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On Kaluza-Klein States from Large Extra Dimensions

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Abstract

We consider the novel Kaluza-Klein (KK) scenario where gravity propagates in the $4+n$ dimensional bulk of spacetime, while gauge and matter fields are confined to the $3+1$ dimensional world-volume of a brane configuration. For simplicity we assume compactification of the extra n dimensions on a torus with a common scale R, and identify the massive KK states in the four-dimensional spacetime. For a given KK level $\vec{n}$ there are one spin-2 state, $(n-1)$ spin-1 states and $n(n-1)/2$ spin-0 states, all mass-degenerate. We construct the effective interactions between these KK states and ordinary matter fields (fermions, gauge bosons and scalars). We find that the spin-1 states decouple and that the spin-0 states only couple through the dilaton mode. We then derive the interacting Lagrangian for the KK states and Standard Model fields, and present the complete Feynman rules. We discuss some low energy phenomenology for these new interactions for the case when $1/R$ is small compared to the electroweak scale, and the ultraviolet cutoff of the effective KK theory is on the order of 1 TeV.

1 Introduction

Kaluza-Klein (KK) reduction [1] has always been an important ingredient in our attempts to relate $d = 4$ physics to $d = 10$ superstrings, as well as to $d = 11$ supergravity, which is now recognized as the low energy effective description of $d = 11$ M-theory [2]. It has become clear, however, that a much more general notion of Kaluza-Klein reduction is applicable in certain regions of the moduli space of consistent superstring/M theory vacua. This occurs when various matter and/or gauge fields are confined to heavy solitonic membranes. These recent developments [3] in superstring theory have led to a radical rethinking of the possibilities for new particles and dynamics arising from extra compactified spatial dimensions [4]-[15].

To appreciate this radical change of view, it is useful to review the conventional Kaluza-Klein scenario [16]. One begins with a $d = 4+n$ dimensional spacetime action, describing a coupled gravity+gauge+matter system. Since field theories of gravity are poorly behaved in the ultraviolet, Kaluza-Klein formulations should be generically regarded as *effective* actions, with an implicit or explicit ultraviolet cutoff Λ. One expands this theory around a vacuum metric which is the product of $d = 4$ Minkowski space with some n-dimensional compact manifold, obtained by stationarizing this higher-dimensional effective action. For consistency, the characteristic length scales R_i of the compact manifold should be larger than $1/\Lambda$. In the shifted vacuum all fields are expanded in normal modes of the n-dimensional compact manifold; the coefficients of this harmonic expansion are conventional $d = 4$ fields. This Kaluza-Klein reduction results in an effective $d = 4$ theory of gravity+gauge+massless matter coupled to towers of massive Kaluza-Klein states, where the massive spectrum is cutoff at the high scale Λ.

Letting E denote the energy scale of some experiment, and assuming for simplicity that the compactification scales $R_i \sim R$ are all roughly equal, one can distinguish three general phenomenological regimes:

1. $E \ll 1/R \lesssim \Lambda$. This case is relevant to compactifications of the weakly coupled heterotic string, with Λ equal to the string scale, approximately 10^{18} GeV. In such a case massive Kaluza-Klein modes only impact low energy physics indirectly, through threshold effects on couplings at the high scale.

2. $E < 1/R \ll \Lambda$. This encompasses Kaluza-Klein scenarios where the cutoff scale Λ is still very high, but some dynamics fixes $1/R$ to a much lower scale, perhaps as low as a few TeV. In this case a very large number $\sim (\Lambda R)^n$ of massive KK states are integrated out in evolving the effective action from the high scale to the low scale. Thus, although the couplings of individual massive KK modes are Planck suppressed, they may contribute non-negligible higher dimension operators to the effective low energy theory [4]. Furthermore, they may have strong effects on the running of the renormalizable Standard Model (SM) couplings [5] above the scale $1/R$.

3. $1/R \ll E < \Lambda$. In this case a large number $\sim (E\,R)^n$ of massive KK states are kinematically accessible. This effectively makes physics look $(4+n)$-dimensional at the energy scale $E \gg 1/R$. There are severe constraints from experiment on such scenarios. We know that $d = 4$ electrodynamics can be distinguished in collider experiments from $d = 4+n$ electrodynamics down to very short length scales. There is also a strong bound from the non-observance of mirror copies of the Standard Model chiral fermions. Consider for example $d = 5$ fermions, which are pseudo-Majorana and have four (on-shell) real degrees of freedom. When dimensionally reduced to $d = 4$, each splits into two Weyl fermions with opposite chirality but the same gauge group representation; therefore one expects mirror fermions with masses $\lesssim 1/R$.

Recently it was observed [6] that this last case can be phenomenologically viable if we assume that the fields of the Standard Model are confined to a three-dimensional membrane or intersection of membranes in the larger dimensional space. Assuming further that the scale of the membrane tension is on the order of the cutoff Λ or larger, the resulting effective theory consists of $(3+1)$-dimensional Standard Model fields coupled to $4+n$ gravity and, perhaps, other $(4+n)$-dimensional "bulk" fields. With these assumptions the phenomenological constraints from gravity experiments, collider physics, and astrophysics are much weaker [6], allowing $1/R$ scales as low as 10^{-4} eV (~ 1 mm^{-1}), for cutoff scales Λ in the range $1 - 10$ TeV.

In superstring theory there are regions of moduli space where compactification radii become large while the string coupling, gauge couplings, and Newton's constant remain fixed [7, 8]. The scale of these large extra dimensions is related to the string scale M_S:

$$\frac{1}{G_N} \sim M_S^{n+2}\, R^n \;, \tag{1}$$

where G_N is the Newton constant. Roughly speaking, M_S plays the role of the ultraviolet cutoff Λ. This reproduces the relationship of scales assumed in the scenario just described.

It has also been shown in superstring theory that it is possible to obtain $d = 4$ N=1 supersymmetric chiral gauge theories confined to the world-volumes of stable configurations of intersecting D-branes [9]. The regions of string moduli space where such configurations have a perturbative description is not necessarily incompatible with the region where large extra dimensions may occur. Thus within our current knowledge (or ignorance) of superstrings it is not implausible to imagine that the Standard Model is confined to a brane configuration [10, 11], while large compactified dimensions are probed only by gravity and other bulk fields [6, 12].

In this paper we will consider the simplest case where gravity is the only $d = 4+n$ bulk field. The couplings of gravity to $d = 4$ gauge and matter fields are completely fixed by general coordinate invariance in the $d = 4+n$ spacetime and the $d = 4$ world-volume. This allows us to deduce the complete Feynman rules for the couplings of

Standard Model particles to the massive KK states. The low energy phenomenology is then calculable modulo the details of how to treat the cutoff Λ, which truncates the KK mode sums.

In the following, we will use hatted letters to denote the $(4+n)$-dimensional quantities, *e.g.*, $\hat{g}_{\hat{\mu}\hat{\nu}}$ denotes the metric tensor in $d = 4+n$. Un-hatted Greek letters $(\mu, \nu, \cdots)$, Roman letters from the beginning $(a, b, \cdots)$ and in the middle $(i, j, \cdots)$ of the alphabet will be used to label four-dimensional Einstein, Lorentz and (the compactified) n-dimensional indices respectively. Repeated indices are summed. Our convention for the signature is $(+, -, -, \cdots)$.

The rest of the paper is organized as follows: In Section 2, we compactify $d = 4+n$ gravity on an n-dimensional torus T^n and perform a mode expansion. A torus compactification is perhaps not realistic, since the bulk fields which we are ignoring are potential sources of n-dimensional curvature, as are the branes themselves. However the torus has the great advantage of conceptual and calculational simplicity. We find that the massive KK modes have a simple physical interpretation. For each KK level, there are one massive spin-2, $(n-1)$ massive spin-1 and $n(n-1)/2$ massive spin-0 particles. We find the general form for the interactions between matter (scalars, gauge bosons and fermions) and the massive KK states. In Section 3, we examine a few physical processes involving the KK states. We calculate their decay widths to the light SM particles; this could have important cosmological consequences. We then construct effective four-fermion and $\bar{f}fVV$ interactions; this provides a useful formalism for studying some high energy processes. We next study the process $e^+e^- \to \gamma + KK$, where KK are spin-0 and 2 massive KK states. In the final example, we calculate the one-loop corrections to the scalar boson masses due to virtual KK states; we find that the corrections are proportional to the scalar mass, instead of the ultraviolet cutoff M_S. Section 4 is reserved for the discussions and conclusions. We list some useful formulae in two appendices. In Appendix A, we present the propagators and polarizations for the physical KK states, and show the complete leading-order $(\mathcal{O}(\kappa))$ vertex Feynman rules. In Appendix B, we discuss the summation over KK states which appears in many physical processes.

2 General Formalism

2.1 Decomposition of the Massive KK States

The starting point for our analysis is the linearized gravity Lagrangian, *i.e.*, the Fierz-Pauli Lagrangian [17]:

$$\frac{1}{\hat{\kappa}^2}\sqrt{|\hat{g}|}\hat{R} = \frac{1}{4}\Big(\partial^{\hat{\mu}}\hat{h}^{\hat{\nu}\hat{\rho}}\partial_{\hat{\mu}}\hat{h}_{\hat{\nu}\hat{\rho}} - \partial^{\hat{\mu}}\hat{h}\partial_{\hat{\mu}}\hat{h} - 2\hat{h}^{\hat{\mu}}\hat{h}_{\hat{\mu}} + 2\hat{h}^{\hat{\mu}}\partial_{\hat{\mu}}\hat{h}\Big) + \mathcal{O}(\hat{\kappa}) , \qquad (2)$$

where $\hat{h} \equiv \hat{h}^{\hat{\mu}}{}_{\hat{\mu}}, \hat{h}_{\hat{\nu}} \equiv \partial^{\hat{\mu}} \hat{h}_{\hat{\mu}\hat{\nu}}$ and we have used $\hat{g}_{\hat{\mu}\hat{\nu}} = \eta_{\hat{\mu}\hat{\nu}} + \hat{\kappa}\hat{h}_{\hat{\mu}\hat{\nu}}$, $\hat{\kappa}^2 = 16\pi G_N^{(4+n)}$, with $G_N^{(4+n)}$ the Newton constant in $d = 4+n$. This Lagrangian is invariant under the general coordinate transformation

$$\delta \hat{h}_{\hat{\mu}\hat{\nu}} \;=\; \partial_{\hat{\mu}}\zeta_{\hat{\nu}} + \partial_{\hat{\nu}}\zeta_{\hat{\mu}} \; . \tag{3}$$

After imposing the de Donder gauge condition*

$$\partial^{\hat{\mu}}(\hat{h}_{\hat{\mu}\hat{\nu}} - \frac{1}{2}\eta_{\hat{\mu}\hat{\nu}}\hat{h}) = 0 \; , \tag{4}$$

the equation of motion is the d'Alembert equation

$$\Box_{(4+n)} \, (\hat{h}_{\hat{\mu}\hat{\nu}} - \frac{1}{2}\eta_{\hat{\mu}\hat{\nu}}\hat{h}) \;=\; 0 \; . \tag{5}$$

The gauge condition, along with the tracelessness condition $\hat{h}^{\hat{\mu}}{}_{\hat{\mu}} = 0$, and the residual general coordinate transformation Eq. (3), with the gauge parameter satisfying $\Box_{(4+n)}\zeta_{\hat{\mu}} = 0$, fixes all but the $(2+n)(3+n)/2 - 1$ physical degrees of freedom for a massless graviton in $4+n$ dimensions.

Now we proceed to perform the KK reduction of the Fierz-Pauli Lagrangian to $d = 4$. We shall assume

$$\hat{h}_{\hat{\mu}\hat{\nu}} \;=\; V_n^{-1/2} \begin{pmatrix} h_{\mu\nu} + \eta_{\mu\nu}\phi & A_{\mu i} \\ A_{\nu j} & 2\phi_{ij} \end{pmatrix} \; , \tag{6}$$

where V_n is the volume of the $d = n$ compactified space, $\phi \equiv \phi_{ii}$, $\mu, \nu = 0, 1, 2, 3$ and $i, j = 5, 6, \cdots, 4+n$, and the $\eta_{\mu\nu}\phi$ term in the (11)-entry is a Weyl rescaling. These fields are compactified on an n-dimensional torus T^n and have the following mode expansions:

$$h_{\mu\nu}(x,y) \;=\; \sum_{\vec{n}} h^{\vec{n}}_{\mu\nu}(x) \; \exp\left(i\frac{2\pi\vec{n}\cdot\vec{y}}{R} \right) \; , \tag{7}$$

$$A_{\mu i}(x,y) \;=\; \sum_{\vec{n}} A^{\vec{n}}_{\mu i}(x) \; \exp\left(i\frac{2\pi\vec{n}\cdot\vec{y}}{R} \right) \; , \tag{8}$$

$$\phi_{ij}(x,y) \;=\; \sum_{\vec{n}} \phi^{\vec{n}}_{ij}(x) \; \exp\left(i\frac{2\pi\vec{n}\cdot\vec{y}}{R} \right) \; , \qquad \vec{n} = \{n_1, n_2, \cdots, n_n\} \; , \tag{9}$$

where the modes of $\vec{n} \neq 0$ are the KK states, and all the compactification radii are assumed to be the same. The generalization to an asymmetric torus with different radii is straightforward. ¿From the transformation properties under the general coordinate transformation $\zeta_{\hat{\mu}} = \{\zeta_\mu, \zeta_i\}$, it should be clear that the zero modes, $\vec{n} = \vec{0}$, correspond to the massless graviton, U(1) gauge bosons and scalars in $d = 4$.

*Here we choose the gauge condition for the sake of clarity; the definitions of physical fields in Eq. (17) do not depend on the gauge choice.

The KK modes satisfy the following equation of motions, from Eq. (5),

$$(\Box + m_{\vec{n}}^2)\,(h_{\mu\nu}^{\vec{n}} - \frac{1}{2}\eta_{\mu\nu}h^{\vec{n}}) \;=\; 0\;, \qquad (\Box + m_{\vec{n}}^2)\,A_{\mu i}^{\vec{n}} \;=\; 0\;,$$
$$(\Box + m_{\vec{n}}^2)\,\phi_{ij}^{\vec{n}} \;=\; 0\;, \qquad \text{where } m_{\vec{n}}^2 \;=\; \frac{4\pi^2\vec{n}^2}{R^2}\;, \tag{10}$$

and $\Box$ is the four-dimensional d'Alembert operator. The gauge condition in Eq. (4) reduces to the following two equations

$$\partial^\mu h_{\mu\nu}^{\vec{n}} - \frac{1}{2}\partial_\nu h^{\vec{n}} + i\frac{2\pi n_i}{R}A_{\nu i}^{\vec{n}} \;=\; 0\;, \tag{11}$$
$$\partial^\mu A_{\mu i}^{\vec{n}} + i\frac{4\pi n_j}{R}\phi_{ij}^{\vec{n}} + i\frac{\pi n_i}{R}h^{\vec{n}} + i\frac{2\pi n_i}{R}\phi^{\vec{n}} \;=\; 0\;. \tag{12}$$

From Eq. (12), it follows

$$\phi^{\vec{n}} + \frac{2n_in_j}{\vec{n}^2}\phi_{ij}^{\vec{n}} + \frac{1}{2}h^{\vec{n}} - i\frac{n_iR}{2\pi\vec{n}^2}\partial^\mu A_{\mu i}^{\vec{n}} \;=\; 0\;, \tag{13}$$
$$P_{ik}^{\vec{n}}(\partial^\mu A_{\mu i}^{\vec{n}} + i\frac{4\pi n_j}{R}\phi_{ij}^{\vec{n}}) \;=\; 0\;, \tag{14}$$

where we have defined projectors

$$P_{ij}^{\vec{n}} \;=\; \delta_{ij} - \frac{n_in_j}{\vec{n}^2}\;, \qquad \tilde{P}_{ij}^{\vec{n}} \;=\; \frac{n_in_j}{\vec{n}^2}\;, \tag{15}$$

they satisfy

$$P_{ij}^{\vec{n}}P_{jk}^{\vec{n}} \;=\; P_{ik}^{\vec{n}}\;, \quad \tilde{P}_{ij}^{\vec{n}}\tilde{P}_{jk}^{\vec{n}} \;=\; \tilde{P}_{ik}^{\vec{n}}\;, \quad P_{ij}^{\vec{n}}\tilde{P}_{jk}^{\vec{n}} \;=\; 0\;, \quad P_{ij}^{\vec{n}} + \tilde{P}_{ij}^{\vec{n}} \;=\; \delta_{ij}\;,$$
$$P_{ii}^{\vec{n}} \;=\; n-1\;, \quad \tilde{P}_{ii}^{\vec{n}} \;=\; 1\;, \quad P_{ij}^{\vec{n}}\,n_i \;=\; 0\;, \quad \tilde{P}_{ij}^{\vec{n}}\,n_i \;=\; n_j\;. \tag{16}$$

We then redefine the fields

$$\tilde{h}_{\mu\nu}^{\vec{n}} \;=\; h_{\mu\nu}^{\vec{n}} - i\frac{n_iR}{2\pi\vec{n}^2}(\partial_\mu A_{\nu i}^{\vec{n}} + \partial_\nu A_{\mu i}^{\vec{n}}) - (P_{ij}^{\vec{n}} + 3\tilde{P}_{ij}^{\vec{n}})\Big(\frac{2}{3}\frac{\partial_\mu\partial_\nu}{m_{\vec{n}}^2} - \frac{1}{3}\eta_{\mu\nu}\Big)\phi_{ij}^{\vec{n}}\;,$$
$$\tilde{A}_{\mu i}^{\vec{n}} \;=\; P_{ij}^{\vec{n}}(A_{\mu j}^{\vec{n}} - i\frac{n_kR}{\pi\vec{n}^2}\partial_\mu\phi_{jk}^{\vec{n}})\;, \qquad \tilde{\phi}_{ij}^{\vec{n}} \;=\; \sqrt{2}(P_{ik}^{\vec{n}}P_{jl}^{\vec{n}} + aP_{ij}^{\vec{n}}P_{kl}^{\vec{n}})\phi_{kl}^{\vec{n}}\;, \tag{17}$$

where a is the solution of the equation $3(n-1)a^2 + 6a = 1$. This form of $\tilde{\phi}_{ij}^{\vec{n}}$ is chosen to make its kinetic term canonical, as will be seen in Eq. (24). It is obvious that tilded fields satisfy the same equations of motion as untilded fields. Furthermore from Eqs. (11), (13), (14) and (17), we have

$$\partial^\mu\tilde{h}_{\mu\nu}^{\vec{n}} \;=\; 0\;, \qquad \tilde{h}^{\vec{n}} \;=\; 0\;, \tag{18}$$
$$\partial^\mu\tilde{A}_{\mu i}^{\vec{n}} \;=\; 0\;, \qquad n_i\tilde{A}_{\mu i}^{\vec{n}} \;=\; 0\;, \qquad n_i\tilde{\phi}_{ij}^{\vec{n}} \;=\; 0\;. \tag{19}$$

This verifies that $\widetilde{h}_{ij}^{\vec{n}}$ are massive spin-2 particles, $\widetilde{A}_{\mu i}^{\vec{n}}$ are $(n-1)$ massive spin-1 particles, and $\widetilde{\phi}_{ij}^{\vec{n}}$ are $n(n-1)/2$ massive spin-0 particles, all with the same mass $m_{\vec{n}}$.

This redefinition of fields is associated with spontaneous symmetry breaking. It was shown for $n=1$ there is an infinite-dimensional symmetry (the loop algebra on S^1) at the Lagrangian level [18], but it is broken by the vacuum configuration, $\hat{g}_{\hat{\mu}\hat{\nu}} = \eta_{\hat{\mu}\hat{\nu}}$. Similar to the Higgs mechanism, the massless spin-2 fields $h_{\mu\nu}^{\vec{n}}$ absorb the spin-1 and spin-0 fields at the same KK level $\vec{n}$ and become massive. It is remarkable that this mechanism is geometrical in nature and does not need any scalar Higgs field. We here explicitly find the composition for massive spin-2, 1 and 0 fields for $n \geq 2$.

One can further show that $\widetilde{h}_{ij}^{\vec{n}}$, $\widetilde{A}_{\mu i}^{\vec{n}}$ and $\widetilde{\phi}_{ij}^{\vec{n}}$ are invariant under the general coordinate transformation, which has the following linearized form

$$\begin{aligned} \delta h_{\mu\nu}^{\vec{n}} &= \partial_\mu \zeta_\nu^{\vec{n}} + \partial_\nu \zeta_\mu^{\vec{n}} + i\eta_{\mu\nu}\frac{2\pi n_i}{R}\zeta_i^{\vec{n}} \;, & (20)\\ \delta A_{\mu i}^{\vec{n}} &= -i\frac{2\pi n_i}{R}\zeta_\mu^{\vec{n}} + \partial_\mu \zeta_i^{\vec{n}} \;, & (21)\\ \delta \phi_{ij}^{\vec{n}} &= -i\frac{\pi n_i}{R}\zeta_j^{\vec{n}} - i\frac{\pi n_j}{R}\zeta_i^{\vec{n}} \;, & (22) \end{aligned}$$

where we have assumed the transformation parameters $\zeta_\mu^{\vec{n}}, \zeta_i^{\vec{n}}$ to have the same mode expansion as in Eq. (9).

We should note that the field redefinition in Eq. (17) does not depend on the particular gauge choice. To see this, we rewrite the Lagrangian in Eq. (2) without imposing the de Donder gauge. For the zero modes, it simply follows from Eq. (6),

$$\begin{aligned} \mathcal{L}^{\vec{0}} &= \frac{1}{4}\Big(\partial^\mu h^{\nu\rho}\partial_\mu h_{\nu\rho} - \partial^\mu h \partial_\mu h - 2h^\mu h_\mu + 2h^\mu \partial_\mu h\Big) \\ &\quad - \sum_{i=1}^{n}\frac{1}{4}F_i^{\mu\nu}F_{\mu\nu i} + \frac{1}{2}\partial^\mu\phi\partial_\mu\phi + \sum_{(ij)=1}^{n(n+1)/2}\partial^\mu\phi_{ij}\partial_\mu\phi_{ij} \;, \qquad (23) \end{aligned}$$

where $F_{\mu\nu i} = \partial_\mu A_{\nu i} - \partial_\nu A_{\mu i}$. We see it indeed describes massless graviton, vectors and scalars.

The Lagrangian for the massive KK modes can be rewritten in terms of the tilded fields according to Eq. (17). After a tedious calculation, we find

$$\begin{aligned} \mathcal{L}^{\vec{n}} &= \frac{1}{2}\Big(\partial^\mu \widetilde{h}^{\nu\rho,\vec{n}}\partial_\mu \widetilde{h}_{\nu\rho}^{-\vec{n}} - \partial^\mu \widetilde{h}^{\vec{n}}\partial_\mu \widetilde{h}^{-\vec{n}} - 2\widetilde{h}^{\mu,\vec{n}}\widetilde{h}_\mu^{-\vec{n}} + 2\widetilde{h}^{\mu,\vec{n}}\partial_\mu \widetilde{h}^{-\vec{n}} \\ &\qquad - m_{\vec{n}}^2 \widetilde{h}^{\mu\nu,\vec{n}}\widetilde{h}_{\mu\nu}^{-\vec{n}} + m_{\vec{n}}^2 \widetilde{h}^{\vec{n}}\widetilde{h}^{-\vec{n}}\Big) + \sum_{i=1}^{n}\Big(-\frac{1}{2}\widetilde{F}_i^{\mu\nu,\vec{n}}\widetilde{F}_{\mu\nu i}^{-\vec{n}} + m_{\vec{n}}^2\widetilde{A}_i^{\mu,\vec{n}}\widetilde{A}_{\mu i}^{-\vec{n}}\Big) \\ &\quad + \sum_{(ij)=1}^{n(n+1)/2}\Big(\partial^\mu\widetilde{\phi}_{ij}^{\vec{n}}\partial_\mu\widetilde{\phi}_{ij}^{-\vec{n}} - m_{\vec{n}}^2\widetilde{\phi}_{ij}^{\vec{n}}\widetilde{\phi}_{ij}^{-\vec{n}}\Big) \;, \qquad (24) \end{aligned}$$

the fields $\widetilde{A}^{\vec{n}}_{\mu i}$ and $\widetilde{\phi}^{\vec{n}}_{ij}$ are subject to the constraints in Eq. (19).

The equation of motion of $\widetilde{h}^{\vec{n}}_{\mu\nu}$ from Eq. (24) is the Fierz-Pauli equation for massive spin-2 particles

$$\partial^\mu\partial_\mu\widetilde{\chi}^{\vec{n}}_{\nu\rho} - \partial_\nu\widetilde{\chi}^{\vec{n}}_\rho - \partial_\rho\widetilde{\chi}^{\vec{n}}_\nu + \partial^\mu\widetilde{\chi}^{\vec{n}}_\mu\eta_{\nu\rho} + m^2_{\vec{n}}(\widetilde{h}^{\vec{n}}_{\nu\rho} - \eta_{\mu\nu}\widetilde{h}^{\vec{n}}) \;=\; 0 \;, \tag{25}$$

where

$$\widetilde{\chi}^{\vec{n}}_{\mu\nu} \;=\; \widetilde{h}^{\vec{n}}_{\mu\nu} - \frac{1}{2}\widetilde{h}^{\vec{n}}\eta_{\mu\nu} \;, \quad \widetilde{\chi}^{\vec{n}}_\mu \;=\; \partial^\nu\widetilde{\chi}^{\vec{n}}_{\mu\nu} \;, \tag{26}$$

and $\widetilde{A}^{\vec{n}}_{\mu i}$ and $\widetilde{\phi}^{\vec{n}}_{ij}$ satisfy

$$\partial^\mu\widetilde{F}^{\vec{n}}_{\mu\nu i} + m^2_{\vec{n}}\widetilde{A}^{\vec{n}}_{\nu i} \;=\; 0 \;, \qquad (\Box + m^2_{\vec{n}})\widetilde{\phi}^{\vec{n}}_{ij} \;=\; 0 \;. \tag{27}$$

These equations can be recast into the form in Eqs. (10), (18) and (19).

The propagators and polarizations of the physical (tilded) fields will be given in Appendix A.1.

2.2 Coupling of the KK States to Matter

The basic picture for our physical world, as considered in this paper, is that all Standard Model fields are confined to a four-dimensional brane world-volume. As we showed in the previous section, from the four-dimensional perspective, the zero modes of the $(4+n)$-dimensional graviton become the graviton, n massless U(1) gauge bosons and $n(n+1)/2$ massless scalar bosons, while the KK modes in each level reorganize themselves to a massive spin-2 particle, $(n-1)$ massive vector bosons and $n(n-1)/2$ massive scalar bosons. In the following, we will formulate the coupling of these physical KK modes to the matter. Although these interactions only have gravitational strength, they can be enhanced in the case of large size extra dimensions, due to the many available KK states.

We begin with the minimal gravitational coupling of the general scalar S, vector V, and fermion F[†],

$$\int d^4x\sqrt{-\hat{g}}\;\mathcal{L}(\hat{g}, S, V, F) \;, \tag{28}$$

where $\hat{g}$ is the induced metric in $d=4$, $\hat{g}_{\mu\nu} = \eta_{\mu\nu} + \kappa(h_{\mu\nu} + \eta_{\mu\nu}\phi)$, $\phi \equiv \phi_{ii}$. The $d=4$ Newton constant $\kappa = \sqrt{16\pi G_N}$ is related to $\hat{\kappa}$ by $\kappa = V_n^{-1/2}\hat{\kappa}$, where $V_n = R^n$ for the torus T^n.

The $\mathcal{O}(\kappa)$ term of Eq. (28) can be easily shown to be

$$-\frac{\kappa}{2}\int d^4x(h^{\mu\nu}T_{\mu\nu} + \phi T^\mu{}_\mu) \;, \tag{29}$$

where

$$T_{\mu\nu}(S, V, F) \;=\; \Big(-\eta_{\mu\nu}\mathcal{L} + 2\frac{\delta\mathcal{L}}{\delta\hat{g}^{\mu\nu}}\Big)\Big|_{\hat{g}=\eta} \;, \tag{30}$$

[†]For the fermion, one should use the vierbein formalism, but our result in Eq. (33) is still true.

and we have used

$$\sqrt{-\hat{g}} \;=\; 1+\frac{\kappa}{2}h+2\kappa\phi\;,\quad \hat{g}^{\mu\nu} \;=\; \eta^{\mu\nu}-\kappa h^{\mu\nu}-\kappa\eta^{\mu\nu}\phi\;. \tag{31}$$

For the KK modes, we should replace $h^{\vec{n}}_{\mu\nu}$ and $\phi^{\vec{n}}$ by the physical fields $\widetilde{h}^{\vec{n}}_{\mu\nu}$ and $\widetilde{\phi}^{\vec{n}}$ according to Eq. (17). Using

$$P^{\vec{n}}_{ij}\phi^{\vec{n}}_{ij} \;=\; \frac{3\omega}{2}\widetilde{\phi}^{\vec{n}}\;, \tag{32}$$

where $\widetilde{\phi}^{\vec{n}} \;\equiv\; \widetilde{\phi}^{\vec{n}}_{ii}$, $\omega=\sqrt{\frac{2}{3(n+2)}}$, and the conservation of the energy-momentum tensor, we obtain

$$-\frac{\kappa}{2}\sum_{\vec{n}}\int d^4x(\widetilde{h}^{\mu\nu,\vec{n}}T_{\mu\nu}+\omega\widetilde{\phi}^{\vec{n}}T^{\mu}{}_{\mu})\;. \tag{33}$$

It is remarkable that the vector KK modes $\widetilde{A}^{\vec{n}}_{\mu i}$ decouple and the scalar KK modes $\widetilde{\phi}^{\vec{n}}_{ij}$ only couple through their trace $\widetilde{\phi}^{\vec{n}}$, the dilaton mode.

We now present the Lagrangian to the order of $\mathcal{O}(\kappa)$; a complete list of vertex functions will be given in Appendix A.2.

2.2.1 Coupling to Scalar Bosons

For a general complex scalar field Φ, we have the conserved energy-momentum tensor

$$T^{\rm S}_{\mu\nu} \;=\; -\eta_{\mu\nu}D^{\rho}\Phi^{\dagger}D_{\rho}\Phi+\eta_{\mu\nu}m^2_{\Phi}\Phi^{\dagger}\Phi+D_{\mu}\Phi^{\dagger}D_{\nu}\Phi+D_{\nu}\Phi^{\dagger}D_{\mu}\Phi\;, \tag{34}$$

where the gauge covariant derivative is defined as

$$D_{\mu}=\partial_{\mu}+igA^a_{\mu}T^a, \tag{35}$$

with g the gauge coupling, A^a_{μ} the gauge fields and T^a the Lie algebra generators. The gauge-invariant Lagrangian for a level-$\vec{n}$ KK state coupled to the scalar bosons is

$$\begin{aligned}\kappa^{-1}\mathcal{L}^{\vec{n}}_{\rm S}(\kappa) \;&=\; -(\widetilde{h}^{\mu\nu,\vec{n}}-\frac{1}{2}\eta^{\mu\nu}\widetilde{h}^{\vec{n}})D_{\mu}\Phi^{\dagger}D_{\nu}\Phi-\frac{1}{2}\widetilde{h}^{\vec{n}}m^2_{\Phi}\Phi^{\dagger}\Phi\\ &\quad+\omega\widetilde{\phi}^{\vec{n}}(D^{\mu}\Phi^{\dagger}D_{\mu}\Phi-2m^2_{\Phi}\Phi^{\dagger}\Phi)\;.\end{aligned} \tag{36}$$

¿From this, one finds the Feynman rules for KK-$\Phi\Phi$ vertices as well as the contact interactions of KK-$\Phi\Phi$ with additional gauge bosons. They are listed in Appendix A.2.

2.2.2 Coupling to Gauge Bosons

The conserved energy-momentum tensor for a gauge vector boson is

$$\begin{aligned}T^{\rm V}_{\mu\nu} \;&=\; \eta_{\mu\nu}\Big(\frac{1}{4}F^{\rho\sigma}F_{\rho\sigma}-\frac{m^2_A}{2}A^{\rho}A_{\rho}\Big)-\Big(F_{\mu}{}^{\rho}F_{\nu\rho}-m^2_AA_{\mu}A_{\nu}\Big)\\ &\quad-\frac{1}{\xi}\eta_{\mu\nu}\Big(\partial^{\rho}\partial^{\sigma}A_{\sigma}A_{\rho}+\frac{1}{2}(\partial^{\rho}A_{\rho})^2\Big)+\frac{1}{\xi}(\partial_{\mu}\partial^{\rho}A_{\rho}A_{\nu}+\partial_{\nu}\partial^{\rho}A_{\rho}A_{\mu})\;,\end{aligned} \tag{37}$$

where the ξ-dependent terms correspond to adding a gauge-fixing term $-(\partial^\mu A_\mu - \Gamma^{\mu\nu}{}_\nu A^\mu)^2/2\xi$, with $\Gamma^{\mu\nu}{}_\nu = \eta^{\nu\rho}\Gamma^\mu{}_{\nu\rho}$ the Christoffel symbol (affine connection). The Lagrangian for a level-$\vec{n}$ KK state coupled to the gauge bosons is

$$\begin{aligned}\kappa^{-1}\mathcal{L}_{\rm V}^{\vec{n}}(\kappa) \;=\; & -\frac{1}{8}(\widetilde{h}^{\vec{n}}\eta^{\mu\nu} - 4\widetilde{h}^{\mu\nu,\vec{n}})F_\mu{}^\rho F_{\nu\rho} + \frac{1}{4}(\widetilde{h}^{\vec{n}}\eta^{\mu\nu} - 2\widetilde{h}^{\mu\nu,\vec{n}})m_A^2 A_\mu A_\nu \\ & + \frac{\widetilde{h}^{\vec{n}}}{2\xi}\Big(\partial^\rho\partial^\sigma A_\sigma A_\rho + \frac{1}{2}(\partial^\rho A_\rho)^2\Big) - \frac{\widetilde{h}^{\mu\nu,\vec{n}}}{\xi}\partial_\mu\partial^\rho A_\rho A_\nu \\ & + \frac{\omega}{2}m_A^2\widetilde{\phi}^{\vec{n}}A^\mu A_\mu - \frac{\omega}{\xi}\partial^\mu\widetilde{\phi}^{\vec{n}}\partial^\nu A_\nu A_\mu \;. \end{aligned} \tag{38}$$

The corresponding Feynman rules for three-point KK-AA vertices as well as the contact interactions of KK-AAA and KK-$AAAA$ are given in Appendix A.2.

2.2.3 Coupling to Fermions

To describe a fermion in the gravitation theory, one needs to use the vierbein formalism. The fermion Lagrangian is

$$\mathcal{L}_{\rm F} \;=\; e\overline{\psi}(i\gamma^\mu\mathcal{D}_\mu - m_\psi)\psi \;, \tag{39}$$

where $e = \det(e_\mu{}^a)$, $e_\mu{}^a e_\nu{}^b\eta_{ab} = g_{\mu\nu}$, $\gamma^\mu = e^\mu{}_a\gamma^a$, and a, b are Lorentz indices. The covariant derivative on the fermion field is defined by

$$\mathcal{D}_\mu\psi \;=\; (D_\mu + \frac{1}{2}\omega_\mu^{ab}\sigma_{ab})\psi \;, \tag{40}$$

where $\sigma_{ab} = \frac{1}{4}[\gamma_a, \gamma_b]$. In the absence of a spin-3/2 field, the spin connection ω_μ^{ab} can be solved in terms of the vierbein,

$$\omega_{\mu ab} \;=\; \frac{1}{2}(\partial_\mu e_{b\nu} - \partial_\nu e_{b\mu})e_a{}^\nu - \frac{1}{2}(\partial_\mu e_{a\nu} - \partial_\nu e_{a\mu})e_b{}^\nu - \frac{1}{2}e_a{}^\rho e_b{}^\sigma(\partial_\rho e_{c\sigma} - \partial_\sigma e_{c\rho})e^c{}_\mu \;. \tag{41}$$

We find the conserved energy-momentum tensor

$$\begin{aligned} T^{\rm F}_{\mu\nu} \;=\; & -\eta_{\mu\nu}(\overline{\psi}i\gamma^\rho D_\rho\psi - m_\psi\overline{\psi}\psi) + \frac{1}{2}\overline{\psi}i\gamma_\mu D_\nu\psi + \frac{1}{2}\overline{\psi}i\gamma_\nu D_\mu\psi \\ & + \frac{\eta_{\mu\nu}}{2}\partial^\rho(\overline{\psi}i\gamma_\rho\psi) - \frac{1}{4}\partial_\mu(\overline{\psi}i\gamma_\nu\psi) - \frac{1}{4}\partial_\nu(\overline{\psi}i\gamma_\mu\psi) \;, \end{aligned} \tag{42}$$

where we have used the linearized vierbein

$$e_\mu{}^a \;=\; \delta_\mu{}^a + \frac{\kappa}{2}(h_\mu{}^a + \delta_\mu{}^a\phi) \;. \tag{43}$$

The Lagrangian for a level-$\vec{n}$ KK state coupled to fermions is

$$\begin{aligned} \kappa^{-1}\mathcal{L}_{\rm F}^{\vec{n}}(\kappa) \;=\; & \frac{1}{2}\Big[(\widetilde{h}^{\vec{n}}\eta^{\mu\nu} - \widetilde{h}^{\mu\nu,\vec{n}})\overline{\psi}i\gamma_\mu D_\nu\psi - m_\psi\widetilde{h}^{\vec{n}}\overline{\psi}\psi + \frac{1}{2}\overline{\psi}i\gamma^\mu(\partial_\mu\widetilde{h}^{\vec{n}} - \partial^\nu\widetilde{h}^{\vec{n}}_{\mu\nu})\psi\Big] \\ & + \frac{3\omega}{2}\widetilde{\phi}^{\vec{n}}\overline{\psi}i\gamma^\mu D_\mu\psi - 2\omega m_\psi\widetilde{\phi}^{\vec{n}}\overline{\psi}\psi + \frac{3\omega}{4}\partial_\mu\widetilde{\phi}^{\vec{n}}\overline{\psi}i\gamma^\mu\psi \;. \end{aligned} \tag{44}$$

The Feynman rules for KK-$\psi\psi$ vertices as well as contact interactions of KK-$\psi\psi$ with additional gauge bosons are listed in Appendix A.2.

3 Application to Physical Processes

We are interested in a scenario in which the experimentally accessible energy is larger than the compactification scale $1/R$ (from $\sim 10^{-4}$ eV to 100 MeV for $n = 2$ to 7) but lower than the ultraviolet cutoff Λ. We first consider how the KK states decay to the SM particles. We then outline some low energy phenomenology and formulate effective amplitudes relevant to further studies at colliders. Finally, we evaluate typical one-loop corrections from virtual KK states to a scalar propagator. For simplicity, we will take the ultraviolet cutoff Λ to be the string scale M_S. More general choice of Λ can be obtained by simple scaling.

3.1 Decay of the Massive KK States

A massive KK state may decay to a pair of SM particles, beside its normal decay modes to massless gravitons and the lighter KK states. Depending on its mass, it can go to $\gamma\gamma, f\bar{f}, WW, ZZ$ and hh. While the decay of an individual massive KK state may not be much of interest for the current high energy experiments since it must be gravitationally suppressed, cosmological considerations of their lifetimes may have significant implications for their masses and interactions. Without speculating on the production and freeze-out of the KK modes at the early Universe with extra dimensions, we simply evaluate their decay widths and lifetimes to SM particles.

3.1.1 Spin-2 KK States

We first consider a massive spin-2 KK state ($\tilde{h}$) decay to gauge bosons

$$\tilde{h} \to VV \ . \tag{45}$$

It is straightforward to work out the partial decay width to massless gauge bosons,

$$\Gamma(\tilde{h} \to VV) = N \frac{\kappa^2 m_{\tilde{h}}^3}{320\pi}, \tag{46}$$

where $N = 1$ (8) for photons (gluons).

Due to the universal $\tilde{h}$ coupling to all gauge bosons, the two-photon mode $\tilde{h} \to \gamma\gamma$ is kinematically most favored for the lower-lying KK states. The lifetime is estimated to be

$$\tau_{\gamma\gamma} \approx \frac{10^3}{\kappa^2 m_{\tilde{h}}^3} \approx 6 \times 10^9 \text{ yr} \left(\frac{100 \text{ MeV}}{m_{\tilde{h}}}\right)^3 , \tag{47}$$

where we have taken the reduced Planck mass $M_{\rm pl}^* = \sqrt{2}\kappa^{-1} = 2.4 \times 10^{18}$ GeV. It is very long-lived via this decay mode. For a KK state heavier than the lower-lying hadrons, its lifetime via $\tilde{h} \to gg$ would be shorter

$$\tau_{gg} \approx 7 \times 10^5 \text{ yr} \left(\frac{1 \text{ GeV}}{m_{\tilde{h}}}\right)^3 . \tag{48}$$

If kinematically allowed, the KK mode can decay to massive gauge bosons and the decay width is

$$\Gamma(\tilde{h} \to VV) = \delta \frac{\kappa^2 m_{\tilde{h}}^3}{160\pi}(1-4r_V)^{1/2}(\frac{13}{12}+\frac{14}{39}r_V+\frac{4}{13}r_V^2), \tag{49}$$

where $\delta = 1/2$ for identical particles. Here and henceforth, we will use a notation for the mass ratio $r_i = m_i^2/m_{\tilde{h}}^2$ or $m_i^2/m_{\tilde{\phi}}^2$. The lifetime through this decay channel is

$$\tau_{VV} \approx \frac{5\times 10^2}{\kappa^2 m_{\tilde{h}}^3} \approx 30 \text{ yr} \left(\frac{100 \text{ GeV}}{m_{\tilde{h}}}\right)^3 . \tag{50}$$

The other decay channel goes through fermions,

$$\tilde{h} \to f\bar{f}. \tag{51}$$

The decay width is

$$\Gamma(\tilde{h} \to f\bar{f}) = N_c \frac{\kappa^2 m_{\tilde{h}}^3}{640\pi}(1-4r_f)^{3/2}(1+\frac{8}{3}r_f), \tag{52}$$

where the color factor N_c is three for the quark pair mode. The lifetime for this channel is of the same order of magnitude as that of Eq. (47).

Finally, the decay width to a pair of Higgs bosons is

$$\Gamma(\tilde{h} \to H\overline{H}) = \frac{\kappa^2 m_{\tilde{h}}^3}{1920\pi}(1-4r_H)^{5/2}. \tag{53}$$

We notice the threshold effects for the above three modes as S, P and D waves.

3.1.2 Spin-0 KK States

The spin-0 KK state ($\tilde{\phi}$) couplings to massless gauge bosons vanish at tree level, so that a $\tilde{\phi}$ does not decay to photons nor to gluons at the leading order. If kinematically allowed, a massive $\tilde{\phi}$ can decay to massive gauge bosons

$$\tilde{\phi} \to VV \ . \tag{54}$$

The partial decay width is calculated to be

$$\Gamma(\tilde{\phi} \to VV) = \frac{\delta}{n+2}\,\frac{\kappa^2 m_{\tilde{\phi}}^3}{96\pi}(1-4r_V)^{1/2}(1-4r_V+12r_V^2), \tag{55}$$

where, again, $\delta = 1/2$ for identical particles. The lifetime based on this decay channel is about the same order of magnitude as that of Eq. (50).

On the other hand, a light $\tilde{\phi}$ can still decay to a pair of light fermions

$$\tilde{\phi} \to f\bar{f}. \tag{56}$$

The decay width is given by

$$\Gamma(\tilde{\phi} \to f\bar{f}) = \frac{N_c}{n+2}\,\frac{\kappa^2 m_f^2 m_{\tilde{\phi}}}{48\pi}(1-4r_f)^{1/2}(1-2r_f). \tag{57}$$

The width for this channel is rather different from $\tilde{h}$ decay, being proportional linearly to $m_{\tilde{\phi}}$ and quadratically to m_f. This is because of the fermion spin-flip interactions by a scalar. The lifetime of $\tilde{\phi}$ for this channel is estimated to be

$$\tau \approx \frac{6\times 10^2}{\kappa^2 m_f^2 m_{\tilde{\phi}}} \approx 4\times 10^{10}\ \mathrm{yr}\ \frac{(100\ \mathrm{MeV})^3}{m_f^2 m_{\tilde{\phi}}}\ . \tag{58}$$

The decay width to a pair of Higgs bosons is given by

$$\Gamma(\tilde{\phi} \to H\overline{H}) = \frac{\delta}{n+2}\,\frac{\kappa^2 m_{\tilde{\phi}}^3}{96\pi}(1-4r_H)^{1/2}(1+2r_H)^2. \tag{59}$$

3.2 Effective 4-fermion Interactions

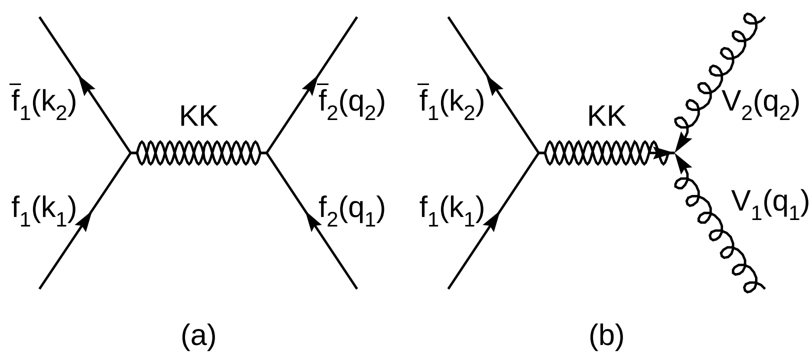

Figure 1: Feynman diagrams for (a) four-fermion interactions and (b) $\overline{f}fVV$ interactions. We represent KK states by double-sinusoidal curves.

The most basic contribution for KK states to current high energy phenomenology would be the effects on four-fermion interactions. Consider a generic four-fermion process

$$f_1(k_1)\ \bar{f}_1(k_2) \to f_2(q_1)\ \bar{f}_2(q_2) \tag{60}$$

in Fig. 1a, where the fermion momenta are chosen to be along the fermion line direction. The effective amplitudes are calculated to have the forms

$$
\begin{aligned}
i\mathcal{M}_4(\tilde{h}) &= -\frac{C_4}{32}\Big[(k_1+k_2)\cdot(q_1+q_2)\overline{f}_2\gamma^\mu f_2\overline{f}_1\gamma_\mu f_1 \\
&\quad +\overline{f}_2(\not{k}_1+\not{k}_2)f_2\overline{f}_1(\not{q}_1+\not{q}_2)f_1-\frac{8}{3}m_{f_1}m_{f_2}\overline{f}_2f_2\overline{f}_1f_1\Big], \qquad (61)\\
i\mathcal{M}_4(\tilde{\phi}) &= \left(\frac{n-1}{n+2}\right)\frac{C_4}{6}m_{f_1}m_{f_2}\overline{f}_2f_2\overline{f}_1f_1\ , \qquad (62)
\end{aligned}
$$

where

$$C_4=\kappa^2 D(s), \qquad (63)$$

and $s=(k_1-k_2)^2=(q_2-q_1)^2$. The function $D(s)$ counts for the exchange of virtual KK states. In principle, all the contributing KK modes in a tower should be summed coherently. However, the summation would be ultravioletly divergent for $n\geq 2$. We have chosen to introduce an explicit cutoff M_S in the summation. The full derivation and expression of $D(s)$ is given in Appendix B. Taking the leading contribution in $M_S \gg s$, combining with the coupling by taking

$$\kappa^2R^n=(4\pi)^{n/2}\Gamma(n/2)M_S^{-(n+2)}, \qquad (64)$$

the coefficient C_4 reads

$$
\begin{aligned}
C_4 &\approx -iM_S^{-4}\log(M_S^2/s) \quad (n=2), \qquad (65)\\
&\approx \frac{-2iM_S^{-4}}{(n-2)} \quad (n>2). \qquad (66)
\end{aligned}
$$

We see that the amplitude has the dimensionful pre-factor M_S^{-4}, instead of the Planck mass suppression. We also note that C_4 remains the same with $s\to|t|$ or $|u|$ for t, u channels. Thus Eqs. (61) and (62) are indeed the appropriate low energy effective Lagrangians. On the other hand, if the cutoff scale is not too far away from the c. m. energy $\sqrt{s}$, then the resonant contribution in the s-channel should be included, as given by the real part in Eq. (B.6) of Appendix B.

These interactions would lead to modifications to decays of quarkonia via

$$(q\bar{q})\to\ell\bar{\ell},\ m\bar{m}, \qquad (67)$$

where $(q\bar{q})$ denotes a quarkonium such as $\Upsilon, J/\psi, \phi^0, \pi^0, \rho^0$ etc., $\ell=e,\mu,\tau$ and $m\bar{m}$ are light meson pairs. They would also modify the scattering cross sections such as

$$
\begin{aligned}
e^+e^- &\to \ell\bar{\ell},\ q\bar{q} \qquad (68)\\
q\bar{q} &\to \ell\bar{\ell},\ q\bar{q}. \qquad (69)
\end{aligned}
$$

Due to the particular structure of the contact interactions in Eq. (61), analyses on the final state angular distributions may reveal deviations from the Standard Model predictions.

3.3 Effective $\overline{f}f\ VV$ Interactions

Exchanges of virtual KK states can also contribute to processes like

$$f_1(k_1)\ \bar{f}_1(k_2) \to V_1(q_1)\ \overline{V}_2(q_2), \tag{70}$$

as in Fig. 1b, where the fermion momenta are chosen to be along the fermion line direction, and the gauge boson momenta are incoming to the vertex. The effective amplitudes for fermion-gauge bosons should have the general form of

$$\begin{aligned} i\mathcal{M}_V(\tilde{h}) &= -\frac{C_4}{8}\Big[2m_f(q_1\cdot V_2)(q_2\cdot V_1)\overline{f}f + (\frac{4}{3}m_V^2 m_f - s m_f)(V_1\cdot V_2)\overline{f}f \\ &\quad +2(k_1\cdot q_2 - k_1\cdot q_1)(V_1\cdot V_2)\overline{f}\not{q}_1 f + 2(k_1\cdot V_1)(q_1\cdot V_2)\overline{f}\not{q}_1 f \\ &\quad -2(k_1\cdot V_2)(q_2\cdot V_1)\overline{f}\not{q}_1 f - 2(k_1\cdot q_2)(q_1\cdot V_2)\overline{f}\not{V}_1 f + s(k_1\cdot V_2)\overline{f}\not{V}_1 f \\ &\quad -2(k_1\cdot q_1)(q_2\cdot V_1)\overline{f}\not{V}_2 f + s(k_1\cdot V_1)\overline{f}\not{V}_2 f\Big] \qquad (71) \\ i\mathcal{M}_V(\tilde{\phi}) &= -\left(\frac{n-1}{n+2}\right)\frac{C_4}{3}m_V^2 m_f(V_1\cdot V_2)\overline{f}f\ , \qquad (72) \end{aligned}$$

where C_4 is the same as in Eq. (63), $s=(q_1+q_2)^2=(k_1-k_2)^2$ and V_1, V_2 represent polarization vectors of the external gauge bosons. Examples for the induced physical processes include

$$\begin{aligned} e^+e^-,\ q\bar{q} &\to \gamma\gamma,\ W^+W^-,\ ZZ \text{ and } gg \qquad (73)\\ \gamma\gamma,\ gg &\to \ell\bar{\ell},\ q\bar{q}. \qquad (74) \end{aligned}$$

3.4 KK State Real Emission

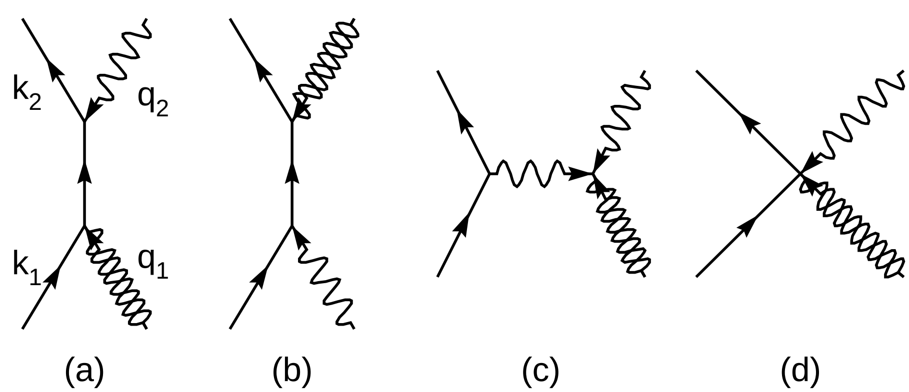

Figure 2: Feynman diagrams for $e^-e^+ \to \gamma + KK$.

Since the KK states couple to all the SM particles, they may be radiated from quarkonium decays if kinematically allowed, or be copiously produced at high energy colliders. Consider the process

$$f\bar{f} \to V + KK, \tag{75}$$

where V is a SM gauge boson. There are four diagrams to contribute to the process: s, t, u channels plus a four-point contact diagram as shown in Fig. 2. For simplicity, we consider a massless gauge boson (a photon or a gluon).

For the $\tilde{\phi}$ emission, it is interesting to note that only the fermion-mass dependent terms survive from the t and u diagrams. The amplitude for the emission of $\tilde{\phi}$ of mass $m_{\vec{n}}$ is

$$i\mathcal{M}(\tilde{\phi}) = \delta_{ij}\frac{-i}{2}\omega g_V m_f \kappa\ \overline{u}(k_2)\ (\frac{\not{\ell}\gamma_\rho}{t} + \frac{\gamma_\rho \not{j}}{u})u(k_1)\epsilon^\rho(q_2), \tag{76}$$

where ω is the normalization factor in Eq. (33) and $g_V = eQ_f$ for a photon and $g_s T^a_{nm}$ for a gluon, and $\ell = k_1 + q_2$, $j = k_1 + q_1$. Again, our momentum convention is that the fermion momenta follow the fermion line and the gauge boson and the KK state have their momenta incoming to the vertices. The amplitude for $\tilde{h}$ emission is calculated to be

$$\begin{aligned} i\mathcal{M}(\tilde{h}) \ &= \ \frac{-i}{2} g_V \kappa\ \overline{u}(k_2)\ \Big[\frac{1}{u}\gamma_\rho \not{j}\gamma_\mu k_{1\nu} + \frac{1}{t}\gamma_\mu k_{2\nu}\not{\ell}\gamma_\rho + \frac{2}{s}\gamma^\sigma(q_1\cdot q_2\eta_{\mu\sigma}\eta_{\nu\rho} + \eta_{\mu\rho}k_\nu q_{2\sigma} \\ &\quad -\eta_{\mu\sigma}q_{1\rho}q_{2\nu} - \eta_{\rho\sigma}k_\mu q_{2\nu}) - \gamma_\mu\eta_{\nu\rho}\Big]\ u(k_1)\epsilon^\rho(q_2)\epsilon^{\mu\nu}(q_1), \end{aligned} \tag{77}$$

with $k = k_1 - k_2$. The amplitudes of Eqs. (76)-(77) are directly applicable to physical processes like quarkonium radiative decays and e^+e^-, $q\bar{q} \to \gamma\ (g) + KK$, or $e\gamma \to e + KK$ and $qg \to q + KK$. Similar calculations can be carried out for $W, Z + KK$ processes.

Unlike the processes with internal KK exchanges, the diagrams for the external emission of KK modes with different masses do not interfere. Instead, contributions from different KK modes will have to be summed up at the cross-section level. A general discussion of the KK state summation is presented in Appendix B. As an illustration, we calculate the cross-section for Eq. (76). The cross-section is given by

$$\sigma = \left(\frac{n-1}{n+2}\right)\frac{c^2}{12N_c}\ \frac{m_f^2}{s^2}\ (s/M_S^2)^{n/2+1}\ I_\theta\ I_y(n), \tag{78}$$

where $c^2 = Q_f^2\alpha$ for a photon and $(N_c^2-1)\alpha_s$ for a gluon and N_c is the number of colors. The integrals are

$$I_\theta = \int_{-1+\delta}^{1-\delta}\frac{d\cos\theta}{1-\cos^2\theta} = \log\left(\frac{2-\delta}{\delta}\right), \quad I_y(n) = \int_0^1 dy^2\frac{y^{n-2}(1+y^4)}{(1-y^2)^{1/2}}, \tag{79}$$

where θ is the photon scattering angle in the c. m. frame with respect to the beam direction and $y^2 = m_{\vec{n}}^2/s$. The integral I_θ is logarithmically divergent, corresponding to the collinear singularity ($\delta \to 0$) associated with the massless gauge boson emission. From Eq. (78), we see once again that the cross-section rate is not suppressed by the Planck scale rather by a power of s/M_S^2, due to the summation over the large number of KK states. However, the additional factor m_f^2/s significantly suppresses the $\tilde{\phi}$ emission off light fermions. On the other hand the $\tilde{h}$ emission would not have this suppression and may be phenomenologically more interesting to study.

3.5 One-loop Corrections from Virtual KK States

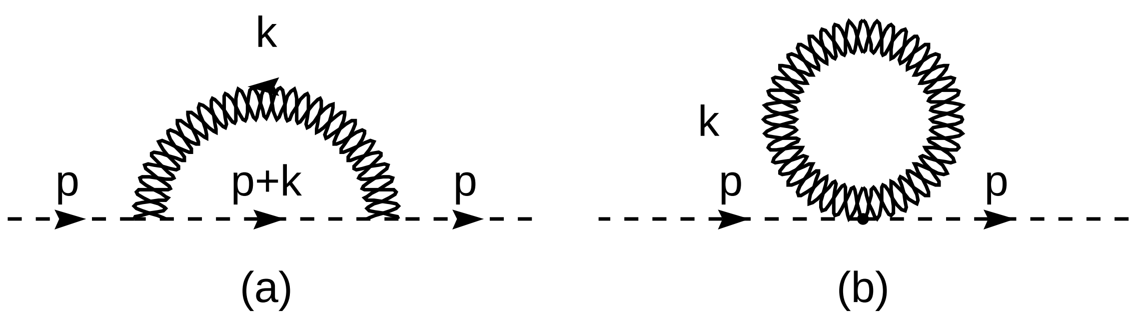

Figure 3: One-loop self-energy diagrams of the scalar particle.

It is of great interest to ask what radiative effects the SM fields may receive from the virtual KK states. As an example, we calculate the massive spin-2 KK state $\tilde{h}_{\mu\nu}^{\vec{n}}$ contribution to the one-loop self-energy for a scalar boson. The momentum integrals involved have much worse ultraviolet behavior than their four-dimensional counterparts, we need to introduce an explicit cutoff M_S to regularize the ultraviolet divergence.

There are two contributing diagrams, as shown in Fig. 3. The first one (Fig. 3a) originates from the KK-$\Phi\Phi$ vertex. The complete expression for this diagram is very complicated. However, to see the leading behavior, it is sufficient to evaluate the self-energy at zero external momentum. After some algebra, it can be simplified to

$$-i\Pi(0) \;=\; i\frac{\kappa^2}{16\pi^2}\int_0^\infty dk^2\, k^2 \left(\frac{1}{k^2+m_\Phi^2}\right)\sum_{\vec{n}}\left[\left(\frac{1}{k^2+m_{\vec{n}}^2}\right)\left(\frac{2m_\Phi^4}{3}-\frac{k^4 m_\Phi^2}{2m_{\vec{n}}^2}+\frac{k^2 m_\Phi^2}{2}\right)\right], \tag{80}$$

where we have performed the Wick rotation.

Since the spacing between adjacent KK states is of order $\mathcal{O}(1/R)$ and small, one can approximate the summation over the KK states by an integration, as shown in the Appendix B ‡. This reduces the above self-energy to

$$-i\Pi(0) \;=\; \frac{i}{16\pi^2}\Big(I_1(n) - \frac{I_2(n)}{2}\Big)m_\Phi^2 \;, \tag{81}$$

where we have introduced an explicit ultraviolet cutoff M_S for the momentum integration and used the relation Eq. (64). The integrals $I_1(n)$ and $I_2(n)$ are

$$I_1(n) \;=\; \int_0^1\int_0^1 dxdy\; \frac{(x+\frac{2}{3}r_\Phi)xy^{n/2-1}}{(x+r_\Phi)(x+y)} \;, \tag{82}$$

$$I_2(n) \;=\; \int_0^1\int_0^1 dxdy\; \left(\frac{x^2y^{n/2-2}}{x+r_\Phi}\right) \;, \tag{83}$$

where $r_\Phi = m_\Phi^2/M_S^2$.

The second diagram (Fig. 3b) comes from the four-point KK-KK-ΦΦ (seagull) vertex. To derive the Feynman rule for this vertex, one has to expand the interaction Lagrangian to the order of κ^2. After some tedious algebra, it can be shown that the Feynman rule is

$$i\frac{\kappa^2}{4}\delta_{ij}\Big(C_{\mu\nu,\rho\sigma}m_\Phi^2 + C_{\mu\nu,\rho\sigma|\lambda\eta}k_1^\lambda k_2^\eta\Big) \;, \tag{84}$$

where k_1, k_2 are four-momentum of the scalars, $C_{\mu\nu,\rho\sigma}$ is defined in Eq. (A.10) and

$$C_{\mu\nu,\rho\sigma|\lambda\eta} \;=\; \frac{1}{2}\Big[\eta_{\mu\lambda}C_{\rho\sigma,\nu\eta} + \eta_{\sigma\lambda}C_{\mu\nu,\rho\eta} + \eta_{\rho\lambda}C_{\mu\nu,\sigma\eta} + \eta_{\nu\lambda}C_{\mu\eta,\rho\sigma} - \eta_{\lambda\eta}C_{\mu\nu,\rho\sigma} + (\lambda\leftrightarrow\eta)\Big] \,. \tag{85}$$

The one-loop self-energy is then

$$-i\Pi(p^2) \;=\; i\frac{\kappa^2}{16\pi^2}\int_0^\infty dk^2k^2\sum_{\vec{n}}\Bigg[\Big(\frac{1}{k^2+m_{\vec{n}}^2}\Big)\Big(\frac{14m_\Phi^2}{3} + \frac{4k^2m_\Phi^2}{3m_{\vec{n}}^2} + \frac{k^4p^2}{3m_{\vec{n}}^4} - \frac{2k^2p^2}{m_{\vec{n}}^2}\Big)\Bigg] \,. \tag{86}$$

Again we replace the summation by integration and introduce a cutoff M_S, the above equation then becomes

$$-i\Pi(p^2) \;=\; \frac{i}{16\pi^2}\Bigg[\Big(\frac{10m_\Phi^2}{3} + \frac{7p^2}{3}\Big)I_3(n) + \Big(\frac{2m_\Phi^2}{3} - \frac{7p^2}{6}\Big)I_4(n) + \frac{p^2}{9}I_5(n)\Bigg] \;, \tag{87}$$

where

$$I_3(n) \;=\; \int_0^1\int_0^1 dxdy\; \left(\frac{xy^{n/2-1}}{x+y}\right) \;, \tag{88}$$

$$I_4(n) \;=\; \int_0^1 dx\; x^{n/2-2} \;, \qquad I_5(n) \;=\; \int_0^1 dx\; x^{n/2-3} \,. \tag{89}$$

‡The summation over the KK states can also be calculated using the Jacobi theta function [5].

Integrals $I_4(n)$ and $I_5(n)$ are infrared divergent when $n \leq 2$ and 4 respectively[§], this is unphysical since the summation should really start at the first nonzero mode. Therefore a natural infrared cutoff $1/(RM_S)^2$ can be included when necessary.

It is important to note that the leading one-loop correction to the scalar-boson mass is proportional to m_Φ^2, as opposed to the usual cutoff (M_S^2) dependent corrections from other particles in loops. We expect this fact to hold as well for the gauge bosons.

4 Conclusions

We have identified the massive KK states in the four-dimensional spacetime from the $(4+n)$-dimensional Kaluza-Klein (KK) theory, assuming compactification of the extra n dimensions on a torus. For a given KK level $\vec{n}$, we find that there are one spin-2 state, $(n-1)$ spin-1 states and $n(n-1)/2$ spin-0 states and they are all mass-degenerate.

We have constructed the effective interactions among these KK states and ordinary matter fields (fermions, gauge bosons and scalars). We find that the spin-1 states decouple and the spin-0 states only couple through the dilaton mode. We derived the interacting Lagrangian for the KK states and Standard Model fields. These interactions are flavor-diagonal and thus have no new flavor-changing neutral currents, nor baryon and lepton number violation. We also obtained the corresponding Feynman rules, as given in Appendix A, based on which further phenomenological applications can be carried out.

For the interesting scenario when the compactification scale $1/R$ is small compared to experimentally accessible energies, and the cutoff scale is on the order of 1 TeV, we outlined some low energy phenomenology for further studies. Examples include quarkonium radiative decays, four-fermion interactions and the associated production of gauge bosons and KK states for those new interactions resulting from the massive KK modes. Although formally suppressed by the Planck mass, the typical physical processes are only suppressed by powers of s/M_S^2 after summing over the contributing KK states. This implies possibly significant experimental signatures. It also recovers the "decoupling theorem" in the limit $M_S \to \infty$.

We also found that radiative corrections to the scalar self-energy via virtual KK modes are proportional to the scalar mass-squared. Finally, based on our dicussions for the KK decays, cosmology at the early Universe should be carefully examined with the existence of KK states in the extra large dimensions.

Notes added: When we are finishing this current work, another article dealing with the same subject appeared [19].

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[§] I_4 and I_5 come from the summations $\sum \frac{1}{m_{\vec{n}}^2}$ and $\sum \frac{1}{m_{\vec{n}}^4}$, they can be regularized by the Epstein ζ-function instead of by the explicit cutoffs.

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Appendix A: Feynman Rules

A.1 Propagators and Polarizations

The propagator for the massive spin-2 KK states $\widetilde{h}^{\vec{n}}_{\mu\nu}$ is

$$i\Delta^{\widetilde{h}}_{\{\mu\nu,\vec{n}\},\{\rho\sigma,\vec{m}\}}(k) \;=\; \frac{\frac{i}{2}\delta_{\vec{n},-\vec{m}}\; B_{\mu\nu,\rho\sigma}(k)}{k^2 - m_{\vec{n}}^2 + i\varepsilon} \;, \tag{A.1}$$

where

$$\begin{aligned} B_{\mu\nu,\rho\sigma}(k) \;=\;& \left(\eta_{\mu\rho} - \frac{k_\mu k_\rho}{m_{\vec{n}}^2}\right)\left(\eta_{\nu\sigma} - \frac{k_\nu k_\sigma}{m_{\vec{n}}^2}\right) + \left(\eta_{\mu\sigma} - \frac{k_\mu k_\sigma}{m_{\vec{n}}^2}\right)\left(\eta_{\nu\rho} - \frac{k_\nu k_\rho}{m_{\vec{n}}^2}\right) \\ & -\frac{2}{3}\left(\eta_{\mu\nu} - \frac{k_\mu k_\nu}{m_{\vec{n}}^2}\right)\left(\eta_{\rho\sigma} - \frac{k_\rho k_\sigma}{m_{\vec{n}}^2}\right) \;. \end{aligned} \tag{A.2}$$

It is obvious that $k^\mu B_{\mu\nu,\rho\sigma} = 0$ and $B^\mu{}_{\mu,\rho\sigma} = 0$ if $\widetilde{h}^{\vec{n}}_{\mu\nu}$ is on shell, $k^2 = m_{\vec{n}}^2$.

The polarization tensors for $\widetilde{h}^{\vec{n}}_{\mu\nu}$ can be constructed from the polarization vectors of the massive vector bosons, $\epsilon^{\pm,0}_\mu$, as follows:

$$\epsilon^s_{\mu\nu} \;=\; \left\{ \epsilon^+_\mu\epsilon^+_\nu, \frac{1}{\sqrt{2}}(\epsilon^+_\mu\epsilon^0_\nu + \epsilon^0_\mu\epsilon^+_\nu), \frac{1}{\sqrt{6}}(\epsilon^+_\mu\epsilon^-_\nu + \epsilon^-_\mu\epsilon^+_\nu - 2\epsilon^0_\mu\epsilon^0_\nu), \frac{1}{\sqrt{2}}(\epsilon^-_\mu\epsilon^0_\nu + \epsilon^0_\mu\epsilon^-_\nu), \epsilon^-_\mu\epsilon^-_\nu \right\} . \tag{A.3}$$

These polarization tensors are traceless, transverse and orthogonal,

$$(\epsilon^s)^\mu{}_\mu \;=\; 0 \;, \qquad k^\mu\epsilon^s_{\mu\nu} \;=\; 0 \;, \qquad \epsilon^{s,\mu\nu}\epsilon^{s'*}_{\mu\nu} \;=\; \delta^{ss'} \;. \tag{A.4}$$

The completeness condition then follows from that of ϵ^s_μ and the definition, Eq. (A.3),

$$\sum_{s=1}^{5}\epsilon^s_{\mu\nu}\epsilon^{s*}_{\rho\sigma} \;=\; \frac{1}{2}B_{\mu\nu,\rho\sigma}(k) \;. \tag{A.5}$$

The propagators for $\widetilde{\phi}^{\vec{n}}_{ij}$ and $\widetilde{A}^{\vec{n}}_{\mu i}$ have the following forms

$$i\Delta^{\widetilde{\phi}}_{\{ij,\vec{n}\},\{kl,\vec{m}\}}(k) \;=\; \frac{\frac{i}{2}(P^{\vec{n}}_{ik}P^{\vec{n}}_{jl} + P^{\vec{n}}_{il}P^{\vec{n}}_{jk})\delta_{\vec{n},-\vec{m}}}{k^2 - m_{\vec{n}}^2 + i\varepsilon} \;, \tag{A.6}$$

$$i\Delta^{\widetilde{A}}_{\{\mu i,\vec{n}\},\{\nu j,\vec{m}\}}(k) \;=\; -\frac{iP^{\vec{n}}_{ij}\delta_{\vec{n},-\vec{m}}(\eta_{\mu\nu} - k_\mu k_\nu/m_{\vec{n}}^2)}{k^2 - m_{\vec{n}}^2 + i\varepsilon} \;, \tag{A.7}$$

where $P_{ij}^{\vec{n}}$ are the projectors defined in Eq. (15). Their appearance can be understood from the fact that $\widetilde{\phi}_{ij}^{\vec{n}}$ and $\widetilde{A}_{ij}^{\vec{n}}$ only couple to the sources which are dressed up by the projectors.

Since $\widetilde{A}_{\mu i}^{\vec{n}}$ and $\widetilde{\phi}_{ij}^{\vec{n}}$ satisfy the divergencelessness condition in Eq. (19), each external state of these particles should be accompanied by an extra-dimension "polarization" vector (ϵ_i) or tensor (ϵ_{ij}), which satisfies

$$n_i \epsilon_i^s = 0 \;, \quad \epsilon_i^s \epsilon_i^{s'*} = \delta^{ss'} \;, \quad \sum_{s=1}^{n-1} \epsilon_i^s \epsilon_j^{s*} = P_{ij}^{\vec{n}} \;, \tag{A.8}$$

$$n_i \epsilon_{ij}^s = 0 \;, \quad \epsilon_{ij}^s \epsilon_{ij}^{s'*} = \delta^{ss'} \;, \quad \sum_{s=1}^{n(n-1)/2} \epsilon_{ij}^s \epsilon_{kl}^{s*} = \frac{1}{2} P_{ik}^{\vec{n}} P_{jl}^{\vec{n}} + \frac{1}{2} P_{il}^{\vec{n}} P_{jk}^{\vec{n}} \;, \tag{A.9}$$

for each KK level.

A.2 Vertex Feynman Rules

In the following we list the complete leading order Feynman rules in three figures, Figs. 4, 5 and 6. Some of the symbols used are defined as follows:

$$C_{\mu\nu,\rho\sigma} = \eta_{\mu\rho}\eta_{\nu\sigma} + \eta_{\mu\sigma}\eta_{\nu\rho} - \eta_{\mu\nu}\eta_{\rho\sigma} \;, \tag{A.10}$$

$$D_{\mu\nu,\rho\sigma}(k_1, k_2) = \eta_{\mu\nu} k_{1\sigma} k_{2\rho} - \Big[\eta_{\mu\sigma} k_{1\nu} k_{2\rho} + \eta_{\mu\rho} k_{1\sigma} k_{2\nu} - \eta_{\rho\sigma} k_{1\mu} k_{2\nu} + (\mu \leftrightarrow \nu)\Big] \;, \tag{A.11}$$

$$\begin{aligned} E_{\mu\nu,\rho\sigma}(k_1, k_2) &= \eta_{\mu\nu}(k_{1\rho}k_{1\sigma} + k_{2\rho}k_{2\sigma} + k_{1\rho}k_{2\sigma}) \\ &\quad - \Big[\eta_{\nu\sigma} k_{1\mu} k_{1\rho} + \eta_{\nu\rho} k_{2\mu} k_{2\sigma} + (\mu \leftrightarrow \nu)\Big] \;, \end{aligned} \tag{A.12}$$

$$\begin{aligned} F_{\mu\nu,\rho\sigma\lambda}(k_1, k_2, k_3) &= \eta_{\mu\rho}\eta_{\sigma\lambda}(k_2 - k_3)_\nu + \eta_{\mu\sigma}\eta_{\rho\lambda}(k_3 - k_1)_\nu \\ &\quad + \eta_{\mu\lambda}\eta_{\rho\sigma}(k_1 - k_2)_\nu + (\mu \leftrightarrow \nu) \;, \end{aligned} \tag{A.13}$$

$$\begin{aligned} G_{\mu\nu,\rho\sigma\lambda\delta} &= \eta_{\mu\nu}(\eta_{\rho\sigma}\eta_{\lambda\delta} - \eta_{\rho\delta}\eta_{\sigma\lambda}) + \Big(\eta_{\mu\rho}\eta_{\nu\delta}\eta_{\lambda\sigma} + \eta_{\mu\lambda}\eta_{\nu\sigma}\eta_{\rho\delta} \\ &\quad - \eta_{\mu\rho}\eta_{\nu\sigma}\eta_{\lambda\delta} - \eta_{\mu\lambda}\eta_{\nu\delta}\eta_{\rho\sigma} + (\mu \leftrightarrow \nu)\Big) \;. \end{aligned} \tag{A.14}$$

All of them are symmetric in $\mu \leftrightarrow \nu$. $C_{\mu\nu,\rho\sigma}$ is the symbol that appears in the massless graviton propagator in the de Donder gauge.

Appendix B. Summation of the KK States

Since the KK states are nearly degenerate in mass, one would encounter the summation over those modes that are contributing to a given physical process. Consider the number of KK states within a mass scale $m_{\vec{n}}^2$. This is equivalent to counting the n-dimensional hyper-cubic lattice sites in $\vec{n} = (n_1, n_2 ..., n_n)$ with a relation to the mass

$$m_{\vec{n}}^2 = \frac{4\pi^2 \vec{n}^2}{R^2}, \quad \text{or} \quad r^2 \equiv \vec{n}^2 = \frac{m_{\vec{n}}^2 R^2}{4\pi^2}. \tag{B.1}$$

Since the mass separation of $\mathcal{O}(1/R)$ is much smaller than any other physical scale involved in the problem, it is much more convenient to consider the discrete $\vec{n}$ in the continuum limit. Therefore, the number of states in the mass interval $dm_{\vec{n}}^2$ can be obtained by

$$\Delta \vec{n}^2 \approx d^n r = \rho(m_{\vec{n}}) dm_{\vec{n}}^2, \tag{B.2}$$

where the KK state density as a function of $m_{\vec{n}}$ is given by

$$\rho(m_{\vec{n}}) = \frac{R^n \ m_{\vec{n}}^{n-2}}{(4\pi)^{n/2} \ \Gamma(n/2)}. \tag{B.3}$$

This is the state density function that is to be convoluted with a physical amplitude or cross-section for a KK state with a given mass $m_{\vec{n}}$.

A less trivial example is when constructing the effective interactions due to virtual KK state exchanges, one has to sum over them in the propagator

$$D(s) \ = \ \sum_{\vec{n}} \frac{i}{s - m_{\vec{n}}^2 + i\varepsilon} \ = \int_0^\infty dm_{\vec{n}}^2 \ \rho(m_{\vec{n}}) \frac{i}{s - m_{\vec{n}}^2 + i\varepsilon} \ , \tag{B.4}$$

which may be singular near a real KK state production. Using

$$\frac{1}{s - m^2 + i\varepsilon} \ = \ P\left(\frac{1}{s - m^2}\right) - i\pi\delta(s - m^2) \ , \tag{B.5}$$

we find

$$D(s) \ = \ \frac{s^{n/2-1}}{\Gamma(n/2)} \frac{R^n}{(4\pi)^{n/2}} \Big[\pi + 2iI(M_S/\sqrt{s})\Big] \ , \tag{B.6}$$

where

$$I(M_S/\sqrt{s}) \ = \ P \int_0^{M_S/\sqrt{s}} dy \ \frac{y^{n-1}}{1 - y^2} \ . \tag{B.7}$$

We have introduced an explicit ultraviolet cutoff $M_S/\sqrt{s}$ in the integral. It should be understood that a point $y = 1$ has been removed from the integration path.

The real part proportional to π in Eq. (B.6) is from the narrow resonant production of a single KK mode with $m_{\vec{n}}^2 = s$ and the imaginary part $I(M_S/\sqrt{s})$ is from the summation over the many non-resonant states. This principal integration of Eq. (B.7) can be easily carried out, it gives

$$\begin{aligned} I(M_S/\sqrt{s}) \ &= \ -\sum_{k=1}^{n/2-1} \frac{1}{2k} \left(\frac{M_S}{\sqrt{s}}\right)^{2k} - \frac{1}{2} \log\left(\frac{M_S^2}{s} - 1\right) \qquad n = \text{even}, \\ &= \ -\sum_{k=1}^{(n-1)/2} \frac{1}{2k-1} \left(\frac{M_S}{\sqrt{s}}\right)^{2k-1} + \frac{1}{2} \log\left(\frac{M_S + \sqrt{s}}{M_S - \sqrt{s}}\right) \qquad n = \text{odd}. \end{aligned} \tag{B.8}$$

For $M_S \gg \sqrt{s}$, the leading contribution comes from the non-resonant states and yields

$$
\begin{aligned}
D(s) &\approx \frac{-i}{4\pi} R^2 \log(M_S^2/s) \quad (n=2), \\
&\approx \frac{-2i}{(n-2)\Gamma(n/2)} \frac{R^n M_S^{(n-2)}}{(4\pi)^{n/2}} \quad (n>2).
\end{aligned}
\tag{B.9}
$$

The summation of space-like propagators can be evaluated similarly, and it gives

$$
D_E(t) = \sum_{\vec{n}} \frac{i}{t - m_{\vec{n}}^2} = \sum_{\vec{n}} \frac{-i}{|t| + m_{\vec{n}}^2} = \frac{|t|^{n/2-1}}{\Gamma(n/2)} \frac{R^n}{(4\pi)^{n/2}} (-2i) I_E(M_S/\sqrt{|t|}\,) \, ,
\tag{B.10}
$$

where the integral I_E is

$$
\begin{aligned}
I_E(M_S/\sqrt{|t|}) &= \int_0^{M_S/\sqrt{|t|}} dy \, \frac{y^{n-1}}{1+y^2} \\
&= (-)^{n/2+1} \left[\sum_{k=1}^{n/2-1} \frac{(-)^k}{2k} \left(\frac{M_S}{\sqrt{|t|}} \right)^{2k} + \frac{1}{2} \log \left(\frac{M_S^2}{|t|} + 1 \right) \right] \qquad n = \text{even} \, , \\
&= (-)^{(n-1)/2} \left[\sum_{k=1}^{(n-1)/2} \frac{(-)^k}{2k-1} \left(\frac{M_S}{\sqrt{|t|}} \right)^{2k-1} + \tan^{-1}(M_S^2/|t|+1) \right] \qquad n = \text{odd} \, .
\end{aligned}
\tag{B.11}
$$

We note that leading terms in $D_E(t)$ for $M_S^2 \gg |t|$ are exactly of the same form as in Eq. (B.9) and lead to $D_E(t) = D(s \to |t|)$. This shows that the low energy effective interactions for s and t channels are equivalent.

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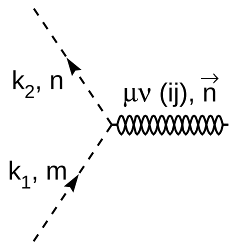
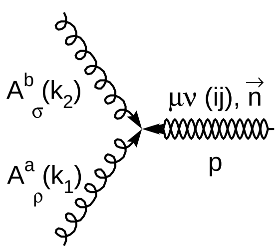


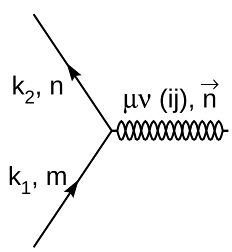
$\tilde{h}^{\vec{n}}_{\mu\nu}\Phi\Phi$: $\quad -i\,\kappa/2\;\delta_{mn}\,(\,m^2_\Phi\,\eta_{\mu\nu} + C_{\mu\nu,\rho\sigma}\,k_1^\rho\,k_2^\sigma\,)$

$\tilde{\phi}^{\vec{n}}_{ij}\Phi\Phi$: $\quad i\,\omega\,\kappa\;\delta_{ij}\,\delta_{mn}\,(\,k_1\cdot k_2 - 2\,m^2_\Phi\,)$



$\tilde{h}^{\vec{n}}_{\mu\nu}AA$: $\quad -i\,\kappa/2\;\delta^{ab}\,(\,(\,m^2_A + k_1\cdot k_2\,)\,C_{\mu\nu,\rho\sigma} + D_{\mu\nu,\rho\sigma}\,(k_1, k_2) + \xi^{-1}\,E_{\mu\nu,\rho\sigma}\,(k_1, k_2)\,)$

$\tilde{\phi}^{\vec{n}}_{ij}AA$: $\quad i\,\omega\,\kappa\;\delta_{ij}\,\delta^{ab}\,(\,\eta_{\rho\sigma}\,m^2_A + \xi^{-1}\,(k_{1\rho}\,p_\sigma + k_{2\sigma}\,p_\rho)\,)$



$\tilde{h}^{\vec{n}}_{\mu\nu}\psi\psi$: $\quad -i\,\kappa/8\;\delta_{mn}\,(\,\gamma_\mu\,(k_{1\nu} + k_{2\nu}) + \gamma_\nu\,(k_{1\mu} + k_{2\mu}) - 2\,\eta_{\mu\nu}\,(\not{k}_1 + \not{k}_2 - 2\,m_\psi)\,)$

$\tilde{\phi}^{\vec{n}}_{ij}\psi\psi$: $\quad i\,\omega\,\kappa\;\delta_{ij}\,\delta_{mn}\,(\,3/4\,\not{k}_1 + 3/4\,\not{k}_2 - 2\,m_\psi)$

Figure 4: Three-point vertex Feynman rules. The KK states are plot in double-sinusoidal curves. The symbols $C_{\mu\nu,\rho\sigma}$, $D_{\mu\nu,\rho\sigma}(k_1,k_2)$ and $E_{\mu\nu,\rho\sigma}(k_1,k_2)$ are defined in Eqs. (A.10), (A.11) and (A.12) respectively. m_Φ, m_A and m_ψ are masses of the scalar, vector and fermion. $\omega=\sqrt{\frac{2}{3(n+2)}}$, $\kappa=\sqrt{16\pi G_N}$ and ξ is the gauge-fixing parameter.

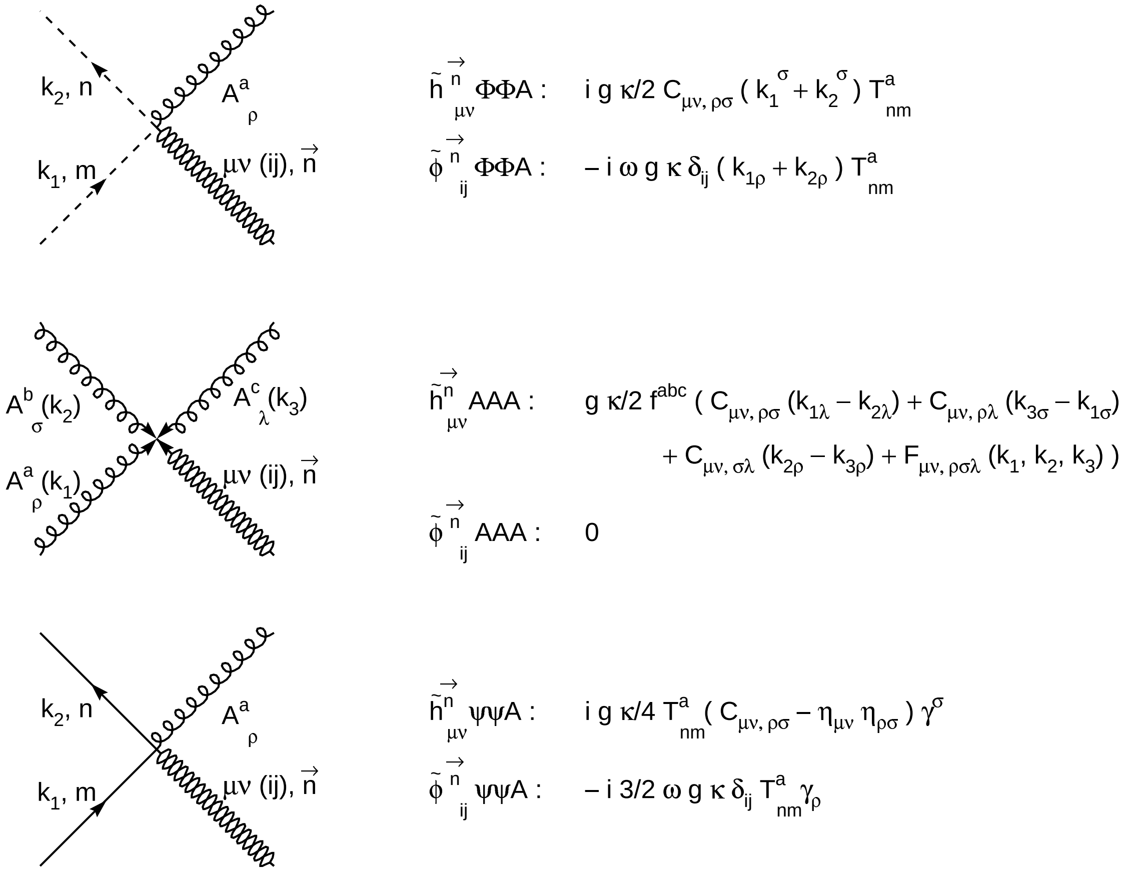
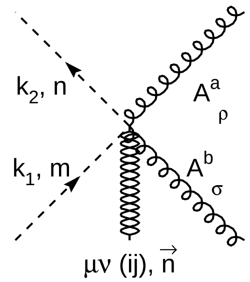
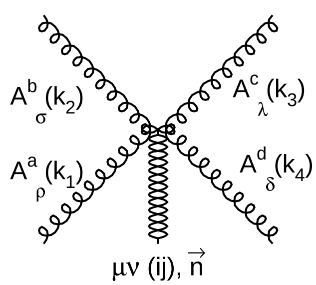


Figure 5: Four-point vertex Feynman rules. g is the gauge coupling and f^{abc} the structure constant of the Lie algebra, $gT^a \to \epsilon Q_f$ for QED. The symbols $C_{\mu\nu,\rho\sigma}$ and $F_{\mu\nu,\rho\sigma\lambda}(k_1, k_2, k_3)$ are defined in Eqs. (A.10) and (A.13).



$\tilde{h}^{\vec{n}}_{\mu\nu}\,\Phi\Phi AA$: $-\, i\, g^2\, \kappa/2\; C_{\mu\nu,\,\rho\sigma}\, \{ T^a, T^b \}_{mn}$

$\tilde{\phi}^{\vec{n}}_{ij}\,\Phi\Phi AA$: $i\, \omega\, g^2\, \kappa\, \delta_{ij}\, \eta_{\rho\sigma}\, \{ T^a, T^b \}_{mn}$



$\tilde{h}^{\vec{n}}_{\mu\nu}\, AAAA$: $i\, g^2\, \kappa/2\, (f^{eac} f^{edb}\, G_{\mu\nu,\,\rho\sigma\lambda\delta} + f^{eab} f^{ecd}\, G_{\mu\nu,\,\rho\lambda\sigma\delta} + f^{ead} f^{ebc}\, G_{\mu\nu,\,\rho\sigma\delta\lambda})$

$\tilde{\phi}^{\vec{n}}_{ij}\, AAAA$: 0

Figure 6: Five-point vertex Feynman rules. $g^2\{T^a, T^b\} \to 2e^2 Q_f^2$ for QED. The symbols $C_{\mu\nu,\rho\sigma}$ and $G_{\mu\nu,\rho\sigma\lambda\delta}$ are defined in Eqs. (A.10) and (A.14).