRACT

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Early data taken during commissioning of the SDSS have resulted in the discovery of a very cool white dwarf. It appears to have stronger collision induced absorption from molecular hydrogen than any other known white dwarf, suggesting it has a cooler temperature than any other. While its distance is presently unknown, it has a surprisingly small proper motion, making it unlikely to be a halo star. An analysis of white dwarf cooling times suggests that this object may be a low-mass star with a helium core. The SDSS imaging and spectroscopy also recovered LHS 3250, the coolest previously known white dwarf, indicating that the SDSS will be an effective tool for identifying these extreme objects.

Subject headings: Stars: atmospheres — Stars: individual (SDSS 1337+00) — White Dwarfs

1. INTRODUCTION

The identification and study of very cool white dwarf (WD) stars is important for understanding the evolution of WDs, their luminosity function, their space density, and their total formation rate. This study is particularly important for WDs in the Galactic halo, because they are sufficiently old that most are expected to have cooled to very low temperatures (Hansen 1998; Isern et al. 1998; Chabrier 1999). Until recently, the coolest WDs known had temperatures ~ 4000 K (Bergeron et al. 1997; Leggett et al. 1998). The discovery (Harris et al. 1999; Hodgkin et al. 2000) of two WDs that definitely have temperatures below 4000 K has shown that such cool WDs do exist and has added impetus for further searches.

Very cool WDs will have molecular hydrogen in their atmospheres if any hydrogen is present. In these high-density atmospheres, collisions can induce a temporary electric dipole moment in the hydrogen molecules. Calculations predict (Borosow et al. 1989; Lenzuni et al. 1991; Saumon et al. 1994; Borosow et al. 1997) a high opacity at infrared wavelengths, referred to as collision-induced absorption (CIA). This opacity produces a spectral energy distribution with greatly depressed red/infrared flux that should result in distinctive colors (Bergeron et al. 1995a; Bergeron et al. 1997; Hansen & Phinney 1998; Hansen 1998; Hansen 1999; Saumon & Jacobson 1999; Chabrier 1999). The Sloan Digital Sky Survey (SDSS) (York et al. 2000) should be especially effective at identifying such stars (Harris et al. 1999; Hansen 2000). This survey is obtaining deep CCD images of 10,000 deg² of the north Galactic cap in five photometric bands (u' , g' , r' , i' , and z') to a limiting magnitude $r' \sim 23$ – see Fukugita et al. (1996) and Appendix A of Fan et al. (2001) for details of the photometric system.

This paper describes the first such star found by SDSS in its commissioning data. This star appears to have stronger CIA and, therefore, is probably cooler than the two previously known sub-4000 K WDs. In addition, the rediscovery of LHS 3250, previously the WD with strongest CIA, is reported. Both (re)discoveries indicate that SDSS can find other similar stars.

2. OBSERVATIONS

Imaging data from the SDSS 2.5 m telescope and its survey camera (Gunn et al. 1998) now include most of the equatorial stripe with $-1.25^\circ < \text{Dec} < 1.25^\circ$ and outside of the Galactic plane, as well as a number of scans away from the equator. The two stars reported here are a new white dwarf, SDSSp J133739.40+000142.8 (hereafter referred to as SDSS 1337+00; “p” indicates that the coordinates are preliminary), and LHS 3250 (Luyten 1976). They were imaged on 1999 March 21 and 2000 April 04, respectively. Their positions, proper motions, and magnitudes are given in Table 1²¹. A finding chart for SDSS 1337+00 is shown in Figure 1. The proper motion is derived from the SDSS position and the position in the USNO-A2.0 catalog (Monet et al. 1998) measured on the Palomar Observatory Sky Survey O and E plates taken in 1952. For comparison, Table 1 also includes data taken from the literature (Hambly et al. 1999; Hodgkin et al. 2000) for the cool WD WD0346+246. Magnitudes in the SDSS system and in the Johnson/Cousins/CIT system are included in Table 1, where some of the values not observed directly have been transformed from the other system using predicted relations for normal stars (Fukugita et al. 1996). These estimated values are probably accurate to better than 0.1 mag. The J magnitude limit for SDSS 1337+00 was kindly obtained by F. Vrba with the USNO 1.55 m telescope and ALADDIN InSb array detector.

The colors of SDSS 1337+00 and LHS 3250 are highly unusual, as shown in Figure 2. They are plotted along with a sample of normal WDs with synthesized SDSS colors (Lenz et al. 1998), and with observed J magnitudes (Bergeron et al. 1997; Leggett et al. 1998). The two curves show the colors of H and He WD model atmospheres (Bergeron et al. 1995b) with $\log g = 8$ (kindly calculated and made available to us by Pierre Bergeron). The deviation of the spectral energy distributions of these two stars from normal white dwarfs and non-degenerate stars is dramatic. SDSS 1337+00 has colors in Figure 2(b) and 2(c) even more extreme than those for LHS 3250.

Spectroscopic “plates” (exposures with fiber configurations covering the 3-degree field of view) are taken by SDSS after imaging data are available — objects (primarily galaxies and QSO candidates) are selected for observation based on their image morphology and colors. QSOs with redshift $3 < z < 4$ have gri colors (Fan 1999; Fan et al. 2001) similar to those predicted for very cool WDs. Objects with such colors will be selected as high-redshift QSO candidates, and those brighter than $i' \approx 20$ will be given high priority for spectroscopic observation as QSO targets. They also will be selected as cool-WD candidates, but given lower priority for spectra. Both SDSS 1337+00 and LHS 3250 were allocated fibers as QSO candidates.

The two SDSS fiber spectrographs²² cover 3800-9200 Å at a spectral resolution of 1800. The

²¹The quoted SDSS *asinh* magnitudes are on the AB system (Lupton et al. 1999), and errors are internal. Only preliminary colors and magnitudes are available, until the SDSS photometric system has been established; u^* , g^* , r^* , i^* , and z^* are used to designate these preliminary values. Final magnitudes and colors are not likely to change by much more than a few hundredths. Note, the difference between *asinh* magnitudes and traditional logarithmic magnitudes is negligible at these magnitudes well above the survey limits.

²²See York et al. (2000) and <http://www.astro.princeton.edu/PBOOK/spectro/spectro.htm> for a more complete

exposure time is 45 min under optimum observing conditions, or more as necessary to reach a target S/N. The spectra are extracted and calibrated with an automated software pipeline (Frieman et al. 2001, in preparation). The spectra of SDSS 1337+00 and LHS 3250 were taken on 2000 May 07 (plate 0299, 75 min exposure) and 2000 May 28 (plate 0349, 90 min exposure), respectively. They have been smoothed to 6 Å resolution, and are shown in Figure 3. The spectrum of LHS 3250 has been scaled to match that of SDSS 1337+00 for ease of comparison; it agrees quite well with the spectrum previously published (Harris et al. 1999). There are no significant features in the spectrum of either star. The spectra show that SDSS 1337+00 has relatively less flux at red-infrared wavelengths, consistent with the colors shown in Figure 2.

3. ATMOSPHERIC PARAMETERS

The low flux at red-infrared wavelengths in SDSS 1337+00 appears similar to that in LHS 3250 but more pronounced. The most viable explanation is that we are seeing CIA from molecular hydrogen (Harris et al. 1999). Less extreme CIA is also seen in WD0346+246 (Hodgkin et al. 2000), and possibly in LHS 1126 (Bergeron et al. 1994) and F351-50 (Ibata et al. 2000). The discovery of CIA in these stars represents a striking confirmation of the qualitative trends expected from theoretical model atmospheres (Bergeron et al. 1995a; Hansen 1998; Hansen 1999; Saumon & Jacobson 1999). Figure 4 compares the spectrum of SDSS 1337+00 with models of pure hydrogen atmospheres at three cool temperatures. The discrepancy between the models and the observed spectrum is most likely the result of missing opacities in the theoretical models, as well as the likelihood of the admixture of helium into the hydrogen atmosphere to provide additional H₂-He CIA (Bergeron et al. 1995a). Improvements to both the atmosphere models and the theoretical opacity inputs are underway. For example, new models with low temperatures and mixed (helium-dominated) composition (Jørgensen et al. 2000) show the sensitivity of CIA and the output spectrum to the input assumptions. However, these new models only explore a few compositions out of many plausible possibilities; they still do not give improved fits to the spectra in Figure 2. Until further improved models are available, we can only deduce broad constraints on the effective temperature. The fact that the H₂ CIA is strong suggests the objects are somewhat cooler than 4000 K. While temperatures below 3000 K cannot be ruled out, such low temperatures imply cooling ages for some ranges of mass that exceed reasonable estimates for the age of the Galaxy (see Section 4), so a temperature range 3000-4000 K is suggested.

description of this instrument. A full description is in preparation (Uomoto et al. 2001).

4. THE NATURE OF SDSS 1337+00 AND LHS 3250

Much recent attention has been devoted to the existence of halo WDs, motivated by microlensing (Alcock et al. 1997) and optical (Ibata et al. 1999) observations, and a few WDs are known to be halo stars (Liebert et al. 1989). Are these two very cool WDs part of the halo population? The proper motion and distance of WD0346+246 give a tangential velocity of 175 km s^{-1} , indicating that star is indeed a halo object (Hambly et al. 1999). However, the proper motion of LHS 3250 is smaller ($v_{\text{tan}} = 81 \text{ km s}^{-1}$) and is consistent with membership in either the disk or the halo; meanwhile, its luminosity is not as low as expected for a presumably very old halo WD. The proper motion of SDSS 1337+00 is smaller yet. If it has an absolute magnitude similar to that of LHS 3250 ($M_V = 15.7$), then its distance will be about 54 pc and v_{tan} will be 46 km s^{-1} . Alternatively, if it has the lower luminosity ($M_V \sim 17.5 - 18$) expected for a conventional, $0.6 M_{\odot}$ WD cooling to a temperature where CIA becomes strong, then its distance will be about 20 pc and v_{tan} will be only 17 km s^{-1} . In either case (pending a parallax measurement), these two coolest WDs do not have the properties expected for halo stars.

To properly constrain the nature of these objects, we must consider not only WDs with conventional C/O cores, but also the known population of low mass ($\sim 0.45 M_{\odot}$ or less) helium core WDs found in binaries (Bergeron et al. 1992; Marsh et al. 1995). Such objects are the result of truncated stellar evolution in close binaries (Kippenhahn et al. 1967) and cool more slowly (for similar masses) as a result of the increased heat capacity of the helium core. Figure 5 shows the cooling time for WDs with hydrogen atmospheres (Hansen 1999; see also Benvenuto & Althaus 1999, Chabrier et al. 2000, Salaris et al. 2000). Conventional WDs ($\sim 0.6 M_{\odot}$) require ages > 8.5 Gyr to cool to 4000 K. WDs more massive than average can cool somewhat faster at late times due to the earlier onset of core crystallization. The aforementioned low mass helium core WDs can potentially cool below 4000 K much faster. A $0.23 M_{\odot}$ helium core WD may cool below 4000 K in ~ 6 Gyr. (Note that very low mass WDs may possess thick hydrogen envelopes and thus will cool more slowly, with a contribution to their luminosity from residual hydrogen burning (Driebe et al. 1998). If true, this will cause the curves shown in Figure 5 to turn upwards again below $\sim 0.25 M_{\odot}$.)

Further progress requires that we determine a temperature or a radius, because the luminosity is known (or soon will be, after the parallax of SDSS 1337+00 is measured). The radius varies from $1.4 \times 10^9 \text{ cm}$ for the $0.23 M_{\odot}$ model, to $5.3 \times 10^8 \text{ cm}$ for the $1.0 M_{\odot}$ model, so the luminosity at fixed temperature varies by a factor of 7 between these two possibilities. Fitting the spectrum with models may give very accurate temperatures when the models are improved (Section 3). The absolute magnitude of LHS 3250 ($M_V = 15.72$) suggests a low mass, helium core WD as the most likely of the above options. In many respects SDSS 1337+00 is similar — the proper motion and apparent magnitude are both consistent with it being an LHS 3250 analog, but slightly cooler, at two to three times the distance. Both stars are more likely to belong to the old disk or thick disk populations than to the halo.

Most WDs have masses $\sim 0.6 M_{\odot}$. Both low mass and high mass WDs are minority con-

stituents. Thus it seems surprising that such potentially anomalous WDs are among the first cool WDs found in the SDSS survey. This may, in part, be due to the strong color selection involved. WDs are only selected for spectroscopic follow-up if they appear to lie outside the main stellar locus in the *griz* passbands. Thus, the accelerated evolution to low temperatures at the high- and low-mass ends of the distribution may favor their detection. Identifying cool WDs with temperatures about 3500-4500 K that do not have such strong CIA is more difficult. Transition stars like WD0346+246 may be found using a combination of visible and near-infrared colors, or using an intermediate-band filter to find stars with no MgH absorption feature (Claver 1995; Metcalfe et al. 2000), and then doing follow-up spectroscopy. With spectroscopic plates taken and examined covering roughly 400 square degrees of sky, we have found two cool WDs (one previously known). Thus we might expect to find at least several tens in the full SDSS survey.

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Fig. 1.— Finding chart for SDSSp J133739.40+000142.8 taken from the SDSS r' image observed on 1999 Mar 21. The frame is $160''$ on a side.

Fig. 2.— Three color-color diagrams showing a sample of normal WDs and the two very cool stars reported here. The curves show the colors of model atmospheres (Bergeron et al. 1995b) of pure H (solid curves) and pure He (dashed curves) with $\log g = 8$.

Fig. 3.— Spectra taken with the SDSS 2.5-m telescope multifiber spectrograph of the known cool WD LHS 3250 and the new, more extreme star SDSS 1337+00. The flux of LHS 3250 has been scaled by a factor of 0.3 to show its close match to SDSS 1337+00 at the blue end of the spectrum.

Fig. 4.— Spectrum of SDSS 1337+00 compared with preliminary models of cool, pure hydrogen atmospheres. None of the models in this temperature sequence adequately fits the data (see text).

Fig. 5.— Cooling times for pure hydrogen models of a given mass to reach a temperature of 4000 K, 3500 K, and 3000 K. Stars with $M \geq 0.50M_{\odot}$ and a C/O core are shown by filled circles; stars with $M \leq 0.45M_{\odot}$ and a He core are shown by open circles.

Table 1. Observational Data

Parameter	SDSS 1337+00	LHS 3250	WD0346+24 ^a
RA	13 37 39.4	16 54 01.3	03 46 46.5
Dec	00 01 43	62 53 55	24 56 04
μ (mas yr ⁻¹)	182±10	566	1302
PA (degrees)	185±3	286	155
u^*	20.67±0.06	19.52±0.04	...
g^*	19.50±0.01	18.37±0.02	19.7 ^d
r^*	19.18±0.01	17.87±0.02	18.5 ^d
i^*	19.53±0.02	18.09±0.01	18.0 ^d
z^*	19.98±0.09	18.55±0.04	...
B	19.8 ^d	18.85±0.02	>20.4
V	19.3 ^d	18.07±0.01	19.06±0.01
R^b	19.1 ^d	17.74±0.02	18.30±0.06
I^b	19.2 ^d	17.87±0.02	17.54±0.02
J^c	>19.9	18.33±0.05	17.60±0.05

Note. — Units of right ascension are hours, minutes, and seconds, and units of declination are degrees, arcminutes, and arcseconds. Coordinates are given for equinox and epoch 2000.

^aValues taken from Hodgkin et al. (2000) and Hambly et al. (1999).

^b R and I magnitudes are on the Cousins system.

^c J magnitudes are on the CIT system.

^dValues estimated from transformations in Fukugita et al. (1996).









