

**Fermi National Accelerator Laboratory**

**FERMILAB-Pub-99/010-A**

**Constraints on the Cosmological Constant from Flows and  
Supernovae**

Idit Zehavi and Avishai Dekel

*Fermi National Accelerator Laboratory  
P.O. Box 500, Batavia, Illinois 60510*

April 1999

To be Submitted as a Letter to *Nature*

## **Disclaimer**

*This report was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency thereof, nor any of their employees, makes any warranty, expressed or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise, does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.*

## **Distribution**

*Approved for public release; further dissemination unlimited.*

## **Copyright Notification**

*This manuscript has been authored by Universities Research Association, Inc. under contract No. DE-AC02-76CHO3000 with the U.S. Department of Energy. The United States Government and the publisher, by accepting the article for publication, acknowledges that the United States Government retains a nonexclusive, paid-up, irrevocable, worldwide license to publish or reproduce the published form of this manuscript, or allow others to do so, for United States Government Purposes.*

# Constraints on the cosmological constant from flows and supernovae

Idit Zehavi<sup>\*,†</sup> & Avishai Dekel<sup>\*</sup>

<sup>\*</sup>Racah Institute for Physics, The Hebrew University, Jerusalem 91904, Israel

<sup>†</sup>NASA/Fermilab Astrophysics Group, Fermi National Accelerator Laboratory, Box 500, Batavia, IL 60510-0500, USA

Recent measurements of the global curvature of space-time, using distant supernovae as distance indicators, favor a positive cosmological constant, associated with an accelerating universal expansion<sup>1,2</sup>. However, these results by themselves still allow an open universe with low mass density and zero cosmological constant. We show here that this degeneracy is removed by independent constraints from galaxy peculiar velocities in our cosmological neighborhood, which provide a lower bound on the permissible mean mass density<sup>3-7</sup>. The joint constraints from the two independent sources thus favor an unbound and nearly flat universe with comparable contributions from cosmological constant and mass density, and they rule out a low-density open universe of vanishing cosmological constant. The indicated conspiracy between the values of the cosmological constant and the mass density seems to require fine tuning that is in conflict with our common wisdom concerning the early universe.

The standard cosmological model based on Einstein's gravity and the anzats of global homogeneity is characterized by two fundamental dimensionless parameters that measure the main contributions to the total energy density (relative to a "critical" density): the mean mass density  $\Omega_m$  and the cosmological constant  $\Omega_\Lambda$ . The latter, an intrinsic part of the theory of General Relativity, represents a uniform energy density that is associated with the vacuum (as opposed to the mass), and, if positive, acts like a repulsive component of the global gravitational force. The values of these parameters determine the type of the universe we live in as follows. The  $\Omega_m$ - $\Omega_\Lambda$  parameter plane shown in Figure 1 is divided by three dotted lines into six permissible regions. The line  $\Omega_m + \Omega_\Lambda = 1$  defines a "flat", Euclidean geometry, favored by theories of Inflation in the early universe. It separates curved models of "closed", finite space (above) and "open", infinite space (below). The horizontal line  $\Omega_\Lambda = 0$  roughly distinguishes between an "unbound" universe that will expand forever (above) and a "bound" universe that will eventually re-collapse (below). Along the line  $\Omega_\Lambda = 0$  itself, the

point  $\Omega_m = 1$  corresponds to the “simplest”, Einstein-deSitter model, which borders between a bound closed universe (right) and an unbound open universe (left). Unless the universe is exactly Einstein-deSitter, it evolves away from this point as it expands (e.g., along the line  $\Omega_m + \Omega_\Lambda = 1$  or  $\Omega_\Lambda = 0$ ). Finally, the line  $\Omega_\Lambda = 0.5\Omega_m$  corresponds to a temporary balance between attraction and repulsion; it separates between a universe whose expansion decelerates (below) or accelerates (above). A major goal of cosmology is to determine in which of these regions we live, and thus reveal the global geometry and ultimate fate of our universe.

New techniques to measure distances of supernovae (type Ia) at large distances (cosmological redshifts 0.5 to 1), where the large-scale curvature of space-time plays a noticeable role, enable an application of a classical cosmological test based on how the “Hubble” relation between velocity and distance depends on the cosmological parameters.<sup>8</sup> The corresponding, inclined confidence limits in Fig. 1 are based on the results of the Supernova Cosmology Project<sup>1</sup>, which are fully consistent with the findings of the competing High-z Supernova Search Team<sup>2</sup>. The allowed region in the parameter plane is an elongated stripe, crudely approximated by  $0.8\Omega_m - 0.6\Omega_\Lambda = -0.2 \pm 0.1$  ( $1\sigma$  errors). Based on this result alone, a flat universe is likely, with maximum probability near  $\Omega_\Lambda \simeq 0.7$  and  $\Omega_m \simeq 0.3$ , but an open model, of  $\Omega_\Lambda = 0$  and  $\Omega_m \sim 0.1$ , say, is still allowed at the 2% confidence level. An orthogonal constraint is required in order to remove the degeneracy between  $\Omega_\Lambda$  and  $\Omega_m$ .

We show in this Letter that current data of large-scale peculiar velocities provide such a constraint. These velocities, which correspond via gravity (and mass conservation) to mass density fluctuations about the mean, depend also on the mean density itself (relative to the critical density) and can thus provide direct constraints on  $\Omega_m$ . These constraints from velocities are practically independent of  $\Omega_\Lambda$ , but combined with the inclined supernova high-likelihood ridge, any bound on  $\Omega_m$  effectively becomes a bound on  $\Omega_\Lambda$ .

The back-bone of the current velocity data consists of the Mark III catalog<sup>9</sup> and the SFI catalog (Haynes *et al.*, in preparation). Mark III samples  $\sim 3000$  galaxies within a distance of  $\sim 70 h^{-1}\text{Mpc}$  around us, and SFI consists of  $\sim 1300$  spiral galaxies with a more uniform spatial coverage in a similar volume. In order to obtain the peculiar velocity of a galaxy, one measures its total velocity via redshift, and separately infers its distance by the so-called Tully-Fisher (or Fundamental Plane) method, with errors of 15 – 21%. The desired peculiar velocity along the line of sight is obtained by subtracting the Hubble velocity (corresponding to the inferred distance) from the total velocity. The peculiar velocities are carefully corrected for systematic errors such as Malmquist bias<sup>7,10</sup>.

The POTENT method<sup>10,11</sup> makes the assertion that the peculiar velocities were generated by gravity and they therefore represent a potential flow on large scales. This allows a recovery of the potential and three-dimensional velocity fields in our local cosmological neighborhood. For an assumed value of  $\Omega_m$ , one can then extract the underlying field of (mostly dark) mass-density fluctuations, using mildly nonlinear approximations to the relation between velocity and density fluctuations within the theory of gravitational instability. Under the

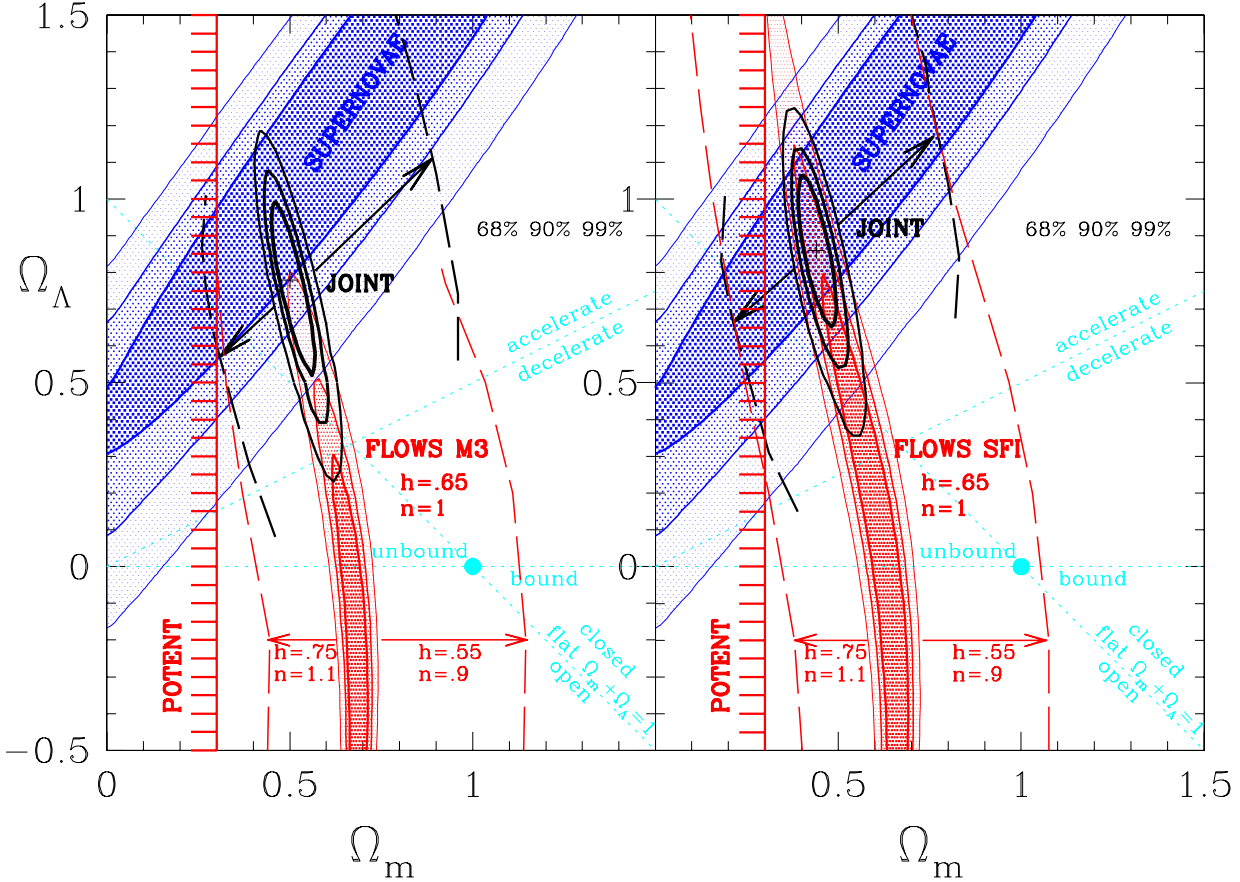


Figure 1: Constraints in the  $\Omega_m - \Omega_\Lambda$  plane, showing confidence limits of 68%, 90% and 99%. The inclined contours (blue, same in both panels), that roughly constrain  $0.8\Omega_m - 0.6\Omega_\Lambda$ , arise from the global geometry of space-time based on supernovae as distance indicators<sup>1</sup>. The lower bound  $\Omega_m > 0.3$  (red) represents a 99% confidence limit based on several different studies of peculiar velocities via POTENT reconstruction, independent of the “biasing” relation between galaxies and mass.<sup>3-5</sup> The almost vertical contours (red) arise from the peculiar velocities of the Mark III (left) and SFI (right) catalogs, based on a likelihood analysis which assumes a parametric mass power spectrum of the CDM family with COBE normalization on large scales<sup>6,7</sup>. The central contours shown are for a “standard” case of fixed  $h = 0.65$  and  $n = 1$ , and the uncertainties in these values are represented by the outer 99% contours on both sides (long dashed), computed for the extreme cases of fixed  $(h, n) = (0.55, 0.9)$  and  $(0.75, 1.1)$ . The strong bounds from flows are on  $\Omega_m$ ; the apparent upper bound on  $\Omega_\Lambda$  arises indirectly from the COBE normalization and is very uncertain. The corresponding joint (relative) confidence limits from supernovae and flows are shown (black); they are almost the same for Mark III and SFI. The combined constraints favor an unbound and roughly flat universe with comparable contributions from cosmological constant and mass density. A universe with very low  $\Omega_m$  and zero cosmological constant is ruled out.

natural assumption that the initial fluctuations were Gaussian, these fields provide direct dynamical constraints on  $\Omega_m$  without appealing to the spatial distribution of galaxies, and thus independent of the unknown “biasing” relation between galaxies and mass. The bottom line is that several different methods of analysis, which have been applied to the POTENT Mark III fields, consistently yield a lower bound of  $\Omega_m > 0.3$  at roughly the 99% confidence level, marked by a vertical line in both panels of Fig. 1. One method constrains  $\Omega_m$  based on the large diverging flow recovered at the vicinity of a large nearby void, using the fact that even a void empty of matter cannot induce outflows as large as observed if the mean mass density outside it is too low<sup>4</sup>. Another method uses the implied deviations from Gaussianity in the probability distribution of the initial density fluctuations as derived from the velocity data when the assumed value of  $\Omega_m$  is too low<sup>3</sup>. A third method derives  $\Omega_m$  based on the gravitational deviations from Gaussianity of the present velocity-divergence probability distribution<sup>5</sup>. Similar constraints, so far confirming the lower bound of  $\Omega_m > 0.3$ , are currently being obtained from the SFI data (in progress).

Recently, we have applied a maximum-likelihood analysis separately to the Mark III and SFI peculiar velocities, to determine cosmological parameters via a parametric model for the mass-density fluctuation power spectrum  $P_k$ <sup>6,7</sup>. The analysis uses linear theory and assumes that both the fluctuations and the errors are Gaussian. The model assumed is of the general Cold Dark Matter (CDM) family, normalized<sup>12</sup> by the fluctuations in the Cosmic Microwave Background (CMB) as measured on very large scales by the COBE satellite. This provides constraints on permissible combinations of 3 parameters:  $\Omega_m$ , the initial power index  $n$  (where  $P_k \propto k^n$  on large, linear scales), and the Hubble constant  $h$ . These parameters enter via the shape of  $P_k$  as well as the geometry and dynamics of space-time. The constraints obtained from the two data sets turn out to be very similar; they define a two-dimensional surface of high likelihood in the  $\Omega_m$ - $h$ - $n$  space. In the case of a flat cosmology this surface can be crudely approximated by  $\Omega_m h_{65}^{1.3} n^2 \simeq 0.58 \pm 0.12$ , and for an  $\Omega_\Lambda = 0$  universe it is  $\Omega_m h_{65}^{0.9} n^{1.4} \simeq 0.66 \pm 0.12$  (where  $h_{65}$  is the Hubble constant in units of  $65 \text{ km s}^{-1} \text{ Mpc}^{-1}$ ). The quoted errors refer to 90% formal likelihood uncertainty (plus the small difference between the two catalogs). The inclusion of a tensor component in the initial fluctuations, assumed in some Inflation models to have a quadrupole-moment tensor to scalar ratio of  $7(1-n)$ , is found not to make a major difference except for strengthening the  $n$  dependence.

It is worth noting that the obtained power spectrum is driven by the velocity data (on scales  $\sim 10 - 100 \text{ h}^{-1} \text{ Mpc}$ ) and is only weakly affected by the COBE normalization ( $\sim 1000 \text{ h}^{-1} \text{ Mpc}$ ); a similar  $P_k$  is reproduced to within a few percent when the amplitude on large scales is left free to be determined by the likelihood analysis. Also, the result is not driven by the assumed errors;  $P_k$  is reproduced to better than 10% when an error model with free parameters is incorporated in the likelihood analysis itself<sup>7</sup>. This robustness is within the  $1\sigma$  confidence limits of the likelihood analysis.

It is worth mentioning in passing that the peculiar velocities have also been analyzed by different methods that incorporate the spatial distribution of galaxies. For example, a

comparison of the Mark III velocities and the IRAS 1.2Jy redshift survey yields<sup>13</sup>  $\Omega_m^{0.6}/b_I = 0.89 \pm 0.12$  (where  $b_I \equiv \sigma_{8I}/\sigma_8$  is the biasing ratio between the rms density fluctuations of IRAS galaxies and mass on a scale of  $8 h^{-1} \text{Mpc}$ ). With the observed  $\sigma_{8I} \sim 0.6$ , this is indeed roughly consistent with the COBE-normalized CDM model of  $\Omega_m \sim 0.5$  (and  $\sigma_8 \sim 1$ ) favored by the  $P_k$  analysis from peculiar velocities alone. Our current bounds on  $\Omega_m$  from velocities also partly overlap with the latest constraints from the evolution of galaxy clusters<sup>14,15</sup> ( $\Omega_m = 0.45 \pm 0.2$  and  $\Omega_m = 0.2_{-0.1}^{+0.3}$  respectively). However, a comprehensive joint analysis of all the available constraints on the different parameters is beyond the scope of the present Letter. In particular, a proper interpretation of the various results involving redshift surveys (the estimates for  $\Omega_m^{0.6}/b_I$  actually span the range  $0.5 - 1.0$ ) must include a full discussion of the nontrivial “biasing” relation between galaxies and mass,<sup>16</sup> which we do not attempt here. The different studies involve complex systematic effects that should be addressed in detail when a global comparison is made. We rather focus here on one set of constraints, namely, dynamical constraints from peculiar velocities, and their implications on  $\Omega_\Lambda$  via the supernova constraints.

For the purpose of obtaining the desired constraints in the  $\Omega_m$ - $\Omega_\Lambda$  plane, we have applied here the  $P_k$  likelihood analysis to the Mark III and SFI velocities, separately and combined. We have used as prior the COBE-normalized<sup>17</sup> CDM models with varying  $\Omega_m$  and  $\Omega_\Lambda$  that now span the *whole* parameter plane. The velocities at the present epoch are expected to be insensitive to the value of  $\Omega_\Lambda$ <sup>18</sup>, but a certain  $\Omega_\Lambda$  dependence enters via the COBE normalization imposed. This is responsible for the small difference in the fits quoted above for the cases  $\Omega_\Lambda = 0$  and  $\Omega_m + \Omega_\Lambda = 1$ , and for the slight bending of the vertical likelihood ridges shown in Fig. 1, resulting in apparent upper bounds on  $\Omega_\Lambda$ . The  $1\sigma$  error in the COBE normalization translates to additional uncertainties (not shown) of  $\pm 6\%$  in  $\Omega_m$  and  $\pm 20\%$  in  $\Omega_\Lambda$  — on the order of the uncertainties displayed by the likelihood contours shown. Thus, the COBE errors would tend to stretch the likelihood contours vertically along the  $\Omega_\Lambda$  axis, and in particular weaken the apparent upper bounds on  $\Omega_\Lambda$ .

Since the velocity data favor an extended two-dimensional surface in the  $\Omega_m$ - $h$ - $n$  space, the likelihood analysis cannot determine the three parameters simultaneously; two of them should be fixed a priori. We adopt as our “standard” case the scale-invariant  $n = 1$  initial spectrum, and a Hubble constant of  $h = 0.65$  (as favored, for example, by nearby supernova data<sup>19</sup>). The central likelihood contours shown in Fig. 1 correspond to this standard case. Based on the current literature, we crudely estimate a  $\pm 15\%$  uncertainty in the Hubble constant<sup>20</sup>, and  $\pm 10\%$  in the power index<sup>21</sup>. In order to illustrate the sensitivity of the velocity constraints to these parameters, we have also applied the likelihood analysis to determine  $\Omega_m$  and  $\Omega_\Lambda$  in the two extreme cases:  $(h, n) = (0.55, 0.9)$  and  $(0.75, 1.1)$ . The outer 99% likelihood contours are shifted accordingly in Fig. 1 — these can be considered as encompassing a conservative range of non-negligible likelihood based on the flows data.

The likelihood ridge of SFI is slightly wider than that of Mark III, and it extends further into high values of  $\Omega_\Lambda$ . The independent constraints from flows (via  $P_k$ ) and supernovae

overlap significantly for SFI, but only near the 90% confidence contours for Mark III with  $n = 1$  and  $h = 0.65$  (which improves when the COBE errors are taken into account, or if  $n > 1$  or  $h > 0.65$ ). The large uncertainty in the upper bound on  $\Omega_\Lambda$  from the flows (via COBE normalization) indicates that the degree of overlap should not be interpreted as consistency or inconsistency between the velocity and supernova data. The robustness of the goodness of fit to variations in the  $\Omega_\Lambda$  direction is partly quantified by the fact that the  $\chi^2$  per degree of freedom does not deviate significantly from unity anywhere within the elongated 99% likelihood contours and even further upwards beyond the 99% tip. It does deteriorate rapidly under  $\Omega_m$  variations to the right or left of the high-likelihood ridge, confirming the fact that the flows constraints on  $\Omega_m$  are robust.

A joint parameter estimation from supernovae and flows is therefore meaningful, though it is limited to *relative* likelihoods; a measure of absolute probabilities of model parameters is not straightforward, despite the seemingly acceptable goodness of fit. The joint contours shown in Fig. 1 are computed by multiplying the two likelihood values at each point, under the assumption that the two kinds of data are independent. The most likely joint values for the supernovae and the combined Mark III and SFI dataset are  $\Omega_m \simeq 0.5$  and  $\Omega_\Lambda \simeq 0.8$ , while the conservative joint 99% confidence limits (including the uncertainties in  $n$  and  $h$ ) allow  $\Omega_m$  values in the range  $0.3 - 0.9$  and  $\Omega_\Lambda$  values in the range  $0.1 - 1.4$ . The constraints obtained separately from Mark III and SFI are very similar.

The constraints from local peculiar velocities thus remove the degeneracy in the constraints from the global-geometry test based on supernovae (and vice versa), and help ruling out an open model with zero cosmological constant. A nearly flat universe, with comparable contributions of matter and cosmological constant to the total energy density, is likely. The favored model is thus of an *unbound* universe that will expand forever with increasing acceleration due to a positive cosmological constant, though one cannot tell yet whether the global geometry is flat, open or closed. Such comparable contributions from the mass density and the cosmological constant represent a puzzling fine tuning, e.g., because the two are expected to vary with time in opposite senses. The standard theory expects the cosmological constant to either vanish or be larger by many orders of magnitude<sup>8,22</sup>. Although the dust has not settled yet on the observed constraints, they already seem interesting enough to pose a serious challenge to theoretical physics.

Other constraints in the  $\Omega_m$ - $\Omega_\Lambda$  plane are worth mentioning in perspective. Constraints consistent with the supernova ridge but of larger uncertainty arise from the age of old star clusters<sup>23</sup> versus the Hubble expansion rate. The number of gravitationally lensed images of quasars provides a similar upper limit:  $\Omega_\Lambda \leq 0.7$  at  $\sim 95\%$  for a flat universe<sup>24</sup>. Constraints of orthogonal orientation, roughly on  $\Omega_\Lambda + \Omega_m$ , can be deduced from the acoustic peaks in the sub-degree angular power spectrum of fluctuations in the CMB as observed from balloons and from the ground<sup>21,25-29</sup>. Two CMB satellites planned for the next decade, MAP and Planck, are expected to provide more accurate constraints<sup>27,30</sup>.

In parallel, future peculiar-velocity data are expected to improve the accuracy of the



constraints reported here. In addition to many new velocities of galaxies and clusters based on Tully-Fisher type distance indicators, the most promising sources of peculiar velocities in the long run are probably the siblings of the same supernovae discussed above but at low redshifts; their distances can be measured with 5 – 10% accuracy out to large distances and their sampling density is limited, in principle, only by the patience of the observer. The supernova hunters are encouraged to continue their effort along this route.

## References

- [1] Perlmutter S. *et al.* Measurements of  $\Omega$  and  $\Lambda$  from 42 high-redshift supernovae. *Astrophys. J.* **516** (in the press); also as preprint astro-ph/9812133 (1999).
- [2] Riess A.G. *et al.* Observational evidence from supernovae for an accelerating universe and a cosmological constant. *Astron. J.* **116**, 1009-1038 (1998).
- [3] Nusser, A. & Dekel, A.  $\Omega$  and the initial fluctuations from velocity and density fields. *Astrophys. J.* **405**, 437-448 (1993).
- [4] Dekel, A. & Rees, M.J.  $\Omega$  from velocities in voids. *Astrophys. J.* **422**, L1-L4 (1994).
- [5] Bernardeau, F., Juszkiewicz, R., Dekel, A. & Bouchet, F.R. Omega from the skewness of the cosmic velocity divergence. *Mon. Not. R. Astron. Soc.* **274**, 20-26 (1995).
- [6] Zaroubi, S., Zehavi, I., Dekel, A., Hoffman, Y. & Kolatt T. Large-scale power spectrum from peculiar velocities via likelihood analysis. *Astrophys. J.* **486**, 21-31 (1997).
- [7] Freudling W., Zehavi I., da Costa L.N., Dekel A., Eldar A., Giovanelli R., Haynes M.P., Salzer J.J., Wegner G., Zaroubi S. Large-scale power spectrum and cosmological parameters from SFI peculiar velocities. *Astrophys. J.* (in the press); also as preprint astro-ph/9904118 (1999).
- [8] Carroll, S.M., Press, W.H. & Turner, E.L. The cosmological constant. *Annu. Rev. Astron. Astrophys.* **30**, 499-542 (1992).
- [9] Willick, J.A., Courteau S., Faber S.M., Burstein D., Dekel A., Strauss M.A. Homogeneous velocity-distance data for peculiar velocity analysis. III. The Mark III catalog of galaxy peculiar velocities *Astrophys. J. S.* **109**, 333-366 (1997).
- [10] Dekel, A., Eldar, A., Kolatt, T., Yahil, A., Willick, J.A., Faber, S.M., Courteau, S. & Burstein, D. POTENT reconstruction from Mark III velocities. *Astrophys. J.* (in the press); also as preprint astro-ph/9812197 (1999).
- [11] Dekel, A., Bertchinger, E. & Faber, S.M. Potential, velocity, and density fields from sparse and noisy redshift-distance samples: Method. *Astrophys. J.* **364**, 349-369 (1990).
- [12] Sugiyama, N. Cosmic background anisotropies in cold dark matter cosmology. *Astrophys. J. S.* **100**, 281-305 (1995).

- [13] Sigad, Y., Eldar, A., Dekel, A., Strauss, M.A. & Yahil, A. IRAS versus POTENT Density Fields on Large Scales: Biasing and Omega. *Astrophys. J.* **495**, 516-532 (1998).
- [14] Eke, V.R., Cole, S., Frenk, C.S. & Henry, P.J. Measuring  $\Omega_0$  using cluster evolution. *Mon. Not. R. Astron. Soc.* **298**, 1145-1162 (1998).
- [15] Bahcall, N.A., & Fan, X. The Most Massive Distant Clusters: Determining  $\Omega$  and  $\sigma_8$ . *Astrophys. J.* **504**, 1-6 (1998).
- [16] Dekel, A. & Lahav, O. Stochastic nonlinear galaxy biasing. *Astrophys. J.* (in the press); also as preprint astro-ph/9806193 (1999).
- [17] Bunn, E.F. & White, M. The four-year COBE normalization and large-scale structure. *Astrophys. J.* **480**, 6-21 (1997).
- [18] Lahav, O., Lilje, P.B., Primack, J.R. & Rees, M.J. Dynamical effects of the cosmological constant *Mon. Not. R. Astron. Soc.* **251**, 128-136 (1991).
- [19] Riess, A.G., Press, W.H. & Kirshner, R.P. A precise distance indicator: type Ia supernova multicolor light-curve shape. *Astrophys. J.* **473**, 88-109 (1996).
- [20] Freedman, W.L. Determination of the Hubble constant. in *Critical Dialogues in Cosmology* (ed Turok, N.) 92-129 (Singapore, World Scientific, 1997).
- [21] Bond, J.R. & Jaffe A.H. Constraining large scale structure theories with the cosmic background radiation. *Phil. Trans. R. Soc. Lond. A* (in the press); also as preprint astro-ph/9809043 (1998).
- [22] Weinberg, S. The cosmological constant problem. *Rev. Mod. Phys.* **61**, 1-23 (1989).
- [23] Chaboyer, B., Demarque, P. Kernan, P.J. & Krauss, L.M. The age of globular clusters in light of Hipparcos: Resolving the age problem? *Astrophys. J.* **494**, 96-110 (1998).
- [24] Kochanek, C.S. Is there a cosmological constant? *Astrophys. J.* **466**, 638-659 (1996).
- [25] Efstathiou, G., Bridle, S.L., Lasenby, A.N., Hobson, M.P. & Ellis, R.S., Constraints on  $\Omega_\Lambda$  and  $\Omega_m$  from distant type Ia supernovae and cosmic microwave background anisotropies. *Mon. Not. R. Astron. Soc.* **303**, L47-L52 (1999).
- [26] Lineweaver, C.H. The cosmic microwave background and observational convergence in the  $\Omega_m - \Omega_\Lambda$  plane. *Astrophys. J.* **505**, L69-L73 (1998).
- [27] Tegmark, M., Eisenstein, D.J., Hu, W. & Kron, R. Cosmic complementarity: probing the acceleration of the universe. *Astrophys. J.* submitted; also as preprint astro-ph/9805117 (1998).
- [28] White, M. Complementary measures of the mass density and cosmological constant. *Astrophys. J.* **506**, 495-501 (1998).
- [29] Lasenby, A.W., Jones, A.W. & Dabrowski, Y. Observations of the cosmic microwave background and implications for cosmology and large scale structure. *Phil. Trans. R. Soc. Lond. A* (in the press); also as preprint astro-ph/9810196 (1998).

- [30] Zaldarriaga, M., Spergel, D.N. & Seljak, U. Microwave background constraints on cosmological parameters. *Astrophys. J.* **488**, 1-13 (1997).

**Acknowledgements.** We thank the Mark III and SFI teams, including D. Burstein, S. Courteau, L.N. daCosta, A. Eldar, S.M. Faber, W. Freudling, R. Giovanelli, M.P. Haynes, T. Kolatt, J.J. Salzer, M.A. Strauss, G. Wagner, J.A. Willick, A. Yahil, & S. Zaroubi. We thank G.R. Blumenthal and O. Lahav for stimulating discussions. This research was supported by US-Israel Binational Science Foundation grant 95-00330 and Israel Science Foundation grant 546/98 at HU, and by the DOE and the NASA grant NAG 5-7092 at Fermilab.

**Correspondence** and requests for materials should be addressed to I.Z. (email: iditz@fnal.gov) and A.D. (email: dekel@astro.huji.ac.il).