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$\mathcal{F}(1)$ for $B \to D^* l\nu$ from Lattice QCD*

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We would like to determine $|V_{cb}|$ from the exclusive semi-leptonic decay $B \to D^* l\nu$. The differential decay rate is

$$\frac{d\Gamma}{dw} = \frac{G_F^2}{4\pi^3}(w^2-1)^{1/2} m_{D^*}^3 (m_B - m_{D^*})^2 \mathcal{G}(w) |V_{cb}|^2 |\mathcal{F}_{B\to D^*}(w)|^2, \tag{1}$$

where $w = v \cdot v'$ and $\mathcal{G}(1) = 1$. At zero recoil ($w = 1$) heavy-quark symmetry requires $\mathcal{F}_{B\to D^*}(1)$ to be close to 1. So, $|V_{cb}|$ is determined by dividing measurements of $d\Gamma/dw$ by the phase space and well-known factors, and extrapolating to $w \to 1$. This yields $|V_{cb}|\mathcal{F}_{B\to D^*}(1)$, and $\mathcal{F}_{B\to D^*}(1)$ is taken from "theory." To date models [1] or a combination of a rigorous inequality plus judgment [2] have been used to estimate $\mathcal{F}_{B\to D^*}(1) - 1$. In this work [3] we calculate $\mathcal{F}_{B\to D^*}(1)$ with lattice gauge theory, in the so-called quenched approximation, but the uncertainty from quenching is included in the error budget.

The "form factor" $\mathcal{F}_{B\to D^*}(w)$ is a linear combination of several form factors of the matrix elements $\langle D^*|\mathcal{V}^\mu|B\rangle$ and $\langle D^*|\mathcal{A}^\mu|B\rangle$. At zero recoil all form factors but h_{A_1} are suppressed by phase space, so

$$\mathcal{F}_{B\to D^*}(1) = h_{A_1}(1) = \langle D^*(v)|\mathcal{A}^\mu|B(v)\rangle, \tag{2}$$

which should be "straightforward" to calculate in lattice QCD. But a brute force calculation of $\langle D^*|\mathcal{A}^\mu|B\rangle$ would not be interesting: similar matrix elements like $\langle 0|\mathcal{A}^\mu|B\rangle$ and $\langle \pi|\mathcal{V}^\mu|B\rangle$ have 15–20% errors [4,5].

Thus, we have to involve heavy-quark symmetry from the outset: if we can focus on $h_{A_1} - 1$, we have a chance of success, because a 20% error on $h_{A_1} - 1$ is interesting: $0.2 \times 0.1 = 0.02$. There are three specific obstacles to overcome: (i) statistical uncertainties, (ii) normalization

*presented by A. S. Kronfeld at *Flavor Physics and CP Violation*, May 16–18, Philadelphia, Pennsylvania uncertainties in the lattice axial vector current, and (iii) how to treat heavy quarks since $m_b a \not\ll 1$. The first two need computational insight; the last two theoretical insight. In the last several years, we have developed tools to attack these problems [6–11].

At zero recoil heavy-quark symmetry implies [12,13]

$$h_{A_1}(1) = \eta_A \left[1_{\mathrm{Isgur-Wise}} + 0_{\mathrm{Luke}} + \delta_{1/m^2} + \delta_{1/m^3}\right] \tag{3}$$

where η_A is a short-distance coefficient of HQET, and the δ_{1/m^n} contain long-distance matrix elements. The structure of the $1/m_Q^n$ corrections is

$$\delta_{1/m^2} = -\frac{\ell_V}{(2m_c)^2} + \frac{2\ell_A}{(2m_c)(2m_b)} - \frac{\ell_P}{(2m_b)^2} \tag{4}$$

$$\delta_{1/m^3} = -\frac{\ell_V^{(3)}}{(2m_c)^3} + \frac{\ell_A^{(3)}\Sigma + \ell_D^{(3)}\Delta}{(2m_c)(2m_b)} - \frac{\ell_P^{(3)}}{(2m_b)^3} \tag{5}$$

where $\Sigma = 1/(2m_c) + 1/(2m_b)$ and $\Delta = 1/(2m_c) - 1/(2m_b)$. One must make sure to calculate η_A and the ℓs in the same renormalization scheme.

Lattice gauge theory with Wilson fermions has the same heavy-quark symmetries as continuum QCD, for all $m_Q a$. It therefore admits a description with HQET, provided $m_Q \gg \Lambda$ [7–10]. In this description, HQET matrix elements, such as the ℓs in Eqs. (4) and (5), are essentially the same as for continuum QCD. So, one needs some quantities with small statistical and normalization errors, whose heavy-quark expansion contains the ℓs. Then, one calculates the short-distance part in perturbation theory [6,14], extracts the ℓs from a fit, and reconstitutes $h_{A_1}(1)$.

In our work on the $B \to D$ form factor [11], we found that certain ratios have the desired low level of uncertainty:

$$\frac{\langle D|\bar{c}\gamma^4 b|B\rangle\langle B|\bar{b}\gamma^4 c|D\rangle}{\langle D|\bar{c}\gamma^4 c|D\rangle\langle B|\bar{b}\gamma^4 b|B\rangle} = \left\{\eta_V^{\mathrm{lat}}\left[1 - \ell_P\Delta^2 - \ell_P^{(3)}\Delta^2\Sigma\right]\right\}^2, \tag{6}$$

$$\frac{\langle D^*|\bar{c}\gamma^4 b|B^*\rangle\langle B^*|\bar{b}\gamma^4 c|D^*\rangle}{\langle D^*|\bar{c}\gamma^4 c|D^*\rangle\langle B^*|\bar{b}\gamma^4 b|B^*\rangle} = \left\{\eta_V^{\mathrm{lat}}\left[1 - \ell_V\Delta^2 - \ell_V^{(3)}\Delta^2\Sigma\right]\right\}^2, \tag{7}$$

$$\frac{\langle D^*|\bar{c}\gamma^j\gamma_5 b|B\rangle\langle B^*|\bar{b}\gamma^j\gamma_5 c|D\rangle}{\langle D^*|\bar{c}\gamma^j\gamma_5 c|D\rangle\langle B^*|\bar{b}\gamma^j\gamma_5 b|B\rangle} = \left\{\check{\eta}_A^{\mathrm{lat}}\left[1 - \ell_A\Delta^2 - \ell_A^{(3)}\Delta^2\Sigma\right]\right\}^2. \tag{8}$$

For lattice gauge theory, the heavy-quark expansions in Eqs. (6)–(8) have been derived in Ref. [8], leaning heavily on Refs. [15]. The one-loop expansions of η_V^{lat} and $\check{\eta}_A^{\mathrm{lat}}$ are in Ref. [10]. Thus, these ratios yield all terms in δ_{1/m^3} except $\ell_D^{(3)}$.

We wish to obtain the $1/m_Q^2$ corrections to the double ratios, but the lattice action and currents do not normalize all such terms correctly. HQET reveals several sources of such contributions, in a systematic way [15,8]. The most crucial are the $1/m_Q^2$ corrections to the currents, which enter the double ratios as follows:

$$\frac{[1 - \lambda(X_b/m_b^2 - 1/m_c m_b + X_c/m_c^2)]^2}{[1 - \lambda(2X_c - 1)/m_c^2][1 - \lambda(2X_b - 1)/m_b^2]} = 1 - \lambda\left(\frac{1}{m_c} - \frac{1}{m_b}\right)^2, \tag{9}$$

where λ is proportional to λ_1 or λ_2, and X_Q/m_Q^2 indicates incorrect normalization. The correct normalization of the $1/m_c m_b$ terms is built into the current we used. The cancellation of the others is a key feature of the double ratios. Other contributions either vanish or are correctly normalized to order α_s [8]. This, and other matching uncertainties of order α_s^2 and $(\bar{\Lambda}/m_Q)^3$, are put into the error budget.

Fig. 1(a) shows the heavy-quark mass dependence from Ref. [3]. As expected, ℓ_V is the largest of the $1/m_Q^2$ matrix elements. Because of the fit, the value of ℓ_V is highly correlated with that of $\ell_V^{(3)}$, but the physical combination is better determined.

We have also studied the dependence of the calculation on the mass of the light spectator quark, over the range $0.4 \le m_q/m_s \le 1$. As seen in Fig. 1(b), there is a slight linear dependence on m_π^2, which is proportional to m_q. The points are correlated, so the trend is significant. The main effect of extrapolating in m_π^2 is to increase the statistical error. In addition, there must be a pion loop contribution [16], which is mistreated in the quenched approximation. We treat the omission of this effect as a systematic error.

After putting everything back together again, we find [3]

$$\mathcal{F}_{B\to D^*}(1) = 0.913^{+0.024}_{-0.017} \pm 0.016^{+0.003}_{-0.014}{}^{+0.000}_{-0.016}{}^{+0.006}_{-0.014}, \tag{10}$$

where the uncertainties stem, respectively, from statistics and fitting, HQET matching, lattice

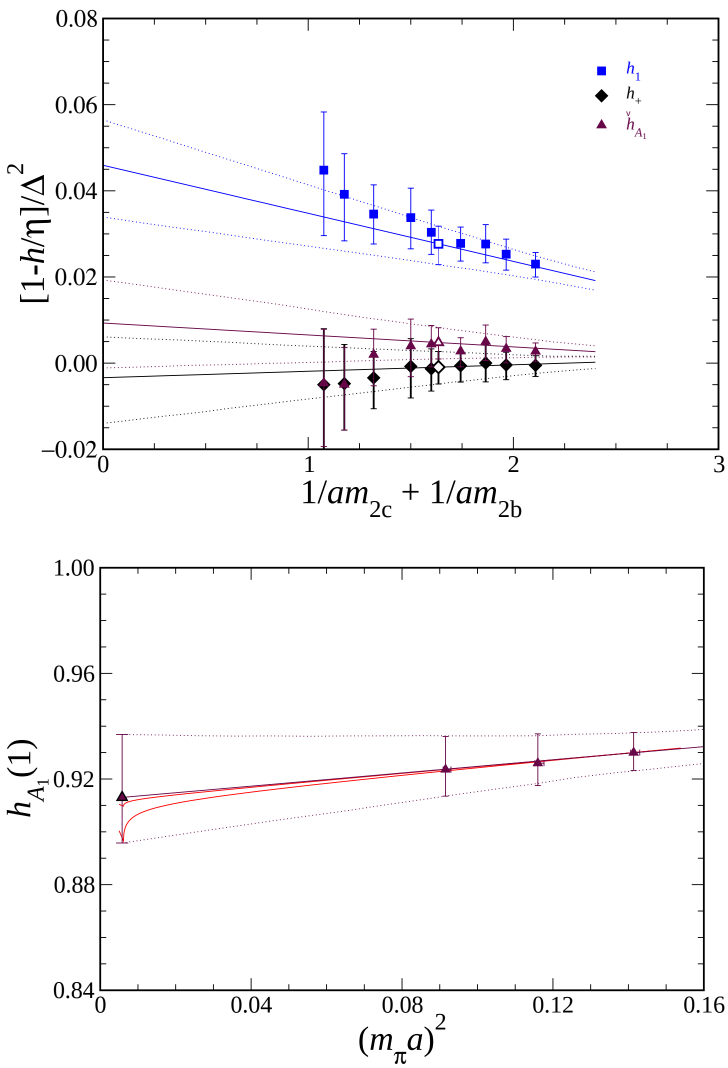

Figure 1: (a) Heavy-quark mass dependence of the double ratios. (b) Chiral extrapolation of $h_{A_1}(1)$.

spacing dependence, the chiral extrapolation, and the effect of the quenched approximation. In Fig. 2(a) we compare our result for $\mathcal{F}_{B\to D^*}(1)$ against the estimate based on the quark model [1], and on the sum rule [17]. The defects are as follows: The quark model omits some dynamics (more than quenching), and it is not clear that it gives the ℓs in the same scheme as η_A. The sum rule has an incalculable contribution from excitations with $(M - m_{D^*})^2 < \mu^2$, which can only be estimated. The present lattice result is in quenched approximation, but the error from quenching is the last error bar in Eq. (10).

For using this result in a global fit to the CKM matrix, it is useful to have some idea what values of theoretical quantities are more (or less) likely. A flat distribution would be incorrect, because the first error in Eq. (10) is essentially statistical, and the others are under some control. Also, one cannot rule out a tail for lower values; they are just unlikely. Finally, we know that $\mathcal{F}_{B\to D^*}(1) \leq 1$ [2]. A simple formula that captures these features is the Poisson distribution

$$P(x) = Nx^7 e^{-7x}, \quad x = \frac{1 - \mathcal{F}_{B\to D^*}(1)}{0.087}. \tag{11}$$

In the future one could reduce the uncertainty by a factor of 3, as sketched in Fig. 2(b), and one could provide a distribution stemming from the Monte Carlo calculation, and properly propagated through the systematic analysis.

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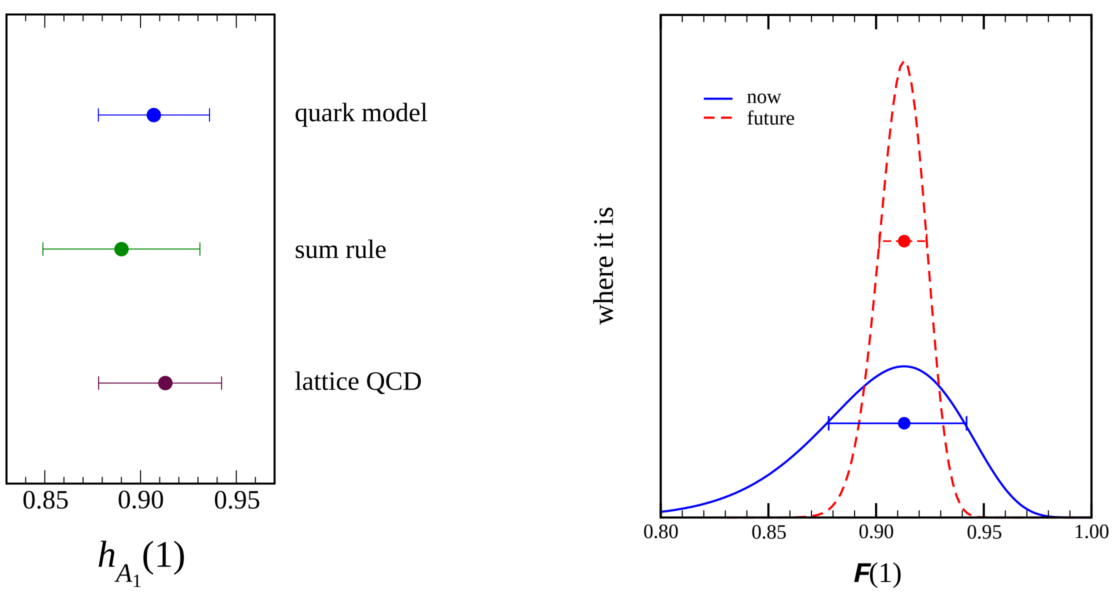

Figure 2: (a) Comparison of methods. (b) Simple Ansatz for more (and less) likely values, now and in the future.

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