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Vector Quark Model and B Meson Radiative Decay

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Abstract

We study the B meson radiative decay $B \to X_s \gamma$ in the vector quark model. Deviation from the Standard Model arises from the non-unitarity of the charged current KM matrix and related new FCNC interactions. We establish the relation between the non-unitarity of charged current mixing matrix and the mixing among the vector quark and the ordinary quarks. We also make explicitly the close connection between this non-unitarity and the flavor changing neutral currents. The complete calculation including leading logarithmic QCD correction is carefully carried out. Using the most updated data and the NLO theoretical calculation, the branching fraction of the observed B meson radiative decay places a limit on the mixing angles as stringent as that from the process $B \to X \mu\bar{\mu}$.

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1 Introduction

A simple extension of Standard Model (SM) is to enlarge the particle content by adding vector quarks, whose right-handed and left-handed components transform in the same way under the weak $SU(2) \times U(1)$ gauge group. This extension is acceptable because the anomalies generated by the vector quarks cancel automatically and vector quarks can be heavy naturally. Vector quarks also arise in some Grand Unification Theory (GUT). For example, in some superstring theories, the E_6 GUT gauge group occurs in four dimensions when we start with $E_8 \times E_8$ in ten dimensions. The fermions are placed in a 27-dimensional representation of E_6. In such model, for each generation one would have new fermions including an isosinglet charge $-\frac{1}{3}$ vector quark.

Recently there is renewed interest in the models with vector quarks partly because of the reported apparent R_b excess, and the R_c deficit in the data [1]. The later one seems to be disappearing as statistics improves [2]. Several authors [3, 4, 5] suggested ways to understand discrepancies by introducing new vector fermions that mix with b and/or c quarks. The mixing will reduce or enhance the couplings of the mixed quarks to Z boson depending on the gauge quantum numbers of the new fermions. For example, in Ref. [3], a vector isosinglet plus a vector isotriplet are introduced. In Ref. [5], a model with vector isodoublet is considered. In Ref.[6], constraints from the precision measurements are analyzed and the result is in favor of the model in Ref. [5].

In this article, we discuss the B meson radiative decay in the context of a generic vector quark model and show that the experimental data can be used to constrain the mixing angles. In vector quark models, due to the mixing of vector quarks with ordinary quarks, the Kobayashi-Maskawa (KM) matrix of the charged current interaction is not unitary. The internal flavor independent contributions in the W exchange penguin diagrams no longer cancel among the various internal up-type quarks. In addition, the mixing also generates non-zero tree level FCNC in the currents of Z boson and that of Higgs boson, which in turn gives rise to new penguin diagrams due to neutral meson exchanges. All these contributions will be carefully analyzed in this paper. Leading logarithmic (LL) QCD corrections are also included by using the effective Hamiltonian formalism. The paper is organized as follows: In section 2, we review the charged current interaction and the FCNC interactions in a generic vector quark model. Through the diagonalization of mass matrix, the non-unitarity of KM matrix and the magnitude of the FCNC can both be related to the mixing angles between vector and ordinary quarks. In section 3, various contributions to B meson radiative decays are discussed in the vector quark model. In section 4, we discuss constraints on the mixing angles from the new data on B radiative decays and from other FCNC effects. There are many previous analyses on the same issue. We shall make detailed comparison at appropriate points (mostly in section 3.) of our discussion. Most vector quark models in the literature are more complicated than the one we considered here.

2 Vector Quark Model

We consider the model in which the gauge structure of SM remains while one charge $-\frac{1}{3}$ and one charge $\frac{2}{3}$ isosinglet vector quarks are introduced. Denote the charge $-\frac{1}{3}$ vector quark as D and the charge $\frac{2}{3}$ vector quark as U. Large Dirac masses of vector quarks, invariant under $SU(2)_L$, naturally arise:

$$M_U(\bar{U}_L U_R + \bar{U}_R U_L) + M_D(\bar{D}_L D_R + \bar{D}_R D_L) \tag{1}$$

All the other Dirac masses can only arise from $SU(2)_L$ symmetry breaking effects. Assume that the weak $SU(2)$ gauge symmetry breaking sector is an isodoublet scalar Higgs field ϕ, denoted as

$$\phi \equiv \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} = \begin{pmatrix} \phi^+ \\ \frac{1}{\sqrt{2}}(v + h^0) \end{pmatrix} \tag{2}$$

We can express the neutral field h in terms of real components:

$$h^0 = H + i\chi. \tag{3}$$

The conjugate of ϕ is defined as

$$\tilde{\phi} \equiv \begin{pmatrix} \phi^{0*} \\ -\phi^- \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{2}}(v + h^{0*}) \\ -\phi^- \end{pmatrix} \tag{4}$$

Masses for ordinary quarks arise from gauge invariant Yukawa couplings:

$$- f_d^{ij}\,\bar{\psi}_L^i d_R^j \phi - f_u^{ij}\,\bar{\psi}_L^i u_R^j \tilde{\phi} - f_d^{ij*}\,\phi^\dagger \bar{d}_R^j \psi_L^i - f_u^{ij*}\,\tilde{\phi}^\dagger \bar{u}_R^j \psi_L^i \tag{5}$$

In addition, gauge invariant Yukawa couplings between vector quarks and ordinary quarks are possible, which give rise to mixing between quarks of the same charge. For the model we are considering, these are:

$$- f_d^{i4}\,\bar{\psi}_L^i D_R \phi - f_u^{i4}\,\bar{\psi}_L^i U_R \tilde{\phi} - f_d^{i4*}\,\phi^\dagger \bar{D}_R \psi_L^i - f_u^{i4*}\,\tilde{\phi}^\dagger \bar{U}_R \psi_L^i \tag{6}$$

In general, U will mix with the up-type quarks and D with down-type quarks. It is thus convenient to put mixing quarks into a four component column matrix:

$$(u_{L,R})_\alpha = \begin{bmatrix} u_{L,R} \\ c_{L,R} \\ t_{L,R} \\ U_{L,R} \end{bmatrix}_\alpha \quad (d_{L,R})_\alpha = \begin{bmatrix} d_{L,R} \\ s_{L,R} \\ b_{L,R} \\ D_{L,R} \end{bmatrix}_\alpha \tag{7}$$

where $\alpha = 1, 2, 3, 4$. All the Dirac mass terms can then be collected into a matrix form:

$$\bar{d}'_L \mathcal{M}_d d'_R + \bar{d}'_R \mathcal{M}_d^\dagger d'_L \quad \text{and} \quad \bar{u}'_L \mathcal{M}_u u'_R + \bar{u}'_R \mathcal{M}_u^\dagger u'_L. \tag{8}$$

In this article, we use fields with prime to denote the weak eigenstates and those without prime to denote mass eigenstates. $\mathcal{M}_{u,d}$ are 4×4 mass matrices. Since all the right-handed quarks, including vector quark, are isosinglet, we can use the right-handed chiral transformation to choose the right handed quark basis so that U_L, D_L do not have Yukawa coupling to the ordinary right-handed quarks. In this basis, $\mathcal{M}_d$ and $\mathcal{M}_u$ can be written as

$$\mathcal{M}_d = \begin{pmatrix} \hat{M}_d & \vec{J}_d \\ 0 & M_D \end{pmatrix}, \quad \mathcal{M}_u = \begin{pmatrix} \hat{M}_u & \vec{J}_u \\ 0 & M_U \end{pmatrix}. \tag{9}$$

with

$$\hat{M}_{u,d} = \frac{v}{\sqrt{2}} f_{u,d}, \quad \vec{J}^i_{u,d} = \frac{v}{\sqrt{2}} f^{i4}_{u,d} \tag{10}$$

$\hat{M}_{d,u}$ (with hats) are the standard 3×3 mass matrices for ordinary quarks. $\vec{J}_{d,u}$ is the three component column matrix which determines the mixings between ordinary and vector quarks. We assume that the bare masses $M_{U,D}$ are much larger M_W. With $M_{U,D}$ factored out, $\mathcal{M}_{d,u}$ can be expressed in terms of small dimensionless parameters a, b:

$$\mathcal{M}_d = M_D \begin{pmatrix} \hat{a}_d & \vec{b}_d \\ 0 & 1 \end{pmatrix}, \quad \mathcal{M}_u = M_U \begin{pmatrix} \hat{a}_u & \vec{b}_u \\ 0 & 1 \end{pmatrix}. \tag{11}$$

The mixing matrix $U_L^{u,d}$ of the left-handed quarks and the corresponding one $U_R^{u,d}$ for right-handed quarks, defined as,

$$u'_{L,R} = U^u_{L,R} u_{L,R}, \quad d'_{L,R} = U^d_{L,R} d_{L,R}, \tag{12}$$

are the matrices that diagonalize $\mathcal{M}_{u,d}\mathcal{M}^\dagger_{u,d}$ and $\mathcal{M}^\dagger_{u,d}\mathcal{M}_{u,d}$ respectively. Hence the mass matrices can be expressed as

$$\mathcal{M}_u = U^u_L m_u U^{u\dagger}_R \quad \mathcal{M}_d = U^d_L m_d U^{d\dagger}_R \tag{13}$$

with $m_{u,d}$ the diagonalized mass matrices. The diagonalization can be carried out order by order in perturbation expansion with respect to small numbers $\hat{a}$ and $\vec{b}$. For isosinglet vector quark model, the right-handed quark mixings are significantly smaller. The reason is that $M_d^\dagger M_d$ is composed of elements suppressed by two powers of a or b except for the $(4,4)$ element. As a result, the mixings of D_R with d_R, s_R, b_R are also suppressed by two powers of a or b. On the other hand, it can be shown that the mixings between D_L and b_L, s_L, d_L are only of first order in a or b. To get leading order results in the perturbation, one can assume that $U_R = I$. For convenience, write U_L as

$$U_L = \begin{pmatrix} \hat{K} & \vec{R} \\ \vec{S}^T & T \end{pmatrix} . \tag{14}$$

where $\hat{K}$ is a 3×3 matrix and $\vec{R}, \vec{S}$ are three component column matrices. To leading order in a and b, T is equal to 1. K equals the unitary matrix that diagonalizes $\hat{a}\hat{a}^\dagger$. The columns $\vec{R}$ and $\vec{S}$, characterizing the mixing, are given by

$$\vec{R} = \vec{b}, \quad \vec{S} = -\hat{K}\vec{b}. \tag{15}$$

Now we can write down the various electroweak interactions in terms of mass eigenstates. The Z coupling to the left-handed mass eigenstates are given by

$$\mathcal{L}_Z = \frac{g}{\cos\theta_W} Z_\mu (J_3^\mu - \sin^2\theta_W J_{\rm em}^\mu), \tag{16}$$

$$J_3^\mu = \bar{u}'_L T_3^w \gamma^\mu u'_L + \bar{d}'_L T_3^w \gamma^\mu d'_L = \frac{1}{2}\bar{u}_L z^u \gamma^\mu u_L - \frac{1}{2}\bar{d}_L z^d \gamma^\mu d_L \tag{17}$$

The 4×4 matrices z are related to the mixing matrices by

$$\begin{aligned} z^u &= U_L^{u\dagger} a_Z U_L^u \\ z^d &= U_L^{d\dagger} a_Z U_L^d. \end{aligned} \tag{18}$$

with $a_Z \equiv {\rm Diag}(1,1,1,0)$. Note that the matrix z is not diagonal. Flavor Changing Neutral Current (FCNC) is generated by the mixings between ordinary and vector quarks[7, 8, 9].

The charged current interaction is given by

$$\mathcal{L}_W = \frac{g}{\sqrt{2}}(W_\mu^- J^{\mu+} + W_\mu^+ J^{\mu-}), \tag{19}$$

$$J^{\mu-} = \bar{u}'_L a_W \gamma^\mu d'_L = \bar{u}_L V \gamma^\mu d_L \tag{20}$$

where $a_W \equiv {\rm Diag}(1,1,1,a)$ is composed of the Clebsch-Gordon coefficients of the corresponding quarks. For an isosinglet vector quark, $a = 0$. The 4×4 generalized KM matrix V is given by:

$$V = U_L^{u\dagger} a_W U_L^d. \tag{21}$$

The standard 3×3 KM matrix $V_{\rm KM}$ is the the upper-left submatrix of V. Neither V nor $V_{\rm KM}$ is unitary. Note that the non-unitarity of V is captured by two matrices

$$\begin{aligned}(V^\dagger V) &= U_L^{d\dagger} a_W^2 U_L^d \\ (VV^\dagger) &= U_L^{u\dagger} a_W^2 U_L^u .\end{aligned} \tag{22}$$

In the model we are considering, these two matrices are identical to $z^{u,d}$ of the FCNC effects in Eq. 18 since a_W^2 is equal to a_z. Indeed

$$V^\dagger V = z^d, \quad VV^\dagger = z^u \tag{23}$$

This intimate relation between the non-unitarity of W charge current and the FCNC of Z boson is important for maintaining the gauge invariance of their combined contributions to any physical process.

The off-diagonal elements of these matrices, characterizing the non-unitarity, is closely related to the mixing of ordinary and vector quarks. The off-diagonal elements are of order a^2 or b^2. To calculate it, in principle, the next-to-leading order expansion of $\hat{K}$, denoted as $\hat{K}_2$, is needed. In fact

$$(V^\dagger V)_{ij} = (\hat{K}_2^d + \hat{K}_2^{d\dagger})_{ij} + a^2(\vec{b}_d)_i(\vec{b}_d)_j^* \tag{24}$$

Fortunately, by the unitarity of the mixing matrix U^d, the combination $\hat{K}_2^d + \hat{K}_2^{d\dagger}$ is equal to $-(\vec{b}_d)(\vec{b}_d)^\dagger$.

$$\hat{K}_2^d + \hat{K}_2^{d\dagger} = -(\vec{b}_d)(\vec{b}_d)^\dagger \tag{25}$$

Thus the off-diagonal elements can be simplified

$$(V^\dagger V)_{ij} = (-1 + a^2)(\vec{b}_d)_i(\vec{b}_d)_j^* \tag{26}$$

For isosinglet vector quark, $a = 0$.

The Yukawa couplings between Higgs fields and quarks in weak eigenstate can be written in a matrix form as

$$-\frac{v}{\sqrt{2}}\left(\bar{\psi}'_L a_z \mathcal{M}_d d'_R \phi + \bar{d}'_R \mathcal{M}_d^\dagger a_z \psi'_L \phi^\dagger + \bar{\psi}'_L a_z \mathcal{M}_u u'_R \tilde{\phi} + \bar{u}'_R \mathcal{M}_u^\dagger a_z \psi'_L \tilde{\phi}^\dagger\right) \tag{27}$$

Note that $\hat{a}_Z$ is added to ensure that the left handed isosinglet vector quarks do not participate in the Yukawa couplings. The Yukawa interactions of quark mass eigenstates and unphysical charged Higgs fields $\phi^\pm$ are given by

$$\mathcal{L}_{\phi^\pm} = \frac{g}{\sqrt{2}M_W}\left[\bar{u}(m_u V L - V m_d R)d\right]\phi^+ + \frac{g}{\sqrt{2}M_W}\left[\bar{d}(-m_d V^\dagger L + V^\dagger m_u R)u\right]\phi^- \tag{28}$$

while those of Higgs boson H and unphysical neutral Higgs field χ by

$$\mathcal{L}_H = -\frac{g}{2M_W}\left[\bar{d}(m_d z^d L + z^d m_d R)d + \bar{u}(m_u z^u L + z^u m_u R)u\right] H \tag{29}$$

$$\mathcal{L}_\chi = -\frac{ig}{2M_W}\left[\bar{d}(-m_d z^d L + z^d m_d R)d + \bar{u}(m_u z^u L - z^u m_u R)u\right] \chi^0 \tag{30}$$

3 B Meson Radiative Decay

The $B \to X_s\gamma$ decay, which already exists via one-loop W-exchange diagram in SM, is known to be extremely sensitive to the structure of fundamental interactions at the electroweak scale and serve as a good probe of new physics beyond SM because new interaction generically can also give rise to significant contribution at the one-loop level.

The inclusive $B \to X_s\gamma$ decay is especially interesting. In contrast to exclusive decay modes, it is theoretically clean in the sense that no specific low energy hadronic model is needed to describe the decays. As a result of the Heavy Quark Effective Theory (HQET), the inclusive B meson decay width $\Gamma(B \to X_s\gamma)$ can be well approximated by the corresponding b quark decay width $\Gamma(b \to s\gamma)$. The corrections to this approximation are suppressed by $1/m_b^2$ [10] and is estimated to contribute well below 10% [11, 12]. This numerical limit is supposed to hold even for the recently discovered non-perturbative contributions which are suppressed by $1/m_c^2$ instead of $1/m_b^2$ [14]. In the following, we focus on the dominant quark decay $b \to s\gamma$.

In SM, $b \to s\gamma$ arises at the one loop level from the various W mediated penguin diagrams as in Fig. 1. The number of diagrams needed to be considered can be reduced by choosing the non-linear R_ξ gauge as in [15]. The gauge fixing term is:

$$\frac{1}{2\alpha}(\partial_\mu A^\mu)^2 + \frac{1}{2\eta}(\partial_\mu Z^\mu + \eta M_Z \chi)^2 + \frac{1}{\xi}|(\partial_\mu - igA_\mu^3)W^{\mu +} - i\xi M_W \phi^+|^2 \tag{31}$$

where A_μ^3 can be expressed in terms of A_μ and Z_μ:

$$A_\mu^3 = A_\mu \sin\theta_W - Z_\mu \cos\theta_W. \tag{32}$$

In this gauge, the tri-linear coupling involving photon, W meson and the unphysical Higgs field ϕ^+ vanishes. Therefore only four diagrams contribute: two of them consist of W meson exchange, with photon emitted respectively from the W meson and the internal quark, and the other two consist of unphysical Higgs field exchange.

For convenience, we choose the gauge parameters $\alpha = \eta = \xi = 1$. The fermion and gauge meson propagators are hence identical to those in the Feynman gauge. The on-shell

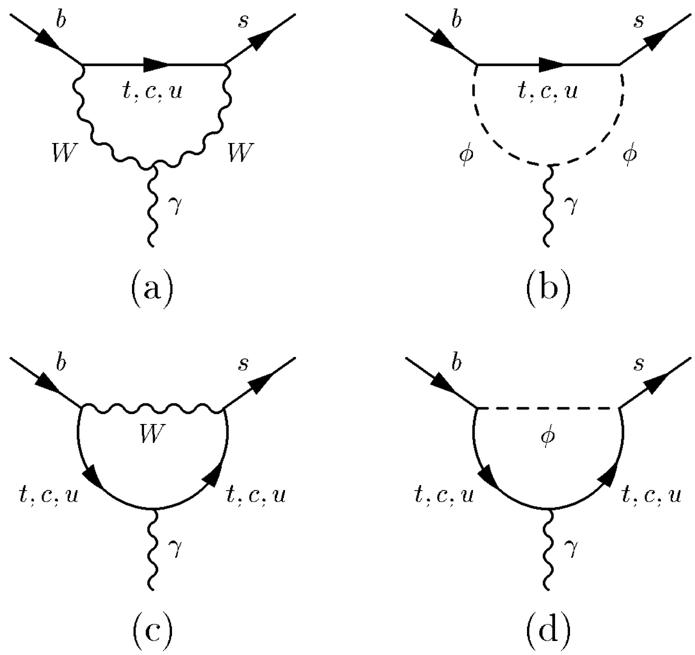

Figure 1: Charged meson mediated penguin.

Feynman amplitude can be written as

$$i\mathcal{M}(b \to s\gamma) = \frac{\sqrt{2}G_F}{\pi}\frac{\epsilon}{4\pi}\sum_i V_{ib}V_{is}^* F_2(x_i) q^\mu \epsilon^\nu \bar{s}\sigma_{\mu\nu}(m_b R + m_s L) b \tag{33}$$

with $x_i \equiv m_i^2/M_W^2$ The sum is over the quarks u, c and t. The contributions to F_2 from the four diagrams are denoted as $f_{1,2}^{W,\phi}$, with the subscript 1 used to denote the contribution of diagrams with photon emitted from internal quark and 2 that of those with photon emitted from W meson. The functions f's are given by

$$\begin{aligned} f_1^W(x) &= Q_i\left[\xi_0(x) - \tfrac{3}{2}\xi_1(x) + \tfrac{1}{2}\xi_2(x)\right] \ , & (34)\\ f_2^W(x) &= \xi_{-1}(x) - \tfrac{5}{2}\xi_0(x) + 2\xi_1(x) - \tfrac{1}{2}\xi_2(x) \ , & (35)\\ f_1^\phi(x) &= \tfrac{1}{4}Q_i x\left[\xi_1(x) + \xi_2(x)\right] \ , & (36)\\ f_2^\phi(x) &= \tfrac{1}{4}x\left[\xi_0(x) - \xi_2(x)\right] \ . & (37)\end{aligned}$$

Here the functions $\xi(x)$ are defined as

$$\xi_n(x) \equiv \int_0^1 \frac{z^{n+1}\mathrm{d}z}{1+(x-1)z} = -\frac{\ln x + (1-x) + \cdots + \frac{(1-x)^{n+1}}{n+1}}{(1-x)^{n+2}} \ , \tag{38}$$

and

$$\xi_{-1}(x) \equiv \int_0^1 \frac{\mathrm{d}z}{1+(x-1)z} = -\frac{\ln x}{1-x} \tag{39}$$

$F_2(x)$ is the sum of these functions and is given by

$$F_2(x) = f_1^W(x) + f_2^W(x) + f_1^\phi(x) + f_2^\phi(x) = \frac{8x^3 + 5x^2 - 7x}{24(1-x)^3} - \frac{x^2(2-3x)}{4(1-x)^4} \ln x + \frac{23}{36} \quad (40)$$

For light quarks such as u and c, with $x_i \to 0$, the first two terms on the right hand side vanish. $F_2(x_{u,c})$ is dominated by the x independent term $\frac{23}{36}$. However these mass-independent terms get canceled among the up-type quarks due to the unitarity of KM matrix in SM

$$\sum_i V_{ib} V_{is}^* = 0 \quad (41)$$

After the cancelation, the remaining contributions are essentially from penguins with internal t quark.

It is convenient to discuss weak decays using the effective Hamiltonian formalism [16, 17], which is crucial for incorporating the QCD corrections to be discussed later. The important dim-6 operators relevant for $b \to s\gamma$ are

$$H_{\text{eff}} = -\frac{4G_F}{\sqrt{2}} V_{ts}^* V_{tb} \sum_{i=1}^{8} C_i(\mu) O_i \quad ; \quad (42)$$

$$\begin{aligned}
O_1 &= (\bar{c}_{L\alpha}\gamma^\mu b_{L\beta})(\bar{s}_{L\beta}\gamma^\mu c_{L\alpha}) \quad ; & (43)\\
O_2 &= (\bar{c}_{L\alpha}\gamma^\mu b_{L\alpha})(\bar{s}_{L\beta}\gamma^\mu c_{L\beta}) \quad ; & (44)\\
O_3 &= (\bar{c}_{L\alpha}\gamma^\mu b_{L\alpha})\sum_q(\bar{q}_{L\beta}\gamma^\mu q_{L\beta}) \quad ; & (45)\\
O_4 &= (\bar{c}_{L\alpha}\gamma^\mu b_{L\beta})\sum_q(\bar{q}_{L\beta}\gamma^\mu q_{L\alpha}) \quad ; & (46)\\
O_5 &= (\bar{c}_{L\alpha}\gamma^\mu b_{L\alpha})\sum_q(\bar{q}_{R\beta}\gamma^\mu q_{R\beta}) \quad ; & (47)\\
O_6 &= (\bar{c}_{L\alpha}\gamma^\mu b_{L\beta})\sum_q(\bar{q}_{R\beta}\gamma^\mu q_{R\alpha}) \quad ; & (48)\\
O_7 &= \frac{e}{(4\pi)^2}\bar{s}_\alpha\sigma^{\mu\nu}(m_b R + m_s L) b_\alpha F_{\mu\nu} & (49)\\
O_8 &= \frac{g_s}{(4\pi)^2}\bar{s}_\alpha\sigma^{\mu\nu}(m_b R + m_s L)(\lambda^A_{\alpha\beta}) b_\alpha G^A_{\mu\nu} \quad . & (50)
\end{aligned}$$

The Wilson coefficients C_i at $\mu = M_W$ are determined by the matching conditions when the W and Z mesons are integrated out. Before QCD evolution, the only non-vanishing Wilson coefficients at $\mu = M_W$ for the above set are $C_{2,7,8}$. C_2 is generated by W gauge boson exchange current-current interaction and

$$C_2(M_W) = -V_{cs}^* V_{cb} / V_{ts}^* V_{tb}. \quad (51)$$

If the KM matrix is unitary, $C_2(M_W)$ is approximately equal to 1, since $V_{us}^* V_{ub}$ can be ignored. In SM, the Wilson coefficient C_7 at the scale M_W is thus given by the earlier penguin calculations,

$$C_7^{\rm SM}(M_W) = \frac{1}{V_{ts}^* V_{tb}} \sum_i V_{ib} V_{is}^* F_2(x_i) = -\frac{1}{2} D_0'(x_t) \simeq -0.193 \qquad . \tag{52}$$

The numerical value is given when $m_t = 180$ GeV. The constant term and the contributions from internal u and c quarks have been removed in the function D_0' defined as [16, 18]

$$D_0'(x) \equiv -\frac{8x^3 + 5x^2 - 7x}{12(1-x)^3} + \frac{x^2(2-3x)}{2(1-x)^4} \ln x. \tag{53}$$

Similarly, in SM the $b \to sg$ transition arises from W exchange penguin diagrams which induce O_8. Since the gluons do not couple to the W mesons, the gluonic W meson penguin consists only of two diagrams, which are given by $f_1^{W,\phi}$ with Q replaced by one. With the mass-independent contribution canceled, the Wilson coefficient C_8 can be written as

$$C_8^{\rm SM}(M_W) = -\frac{1}{2} E_0'(x_t) \simeq 0.096 \qquad , \tag{54}$$

The function E_0' is defined as [18]

$$E_0'(x) \equiv -\frac{x(x^2 - 5x - 2)}{4(1-x)^3} + \frac{3}{2} \frac{x^2}{(1-x)^4} \ln x. \tag{55}$$

It is well known that short distance QCD correction is important for $b \to s\gamma$ decay and actually enhances the decay rate by more than a factor of two. These QCD corrections can be attributed to logarithms of the form $\alpha_s^n(m_b) \log^m(m_b/M_W)$. The Leading Logarithmic Approximation (LLA) resums the LL series ($m \le n$). Working to next-to-leading-log (NLL) means that we also resum all the terms of the form $\alpha_s(m_b)\alpha_s^n(m_b) \log^n(m_b/M_W)$. In the effective Hamiltonian formalism, M_W appears only in the denominators of the operators and as the boundary of the matching calculation. Thus logarithmic dependence of M_W in the QCD corrections can be incorporated simply by running the renormalization scale from the matching scale $\mu = M_W$ down to m_b and then calculate the Feynman amplitude at the scale m_b. The evolution of the Wilson coefficients is determined by the differential equation of Renormalization Group running:

$$\mu \frac{d}{d\mu} C_i = C_j(\mu) \gamma_{ji} \tag{56}$$

where γ is the matrix of anomalous dimensions. The anomalous dimension has been calculated up to NLL order. We refer to [16, 12] for a review and details. The anomalous dimension is scheme dependent even to LL order. For example, it depends on how γ_5 is defined in dimensional regularization. The anomalous dimension is different in, say, Naive Dimension Regularization (NDR) scheme and t'Hooft-Veltman (TV) scheme. This dependence is canceled by the same scheme dependence in the matrix element of effective Hamiltonian operators to render a scheme independent physical result. In the literature it is customary to define and use a set of "effective Wilson coefficients" $C_i^{\rm eff}$ which is certain linear combinations of the original Wilson coefficients but is scheme independent to the leading order [19]. However, since $C_i^{\rm eff}$ are so defined to be identical to C_i in the TV scheme to LL, we can choose the TV scheme and suppress the superscript "eff". The Wilson coefficients at scale m_b can be related to those at scale M_W by integrating this differential equation [16]. As of matrix elements, to leading order in $\alpha_s(m_b)$, only O_7 has a non-vanishing matrix element between b and $s\gamma$. Thus we only need $C_7(m_b)$ to calculate the LLA of $b \to s\gamma$ decay width. For $m_t = 170$ GeV, $m_b = 5$ GeV and $\alpha_s^{(5)}(M_Z) = 0.117$, $C_7(m_b)$ is related to the non-zero Wilson coefficients at M_W by [16, 19, 20]

$$C_7^{(0)}(m_b) = 0.698\, C_7(M_W) + 0.086\, C_8(M_W) - 0.156\, C_2(M_W).$$

The $b \to s\gamma$ amplitude is given by

$$\mathcal{M}(b \to s\gamma) = -V_{tb}V_{ts}^* \frac{4G_F}{\sqrt{2}} C_7(m_b)\langle O_7\rangle_{\rm tree} \tag{57}$$

To avoid the uncertainty in m_b, it is customary to calculate the ratio R between the radiative decay and the dominant semileptonic decay. The ratio R is given, to LLA, by [19]

$$R \equiv \frac{\Gamma(b \to s\gamma)}{\Gamma(b \to ce\bar{\nu}_e)} = \frac{1}{|V_{cb}|^2} \frac{6\alpha}{\pi g(z)} \left|V_{ts}^* V_{tb} C_7(m_b)\right|^2 . \tag{58}$$

In the vector quark model, deviations from SM result come from various sources: (1) charged current KM matrix non-unitarity, (2) Flavor Changing Neutral Current (FCNC) effects in neutral meson mediated penguin diagrams, and (3) the W penguin with internal heavy U vector quark. Since the last one can be incorporated quite straight-forwardly, we do not elaborate on this contribution which will not be relevant for models without the U quark. We concentrate on the first two contributions, which have been discussed in Refs.[21, 22, 23]. Here we make a more careful and complete analysis which supplements or corrects these earlier analyses. Refs.[21] have calculated effects due to non-unitarity of the KM matrix and effects due to the Z mediated penguin in the Feynman gauge, however, their analysis did not include the FCNC contribution from the unphysical neutral Higgs boson, which is necessary for gauge invariance. The Higgs boson mediated penguins were also ignored. On the other hand, Ref.[22], while taking the unphysical Higgs boson into account, did not consider effects due to non-unitarity of the KM matrix, which gives the most important contribution. None of the above treatments, except Ref.[23], included QCD corrections.

For simplicity, we ignore QCD corrections for a moment. As shown in the last section, the KM matrix is not unitary in the presence of an isosinglet vector quark. The mass-independent contributions from the various up-type quarks no longer cancel and could give rise to a significant deviation from SM prediction. The extra contribution to the Wilson coefficient C_7 is given by

$$\frac{(V^\dagger V)_{23}}{V_{ts}^* V_{tb}} \frac{23}{36} = \frac{\delta}{V_{ts}^* V_{tb}} \frac{23}{36} \,. \tag{59}$$

The parameter δ, one of the off-diagonal elements of the matrix $V^\dagger V$, characterizes the non-unitarity:

$$\delta = (V^\dagger V)_{23} = z_{sb} \tag{60}$$

The $b \to s\gamma$ transitions also arise from FCNC Z meson and Higgs boson mediated penguin diagrams as in Fig. 2. The FCNC contribution to $C_7(M_W)$ can be denoted as follows:

$$\frac{z_{sb}}{V_{tb}V_{ts}^*}(f_{s,b}^Z + f_{s,b}^\chi + f_{s,b}^H) + \frac{z_{4b}z_{4s}^*}{V_{tb}V_{ts}^*}(f_D^Z + f_D^\chi + f_D^H) \tag{61}$$

For the sake of gauge invariance, f^Z needs to be considered together with f^χ. The Z meson penguins consist of internal charge $-\frac{1}{3}$ quarks. The contribution from internal $i = b, s$ quark, $f_{s,b}^Z$, is given by ($y_i \equiv m_i^2/M_Z^2$):

$$\begin{aligned} f_b^Z &= -\tfrac{1}{2}Q_d\Big\{(-\tfrac{1}{2} - Q_d \sin^2\theta_W)\,[2\xi_0(y_b) - 3\xi_1(y_b) + \xi_2(y_b)] \\ &\quad + Q_d \sin^2\theta_W\,[4\xi_0(y_b) - 4\xi_1(y_b)]\Big\} \end{aligned} \tag{62}$$

$$f_s^Z = -\tfrac{1}{2}Q_d\Big\{(-\tfrac{1}{2} - Q_d \sin^2\theta_W)\,[2\xi_0(y_s) - 3\xi_1(y_s) + \xi_2(y_s)]\Big\} \tag{63}$$

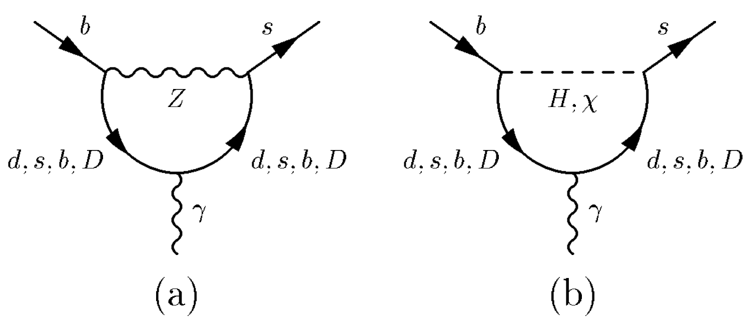

Figure 2: Neutral meson mediated penguin diagrams.

The calculation is similar to that of f^W. For a consistent approximation, the two variables y_b and y_s, which are the ratios of m_b^2, m_s^2 to M_Z^2, are also set to zero. Hence

$$\begin{aligned} f_b^Z + f_s^Z &\approx -\tfrac{1}{2}Q_d\Big\{(-\tfrac{1}{2} - Q_d \sin^2\theta_W)\,[\,4\xi_0(0) - 6\xi_1(0) + 2\xi_2(0)\,] \\ &\qquad +Q_d \sin^2\theta_W\,[\,4\xi_0(0) - 4\xi_1(0)\,]\Big\} \qquad (64) \\ &= -\frac{1}{9} - \frac{1}{27}\sin^2\theta_W \simeq -0.12 \qquad (65) \end{aligned}$$

The Z-mediated penguin diagram with internal D quark can also be calculated.

$$f_D^Z = \frac{1}{4}Q_d\,[\,2\xi_0(y_D) - 3\xi_1(y_D) + \xi_2(y_D)\,] \to -\frac{5}{72 y_D} + O(\frac{1}{y_D^2})\ . \qquad (66)$$

It approaches zero when $y_D \to \infty$ and thus f_D^Z is negligible in large y_D limit. For a gauge invariant result, the unphysical neutral Higgs χ mediated penguin needs to be considered together with the Z meson penguin. In the non-linear Feynman gauge we have chosen, the mass of χ is equal to M_Z. The calculation is very similar to the $\phi^{\pm}$ penguin. For internal s, b, D quarks, the contributions $f_{s,b,D}^{\chi}$ are given by

$$\begin{aligned} f_i^{\chi} &= \frac{Q_d}{8} y_i\,[\,\xi_1(y_i) + \xi_2(y_i)\,] \\ &= -\frac{Q_d}{8} y_i \left[(2 - y_i)\ln y_i - \frac{5}{6}y_i^3 + 4y_i^2 - \frac{13}{2}y_i + \frac{10}{3}\right]\frac{1}{(1-y_i)^4} \qquad (67) \end{aligned}$$

It is obvious that the light quark contributions are suppressed by the light quark masses and thus negligible. The situation is quite different for the heavy D quark. As an approximation, for $y_D \to \infty$, $f_D^{\chi} \to -\frac{5}{144} \sim -0.035$. This contribution, comparable to the Z mediated penguin f^Z from light quarks, has been *overlooked* in previous calculations[21]. Since the quark D may not be much heavier than Z meson, we expand f_D^{χ} in powers of $1/y_D$ and keep also the next leading term.

$$f_D^{\chi} \approx -\frac{5}{144} + \frac{1}{36}\frac{1}{y_D} + O(\frac{1}{y_D^2})\ . \qquad (68)$$

The Higgs boson H mediated penguin is similar to that of unphysical Higgs χ:

$$\begin{aligned} f_i^H &= -\frac{Q_d}{8} w_i\,[3\xi_1(w_i) - \xi_2(w_i)] \\ &= -\frac{Q_d}{8} w_i \left[(-2 + 3w_i)\ln w_i + \frac{7}{6}w_i^3 - 6w_i^2 + \frac{15}{2}w_i - \frac{8}{3}\right]\frac{1}{(1-w_i)^4} \qquad (69) \end{aligned}$$

where $w_i \equiv m_i^2/M_H^2$. Similar to the χ penguin, $f_{s,b}^H$ can be ignored since $m_s, m_b \ll m_H$. For f_D^H, we again expand it in powers of $1/w_D$ and keep up to the next leading term:

$$f_D^H \approx +\frac{7}{144} - \frac{1}{18}\frac{1}{w_D} + O(\frac{1}{w_D^2}) \qquad (70)$$

The leading term is $+0.048$, again comparable to the Z penguin.

Put together, the Wilson coefficient $C_7(M_W)$ in the vector quark model is given by

$$
\begin{aligned}
C_7(M_W) = C_7^{\rm SM}(M_W) + \frac{\delta}{V_{tb}V_{ts}^*}\frac{23}{36} + \frac{z_{bs}}{V_{tb}V_{ts}^*}(f_s^Z + f_s^\chi + f_s^H + f_b^Z + f_b^\chi + f_b^H) \\
+ \frac{z_{4b}z_{4s}^*}{V_{tb}V_{ts}^*}(f_D^Z + f_D^\chi + f_D^H) \\
= C_7^{\rm SM}(M_W) + \frac{z_{bs}}{V_{tb}V_{ts}^*}\left(\frac{23}{36} - \frac{1}{9} - \frac{1}{27}\sin^2\theta_W + \frac{5}{72y_D} + \frac{5}{144} - \frac{1}{36}\frac{1}{y_D} - \frac{7}{144} + \frac{1}{18}\frac{1}{w_D}\right) \\
\to -0.193 + \frac{z_{bs}}{V_{tb}V_{ts}^*} \times 0.506 \quad . \qquad (71)
\end{aligned}
$$

Here we have used the unitarity relations $z_{4b}z_{4s}^* = -|U_{44}|^2 z_{sb} \approx -z_{sb}$ to leading order in FCNC due to the unitarity of U_L^d and $\delta = z_{sb}$ from Eq. (60). In the above numerical estimate we took y_D, w_D to infinity.

Similarly the Wilson coefficient of the gluonic magnetic-penguin operator O_8 is modified by the vector quark. In the vector quark model, the mass-independent term will give an extra contribution $\frac{1}{3}\delta$ if the KM matrix is non-unitary[15]. The FCNC neutral meson mediated gluonic magnetic penguin diagrams are identical to those of the photonic magnetic penguin, except for a trivial replacement of Q_d by color factors, since photon and gluons do not couple to neutral mesons. $C_8(M_W)$ in the vector quark model is given by

$$
\begin{aligned}
C_8(M_W) = C_8^{\rm SM}(M_W) + \frac{\delta}{V_{tb}V_{ts}^*}\frac{1}{3} - 3\frac{z_{bs}}{V_{tb}V_{ts}^*}(f_s^Z + f_s^\chi + f_s^H + f_b^Z + f_b^\chi + f_b^H) \\
-3\frac{z_{4b}z_{4s}^*}{V_{tb}V_{ts}^*}(f_D^Z + f_D^\chi + f_D^H) \\
= C_8^{\rm SM}(M_W) + \frac{z_{bs}}{V_{tb}V_{ts}^*}\left(\frac{1}{3} + \frac{1}{3} + \frac{1}{9}\sin^2\theta_W - \frac{5}{24y_D} - \frac{5}{48} + \frac{1}{12}\frac{1}{y_D} + \frac{7}{48} - \frac{1}{6}\frac{1}{w_D}\right) \\
\to -0.096 + \frac{z_{bs}}{V_{tb}V_{ts}^*} \times 0.942 \quad . \qquad (72)
\end{aligned}
$$

The above deviation from SM does not include QCD evolution. Actually it is trivial to incorporate LL QCD corrections to these deviations in the framework of effective Hamiltonian. The key is that the deviation from vector quark model is a short distance effect at the scale of M_W and M_Q. It can be separated into the Wilson coefficients at the matching scale, as we just did. The evolution of Wilson coefficients, which incorporates the LL QCD corrections, is not affected by the short distance physics of vector quark model and all the anomalous dimensions used in SM calculation still valid here. One only needs to use the corrected Wilson coefficients at $\mu = M_W$ and in so doing we resum all the terms of the form $z_{sb}\alpha_s^n(m_b)\log^n(m_b/M_W)$. The correction to ratio R in the vector quark model, including its

LL QCD corrections, is given by

$$\Delta R = \frac{6\alpha}{\pi g(z)} \times 0.163 \times \mathrm{Re}\left[\frac{V_{cs}^*}{V_{cb}} z_{sb}\right] = 0.123 \mathrm{Re} z_{sb}$$

to leading order in δ. In this result, the difference between $V_{ts}^* V_{tb}$ and $-V_{cs}^* V_{cb}$, i.e.

$$V_{ts}^* V_{tb} = z_{sb} - V_{cs}^* V_{cb}, \tag{73}$$

has been taken into account. It is expressed in terms of V_{cs} and V_{cb} since they can be directly measured without using the unitarity of KM matrix.

4 Constraints

The inclusive $B \to X_s \gamma$ branching ratio has been measured by CLEO with the branching ratio [24]

$$\mathcal{B}(B \to X_s \gamma) = (2.32 \pm 0.67) \times 10^{-4} \tag{74}$$

Recently they report a preliminary update: [25]

$$\mathcal{B}(B \to X_s \gamma)_{\mathrm{EXP}} = (3.15 \pm 0.54) \times 10^{-4} \tag{75}$$

This branching ratio could be used to constrain the mixing in the vector quark model. To discuss the constraint from B meson radiative decay, we treat both $\alpha_s(m_b)$ and z_{sb} as perturbation parameters, while $\alpha_s(m_b)\log(m_b/M_W)$ as of order 1. SM prediction has been calculated up to next-to-leading logarithmic order recently [12]. However, a next-to-leading order calculation of the vector quark model deviation is not necessary since it will be second order in the perturbation expansion. In other words, here we consider LL and all the terms of the form $\alpha_s(m_b)\alpha_s^n(m_b)\log^n(m_b/M_W)$ and $z_{sb}\alpha_s^n(m_b)\log^n(m_b/M_W)$.

SM theoretical prediction up to Next-to-Leading Order (NLO) was first calculated in Ref.[12], with the result

$$\mathcal{B}(B \to X_s \gamma)_{\mathrm{NLO}} = (3.28 \pm 0.33) \times 10^{-4} \tag{76}$$

Ref.[13] later did a new analysis, which discards all corrections beyond NLO by expanding formulas like Eq.(58) in powers of α_s, and reported a slightly higher result:

$$\mathcal{B}(B \to X_s \gamma)_{\mathrm{NLO}} = (3.60 \pm 0.33) \times 10^{-4} \tag{77}$$

The difference between the experimental data and the Standard Model NLO prediction, with the errors added up directly, is

$$\mathcal{B}(B \to X_s\gamma)_{\rm EXP} - \mathcal{B}(B \to X_s\gamma)_{\rm NLO} \;=\; (-0.13 \pm 0.63) \times 10^{-4} \quad [12]$$
$$(-0.45 \pm 0.63) \times 10^{-4} \quad [13] \qquad (78)$$

It gives a range of possible vector quark model deviation and hence on z_{sb} (with the input $\mathrm{B}(B \to X_c e\bar{\nu}) = 0.105$):

$$-0.0057 < \; z_{sb} < \; 0.0038 \quad [12]$$
$$-0.0081 < \; z_{sb} < \; 0.0014 \quad [13] \qquad (79)$$

The previously strongest bound on z_{sb} is from Z-mediated FCNC effect in the mode $B \to X\mu^+\mu^-$ [8]:

$$-0.0012 < z_{sb} < 0.0012 \qquad (80)$$

Our new bound is as strong as that from FCNC. It shows that even though the vector quarks contribute to the radiative decay rate through one loop, as in SM, the data could still put strong bound.

On the other hand, in models like Ref. [5], operators of different chiralities such as

$$O'_7 = \frac{e}{(4\pi)^2}\bar{s}_\alpha\sigma^{\mu\nu}(m_b L + m_s R)b_\alpha F_{\mu\nu}\ , \quad O'_8 = \frac{g_s}{(4\pi)^2}\bar{s}_\alpha\sigma^{\mu\nu}(m_b L + m_s R)(\lambda^A_{\alpha\beta})b_\alpha G^A_{\mu\nu} \qquad (81)$$

occurs via the new interaction. Our study can be extended to these models too. However, the new amplitude for $b \to s\gamma$ belongs to a different helicity configuration in the final state and it will not interfere with the SM contribution. Consequently, the constraint obtained from $b \to s\gamma$ in these models is less stringent than that from $B \to X\mu^+\mu^-$.

In the upcoming years, much more precise measurements are expected from the upgraded CLEO detector, as well as from the B-factories presently under construction at SLAC and KEK. The new experimental result will certainly give us clearer evidence whether the vector quark model is viable.

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