



# Overview of Grand Unified Models and Their Predictions for Neutrino Oscillations <sup>1</sup>

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## Abstract

A brief overview of Grand Unified Models is presented with some attention paid to their predictions for neutrino oscillations. Given the well-known features of the two non-unified standard models, SM and MSSM, a listing of the features of classes of unified models is given, where a GUT flavor symmetry and/or family symmetry are introduced to reduce the number of model parameters. Some general remarks are then made concerning the type of predictions that follow for the neutrino masses and mixings.

*Key words:* Grand Unification; neutrino masses and mixings

## 1 Non-unified Standard Models

We begin with some relevant features of the two non-unified standard models. In the SM of particle physics, where the gauge group is  $SU(3)_c \times SU(2)_L \times U(1)_Y$ , no gauge coupling unification occurs at any scale [1]. Just one Higgs doublet of  $SU(2)_L$  is introduced,  $H_u = \phi = (\phi^+, \phi^0)^T$ , along with its charge conjugate,  $H_d = \tilde{\phi} = (\phi^0, -\phi^-)^T$ , with the symmetry broken by the vacuum expectation value (VEV),  $\langle \phi^0 \rangle = v = 174$  GeV. The left-handed quarks and leptons are placed in doublets, while the right-handed (or left-handed conjugate) quarks and charged leptons are taken to be singlets. The Yukawa Lagrangian can then be written as

$$\mathcal{L}_Y = \lambda_{ij}^u u_i^c Q_j H_u + \lambda_{ij}^d d_i^c Q_j H_d + \lambda_{ij}^e d_i^c L_j H_d \quad (1)$$

where  $Q_j$  and  $L_j$  refer to the  $j$ th family of quark and lepton left-handed doublets. Since no right-handed (or left-handed conjugate) neutrinos are assumed to exist, no renormalizable neutrino mass terms appear in eq. (1).

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In the case of the minimal supersymmetric standard model (MSSM), the same gauge group applies while two independent Higgs doublets,  $H_u$  and  $H_d$  are introduced with VEV's,  $v_u = \langle H_u^0 \rangle$ ,  $v_d = \langle H_d^0 \rangle$ , which give masses to the up-type quarks and down-type quarks, respectively. The constraint  $v = \sqrt{v_u^2 + v_d^2} = 174$  GeV holds, while the parameter  $\tan \beta = v_u/v_d$  remains arbitrary. The quark, lepton, scalar and gauge boson sectors are also doubled by the introduction of superpartners. With these extra SUSY particles present, gauge coupling unification occurs at a scale of  $\Lambda_G = 2 \times 10^{16}$  GeV [1]. The Yukawa superpotential has the same form as for the SM, still with many arbitrary Yukawa couplings and no renormalizable neutrino mass terms.

## 2 Reduction in the Number of Parameters

In order to reduce the number of free parameters in the standard models, one can introduce a flavor (or intra-family) symmetry, a family (or inter-family) symmetry, or both. Such procedures have mainly been carried out in the supersymmetry framework, since that allows the possibility of gauge coupling unification and can sufficiently suppress proton decay.

Flavor symmetry has generally been achieved in the framework of Grand Unified Theories (GUTs) which provide unified treatments of quarks and leptons, as (some) quarks and leptons are placed in the same multiplets. Examples involve  $SU(5)$ ,  $SU(5) \times U(1)$ ,  $SO(10)$ ,  $E_6$ ,  $SU(5) \times SU(5)$ , *etc.*

The introduction of a family or horizontal symmetry, on the other hand, enables one to build in an apparent hierarchy for different family masses belonging to comparable flavors. Such a symmetry may be discrete as in the case of  $Z_2$ ,  $S_3$ ,  $Z_2 \times Z_2$ , *etc.* which results in multiplicative quantum numbers. A continuous symmetry such as  $U(1)$ ,  $U(2)$ ,  $SU(3)$ , *etc.*, on the other hand, results in additive quantum numbers and may be global or local (and possibly anomalous).

Combined flavor and family symmetries will typically reduce the number of model parameters even more effectively. On the other hand, the unification of flavor and family symmetries into one single group such as  $SO(18)$  or  $SU(8)$ , for example, has generally not been successful, as too many extra states are present which must be made superheavy.

We now turn to illustrate various features of the different types of unification. For lack of space only a few selected references are given. A considerably more complete set of references can be found in a more general recent review [2].

## 3 MSSM with Anomalous U(1) Family Symmetry

In 1979 Froggatt and Nielsen [3] added to the SM a scalar singlet “flavon”  $\phi_f$ , which gets a VEV, together with heavy fermions,  $(F, \bar{F})$ , in vector-like representations, all of which carry  $U(1)$  family charges. With  $U(1)$  broken at

a scale  $M_G$  by  $\langle\phi\rangle/M_G \equiv \epsilon \sim (0.01 - 0.1)$ , the light and heavy fermions are mixed; hence  $\epsilon$  can serve as an expansion parameter for the quark and lepton mass matrix entries.

This idea received a revival in the past decade when it was observed by Ibanez [4] that string theories with anomalous  $U(1)$ 's generate Fayet-Iliopoulos D-terms which trigger the breaking of the  $U(1)$  at a scale of  $O(\epsilon)$  below the cutoff, again providing a suitable expansion parameter. The  $\epsilon^n$  structure of the mass matrices can be determined from the corresponding Wolfenstein  $\lambda$  structure of the CKM matrix and the quark and lepton mass ratios, where different  $U(1)$  charges are assigned to each quark and lepton field. But this procedure suffers from the fact that the coefficients (prefactors) of the  $\epsilon$  powers are not accurately determined.

With this scenario, Ramond [5] and many others have shown that maximal  $\nu_\mu \rightarrow \nu_\tau$  mixing of atmospheric neutrinos and the small mixing angle MSW solution for solar  $\nu_e \rightarrow \nu_\mu, \nu_\tau$  can be obtained. Bimaximal mixing solutions, however, are not obtained naturally.

#### 4 Minimal SUSY $SU(5)$

With the minimal SUSY  $SU(5)$  flavor symmetry, the matter fields are placed in  $\bar{\mathbf{5}}$  and  $\mathbf{10}$  representations:  $\bar{\mathbf{5}}_i \supset (d_\alpha^c, \ell, \nu_\ell)_i$ ,  $\mathbf{10}_i \supset (u_\alpha, d_\alpha, u_\alpha^c, \ell^c)_i$ . Higgs fields are placed in the adjoint and fundamental representations:  $\Sigma(24)$ ,  $H_u(5)$ ,  $H_d(\bar{5})$ . The  $SU(5)$  symmetry is broken down to the MSSM at a scale  $\Lambda_G$  with  $\langle\Sigma\rangle = (b, b, b, -\frac{3b}{2}, -\frac{3b}{2})$ , but doublet-triplet splitting must be done by hand. The electroweak breaking occurs when  $H_u$  and  $H_d$  VEV's are generated.

The number of Yukawa couplings has now been reduced in the Yukawa superpotential

$$W_{Yuk} = \lambda_{ij}^u 10_i \cdot 10_j \cdot H_u + \lambda_{ij}^d \bar{5}_i \cdot 10_j \cdot H_d \quad (2)$$

where by the particle assignments, the fermion mass matrices exhibit the symmetries,  $M_U = M_U^T$ ,  $M_D = M_D^T$  which implies at the GUT scale  $m_b = m_\tau$  but also  $m_d/m_s = m_e/m_\mu$ , since minimal  $SU(5)$  is too simplistic and no family symmetry is present.

Proton decay occurs through leptoquark gauge boson exchange with  $\tau_p = 10^{36 \pm 1.5}$  yrs. and through colored Higgsino exchange with  $\tau_p = 10^{32-33}$  yrs. [6] This places it nearly within reach but well above the old non-SUSY  $SU(5)$  limit of  $10^{29}$  years which has long been ruled out.

#### 5 SUSY $SU(5)$ with Anomalous $U(1)$

Kaplan, Lepintre, Masiero, Nelson and Riotto [7] have suggested an extension of the Froggatt-Nielsen idea as applied to  $SU(5)$  by addition of more singlet

Higgs flavons with different VEV's to replace different powers of the same ratio. The effective couplings then appear as  $f^c f H \langle \phi \rangle / \langle \chi \rangle$ , where  $W_{Yuk} = f^c F H + F^c F \chi + F^c f \chi$ . Matter and Higgs supermultiplets are each assigned their own family  $U(1)$  charges. Although there are more restrictions, the model is still not very predictive due to the unknown prefactors.

## 6 Flipped $SU(5) \times U(1)$ with Anomalous $U(1)$

This partially unified  $SU(5) \times U(1)$  GUT [8] has the following unconventional assignments: Matter Fields:  $\bar{5}_i \supset (u_\alpha^c, \nu_\ell, \ell)_i$ ;  $10_i \supset (u_\alpha, d_\alpha, d_\alpha^c, \nu^c)_i$ ;  $1_i \supset (\ell^c)_i$ ; Higgs Fields:  $\Sigma(10)$ ,  $\bar{\Sigma}(\bar{10})$ ,  $H(5)$ ,  $\bar{H}(\bar{5})$ . No  $b - \tau$  unification or seesaw mechanism occurs, while R-parity is broken.

## 7 SUSY $SO(10)$ with Family Symmetry

Here all fermions of one family are placed in a  $\mathbf{16}$  spinor supermultiplet and carry the same family charge assignment:  $\mathbf{16}_i(u_\alpha, d_\alpha, u_\alpha^c, d_\alpha^c, \ell, \ell^c, \nu_\ell, \nu_\ell^c)_i$ ,  $i = 1, 2, 3$ . Massive pairs of  $(\mathbf{16}, \bar{\mathbf{16}})$ 's and  $\mathbf{10}$ 's may also be present. The Higgs Fields:  $\mathbf{45}_H$ 's and  $\mathbf{16}_H, \bar{\mathbf{16}}_H$  break  $SO(10)$  to SM, while  $\mathbf{10}_H$  breaks the electroweak group. A  $\bar{\mathbf{126}}_H$  or  $\bar{\mathbf{16}}_H \cdot \bar{\mathbf{16}}_H \cdot 1_H$  can generate superheavy right-handed Majorana neutrino masses.

$t - b - \tau$  Yukawa coupling unification is possible only for  $\tan \beta = v_u/v_d \simeq 55$  in this minimal case. However, if a  $\mathbf{16}'_H, \bar{\mathbf{16}}'_H$  pair is introduced with the former getting an electroweak-breaking VEV which helps contribute to  $H_d$  [9], Yukawa coupling unification is possible for  $\tan \beta \ll 55$ . Such breaking VEV's can contribute asymmetrically to the down quark and charged lepton mass matrices. This makes it possible to understand large  $\nu_\mu - \nu_\tau$  mixing,  $U_{\mu 3} \simeq 0.707$ , while  $V_{cb} \simeq 0.040$ . Moreover, the Georgi-Jarlskog mass relations [10],  $m_s/m_b = m_\mu/3m_\tau$  and  $m_d/m_b = 3m_e/m_\tau$ , can be generated by the mass matrices with the help of the asymmetrical contributions just mentioned.

Various family symmetries appear in the literature in combination with the  $SO(10)$  group: unspecified [11],  $U(1) \times Z_2 \times Z_2$  [9],  $U(2)$  [12],  $U(2) \times U(1)^n$  [13],  $SU(3)$  [14], to name just a few. Again see [2].

## 8 Mass and Mixings Matrices

Complex symmetric mass matrices can be constructed for each flavor  $f = U, D, N, L$  in the GUT flavor basis

$$\mathcal{B}_f = \{f_{iL}, f_{iL}^c\}, \quad i = 1, 2, 3 : \quad \mathcal{M}_f = \begin{pmatrix} 0 & M_f^T \\ M_f & M_R \end{pmatrix} \quad (3)$$

where  $M_f$  is the  $3 \times 3$  Dirac mass matrix for flavor  $f$  and  $M_R$  is the  $3 \times 3$  superheavy Majorana neutrino mass matrix for  $f = N$ . The  $3 \times 3$  light neutrino mass matrix is determined by the seesaw mechanism,  $M_\nu = -M_N^T M_R^{-1} M_N$ .

The mixing matrices are obtained by diagonalizing the mass matrices according to  $M_f^{diag} = U_f^T M_f U_f$  from which one finds  $V_{CKM} = U_U^\dagger U_D$ ,  $U_{MNS} = U_L^\dagger U_\nu$ . Successful GUT models must essentially generate the CKM mixing matrix and one of the two MNS maximal (SMA MSW) or bimaximal (LMA MSW or Vacuum) mixing matrices corresponding to the still possible solar neutrino solutions:

$$V_{CKM} \simeq \begin{pmatrix} 0.975 & 0.220 & 0.0032e^{-i65^\circ} \\ -0.220 & 0.974 & 0.040 \\ 0.0088 & -0.040 & 0.999 \end{pmatrix} \quad (4)$$

$$U_{MNS}^{(SMA)} \simeq \begin{pmatrix} 0.99 & 0.04 & 0.05 \\ -0.03 & 0.70 & -0.71 \\ -0.03 & 0.71 & 0.70 \end{pmatrix} \quad (5)$$

$$U_{MNS}^{(bimax)} \simeq \begin{pmatrix} 0.71 & -0.70 & \theta_{13} \\ 0.50 & 0.50 & -0.71 \\ 0.50 & 0.50 & 0.70 \end{pmatrix} \quad (6)$$

where  $\theta_{13}$  is bounded to be  $< 0.2$  by the CHOOZ experiment [15]. The effective mixing angles and mass squared differences in question are:

$$\sin^2 2\theta_{atm} \equiv 4|U_{\mu 3}|^2|U_{\tau 3}|^2 \simeq 1.0 \quad (7)$$

$$\sin^2 2\theta_{sol} \equiv 4|U_{e1}|^2|U_{e2}|^2 \simeq \begin{cases} 0.006, & \text{SMA} \\ 1.0, & \text{LMA, Vac} \end{cases} \quad (8)$$

$$\sin^2 2\theta_{reac} \equiv 4|U_{e3}|^2(1 - |U_{e3}|^2) \sim \begin{cases} 0.05, & \text{SMA} \\ 0.001 - 0.1, & \text{LMA, Vac} \end{cases} \quad (9)$$

$$\Delta m_{32}^2 \simeq 3 \times 10^{-3} \text{ eV}^2, \quad \Delta m_{21}^2 \simeq \begin{cases} 6 \times 10^{-6} \text{ eV}^2, & \text{SMA} \\ 5 \times 10^{-5} \text{ eV}^2, & \text{LMA} \\ 4 \times 10^{-10} \text{ eV}^2, & \text{Vac} \end{cases} \quad (10)$$

The SMA MSW solution is readily found in many unified models, if the light neutrino diagonalizing matrix,  $U_\nu$ , is close to the identity matrix and the maximal atmospheric neutrino mixing is provided by the charged lepton diagonalizing matrix. Bimaximal mixing can be obtained with substantial off-diagonal contributions in the  $M_{NR}$  matrix. Of the two, the Vacuum solution tends to be somewhat more natural than the LMA MSW solution, although either can be obtained with careful tuning of the Dirac and Majorana matrices.

## 9 Partial Unification with Light Sterile Neutrinos

Light sterile neutrinos are somewhat of an anomaly in the presence of GUTs. They tend to spoil the seesaw mechanism, so it is not clear why the ordinary and sterile neutrinos remain light. Moreover, if one sterile neutrino is found, one expects many.

In  $SO(10)$ , light sterile neutrinos can be placed into  $\mathbf{1}$  representations, but the number is unexplained. In  $E_6$  one can associate one sterile neutrino with each 16 of  $SO(10)$  in the  $\mathbf{27}$ , since  $\mathbf{27} \supset 16 + 10 + 1$  for each family [16]. But one must insure that the members of the 10 of each  $\mathbf{27}$  become supermassive. Other groups suggested in the literature involve  $SU(5) \times SU(5)$  and  $SO(10) \times SO(10)$ , where the sterile neutrinos live in the second “mirror” groups and remain light while all other particles in the representations decouple from the known interactions [17].

## 10 Concluding Remarks

In order to obtain predictive models of the 12 “light” fermions and their 8 CKM and MNS mixing angles and phases in the framework of this talk, a Grand Unified Model with family symmetry must be introduced.  $SO(10)$  models are more tightly constrained than  $SU(5)$  models and are more economical than larger groups like  $E_6$ , where more fields must be made supermassive. In fact, some  $SO(10)$  models do very well in predicting the 20 “observables” with just 10 or more input parameters [18].

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