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Skewed exponential pairwise velocities from Gaussian initial conditions

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ABSTRACT

Using an Eulerian perturbative calculation, we show that the distribution of relative pairwise velocities which arises from gravitational instability of Gaussian density fluctuations has asymmetric (skewed) exponential tails. The negative skewness is induced by the negative mean streaming velocity of pairs (the infall prevails over expansion), while the exponential tails arise because the relative pairwise velocity is a *number*, not volume weighted statistic. The derived probability distribution is compared with N-body simulations and shown to provide a reasonable fit.

Subject headings: large-scale structure of universe — galaxies: interactions

1. Introduction

Redshift surveys present a distorted picture of the world because peculiar motions displace galaxies from their true spatial positions. This phenomenon, which would make redshift surveys useless for intergalactic spaceship navigators, is extremely useful for cosmologists. It can serve as a probe of the dynamics of gravitational clustering and the cosmological mass density parameter, Ω (Sargent and Turner 1977; Peebles 1980, hereafter LSS; Kaiser 1987; Hamilton 1992; Peebles 1993, hereafter PPC; Regös and Szalay 1995). A convenient statistical measure of the distortion effect is the galaxy two-point correlation function in redshift space. Under certain assumptions it can be expressed as a convolution of the true spatial correlation function, $\xi(r)$, with the distribution of the relative line-of-sight velocities of pairs of galaxies, $p(w|r,\theta)$. Here r and w are respectively, the spatial separation and relative radial velocity of a pair of galaxies, while θ is the angle between the separation vector $\mathbf{r}$ and observer's line of sight (cf. LSS; Fisher 1995, hereafter F95). The purpose of this *Letter* is to derive $p(w|r,\theta)$, using weakly nonlinear gravitational instability theory. This distribution was measured from N-body simulations and estimated indirectly from redshift surveys. At $r \lesssim 1\ h^{-1}$Mpc, where[6] the galaxies are strongly clustered ($\xi \gtrsim 20$), the observations are consistent with an exponential distribution (Peebles 1976; Davis and Peebles 1983; Fisher at al. 1994, hereafter F94; Marzke et al. 1995; Landy, Szalay, & Broadhurst 1997). The fact that $p(w)$ at small separations differs strongly from its initial, Gaussian character, is not surprising: after all, the small-scale velocity field has been 'processed' by strongly-nonlinear dynamics in clusters, and exponential distributions were recently derived from the Press-Schechter (1974) theory (Sheth 1996, Diaferio and Geller 1996). On larger scales, where the fluctuations have small amplitudes, one naïvely expects to see the 'unprocessed' initial conditions. However, N-body experiments suggest

[6]We use the standard parametrisation for the Hubble constant, $H = 100\,h$ km s^{-1}Mpc^{-1}.

that $p(w|r,\theta)$ retains its exponential character even at separations $r \gtrsim 10\ h^{-1}$Mpc, where $\xi \lesssim 0.1$, despite the fact that the initial density and velocity fields in those experiments were drawn from a Gaussian distribution (Efstathiou et al. 1988, hereafter EFWD; Żurek et al. 1994, hereafter ZQSW; F94). At similar separations, an exponential $p(w|r,\theta)$ has also been inferred from observations (F94; Loveday et al. 1996). The simulations also show that the radial component of the distribution, $p(w|r,0°)$ is significantly skewed, in particular at large separations (EFWD; ZQSW; F94). The physical origin of the skewness and exponential shape of $p(w)$ at large separations has until now remained unexplained. We provide the explanation below.

2. The origin of the negative skewness

Let $\mathbf{v}$ and δ be the peculiar velocity of a galaxy and the mass density contrast at comoving position $\mathbf{r}_1$, while $\mathbf{v}'$ and δ' the velocity of another galaxy at position $\mathbf{r}_2$ at a certain fixed separation $\mathbf{r} = \mathbf{r}_2 - \mathbf{r}_1$. In our coordinate system $\mathbf{r} = \{x, y, z\} = r\{\sin\theta\cos\varphi,\ \sin\theta\sin\varphi,\ \cos\theta\}$; the unit vector $\hat{\mathbf{z}}$ points along the observer's line of sight, while v_z and v'_z stand for the line of sight velocity components. The probability that the four considered random fields reach values $\Re = \{\delta, \delta', \mathbf{v}, \mathbf{v}'\}$ in the range $d\Re = d\delta\, d\delta'\, d\mathbf{v}\, d\mathbf{v}'$ is $g(\Re)\, d\Re$, and we will use brackets to denote ensemble averaging, $\langle \ldots \rangle = \int \ldots\, g\, d\Re$. As usual, expectation values $\langle \ldots \rangle$ are assumed to be equal to spatial averages. The latter, however, should not be confused with number-weighted averages carried over galaxy positions (cf. LSS and F95). The N-th moment of the relative velocity, weighted by the density of pairs of galaxies, is given by

$$m_N = \frac{\langle (v'_z - v_z)^N (1+\delta_g)(1+\delta'_g) \rangle}{\langle (1+\delta_g)(1+\delta'_g) \rangle}\ , \tag{1}$$

where $\delta_g = (\delta\rho/\rho)_g$ is the contrast in the number density of galaxies, which may differ from the spatial fluctuations in the mass distribution. Here, we ignore this potential difficulty and implicitly assume $\delta_g(\mathbf{r}) = \delta(\mathbf{r})$. A central moment of order N is given by

$$\mu_N = \langle\, (v'_z - v_z - m_1)^N (1+\delta)\,(1+\delta')\,\rangle\; [1+\xi(r)]^{-1} \tag{2}$$

To calculate the first few moments, we shall expand the random fields in perturbative series, $\mathbf{v} = \mathbf{v}^{(1)} + \mathbf{v}^{(2)} + \ldots$, $\delta = \delta^{(1)} + \delta^{(2)} + \ldots$, where each superscript describes the perturbative order ($\delta^{(2)} = O[\delta^{(1)}]^2$, etc.). We assume that all linear order terms are described by a joint-normal probability distribution. To lowest non-vanishing order, $m_1(r,\theta) = v_{21}(r)\,\cos\theta + O(\xi^2)$; $\mu_2(r,\theta) = {\sigma_{21}}^2(r,\theta) + O(\xi^2)$; $\mu_3(r,\theta) = 6\,\langle\, v_z^{(2)\prime} v_z^{(1)} (v_z^{(1)} - 2v_z^{(1)\prime})\,\rangle + O(\xi^3)$, where $\xi(r) = \langle\, \delta^{(1)}\,\delta^{(1)\prime}\,\rangle$ is the linear correlation function, while the mean streaming velocity, v_{21}, and the pairwise velocity dispersion, ${\sigma_{21}}^2$, are given by

$$v_{21}(r) = -2Hf(\Omega) \int_0^r \xi(s)\,(s/r)^2\,ds\;, \tag{3}$$

$${\sigma_{21}}^2(r,\theta) = 2\,\left[\Pi(0) - \Pi(r)\cos^2\theta - \Sigma(r)\sin^2\theta\right]\;, \tag{4}$$

where $f(\Omega) \approx \Omega^{0.6}$, while Π and Σ are the radial and transverse components of the velocity correlation tensor, related to $\xi(r)$ by equations (21.72)-(21.75) in PPC (see also Górski 1988 and Groth et al. 1989). Note that the leading terms in the expansions for the first two moments come from linear perturbation theory. The third moment is different. In the early universe, linear perturbation theory is sufficient, and to first order, $\mu_3 = 0$, in agreement with the assumed Gaussian initial conditions, symmetric about $\delta = 0$. However, gravitational instability breaks the initial symmetry (LSS; Juszkiewicz et al. 1993, Bernardeau et al. 1995, hereafter BJDB). To calculate μ_3, we need the second-order term for the velocity field. It can be obtained by inverting the expression for $\nabla\cdot\mathbf{v}^{(2)}$, derived by BJDB. For a curl-free flow, the resulting skewness is

$$\mu_3(r,\theta) = {\sigma_{21}}^2 (S_V + S_A)\,\cos\theta\;, \tag{5}$$

where the first term,

$$S_V = 3v_{21}(r)\,(\cos^2\theta + C\,\sin^2\theta);\quad C(\Omega) \approx \tfrac{3}{7}\,\Omega^{-1/21}, \tag{6}$$

is induced by the mean streaming, while the second,

$$S_A = 6\,[\Sigma(r) - \Pi(r)]\;(1+\cos^2\theta)\,(1-C)/(Hfr), \tag{7}$$

comes from the anisotropy of the relative velocity dispersion tensor. When $\mathbf{r} \perp \hat{\mathbf{z}}$ (both galaxies are in the plane of the sky), as well as for zero separation, the skewness vanishes: $\mu_3(r, 90°) = \mu_3(0, \theta) = 0$, in agreement with symmetry requirements. Such requirements do not apply to the radial component of the distribution ($\mathbf{r} \parallel \hat{\mathbf{z}}$) when the mean infall velocity is different from zero. We will now show that if ξ remains positive for separations $< r$, then $\mu_3(r, 0°) < 0$. First, note that a positive ξ implies $v_{21}(r) < 0$ and $\Sigma(r) > \Pi(r)$ (see eq. [3] above and eqs. [21.72]-[21.75] in PPC). Hence, $S_V(r, 0°) < 0$ while $S_A(r, 0°) > 0$: the effect of the negative infall velocity is counterbalanced by the anisotropy of the velocity correlation tensor (in agreement with N-body results; see p. 938 in F94). However, the two effects do not cancel out and for all 'reasonable' models the term, induced by the mean streaming dominates. For a power-law correlation function, $\xi(r) \propto r^{-\gamma}$, the ratio of the two terms is $S_A(r, 0°)/S_V(r, 0°) = -8/7(5-\gamma) \approx -0.35$ for $\gamma = 1.7$.

In order to test our perturbative predictions against fully nonlinear N-body experiments, we used data, generated from the simulations described in Frenk et al. (1990; appropriate codes were kindly provided to us by Marc Davis). The simulations are of a standard CDM model with $\Omega = 1$, $h = 0.5$, and $\sigma_8 = 0.6$ (here σ_8 is the rms density contrast in a 8 h^{-1}Mpc sphere). These simulations contain N= 64^3 particles in a $L = 180$ h^{-1}Mpc box. The first three moments at separation $r = 10.4$ h^{-1}Mpc, and $\theta = 0°$, determined directly from the simulations, are (v_{21}, σ_{21}, ${\mu_3}^{1/3}$) = (-190, 440, -400) km s^{-1}, while equations (3), (4) and (5) give ($-200, 430, -430$) km s^{-1}, respectively. Comparisons with numerical experiments of higher resolution (N = 256^3, Springel et al. 1998) suggest that the accuracy of the N = 64^3 results, quoted above, is $\sim$ 20%. We conclude that the perturbative predictions are in excellent agreement with the N-body experiments.

3. The origin of the exponential tails

Eulerian perturbation theory, truncated at second order, can be used to write $p(w|r,\theta)$ as a marginal probability, obtained by integrating a 14-dimensional Gaussian distribution. Here we will not do that, however. Instead, we will trade accuracy for simplicity of calculations and replace rigorous perturbation theory with a following Ansatz. Suppose that the relation between the pairwise velocity and the random vector $\Re = \{\delta, \delta', \mathbf{v}, \mathbf{v}'\}$ is given by the mapping

$$w = u(1+\delta^{(1)\,\prime})(1+\delta^{(1)}) \; = \; u + u\Delta + O(u^3) \; , \tag{8}$$

where $u \equiv v_z^{(1)\,\prime} - v_z^{(1)}$ and $\Delta \equiv \delta^{(1)\,\prime} + \delta^{(1)}$. The first three moments of this new variable can be readily expressed in terms of v_{21} and σ_{21}. To lowest non-vanishing order, we obtain $\langle w \rangle = v_{21}(r)\,\cos\theta$; $\langle (w - \langle w \rangle)^2 \rangle = {\sigma_{21}}^2(r,\theta)$, and

$$\langle (w - \langle w \rangle)^3 \rangle \; = \; 6\,{\sigma_{21}}^2(r,\theta)\,v_{21}\,\cos\theta \; . \tag{9}$$

Clearly, the transverse ($\theta = 90^\circ$) components of the above moments agree with those obtained from rigorous second-order Eulerian theory in § 2 above. Hence, our Ansatz provides an acceptable approximation of the second-order prediction for $p(w|r,90^\circ)$. For the radial part, we recover the true values of the first two moments only. The third moment, obtained from the approximate distribution is an overestimate of the true $|\mu_3(r,0^\circ)|$. However, the approximate expression (9) correctly reproduces the negative sign of μ_3, the scaling with the cosmological density parameter, $\mu_3 \propto \Omega^{1.8}$, as well as the scaling with the two lower moments, introduced by the dominant, infall term $\propto v_{21}{\sigma_{21}}^2$.

According to equation (8), w is the sum of two variables, u and $\varpi \equiv u\Delta$. The probability distribution for u is

$$p_u(u) = \frac{1}{\sqrt{2\pi}\sigma_{21}} \, \exp\left\{ -\frac{u^2}{2{\sigma_{21}}^2} \right\} \; . \tag{10}$$

The probability distribution for $\varpi \equiv u\Delta$ is readily obtained by integrating the expression

$$p_\varpi(\varpi|r,\theta) = \int\limits_{-\infty}^{+\infty} \frac{du}{|u|}\, p_{u\Delta}\left(u, \frac{\varpi}{u}\right) \ , \tag{11}$$

where $p_{u\Delta}(u, \varpi/u)$ is a standard, joint-normal distribution for u and Δ (eg. F95), with ϖ/u substituted for Δ. This integral gives

$$p_\varpi(\varpi|r,\theta) = \frac{\alpha}{\pi\sigma_{21}}\, e^{\beta\kappa\varpi}\, \mathrm{K}_0\left(\beta|\varpi|\right) \ , \tag{12}$$

where K_0 is the usual modified Bessel function, $\alpha \equiv 1/\sigma_\Delta\sqrt{1-\kappa^2}$, $\beta \equiv 1/\sigma_{21}\sigma_\Delta(1-\kappa^2)$, $\kappa \equiv v_{21}\cos\theta/\sigma_\Delta\sigma_{21}$, and $\sigma_\Delta^2 \equiv \langle\Delta^2\rangle = 2\,[\,\xi(0)+\xi(r)\,]$. There are several remarks worth noting about the properties of the random variable ϖ. Near the origin, its distribution has a cusp; for small values of the argument, $\mathrm{K}_0(|x|) = -[\ln(|x|/2) + 0.577] + O(x^2)$. For large values of the argument, $\mathrm{K}_0(|x|) = \sqrt{\pi/2|x|}\, e^{-|x|}\,[1 + O(1/|x|)]$: this is the exponential behavior in the wings, typical for products of Gaussian fields (eg. the χ^2 distribution; see also Scherrer 1992 or Holzer and Siggia 1993). Finally, note that the exponential in equation (12) is not symmetric about $\varpi = 0$, and its skewness is introduced by cross-correlation between the velocity and density (i.e., by the infall). The distribution of $w = \varpi + u$ is qualitatively similar to p_ϖ; it can be obtained from an expression, similar to eq. (11) with the original integrand, replaced by $|u|^{-1}p_{u\Delta}\,[u, (w/u) - 1]$. The resulting integral can be rewritten as

$$p(w|r,\theta) = \int\limits_0^\infty \frac{d\sigma}{\sigma}\, \exp\left\{-\frac{w^2}{2\sigma^2}\right\} W(w, v_{21}, \sigma_{21})\,, \tag{13}$$

with

$$W = \frac{\alpha}{\pi\sigma_{21}}\, e^{\beta\kappa w - \frac{1}{2}(\beta^2\sigma^2+\alpha^2)} \cosh\left[\alpha\left(\frac{w}{\sigma} - \kappa\beta\sigma\right)\right]. \tag{14}$$

One can approximate the above integral via the method of steepest descent. This yields a convenient analytic formula for the probability distribution,

$$p(w|r,\theta) \approx \frac{\cosh\left(\sqrt{|U|}\,/K\sigma_\Delta\right)}{\sqrt{2\pi\sigma_{21}\sigma_\Delta|w|}}\, \exp\left\{-\frac{|U|}{K} - \frac{\alpha^2}{2}\right\} \tag{15}$$

where $U = w/\sigma_{21}\sigma_\Delta$, $K = 1 + \mathrm{sgn}(w)\kappa$, $\mathrm{sgn}(w) = +1$ for $w \geq 0$, and $\mathrm{sgn}(w) = -1$ for $w < 0$. The characteristic function is given by the Fourier transform of equation (13),

$$\langle e^{iwt} \rangle \ = \ [\phi(t)]^{-1/2} \exp\left(-{\sigma_{21}}^2 t^2 / 2\phi(t)\right) \ , \tag{16}$$

where $\phi(t) = 1 - 2i\kappa\sigma_{21}\sigma_\Delta t + \sigma_\Delta^2(1-\kappa^2){\sigma_{21}}^2 t^2$. In the limit $\sigma_\Delta \to 0$, $\langle e^{iwt} \rangle \to \exp(-{\sigma_{21}}^2 t^2/2)$, and we simply recover the Gaussian distribution $p_u(u)$ (eq. 10), in agreement with F95. This is obvious when we recall that non-Gaussian behavior of the distribution arises from the quadratic nonlinearities introduced by the number weighting; these quadratic terms become small compared to the volume weighted (and Gaussian distributed) velocity difference as the amplitude of the fluctuations decreases. In the opposite limit, when σ_Δ is increased, the distribution rapidly develops a central cusp and exponential wings. It is interesting to compare our results, valid in the transition zone between the linear and nonlinear regime, on separations $r \gtrsim 5 \ h^{-1}$Mpc, with the analysis of Diaferio & Geller (1996) and Sheth (1996), restricted to much smaller scales, where $\sigma_\Delta \gg 1$ and the perturbative approach breaks down. Note that the integral in equation (13) is a weighted sum over Gaussians having a range of dispersions. The weighting factor W is determined by the velocity correlation tensor and the velocity-density cross correlation, v_{21}. This expression is similar to equation (5) of Sheth (1996), valid in the strongly nonlinear regime, where W is related to the Press-Schechter multiplicity function. The outcome of summing Gaussian distributions in both cases is an exponential distribution. A significant difference is that in the strongly nonlinear regime all sources of the velocity skewness vanish: the limit $r \to 0$, $\sigma_\Delta \gg 1$ corresponds to virialized cluster centers; there is no infall ($v_{21} = 0$); the velocity dispersion is isotropic ($\Pi - \Sigma = 0$), and $\mu_3 = 0$ by symmetry.

We will now compare the predicted velocity distribution with direct measurements from simulations. Since the N-body results we have at our disposal assume a CDM spectrum, we need to introduce a shortwave cutoff in the initial power spectrum; otherwise $\xi(0)$ becomes infinite. We use a Gaussian filter of width R_s and multiply the linear power spectrum, $P(k) = \int \xi(r) \exp(i\mathbf{k}\cdot\mathbf{r}) d^3r$ by $\exp(-k^2 R_s^2)$. The resulting $\xi(r)$ is finite at $r = 0$ and remains flat for $r \lesssim R_s$. Existence of such a 'core radius' in any realistic $\xi(r)$ is necessary anyway since galaxies are not point-like objects and have finite sizes. R_s can also be related to the effective dynamical resolution of the simulations; we postpone the discussion of this problem to a later paper (Springel et al. 1998). Finally, the small-scale cutoff can be useful as a makeshift solution for reducing the $|\mu_3|$, overestimated by our Ansatz, and bringing this moment closer to the value predicted by the rigorous perturbative calculation in §2 above. We tried several filtering widths and found that $R_s = 3$ h^{-1}Mpc provides a reasonable fit to N-body simulations. In Figure 1 we compare the probability distribution, $p(w|r, 0°)dw$, calculated from equation (13), with direct measurements from N-body simulations. The upper panel shows the results of measurements from the simulations of Frenk et al. (1990; N $\approx 2.6 \times 10^5$), while the lower panel – those from Żurek et al. (1994; N $\approx 1.7 \times 10^7$), obtained by fitting their Fig. (7d) by a double exponential. The separation is respectively $r = 10.4$ h^{-1}Mpc, and $r = 10.5$ h^{-1}Mpc. The sizes of the velocity bins are $dw = 72$ and 100 km s^{-1} for the upper and lower panel, respectively. The assumed $P(k)$ in both cases is the standard CDM spectrum, described in §2 (the only difference is that here we use $P(k)\exp(-k^2R_s^2)$ with $R_s = 3$ h^{-1}Mpc, while in §2 $R_s = 0$). Clearly, the agreement between the perturbative predictions and N-body measurements improves when the resolution of the simulations is improved.

A possible alternative to the Ansatz, adopted in this section, is to derive $p(w)$ from the Zel'dovich (1970) approximation (hereafter ZA). Like our Ansatz, this approach makes calculations simpler than the rigorous treatment in §2 above, and the simplicity here, too, is bought at the expense of accuracy in estimating μ_3: at second order, the ZA breaks the momentum conservation (Juszkiewicz et al. 1993; BJDB) by implying $C = 0$ in eqs. (6) and (7). As a result, the ZA underestimates $|\mu_3|$ by $\sim 50\%$. At first order, however, the ZA agrees with the rigorous Eulerian perturbation theory, so its predictions for v_{21} and σ_{21} must be identical with ours. The ZA was recently used by Seto and Yokoyama (1998). Qualitatively, their $p(w)$ is similar to ours. However, at the quantitative level we disagree because their method underestimates v_{21} by at least an order of magnitude. As a consequence, Seto and Yokoyama had to readjust their predicted $p(w)$ 'by hand' to achieve agreement with simulations. Their results seem puzzling given the properties of the ZA, discussed above.

Another alternative approach one might consider, is to expand $p(w)$ in orthogonal polynomials (eg. Juszkiewicz et al. 1995; Lifshitz and Pitaevskii 1981, p. 31). At the end, only direct applications to future redshift surveys, like the SDSS or 2dF, will decide which of the discussed physical models of $p(w)$ provides the optimal combination of simplicity and accuracy.

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REFERENCES

Bernardeau, F., Juszkiewicz, R., Dekel, A. & Bouchet, F.R. 1995, MNRAS, 274, 20 (BJDB)

Davis, M. & Peebles, P.J.E 1983, ApJ, 267, 465

Diaferio, A. & Geller, J.M. 1996, ApJ, 467, 19

Efstathiou, G., Frenk, C., White, S., & Davis, M. 1988, MNRAS, 235, 715 (EFWD)

Fisher, K.B. 1995, ApJ, 448, 494 (F95)

Fisher, K.B, et al. 1994, MNRAS, 267, 927 (F94)

Frenk, C., White, S., Efstathiou, G., & Davis, M. 1990, ApJ, 351, 10

Górski, K. 1988, ApJ, 332, L7

Groth, E.J., Juszkiewicz, R., & Ostriker, J.P. 1989, ApJ, 346, 558

Hamilton, A. J. S. 1992, ApJ, 385, L5

Holzer, M., & Siggia, E. 1993, Phys. Fluids A, 5 (10), 2525

Juszkiewicz, R., Bouchet, F.R., & Colombi, S. 1993, ApJ, 412, L9

Juszkiewicz, R., et al. 1995, ApJ, 442, 39

Kaiser, N. 1987, MNRAS, 227, 1

Landy, S.D., Szalay, A.S., Broadhurst, T.J. 1997, ApJ, submitted (astro-ph/9711045)

Lifshitz, E.M., & Pitaevskii, L.P. 1981, Physical Kinetics, (Oxford: Pergamon Press)

Loveday, J., Efstathiou, G., Maddox, S.J., & Peterson, B. A. 1996, ApJ, 468, 1

Marzke, R.O., Geller, M.J., da Costa, L.N., & Huchra, J.P. 1995, AJ, 110, 477

Peebles, P.J.E. 1976, Ap&SS, 45, 3

Peebles, P.J.E. 1980, The Large-Scale Structure of the Universe, (Princeton: Princeton University Press) (LSS)

Peebles, P.J.E. 1993, Principles of Physical Cosmology, (Princeton: Princeton University Press) (PPC)

Press, W., & Schechter, P. 1974, ApJ, 187, 425

Regös, E., & Szalay, A. 1995, MNRAS, 272, 447

Sargent, W. W., & Turner, E. L. 1977, ApJ, 212, L3

Scherrer, R.J. 1992, ApJ, 390, 330

Seto, N., & Yokoyama, J. 1998, ApJ, 492, 421

Sheth, R.K. 1996, MNRAS, 279, 1310

Springel, V., Juszkiewicz, R., & White, S. 1998, paper in preparation

Zel'dovich, Ya.B. 1970, A&A, 5, 84

Żurek, W.H., Quinn, P.J., Salmon, J.K., & Warren, M.S. 1994, ApJ, 431, 559 (ZQSW)

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FIGURE CAPTION

Figure 1

Comparison of the probability distribution, $dP = p(w|r, 0°)dw$, derived from equation (13) (shown as curves) with direct determinations from N-body experiments (shown as histograms). The number of particles in the two sets of simulations was $\mathrm{N} = 2.6 \times 10^5$ (*upper panel*) and $\mathrm{N} = 1.7 \times 10^7$ (*lower panel*). Note that an increase in resolution brings the N-body results closer to the perturbative predictions.