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Weighing the Cosmological Energy Contents with Weak Gravitational Lensing

Lam Hui

Fermi National Accelerator Laboratory
P.O. Box 500, Batavia, Illinois 60510

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Lam Hui*

NASA/Fermilab Astrophysics Group
Fermi National Accelerator Laboratory
Box 500, Batavia, IL 60510-0500

*Electronic mail: lhui@fnal.gov

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ABSTRACT

Bernardeau et al. (1997), using perturbation theory, showed that the skewness of the large-scale lensing-convergence, or projected mass density, could be used to constrain Ω_m, the matter content of the universe. On the other hand, deep weak-lensing field surveys in the near future will likely measure the convergence on small angular scales ($\lesssim 10$ arcmin.), where the signal will be dominated by highly nonlinear fluctuations. We develop a method to compute the small-scale convergence skewness, making use of a prescription for the highly nonlinear three-point function developed by Scoccimarro and Frieman (1998). This method gives predictions that agree well with existing results from ray-tracing N-body simulations, but is significantly faster, allowing the exploration of a large number of models. We demonstrate that the small-scale convergence skewness is insensitive to the shape and normalization of the primordial (CDM-type) power spectrum, making it dependent almost entirely on the cosmological energy contents, through their influence on the global geometrical distances and fluctuation growth rate. Moreover, nonlinear clustering appears to enhance the differences between predictions of the convergence skewness for a range of models. Hence, in addition to constraining Ω_m, the small-scale convergence skewness from future deep several-degree-wide surveys can be used to differentiate between curvature dominated and cosmological constant (Λ) dominated models, as well as to constrain the equation of state of a quintessence component, thereby distinguishing Λ from quintessence as well. Finally, our method can be easily generalized to other measures such as aperture mass statistics.

Subject headings: cosmology: theory — gravitational lensing — large-scale structure of universe

1. Introduction

The correlated shear of images of distant galaxies provides a promising way to probe the intervening large scale structure of the universe (e.g. Blandford et al. 1991; Miralda-Escudé 1991; Kaiser 1992; see also Jain et al. 1999 [JSW hereafter] and ref. therein). The convergence can be constructed from a shear map (Kaiser & Squires 1993), which can be interpreted as a form of projected mass density (Kaiser 1992; our notation follows that of

Jain & Seljak 1997):

$$\kappa(\boldsymbol{\theta}) = \int_0^{\chi_s} d\chi \; w(\chi) \; \delta(r(\chi)\boldsymbol{\theta}, \chi) \tag{1}$$

where δ is the mass overdensity as a function of spatial position (and implicitly as a function of time as well, where the time and space coordinates fall on the photon null geodesic), $\boldsymbol{\theta}$ is the angular position on the sky, χ is the comoving distance along the line of sight, $r(\chi)$ is the angular diameter distance, and $w(\chi)$ is a weight function which depends on a combination of global geometrical distances and is proportional to the total matter density of the universe. The coordinates $\chi = 0$, χ_s denote respectively the positions of the observer and the sources or background galaxies. The so-called Born approximation has been assumed (see Bernardeau et al. 1997 for a discussion).

It is clear that κ is a valuable quantity for cosmology. One can derive important constraints on the cosmological density parameters Ω's through the dependence of κ on the global geometrical distances and the evolution of δ on the line cone. A commonly used statistic is its second moment $\langle\kappa^2\rangle$, or the two-point correlation function $\langle\kappa(\boldsymbol{\theta})\kappa(\boldsymbol{\theta}')\rangle$. However, it is clear that the second moment depends on the mass power spectrum as well (e.g. Jain & Seljak 1997; Kaiser 1998).

Bernardeau et al. (1997), using perturbation theory, showed that this degeneracy could be broken by using the convergence skewness $S_3 \equiv \langle\kappa^3\rangle/\langle\kappa^2\rangle^2$. It is customary to consider S_3 as a function of angular scale θ_R, assuming κ is first smoothed on scale θ_R. However, future weak lensing surveys are likely to yield measurements of S_3 first on small angular scales, $\theta_R < 10'$, both because the small-scale shear signal is stronger and also because of larger sampling fluctuations on large scales (see e.g. Van Waerbeke et al. 1999, JSW) . For sources at a redshift of $z = 1$, the peak contributions to the lensing signal will come from $z \sim 0.5$, which for $\theta_R < 10'$ translates into a comoving length scale of less than a few Mpc that is generally comparable to or smaller than the nonlinear scale (where the rms density fluctuation is of order unity). It is therefore expected that the perturbative treatment of Bernardeau et al. (1997) would not hold for these angular scales of interest. This has in fact been explicitly demonstrated by JSW using the technique of ray-tracing N-body simulations (see their Fig. 18; see also Couchman et al. 1998). Unfortunately, the prediction of skewness from N-body simulations can become prohibitively expensive, if one is interested in exploring a large number of cosmological models.

There is therefore a need for alternative methods to predict accurately and efficiently the small-scale skewness. It is our aim here to develop such a method. We will test it by comparing with existing results from simulations, and show that the small-scale skewness is a sensitive probe of Ω's. We will then apply it to cosmological models that have not been considered before in the context of weak lensing. In particular, we will predict the small angular-scale skewness for a quintessence model, where quintessence is a component of the cosmological fluid that has negative pressure (e.g. Peebles & Ratra 1988; Frieman et al. 1995; Coble et al. 1997; Turner & White 1997; Ferreira & Joyce 1998; Caldwell et al. 1998). Such models, which include the cosmological constant dominated models as a limiting case, are currently in favor in part because of recent Type Ia supernova measurements (Riess 1998; Garnavich 1998; Perlmutter 1998). We will demonstrate that the convergence skewness can provide interesting constraints on them.

2. The Convergence Skewness

Let us first give the expressions for the cosmology dependent geometrical quantities that appear in eq. (1). The comoving distance along the line of sight χ is given by (Peebles 1993)

$$\chi(z) = cH_0^{-1} \int_0^z dz' [\Omega_m (1+z')^3 + \Omega_k (1+z')^2 + \Omega_q (1+z')^{3(1+w_q)}]^{-1/2} \tag{2}$$

where z is the redshift of interest, H_0 is the Hubble parameter today, c is the speed of light, and the Ω's denote the fractions of the critical energy density today in various components: Ω_m for pressureless matter or dust, Ω_k for spatial curvature and Ω_q for quintessence or a fluid with negative pressure (its pressure p is related to its density ρ by $p = w_q \rho$, where $w_q < 0$), with the cosmological constant Λ as a limiting case ($w_q = -1$). The Ω's sum to unity. The angular-diameter distance $r(\chi)$ is given by $r(\chi) = K^{-1/2} \sin K^{1/2}\chi, \chi, (-K)^{-1/2} \sinh(-K)^{1/2}\chi$ for closed, flat and open models respectively, and $K = (\Omega_m - 1)c^{-2}H_0^2$. In other words, the metric is given by $ds^2 = -c^2dt^2 + a(t)^2(d\chi^2 + r(\chi)^2 d^2\theta)$, where $a(t) = 1/(1+z)$ is the expansion scale factor as a function of proper time t. The line-of-sight projection of δ in eq. (1) is weighed by the function $w(\chi)$:

$$w(\chi) = \frac{3}{4a} c^{-2} H_0^2 \Omega_m \frac{r(\chi)\, r(\chi_s - \chi)}{r(\chi_s)} \tag{3}$$

where χ_s is the comoving radial position of the sources. Note that here, as in the rest of the paper, we assume all sources are at the same redshift. Eq. (1) and (3) can be easily generalized to the case of multiple source-redshifts by integrating over contributions from different χ_s's. Statistical measures of κ for sources distributed in a realistic fashion can usually be approximated by having all sources at the same mean redshift (e.g. JSW).

The convergence skewness is defined by

$$S_3(\theta_R) \equiv \frac{\langle {\kappa_{\theta_R}}^3 \rangle}{\langle {\kappa_{\theta_R}}^2 \rangle^2} \,, \quad \kappa_{\theta_R} \equiv \int \kappa(\boldsymbol{\theta}') W_{\theta_R}(\boldsymbol{\theta} - \boldsymbol{\theta}') d^2\theta' \tag{4}$$

where W_{θ_R} is a smoothing kernel of radius θ_R (in this paper, we will use a top-hat). The utility of S_3 derives from, crudely speaking, the fact that its analogue for the mass overdensity ($\langle\delta^3\rangle/\langle\delta^2\rangle$) is quite insensitive to details of the power spectrum, especially on small scales. Hence, as we will see, S_3 is almost purely determined by the cosmological energy contents.

Combining eq. (1) and (4), it can be shown that

$$\begin{aligned} & S_3 = K_3/(K_2)^2 \qquad \text{with} \\ & K_3 \equiv (2\pi)^2 \int_0^{\chi_s} d\chi \frac{w(\chi)^3}{r(\chi)^4} \int d^2\ell_1 d^2\ell_2 B(\boldsymbol{\ell}_1/r(\chi), \boldsymbol{\ell}_2/r(\chi), \boldsymbol{\ell}_3/r(\chi)) \\ & \qquad \tilde{W}(\ell_1\theta_R)\tilde{W}(\ell_2\theta_R)\tilde{W}(\ell_3\theta_R) \\ & K_2 \equiv 2\pi \int_0^{\chi_s} d\chi \frac{w(\chi)^2}{r(\chi)^2} \int d^2\ell P(\ell/r(\chi))\tilde{W}(\ell\theta_R)^2 \end{aligned} \tag{5}$$

where $\tilde{W}(x)$ is the Fourier transform of the two-dimensional top-hat, $\tilde{W}(x) = 2J_1(x)/x$ with J_1 being the first-order Bessel function. The ℓ's represent the Fourier coordinates in angular space, in other words we are taking the small-angle approximation where spherical harmonics can be replaced by plane waves. The combination $\boldsymbol{\ell}_1 + \boldsymbol{\ell}_2 + \boldsymbol{\ell}_3$ vanishes. The three-dimensional mass power spectrum and bispectrum are respectively P and B. Our convention is: $\xi_2(|\mathbf{r}|) = \int d^3k P(k) e^{-i\mathbf{k}\cdot\mathbf{r}}$ and $\xi_3(\mathbf{r_1}, |\mathbf{r_2}|) = \int d^3k_1 d^3k_2 B(\mathbf{k_1}, \mathbf{k_2}, -\mathbf{k_1} - \mathbf{k_2}) e^{-i\mathbf{k_1}\cdot\mathbf{r_1} - i\mathbf{k_2}\cdot\mathbf{r_2}}$ where the $\mathbf{k}$'s denote the Fourier coordinates in three-dimensional space, and ξ_2 and ξ_3 are the two- and three-point correlation functions respectively. (For readers who are used to putting $(2\pi)^3$ under d^3k: simply replace all relevant expressions in this paper by $P \to P/(2\pi)^3$ and $B \to B/(2\pi)^6$.) Note how in eq. (5) the projection forces the $\mathbf{k}$'s to lie in the plane of the sky. The reader is referred to Kaiser (1992), Bernardeau et al. (1997) and Jain & Seljak (1997) for detailed derivations.

To compute S_3 on small angular scales, we need to understand the nonlinear evolution of P and B. The nonlinear behavior of P can be described by a scaling ansatz introduced by Hamilton et al. (1991), which was later extended by Jain et al. (1995) and Peacock & Dodds (1994; 1996). We will employ the latest version set out in the latter. Jain & Seljak (1997) have considered the two-point version of K_2 using this ansatz. Essentially, the ansatz consists of postulating that $4\pi k^3 P(k) = f[4\pi k_L^3 P_L(k_L)]$ where f is some universal function, and $P_L(k_L)$ is the linear power spectrum at the rescaled wave-number defined by $k_L = [1 + 4\pi k^3 P(k)]^{-1/3} k$. The cosmological dependence comes in through the linear fluctuation growth rate $P_L \propto [g(z)/(1+z)]^2$ (fitting formula from Carroll et al. 1992):

$$g(z) = \frac{5}{2}\Omega_m(z)[\Omega_m(z)^{4/7} - \Omega_\Lambda(z) + (1 + \Omega_m(z)/2)(1 + \Omega_\Lambda(z)/70)]^{-1} \tag{6}$$

where $\Omega_m(z) = \Omega_m(1+z)^3/[\Omega_m(1+z)^3 + \Omega_k(1+z)^2 + \Omega_\Lambda]$ and $\Omega_\Lambda(z) = \Omega_\Lambda/[\Omega_m(1+z)^3 + \Omega_k(1+z)^2 + \Omega_\Lambda]$, and Ω's without explicit z dependence denote their values today. For quintessence models with $w_q \neq -1$, we integrate numerically the equation for the linear growth rate, and substitute this in the corresponding expressions given by Peacock & Dodds (1996) (see Wang & Steinhardt 1998 for a useful fitting formula).

For the bispectrum, it has been conjectured for some time that the following scaling approximately holds in the highly nonlinear regime (e.g. Davis & Peebles 1980; Peebles 1980; Fry 1984; Hamilton 1988):

$$B(\mathbf{k_1}, \mathbf{k_2}, \mathbf{k_3}) = Q_3(P(k_1)P(k_2) + P(k_2)P(k_3) + P(k_3)P(k_1)) \tag{7}$$

where the three $\mathbf{k}$'s form a closed triangle, and Q_3 is a weak function of scale but independent of the triangle configuration. The above nonlinear hierarchical form (sometimes called the hierarchical ansatz) is also observed in N-body simulations (see Scoccimarro et al. 1998 and ref. therein). The problem, however, was that, there has been for a long time no way to predict the amplitude of Q_3, other than by examining N-body simulations on a case by case basis. Recently, Scoccimarro & Frieman (1998) introduced a method they named hyperextended perturbation theory which allows one to calculate Q_3 analytically:

$$Q_3(n) = [4 - 2^n]/[1 + 2^{n+1}] \tag{8}$$

where n is the *linear* power spectral index at the scale of interest $(k_1+k_2+k_3)/3$. The above expression implies that Q_3 is insensitive to cosmology, except through n (see Scoccimarro et al. 1998).

Hence, combining eq. (7), (8) and the nonlinear evolution of P given by Peacock & Dodds (1996), together with eq. (5), completely specifies S_3 for any given primordial power spectrum and cosmology. To ease the computation, we find that the following approximation for K_3 agrees with the exact integration to within a few percent for the models considered in this paper:

$$K_3 \sim 3(2\pi)^2 \int_0^{\chi_s} d\chi \frac{w(\chi)^3}{r(\chi)^4} \left[\int d^2\ell \sqrt{Q_3} P(\ell/r(\chi)) \tilde{W}(\ell\theta_R)^2 \right]^2 \tag{9}$$

where Q_3 is evaluated at an n corresponding to the scale $\ell/r(\chi)$. This approximation works in part because Q_3 varies slowly with scale, on the relevant small scales.

In Fig. 1a, we show a comparison of the skewness computed as described above with the skewness obtained from ray-tracing N-body simulations (JSW), for sources at $z = 1$. The error-bars shown are estimated from the dispersions between 5-10 ray-tracing realizations. Three models are shown (see Table 1). They are all normalized to match the cluster abundance today. The agreement is good, to better than 10% for $\theta_R \sim 1' - 5'$, and it remains reasonable at larger angular scales, although its exact level is somewhat uncertain because of the large dispersions of the N-body results. The agreement here is to be contrasted with the as much as 30% discrepancy for the perturbation theory predictions, shown as points on the far left of Fig. 1a. In particular, perturbation theory brings S_3 for OCDM and LCDM much closer than what it should be (the good agreement of the perturbative S_3 with the actual value for LCDM seems to be a coincidence). It is interesting how nonlinear clustering makes it easier to tell them apart. There seems to be a complicated interplay of projection and nonlinear clustering (e.g. Gaztañaga & Bernardeau 1998). Our OCDM predictions seem to be systematically a little higher than the N-body results, but it should be kept in mind that measurements at different scales are correlated and that a measurement-bias due to a division of estimators might be present (Hui & Gaztañaga 1999).

The accuracy of our method is actually somewhat surprising because of the inherent approximate nature of the prescription for Q_3 (eq. [8]) and of the scaling ansatz for the power spectrum evolution. Moreover, the weakly nonlinear fluctuations, which do not obey the hierarchical form with a configuration independent Q_3 as in eq. (7), must contribute at some level to the relevant integrals for S_3. To check this, we perform an alternative integration for K_2 in eq. (5) by including only 'nonlinear' modes: taking the lower limit of integration to be $\ell_{\rm nl}$ instead of 0, where $\ell_{\rm nl}$ satisfies $4\pi(\ell_{\rm nl}/r(\chi))^3 P(\ell_{\rm nl}/r(\chi)) = 1$. Let us call the resulting integral $K_2^{\rm nl}$, and define $\Delta_{K_2} \equiv |K_2^{\rm nl} - K_2|/K_2$. We find that Δ_{K_2} is very similar for all 3 models above, and is about $10 - 30\%$ at $\theta_R \sim 1' - 5'$, reaching about 45% at $\theta_R \sim 10'$. We therefore propose the following self-consistency check: Δ_{K_2} should be less than about 30% for our method to yield reliable estimates of S_3.

We show in Fig. 1b our prediction of S_3 for the same three models, but the points with error-bars now represent measurements from simulated surveys, of a size $3^o \times 3^o$, with 2×10^5 $z = 1$ galaxies per square degree whose intrinsic ellipticities are Gaussian distributed with an rms of 0.4 for each component (attainable with multiple several-hour-long exposures on a 4-meter class telescope using large CCDs; Van Waerbeke et al. 1999, JSW). It is clear that such a survey can separate these 3 models very nicely. Note, however, systematic errors have not been taken into account.

Perhaps more interestingly, we show in the same figure our prediction of S_3 for a cluster-normalized (Wang & Steinhardt 1998) quintessence model (qCDM, see Table 1). The equation of state ($w_q = -0.5$) is motivated by certain models of dynamical supersymmetry breaking (e.g. Binetruy 1999). Fluctuations in the quintessence component have been ignored, which is probably a good approximation on the small scales of interest (Turner & White 1997; Caldwell et al. 1998). Wang et al. (1999) have argued that current observations cannot tell apart qCDM models with $-1 < w_q \lesssim -0.4$ from LCDM models, for $\Omega_m \sim 0.3$. Future microwave background experiments would be able to provide better constraints, but there exist significant degeneracies, especially if H_0 is allowed to vary (Huey et al. 1998). The skewness has the advantage that it is independent of H_0. Fig. 1b shows that the small angular-scale convergence skewness (especially at $\theta_R \sim 1' - 5'$) provides a promising way to disentangle qCDM and LCDM models: fixing Ω_m, S_3 varies smoothly from the qCDM values shown to the LCDM values as w_q changes from -0.5 to -1. Moreover, it is a very clean test, because the small-scale S_3 is almost independent of all cosmological parameters except Ω's and w_q.

To emphasize this point, we show in Fig. 2 S_3's for 3 different qCDM models, with different Γ or σ_8. They all agree to within a few percent. From eq. (5) and (7), it is not hard to see that the normalization of P gets divided out in the combination for S_3. However, this really refers to the normalization of the nonlinear P. The normalization of the linear P, σ_8, should have some effect on S_3 through its impact on the shape of the nonlinear power spectrum. However, we find that at the small scales which dominate the relevant integrals for S_3, the nonlinear power spectra for most models have rather similar shapes: a slope around -1.5 or so. Together with the fact that the nonlinear Q_3 is a weak function of scale, this explains why the small angular-scale S_3 is relatively insensitive to both σ_8 and Γ. By the same reasoning, S_3 is quite independent of the spectral tilt as well. Nonlinear clustering seems to erase memory of the initial conditions in S_3, as far as CDM-type power spectra are concerned. We therefore have in hand a powerful statistical measure: the small-scale S_3 is almost purely determined by the cosmological energy contents.

3. Discussion

How do we understand the cosmological dependence of S_3? The best way is to go back to the definition of the projected mass density κ in eq. (1). Observe that w occurs three times in the numerator of S_3 and four times in the denominator (eq. [5]). This means any overall constant multiplying w is going to show up in S_3 (except for H_0/c which is canceled out in the combination for κ). Among other things, S_3 scales as $1/\Omega_m$, the reciprocal of the total matter content. This means flat matter dominated models generally have lower S_3 compared to low density models, as is seen in Fig. 1. Moreover, a Λ or quintessence dominated universe has a larger volume out to $z = 1$, compared to an open universe, making w larger and S_3 smaller. Finally, the different fluctuation growth rates in different cosmologies also shift the skewness to some extent. We find that the following crude approximation works surprisingly well in reproducing our results from integrating eq. (5):

$$S_3 \sim \quad 3\tilde{Q}_3 \int_0^{\chi_s} d\chi [w(\chi)^3/r(\chi)^4][g(z)/(1+z)]^4 r(\chi)^{-2\tilde{n}} / \tag{10}$$

$$\left[\int_0^{\chi_s} d\chi[w(\chi)^2/r(\chi)^2][g(z)/(1+z)]^2 r(\chi)^{-\tilde{n}}\right]^2$$

where $g(z)$ is given by eq. (6) or its generalization to include quintessence, $\tilde{Q}_3$ is taken to be 2.7, and $\tilde{n}$ is -1.2 i.e. the power spectrum is assumed to obey a power-law with simply linear evolution. The assumptions of a constant Q_3 and a power-law P implies a scale-independent S_3. The results of applying eq. (10) are shown as open circles in Fig. 1b.

For more accurate results, we recommend going back to eq. (5), (8) and (9), which give S_3 accurate to within 10% at $\theta_R \sim 1' - 5'$, assuming the sources are at $z = 1$. We have also suggested in §2 a useful consistency check: Δ_{K_2}, a measure that quantifies the degree of linearity, should be less than about 30%. For models that are too linear, the hierarchical ansatz with a configuration independent Q_3 (eq. 8) breaks down. An interesting example is provided by the τCDM model simulated by JSW, which is exactly the same as SCDM except that $\Gamma = 0.21$ and hence has less power on small scales. As explained before, the highly nonlinear S_3 should be insensitive to Γ, and our method would predict a τCDM S_3 very close to that of SCDM. JSW found that that the τCDM N-body results are in fact about 30% higher than the SCDM results at a few arcminutes. Applying our consistency test shows that Δ_{K_2} is 2 - 3 times higher for τCDM compared to all other models we have considered. Hence, the somewhat large difference between τCDM and SCDM seen in JSW is a reflection of their different levels of nonlinearity. Models with as little small-scale power as τCDM are probably inconsistent with observations of the Lyman-alpha forest (Hui et al. 1997; Croft et al. 1998).

We have argued that deep lensing surveys, with a total area of several square degrees and background galaxies at $z \sim 1$, should be capable of distinguishing between cosmological models with different energy contents. In particular, contrary to what is indicated by perturbation theory, the small-scale skewness can be used to differentiate between curvature and cosmological constant dominated models. Moreover, S_3 also shows sensitivity to the equation of state, w_q, of quintessence models, making them distinguishable from Λ models. In practice, however, since we have only one observable in S_3, one needs to impose extra constraints to restrict the range of models when engaging in model testing. For example, one can assume the class of flat tracker-field models (Zlatev et al. 1998) where w_q is determined by Ω_q, and so the only free parameter is Ω_q. Another example: making use of the fact that for a given Ω_m curvature dominated models yield higher S_3 than Λ models, one can obtain lower limits on Ω_m. Additional constraints from other observations such as the microwave background and large scale structure are obviously useful. It should also be borne in mind that systematic errors, such as those due to the correction of an anisotropic point-spread-function, have not been taken into account (Kaiser et al. 1995). In addition, we have assumed that the galaxy redshifts are known, but this is likely achievable by photometric techniques.

A few issues are worth further investigation. A fitting formula for the three-point function, which smoothly interpolates between the perturbative and the highly nonlinear regimes, could in principle be used to extend our calculation to cover all angular scales. At present, no such formula exist for CDM-type spectra (Scoccimarro & Frieman 1998). Moreover, our method can be easily generalized to S_N for arbitrary N. Such a calculation would be useful for the estimation of measurement errors from lensing surveys (Scoccimarro et al. 1999). Lastly, our expressions are easily generalizable to measures such as the aperture mass (Schneider et al. 1998), which corresponds to using a different smoothing kernel W_{θ_R} in eq. (4), and its Fourier transform $\tilde{W}$ in the rest of our expressions. Eq. (10) should remain roughly valid because it is independent of $\tilde{W}$.

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Model	Ω_m	Ω_k	Ω_q/Ω_Λ	w_q	Γ	σ_8
SCDM	1	0	0	–	0.5	0.6
OCDM	0.3	0.7	0	–	0.21	0.85
LCDM	0.3	0	0.7	-1	0.21	0.9
qCDM	0.3	0	0.7	-0.5	0.21	0.8

Table 1: A list of models.

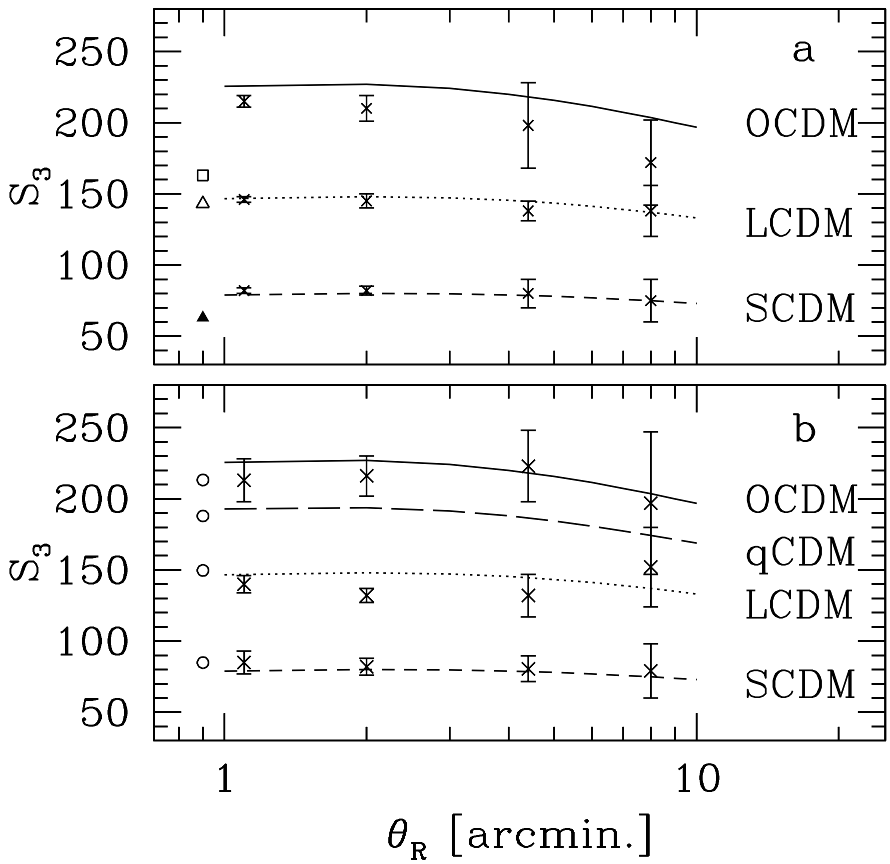

Fig. 1.— Panel a: A comparison of the skewness obtained from N-body simulations (points with error-bars, from 5 - 10 ray-tracing realizations with no noise added; taken from JSW) and from numerical integration of eq. (5) (lines). The points without error-bars on the far left denote, from top to bottom, the perturbation theory predictions of S_3 for O/L/SCDM models at $\theta_R = 1'$ (from JSW). Panel b: The lines are the same as before, with the addition of predictions for a qCDM model. Points with error-bars denote measurements for O/L/SCDM models from simulated $3^o \times 3^o$ surveys with 2×10^5 galaxies per sq. deg. (and random noise added; taken from JSW). The open circles on the far left denote the approximate S_3 from eq. (10). All sources/galaxies are assumed to be at $z = 1$.

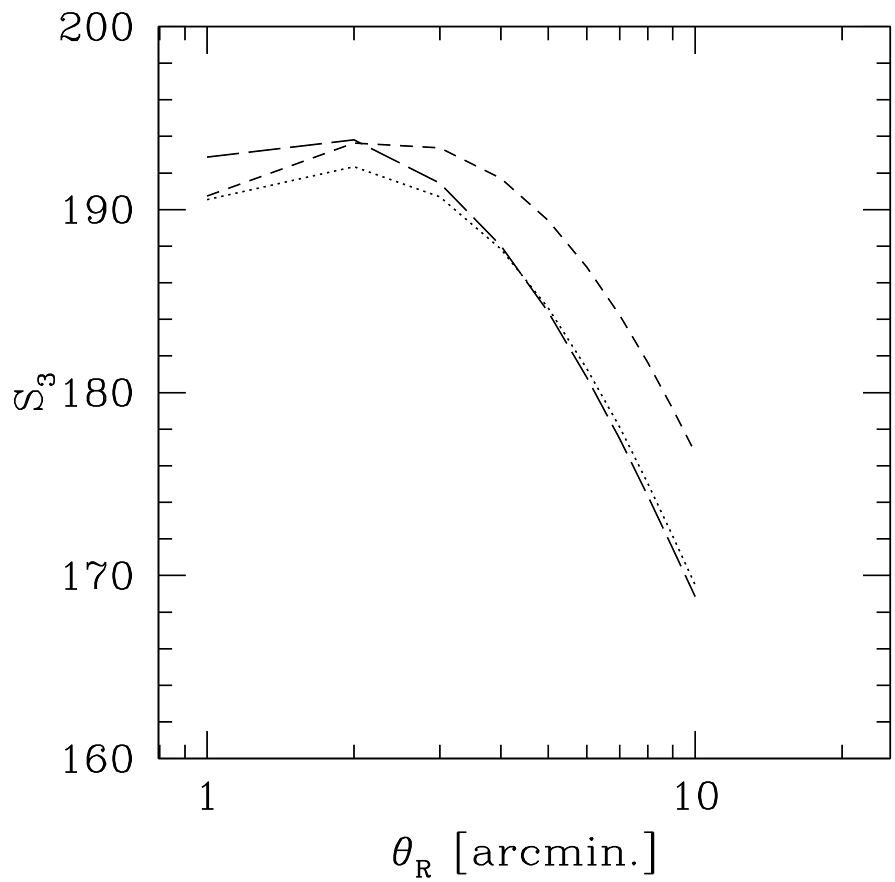

Fig. 2.— Skewness for three qCDM models. Long-dashed line - fiducial qCDM as in Fig. 1b (Table 1); dotted line - same qCDM but with $\Gamma = 0.5$; short-dashed line - same qCDM but with $\sigma_8 = 1.0$