Fermi National Accelerator Laboratory

FERMILAB-Pub-98/226-A

A Geometrical Test of the Cosmological Energy Contents Using the Lyman-alpha Forest

L. Hui, A. Stebbins and S. Burles

Fermi National Accelerator Laboratory
P.O. Box 500, Batavia, Illinois 60510

July 1998

Submitted to *Astrophysical Journal Letters*

Operated by Universities Research Association Inc. under Contract No. DE-AC02-76CH03000 with the United States Department of Energy

Disclaimer

Distribution

Fermi National Accelerator Laboratory

FERMILAB–PUB–98/226-A
astro-ph/9807190
July 17, 1998

A Geometrical Test of the Cosmological Energy Contents Using the Lyman-alpha Forest

Lam Hui*
NASA/Fermilab Astrophysics Center
Fermi National Accelerator Laboratory, Batavia, IL 60510

Albert Stebbins†
NASA/Fermilab Astrophysics Center
Fermi National Accelerator Laboratory, Batavia, IL 60510

Scott Burles‡
Department of Astronomy and Astrophysics
The University of Chicago, Chicago, IL 60637

*Electronic mail: lhui@fnal.gov

†Electronic mail: stebbins@fnal.gov

‡Electronic mail: scott@oddjob.uchicago.edu

ABSTRACT

In this *Letter* we explore a version of the test of cosmological geometry proposed by Alcock & Paczyński (1979), using observations of the Lyman-α forest in the spectra of close quasar pairs. By comparing the correlations in absorption in one quasar spectrum with correlations between the spectra of neighboring quasars one can determine the relation of the redshift distance scale to the angle distance scale at the redshift of the absorbers, $z \sim 2-4$. Since this relationship depends on the parameters of the cosmological model, these parameters may be determined using the Lyman-α forest. While this test is relatively insensitive to the density parameter Ω_m in a dust-dominated universe, it is more sensitive to the presence of a matter component with large negative pressure (such as a cosmological constant Λ) and its equation of state. With only 25 pairs of quasar spectra at angular separations $0.5' - 2'$, one can discriminate between an $\Omega_m = 0.3$ open universe ($\Lambda = 0$) and an $\Omega_m = 0.3$ flat (Λ-dominated) universe at the $4-\sigma$ level. The S/N can be enhanced by considering quasar pairs at smaller angular separations, but requires proper modeling of nonlinear redshift space distortions. Here the correlations and redshift space distortions are modeled using linear theory.

Subject headings: cosmology: theory — intergalactic medium — quasars: absorption lines — large-scale structure of universe

1. Introduction

Recent supernova Ia observations have generated a lot of interest in cosmological models where a significant fraction of the energy contents has negative pressure (Perlmutter et al. 1997; Riess et al. 1998; Garnavich et al. 1998). A common way of parameterizing the equation of state of this component, which we call Q, is $p = w\rho$, where p and ρ denote the pressure and density respectively. The cosmological constant Λ corresponds to the case $w = -1$.

It is important to have independent ways to constrain the abundance and properties of such a component, as different methods suffer from different systematic errors, and, perhaps more importantly, different methods are sensitive to different combinations of parameters. In this paper, we discuss a version of a test proposed by Alcock & Paczyński (1979; AP hereafter), which is particularly sensitive to the presence/absence of Q. They observed that an object placed at a cosmological distance would have a definite relationship between its angular and redshift extents, which is cosmology-dependent.

Consider an object with mean redshift $\bar{z}$, and angular size θ which thus has transverse extent (in velocity units)

$$u_\perp(\theta) = \frac{\bar{H}}{1+\bar{z}} D_A(\bar{z})\theta \ . \tag{1}$$

Here $\bar{H}$ is the Hubble parameter at redshift $\bar{z}$, and $D_A(\bar{z})$ is the angular diameter distance. For spherical objects the radial and transverse extents are equal, but more generally if the object is squashed radially by a factor α_s, then the radial extent is $u_{||} \equiv \frac{c\Delta z}{1+z} = \alpha_s u_\perp$ Here c is the speed of light and $u_\perp$, $u_{||} \ll c$ is assumed. A plot of $u_\perp/\theta$ is shown in Fig. 2. Note how its value for a Q-dominated universe (for $w \lesssim -1/3$) differs significantly from that for a no-Q-universe.

Various incarnations of this test have been discussed in the context of galaxy and quasar surveys (e.g. Ryden 1995; Ballinger et al. 1996; Matsubara & Suto 1996; Popowski et al. 1998; De Laix & Starkman 1998) where the "object" used is the two-point correlation function, whose "shape"(α_s above) need not be spherical because of redshift-anisotropy induced by peculiar motion. In the case of the Lyman-alpha (Lyα) forest, we cannot observe the full three-dimensional (3D) correlation directly. Instead we can measure the one-dimensional (1D) correlation along a line of sight (LOS), and the cross-correlation between two (or multiple) close-by LOS, or their Fourier counterparts: the auto- and the cross-spectra. The two are related to each other through an underlying 3D power spectrum. These relations are spelled out in §2 (for auto-spectrum, see also Kaiser & Peacock 1991). Using a method developed by Hui (1998a) building on earlier work by Croft et al. (1998), one can invert the auto-spectrum to obtain the underlying 3D power spectrum, which then allows one to predict what the cross-spectrum should be, except that the prediction is cosmology-dependent. A comparison between the observed and the predicted cross-spectrum is then our version of the AP test. It is not the only version possible in the context of the forest, but we will use this version to gain some intuition on the sensitivity of the AP test to various cosmological parameters. A hypothetical implementation of this test is studied in §3.2.

An important problem in this application of the AP test is the modeling of the redshift-space distortions (the "shape"). We give a first estimate using linear theory in §3.1. We conclude in §4 with an assessment of the expected S/N of the AP-effect-measurement in the Lyα forest, and remarks on lines of further investigation.

2. Auto- and Cross-spectrum: Definitions and General Formula

Given the observed transmission $f = e^{-\tau}$ as a function of the velocity $u_{||}$ along a LOS and the angular position θ on the sky, let us define $\delta_f = (f - \bar{f})/\bar{f}$ where $\bar{f}$ is the mean transmission. Then the auto-correlation $\xi^f_{||}$ along the LOS and the cross-correlation $\xi^f_\times$ between two close-by LOS of angular separation θ, and their Fourier counterparts the auto-spectrum $P^f_{||}$ and the cross-spectrum $P^f_\times$, are respectively defined by

$$\langle \delta_f(u'_{||},\theta')\delta_f(u'_{||}+u_{||},\theta')\rangle = \xi^f_{||}(u_{||}) \ , \ \langle \delta_f(u'_{||},\theta')\delta_f(u'_{||}+u_{||},\theta'+\theta)\rangle = \xi^f_\times(u_{||},\theta) \quad (2)$$

$$P^f_{||}(k_{||}) = \int \xi^f_{||}(u_{||})e^{-ik_{||}u_{||}}du_{||} \ , \ P^f_\times(k_{||},\theta) = \int \xi^f_\times(u_{||},\theta)e^{-ik_{||}u_{||}}du_{||} \quad (3)$$

where we use $k_{||}$ to denote the wave-vector along the line of sight.

The two quantities are related to an underlying 3D power spectrum $\tilde{P}^f(k_{||},k)$, where we use ˜ to denote the 3D nature, and k is the length of the 3D wave-vector:

$$\begin{aligned} P^f_{||}(k_{||}) &= \int_{k_{||}}^\infty \tilde{P}^f(k_{||},k)k\frac{dk}{2\pi} \\ P^f_\times(k_{||},\theta) &= \int_{k_{||}}^\infty \tilde{P}^f(k_{||},k)J_0[k_\perp u_\perp(\theta)]k\frac{dk}{2\pi} \end{aligned} \quad (4)$$

where $J_0[x] = \int_0^{2\pi} e^{-ix\cos\alpha}d\alpha/(2\pi)$ is the zeroth order Bessel function, $k_\perp = \sqrt{k^2 - k^2_{||}}$, and $u_\perp$ is the transverse velocity-separation for the given θ (eq. [1]).

We deliberately allow $\tilde{P}^f(k_{||},k)$ two arguments to account for the possibility of an anisotropic 3D power spectrum, as for instance in the presence of distortions by peculiar motion, or thermal broadening (bear in mind that line-broadening always acts along, not transverse, to the LOS). The tricky part is how to model this distortion. A general form is

$$\tilde{P}^f(k_{||},k) = W(k_{||}/k,k)\tilde{P}^f(k) \quad (5)$$

where W is a distortion kernel, and $\tilde{P}^f(k)$ with only one argument represents the transmission power spectrum in the absence of peculiar motion and line-broadening.

As we will discuss in §3.1, eq. (5) together with eq. (4) allow us to predict the cross-spectrum, once the auto-spectrum is given and W as well as the relevant cosmological parameters determining the angle-velocity relation (eq. [1]) are specified. A comparison of the predicted and observed cross-spectra then allows one to discriminate between different cosmological models, which is our version of the AP test. We will perform a first estimate of W using perturbation theory.

3. A Perturbative Estimate

3.1. Perturbative Auto- and Cross-spectra

A linear perturbative calculation (i.e. assume $\delta_f \ll 1$) of the auto-spectrum has been carried out in Hui (1998a). The cross-spectrum can be computed in a very similar manner. We simply state the results here, and refer the reader to Hui (1998a) for details.

Essentially, both power spectra follow from the substitution of eq. (5) into eq. (3), with W given by[§]

$$W(k_{||}/k, k) = \left[1 + \beta_f \frac{k_{||}^2}{k^2} + \Delta_b\right]^2 \exp[-k_{||}^2/k_{||}^{s2}] \tag{6}$$

with

$$\beta_f = \frac{f_\Omega}{2 - 0.7(\gamma - 1)} \; , \; \Delta_b = \frac{1 - \gamma}{8 - 2.8(\gamma - 1)} k_{||}^2 b_{T_0}^2 \tag{7}$$

where $f_\Omega = d\,\mathrm{lnD}/d\,\mathrm{lna}$ with D being the linear growth factor and a the expansion scale factor, b_{T_0} is the thermal broadening width associated with a mean temperature of T_0, and γ specifies the temperature-density relation through $T \propto \rho^{\gamma-1}$ where T is gas pressure and ρ is the density. The smoothing scale $k_{||}^s$ is associated with the thermal broadening scale as well as observation resolution. (see Hui 1998a for details).

An important feature of eq. (6) is that on large scales ($k_{||}$, $k \ll k_{||}^s$, $1/b_{T_0}$), it reduces to the famous Kaiser (1987) formula, with β_f playing the role of the galaxy-bias factor:

$$W(k_{||}/k, k) \rightarrow W_{\rm LS}(k_{||}/k, k) = (1 + \beta_f k_{||}^2/k^2)^2 \tag{8}$$

As shown in Hui (1998a), this allows a one-parameter-only (β_f) inversion of the large scale 3D isotropic power spectrum $\tilde{P}^f(k)$ using this linear integral equation, which follows from eq. (4) and (5):

$$P_{||}^f(k_{||}) = \int_{k_{||}}^\infty W(k_{||}/k, k) \tilde{P}^f(k) \frac{kdk}{2\pi} \, . \tag{9}$$

The above, in discretized form, can be viewed as a matrix equation, and one simply inverts a matrix proportional to W (which is in essence upper or lower triangular because of the limits of integration/summation) to obtain $\tilde{P}^f(k)$ from the auto-spectrum. It can be shown that $W_{\rm LS}$ (eq. [8]) could be used instead of the full W in obtaining the large scale $P^f(k)$, with good accuracy. The reader is referred to Hui (1998a) for illustrations of this method.

[§]We have equated $\tilde{P}^f(k)$ with the $A'\tilde{P}^\rho(k)\exp[-k^2/k_F^2]$ in Hui (1998a), where A' is a constant, k_F is roughly the Jean scale, and $\tilde{P}^\rho(k)$ is the 3D real-space mass power spectrum.

An interesting bonus of the distortion kernel given in eq. (6) is that the factor of $\exp[-k_{||}^2/k_{||}^{s2}]$ is commonly used to describe nonlinear redshift distortions of galaxy distributions (see e.g. Peacock & Dodds 1994). This should come as no surprise because nonlinear distortions arise from virialized objects which have a Maxwellian distribution of velocities – precisely the form for thermal broadening as well, which determines $k_{||}^s$. One can then use the perturbative kernel in eq. (6) but allow $k_{||}^s$ to vary to account for nonlinear distortions. We will not pursue that here, but will come back to it in §4. It should be emphasized that even if W turns out to be substantially different from eq. (6) or (8), perhaps due to nonlinear clustering, the procedure of inverting a triangular matrix using eq. (9) should generally work.

Given the 3D isotropic power spectrum $\tilde{P}^f(k)$, one can predict the cross-spectrum using:

$$P_\times^f(k_{||},\theta) = \int_{k_{||}}^\infty W(k_{||}/k,k)\tilde{P}^f(k)J_0[k_\perp u_\perp(\theta)]\frac{kdk}{2\pi} \qquad (10)$$

which follows from eq. (4) and (5). As in the inversion procedure above, we will use $W_{\rm LS}$ instead of the full W, when performing the above integration. We will see in the next section that the induced error in the predicted large scale cross-spectrum is small. Note that the rapid oscillation of J_0 and decay of $\tilde{P}^f$ at high $k_\perp$ or k means the large scale $P_\times^f$ is not sensitive to assumptions made about the small scale distortion kernel.

3.2. The Alcock-Paczyński Test

The AP test for the Lyα forest could be implemented in several different ways. The version adopted here should be seen as a first step towards understanding the sensitivity of the test. We will discuss the issue of other possible implementations in §4.

Suppose one is given a set of idealized observed transmission power spectra, say the auto-spectrum and the cross-spectra for two different angular separations $\theta = 1'$ and $2'$. In real life, one of course does not know *a priori* the underlying cosmological model behind this set of observations. For the sake of our testing here, let us construct these power spectra by assuming an input "true" model: the open Cold-Dark-Matter (OCDM) universe with $\Omega_m = 0.3$ and no Q. The full distortion kernel (eq. [6]) is used to compute these power spectra, with $\gamma = 1.5$, and b_{T_0}, $k_{||}^s$ and k_F corresponding to a gas of temperature 10^4 K (see Hui 1998a for details). The mass power spectrum is assumed to have a shape parameter of $\Gamma = 0.25$. Let $\bar{z} = 3$.

Pretending we have no insider information on the underlying cosmology, our version of the AP test comes in two steps. First, perform the inversion from the "observed" auto-spectrum $P^f_{||}(k_{||})$ to the 3D isotropic power spectrum $\tilde{P}^f(k)$ using eq. (9) and $W_{\rm LS}$ in eq. (8). By using the latter instead of the full kernel as in eq. (6), we will be making an error. However, the error will be small on sufficiently large scales, as we will see. The second step involves predicting the cross-spectra for the corresponding angular separations, using eq. (10), once the 3D isotropic power spectrum $\tilde{P}^f(k)$ is computed.

Both steps require the assumption of a cosmological model. In the first step, the cosmological density parameters determine the amount of redshift-space anisotropy that needs to be accounted for, through the parameter β_f or f_Ω (eq. [8] or [7]). The temperature-density-relation index γ also enters into β_f; we will come back to this below. In the second step, cosmology dictates the angle-velocity relation through the quantity $u_\perp(\theta)$ (eq. [1] & [10]).

To parameterize the cosmological model that we need to assume in carrying out the AP test, we use Ω_m, Ω_k and Ω_Q, which correspond to the mass, curvature and Q density parameters. They sum to 1. The Q component is described by an equation of state of the form $p = w\rho$ where $-1 \leq w \leq 0$. The limiting cases of -1 and 0 correspond to the cosmological constant Λ and matter m respectively. For $w > -1$, the Q component could cluster (e.g. Frieman et al. 1995; Ferreira & Joyce 1998; Caldwell et al. 1997), which would affect the distortion parameter β_f. We consider the simple case of no Q-clustering here (see e.g. Turner & White 1997).

Assuming any set of parameters different from that of the underlying input model (OCDM) will result in predicted cross-spectra different from the "observed" ones. The fractional errors, for a set of 5 cosmological models, are shown in Fig. 1.

In the case where the actual input cosmology is used, the cross-spectra are quite accurately predicted. It is not perfect, especially at smaller scales, because the inversion procedure in the first step of our version of the AP test is inherently approximate by avoiding the modeling of small scale distortions (see Hui 1998a). However, in cases where the "wrong" model-cosmology is assumed, the predicted cross-spectra are systematically different from the "observed" or input cross-spectra.

The effect is most pronounced when Q is present. The canonical example is the cosmological constant with $w = -1$, which gives the strongest departure, among the five models, of the predicted cross-spectra from those of the input cosmology, with differences as large as 200% at sufficiently large $k_{||}$'s. Increasing w while keeping Ω_Q fixed (the $w = -1/3$ model), or decreasing Ω_Q while keeping w fixed (adding in mixture of curvature, as in the somewhat perverse model of $\Omega_m = 0.2$, $\Omega_k = 0.4$ & $\Omega_\Lambda = 0.4$) tends to bring the predictions closer to that of the input open universe, but still with substantial differences. The $\Omega_m = 1$ critical matter density universe is closest to the input model (that is, aside from the input model itself), with a difference of about $10 - 20\%$.

As we discuss before, the cosmological model influences the prediction of the cross-spectra through two parameters: f_Ω (or β_f; see eq. [7]) of the distortion kernel, and $u_\perp(\theta)$ (eq. [1]) of the velocity-distance relation. To understand approximately the size of the effect we are measuring, it is simplest to ignore redshift-distortion first, and consider the influence of the second parameter alone.

Putting $W = 1$ and assuming a power-law $\tilde{P}^f(k) = Bk^n$ in eq. (10), it can be shown that (Abramowitz & Stegun 1964)

$$P^f_\times(k_{||},\theta) = \frac{B}{2\pi}\left[2k_{||}/u_\perp\right]^{1+\frac{n}{2}}\, K_{1+\frac{n}{2}}(k_{||}u_\perp)\,/\,\Gamma(-n/2) \tag{11}$$

where K_ν is the ν-th order modified Bessel function and Γ is the gamma function. $K_\nu(x)$ has an asymptote of $\sqrt{\pi/(2x)}\exp[-x]$ in the large-x limit. (For $\nu = \pm 1/2$ this is exact.) For the scales of interest, n should vary somewhere between -2 and -3. For sufficiently large $k_{||}$ (i.e. $k_{||}u_\perp \gtrsim \pi$), the $u_\perp$ and $k_{||}$ dependence of the cross-spectrum is given by: for $n = -2$, $P^f_\times(k_{||},\theta) \propto \exp[-k_{||}u_\perp]/\sqrt{k_{||}u_\perp}$, while for $n = -3$, $P^f_\times(k_{||},\theta) \propto \exp[-k_{||}u_\perp]/k_{||}$.

This exponential dependence on $u_\perp(\theta)$ explains the large differences between different models at the smaller scales. (Of course, the exponential suppression of power at high $k_{||}$'s also means the cross-spectrum is going to be hard to measure at such scales. See §4.) We show in Fig. 2 two curves depicting how $u_\perp/\theta$ varies with w and with Ω_k. It can be seen that pure matter-plus-curvature models (with no Q) generally have a higher $u_\perp/\theta$ than Q-dominated models. Combining such information with Fig. 1, we can say that the cross-spectra of models with a significant Q component are generally 50% higher, or more, than those of no-Q-models, at $k_{||}u_\perp \gtrsim \pi$. This can be understood using our analytic formula in eq. (11), assuming $n \sim -2$ to -3.

Put in another way, given an observed auto-spectrum, the inferred correlation length in physical distance scale is larger in a universe dominated by Q compared to a no-Q-universe; ignoring redshift-space distortions, this implies a larger cross-correlation length, and manifests itself in a stronger cross-correlation for the Q-dominated model. That is what we see in Fig. 1.

A good illustration of redshift-space anisotropy can be seen by a comparison of the $\Omega_m = 1$ model and the open model with $\Omega_m = 0.3$ & $\Omega_k = 0.7$. According to the dotted line of Fig. 2, the two have very similar $u_\perp$'s with the latter's a little smaller. This means, with no redshift distortions, the latter should have a cross-spectrum close to, but above, that of the $\Omega_m = 1$ model. Exactly the opposite is observed in Fig. 1. This is because peculiar motion induces a stronger anisotropy of the correlation function in the $\Omega_m = 1$ model, in the sense of a greater squashing of the correlation length along the LOS (see eq. [8]). The end-result is a stronger cross-correlation of the critical-matter-density model over the open model. The effect is not large. It shifts the cross-spectrum of the $\Omega_m = 1$ model relative to that of the $\Omega_m = 0.3$ - $\Omega_k = 0.7$ model by about 10%. A similar conclusion holds for most other models in that redshift-space distortions change their fractional differences by about $10 - 20\%$. For reference, the values of f_Ω (eq. [7]) for the 5 models in Fig. 1 are 0.98, 0.77, 0.78, 1.0 and 0.77 from top to bottom .

It can also be seen from our hypothetical AP test that besides constraining the absence or presence of Ω_Q, the predicted cross-spectra can also be used to discriminate between different equations of state in cases where Ω_Q is known. Unfortunately, according to Fig. 2, the quantity $u_\perp/\theta$ takes rather similar values for $w \lesssim -1/3$, which are also values that seem to be allowed by current supernova observations (Garnavich et al. 1998). The test does have good discriminating power among models with larger w's, however.

The distortion parameter β_f depends on, in addition to the various Ω's, the temperature-density-relation index γ (eq.[7]), which while not known precisely, has been shown by Hui & Gnedin (1997) to generally lie in the range $1.3 \lesssim \gamma \lesssim 1.6$. Varying γ within this range changes the cross-spectra by $\lesssim 10\%$.

4. Discussion

At this point, a natural question to ask is: how well can we measure the cross-spectrum? And given the expected signal-to-noise of such a measurement, what is the expected discriminating power among different cosmologies?

The expected variance in the measured cross-spectrum for a single $k_{||}$-mode is approximately given by $(P^f_{||})^2 + (P^f_\times)^2$ (see Hui 1998b). This is obtained by ignoring shot-noise, which is likely to be accurate because the Poisson-variance is known to be sub-dominant compared to sample-variance (at least for the brighter quasars, see e.g. Croft et al. 1998), and assuming Gaussianity, which is probably a good approximation by the central-limit-theorem, especially on large scales (Hui 1998b). Since $P^f_{||} \gg P^f_\times$ in general, we can approximate the signal-to-noise for each wave-mode of the measured cross-spectrum by $P^f_\times/P^f_{||}$.

On the other hand, our calculation in the last section tells us what $\delta P^f_\times/P^f_\times$ (Fig. 1) is between any pair of cosmological models. Let us denote these two models by A and B. As argued in §3.2, the dominant contribution to the difference in $P^f_\times$'s can be estimated by ignoring redshift-space distortions. In that case, the difference in the cross-spectra is due entirely to the difference in the transverse velocity-separations (eq. [1]), let us call them $u^A_\perp$ and $u^B_\perp$ respectively. Assuming $n = -3$, eq. (11) & (9) allows us to compute both $\delta P^f_\times/P^f_\times$ and $P^f_\times/P^f_{||}$, from which we can deduce the following: one can rule out model B, if A is the true model, with a σ-level or signal-to-noise (S/N) of

$$\frac{S}{N} = \sqrt{\sum_{k_{||}} \left[\exp[-k_{||}(u_\perp^B - u_\perp^A)] - 1\right]^2 \exp[-2k_{||}u_\perp^A]} \tag{12}$$

where the sum is over all $k_{||}$ modes for which the AP test can be applied. From Fig. 1, we will take $0.002 \leq k_{||} \leq 0.02\,\mathrm{s/km}$, where the lower limit is set by the scale at which the slowly-fluctuating continuum would contaminate the signal, and the upper limit is set by the scale at which nonlinear distortions would start to become important (see Hui 1998a). One can replace the summation by an integration: $\sum_{k_{||}} \rightarrow (L/\pi)\int dk_{||}$ where L is the length of the absorption spectrum available. Assuming full coverage between Lyα and Lyβ, $L \sim 50000\,\mathrm{km/s}$.

Hence, taking model A to be the $\Omega_m = 0.3$ - $\Omega_k = 0.7$ universe and model B to be the $\Omega_m = 0.3$ - $\Omega_\Lambda = 0.7$ universe, we find that, at $\bar{z} = 3$, the expected S/N is 0.8 and 0.6 respectively for angular separations of $1'$ and $2'$. It turns out S/N peaks at about $0.5'$, reaching 1, but drops off at smaller θ's. (The S/N estimate above should be modified for sufficiently small θ's because $P_{||}^f \gg P_\times^f$ no longer holds. Also, it changes somewhat with $\bar{z}$, n and the $k_{||}$'s we include, but it provides a good rough estimate for a reasonable range of parameters.) There are two competing effects: a larger θ gives a larger difference between models (Fig. 1), but also a smaller $P_\times^f$, hence harder to measure. Note that this is for only one pair of quasar spectra. To reach a $4-\sigma$ level discrimination, something like 25 pairs, at angular separations $0.5' - 2'$, would be required. Note also that in the above estimate, we have ignored the error in the prediction of the cross-spectrum from the observed auto-spectrum. This error is likely to be sub-dominant, particularly because a very large number of LOS is in principle available for measuring the auto-spectrum. There are roughly 10 pairs of quasars with existing spectra at the above angular separations, or slightly larger, for $\bar{z} > 1$ (see e.g. Crotts & Fang 1998 & ref. therein). Upcoming surveys such as the AAT 2dF and SDSS are expected to increase this number by at least an order of magnitude. Some of these will be faint quasars for which shot-noise might be important.

A few issues should be further explored in the application of the AP test. First, it is obvious from the above analysis that we could boost the S/N for the smaller angular separations by extending to higher $k_{||}$'s. This requires, however, the modeling of nonlinear redshift distortions. The finger-of-God on small scales would lower our predicted cross-spectrum. The exponential factor in eq. (6) could be used for modeling this (e.g. Peacock & Dodds 1994), but it probably depends on more than simply the Ω's we are interested in. Moreover, our large-scale linear distortion kernel (eq. [8]) should be checked against simulations for accuracy. It is known in the case of galaxy-surveys, for instance, that the linear prediction for redshift distortions could be modified even on very large scales. Furthermore, the effective bias factor in the distortion kernel ($2 - 0.7(\gamma - 1)$ in β_f; eq. [7]) might also deviate from the linear prediction (Hui 1998a).

Second, we have focused on a particular version of the AP test here, in which we construct a predicted cross-spectrum based on the observed auto-spectrum. In practice, one should test the whole inversion procedure from the auto-spectrum to the cross-spectrum with simulated noisy data, to guard against possible instabilities. An alternative would be, for instance, to parameterize $\tilde{P}^f(k)$ (eq. [9] & [10]) by an amplitude and a slope, and then fit simultaneously these parameters and the various Ω's to match the observed auto- and cross-spectra. This could lead to smaller error-bars for the measured Ω's by restricting the form of $\tilde{P}^f(k)$.

Lastly, we have chosen to focus on a subset of cosmological parameters which are deemed to be currently popular (Fig. 2). However, there exist cosmological models which are less conventional, but nonetheless not necessarily ruled out by current observations, such as a universe closed by a large Ω_Q (e.g. Kamionkowski & Toumbas 1996). Such models could predict a cross-spectrum so strong that even one pair of quasar spectra would be sufficient to rule them out, that is, if they do not describe the actual universe. We will pursue this and other observational issues in a separate paper.

As this work was nearing completion, we became aware of efforts by several groups who considered similar ideas (Croft 1998; McDonald & Miralda-Escude 1998; Seljak 1998). This work was supported by the DOE and the NASA grant NAG 5-7092 at Fermilab.

References

Abramowitz, M. & Stegun, I. A., 1964, *Handbook of Mathematical Functions*, Dover Publications, Inc.

Alcock, C. & Paczynński, B., 1979, Nature **281**, 358

Ballinger, W. E., Peacock, J. A., & Heavens, A. F., 1996, MNRAS **282**, 877+

Caldwell, R. R., Dave, R., & Steinhardt, P. J., 1997, preprint, astroph-9708069

Croft, R. A. C., 1998, private communication

Croft, R. A. C., Weinberg, D. H., Katz, N., & Hernquist, L., 1998, ApJ **495**, 44+

Crotts, A. P. S. & Fang, Y. 1998, ApJ **502**, 16

De Laix, A. A. & Starkman, G., 1998, ApJ **501**, 427+

Ferreira, P. G., & Joyce, M., 1998, Phys. Rev., **D58**, 23503

Frieman, J. A., Hill, C. T., Stebbins, A., & Waga, I., 1995, Phys. Rev. Lett., **75**, 2077

Garnavich, P. M. et al., 1998, preprint, astroph-9806396

Hui, L., 1998a, preprint, astro-ph 9707068

Hui, L., 1998b, in preparation

Hui, L. & Gnedin, N. Y., 1997, MNRAS **292**, 27

Kaiser, N., 1987, MNRAS **227**, 1

Kaiser, N. & Peacock, J. A., 1991, ApJ **379**, 482

Kamionkowski, M. & Toumbas, N., 1996, Phys. Rev. Lett. **77**, 587

Matsubara, T. & Suto, Y., 1996, ApJ **470**, L1

McDonald, P. & Miralda-Escude, J., 1998, preprint

Peacock, J. A. & Dodds, S. J., 1994, MNRAS **267**, 1020+

Perlmutter, S. et al., 1997, Poster at American Astronomical Society Meeting **191**, 8504+

Popowski, P. A., Weinberg, D. H., Ryden, B. S., & Osmer, P. S., 1998, ApJ **498**, 11+

Riess, A. G. et al., 1998, preprint, astroph-9805201

Ryden, B. S., 1995, ApJ **452**, 25+

Seljak, U., 1998, private communication

Turner, M. S. & White, M., 1997, Phys. Rev. **D56**, 4439

This preprint was prepared with the AAS LaTeX macros v4.0.

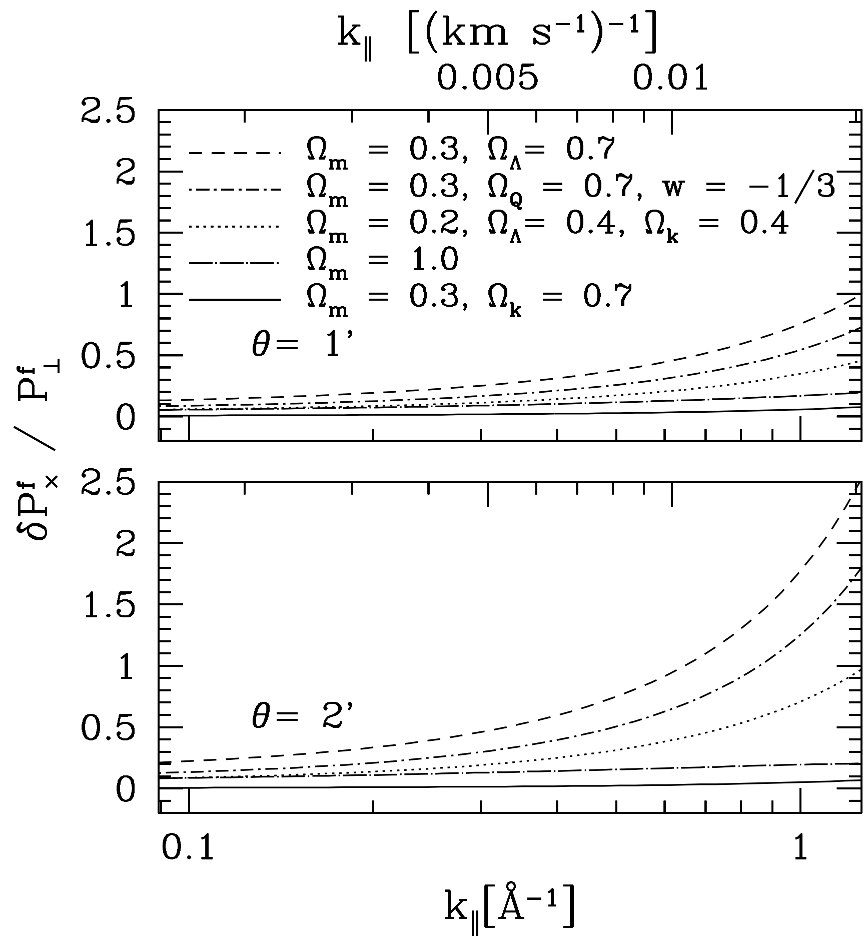

Fig. 1.— The fractional error in the predicted cross-spectra (each panel for the cross-spectrum of the given angular separation) for 5 different assumed cosmologies, labeled according to the order of the curves from top to bottom. The input cosmology coincides with the assumed model of the solid line. The fractional error is defined as $\delta P^f_\times / \left. P^f_\times \right|_{\rm input}$, where $\delta P^f_\times = P^f_\times - \left. P^f_\times \right|_{\rm input}$ and $P^f_\times$ is the predicted/output cross-spectrum. γ is assumed to be the same as the input value (see text).

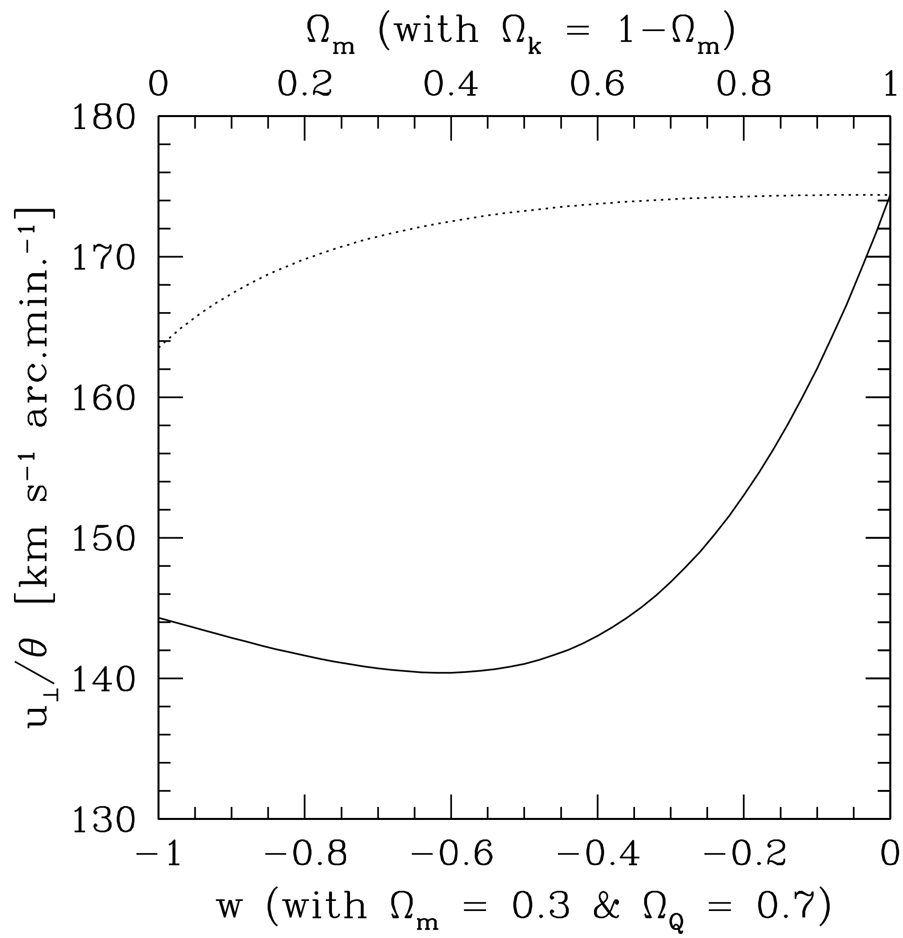

Fig. 2.— The velocity-angle ratio (eq. [1]) at a redshift of $\bar{z} = 3$ as a function of two parameters. The bottom solid curve shows $u_\perp/\theta$ as a function of w (bottom axis) for models in which $\Omega_m = 0.3$ & $\Omega_Q = 0.7$. The upper dotted curve shows $u_\perp/\theta$ as a function of Ω_m (top axis) for models with no Q, i.e. $\Omega_m + \Omega_k = 1$. Models with mixture of Ω_Q and Ω_k smaller than 0.7 generally have $u_\perp/\theta$ somewhere between the dotted and solid lines.