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Abstract:

I calculate the form factors describing semileptonic and penguin induced decays of B mesons into light pseudoscalar mesons. The form factors are calculated from QCD sum rules on the light-cone including contributions up to twist 4, radiative corrections to the leading twist contribution and SU(3) breaking effects. The theoretical uncertainty is estimated to be $\sim$ (10–15)%.

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1. Decays of B mesons into light mesons offer the possibility to access the less well known entries in the CKM quark mixing matrix like V_{ub} and V_{ts}. The measurement of rare penguin induced B decays may also give hints at new physics in the form of loop-induced effects. With new data of hitherto unknown precision from the new experimental facilities BaBar at SLAC and Belle at KEK expected to be available in the near future, the demands at the accuracy of theoretical predictions are ever increasing. The central problem of all such predictions, our failure to solve nonperturbative QCD, is well known and so far prevents a rigorous calculation of form factors from first principles. Theorists thus concentrate on providing various approximations. The maybe most prominent of these, simulations of QCD on the lattice, have experienced considerable progress over recent years; the current status for B decays is summarized in [1]. It seems, however, unlikely that lattice calculations will soon overcome their main restriction in describing $b \to u$ and $b \to s$ transitions, namely the effective upper cut-off that the finite lattice size imposes on the momentum of the final state meson. The cut-off restricts lattice predictions of B decay form factors to rather large momentum transfer q^2 of about $15\,\mathrm{GeV}^2$ or larger. The physical range in B decays, however, extends from 0 to about $20\,\mathrm{GeV}^2$, depending on the process; for radiative decays like $B \to K^*\gamma$ it is exactly $0\,\mathrm{GeV}^2$. Still, one may hope to extract from the lattice data some information on form factors in the full physical range, as their behaviour at large q^2 restricts the shape at small q^2 via the analytical properties of a properly chosen vacuum correlation function. The latter function, however, also contains poles and multi-particle cuts whose exact behaviour is not known, which limits the accuracy of bounds obtained from such unitarity constraints and until now has restricted their application to $B \to \pi$ transitions [2, 3]. The most optimistic overall theoretical uncertainty one may hope to obtain from this method is the one induced by the input lattice results at large q^2, which to date is around (15–20)% [4, 2]. A more model-dependent extension of the lattice form factors into the low q^2 region is discussed in [5].

An alternative approach to heavy-to-light transitions is offered by QCD sum rules on the light-cone. In contrast and complementary to lattice simulations, it is just the fact that the final state meson *does* have large energy and momentum of order $\sim m_B/2$ in a large portion of phase-space that is used as starting point (which restricts the method to not too large momentum transfer, to be quantified below). The key idea is to consider $b \to u$ and $b \to s$ transitions as hard exclusive QCD processes and to combine the well-developed description of such processes in terms of perturbative amplitudes and nonperturbative hadronic distribution amplitudes [6] (see also [7] for a nice introduction) with the method of QCD sum rules [8] to describe the decaying hadron. The idea of such "light-cone sum rules" was first formulated and carried out in [9] in a different context for the process $\Sigma \to p\gamma$, its first application to B decays was given in [10]. Subsequently, light-cone sum rules were considered for many B decay processes, see [11, 12] for reviews.[1] As light-cone sum rules

[1] There also exists an extended literature on a more "direct" extension of QCD sum rules to heavy-to-light transitions, which is based on three-point correlation functions, see e.g. [13]. The conceptional restrictions of these sum rules are discussed in Ref. [14]. They fail to give a viable description of form factors at small and moderate momentum transfer.

are based on the light-cone expansion of a correlation function, they can be systematically improved by including higher twist contributions and radiative corrections to perturbative amplitudes. The first calculation in [10] was done at tree-level and to leading twist 2 accuracy. In [15, 16], twist 3 and 4 contributions were included, and in [17], one-loop radiative corrections to the twist 2 contribution to the form factor f_+^π were calculated. In the present letter, I calculate all semileptonic and penguin $B \to \pi$ and $B \to K$ form factors including one-loop radiative corrections to the twist 2 contribution and using an updated version of the twist 2 distribution amplitude of the K meson.

2. Let me begin with defining the relevant quantities. The semileptonic form factors are defined as ($q = p_B - p$)

$$\langle P(p)|\bar{q}\gamma_\mu b|B(p_B)\rangle = f_+^P(q^2)\left\{(p_B+p)_\mu - \frac{m_B^2-m_P^2}{q^2}\,q_\mu\right\} + \frac{m_B^2-m_P^2}{q^2}\,F_0^P(q^2)\,q_\mu, \qquad (1)$$

where P stands for the pseudoscalar meson π or K and $q = u$ for the π and $q = s$ for the K. The penguin form factor is defined as

$$\langle P(p)|\bar{q}\sigma_{\mu\nu}q^\nu(1+\gamma_5)b|B(p_B)\rangle \equiv \langle P(p)|\bar{s}\sigma_{\mu\nu}q^\nu b|B(p_B)\rangle$$
$$= i\left\{(p_B+p)_\mu q^2 - q_\mu(m_B^2-m_K^2)\right\}\frac{f_T^P(q^2)}{m_B+m_K}. \qquad (2)$$

The physical range in q^2 is $0 \leq q^2 \leq (m_B - m_P)^2$. Although there are of course no semileptonic decays $B \to Ke\nu$, the above form factors contribute to e.g. $B \to K\ell\bar{\ell}$. Recalling the results of perturbative QCD for the π electromagnetic form factor as summarized in [7], one may suppose that the dominant contribution to the above form factors be the exchange of a hard perturbative gluon between e.g. the u quark and the antiquark, which possibility was advocated for instance in [18]. This is, however, not the case, and it was pointed out already in Ref. [10] that the dominant contribution comes from the so-called Feynman mechanism, where the quark created in the weak decay carries nearly all of the final state meson's momentum, while all other quarks are soft, and which bears no perturbative suppression by factors α_s/π. In an expansion in the inverse b quark mass, the contribution from the Feynman mechanism is of the same order as the gluon exchange contribution with momentum fraction of the quark of order $1-1/m_b$, but it dies off in the strict limit $m_b \to \infty$ due to Sudakov effects. This means that — unlike in the case of the electromagnetic π form factor — knowledge of the hadron distribution amplitudes

$$\phi(u,\mu^2) \sim \int_0^{\mu^2} dk_\perp^2\,\Psi(u,k_\perp),$$

where Ψ is the full Fock-state wave function of the B and $\pi(K)$, respectively, u is the longitudinal momentum fraction carried by the (b or $u(s)$) quark, $k_\perp$ is the transverse quark momentum, is not sufficient to calculate the form factors in the form of overlap integrals

$$F \sim \int_0^1 du\,dv\,\phi^*_{\pi(K)}(u)\,T_{\rm hard}(u,v;q^2)\,\phi_B(v)$$

(with $T_{\rm hard} \propto \alpha_s$).[2] Instead, in the method of light-cone sum rules, only the light meson is described by distribution amplitudes. Logarithms in $k_\perp$ are taken into account by the evolution of the distribution amplitudes under changes in scale, powers in $k_\perp$ are taken into account by higher twist distribution amplitudes. The B meson, on the other hand, is described like in QCD sum rules by the pseudoscalar current $\bar{d}i\gamma_5 b$ in the unphysical region with virtuality $p_B^2 - m_b^2 \sim O(m_b)$, where it can be treated perturbatively. The real B meson, residing on the physical cut at $p_B^2 = m_B^2$, is then traced by analytical continuation, supplemented by the standard QCD sum rule tools to enhance its contribution with respect to that of higher single- or multi-particle states coupling to the same current.

The starting point for the calculation of the form factors in (1) and (2) are thus the correlation functions ($j_B = \bar{d}i\gamma_5 b$):

$$\mathrm{CF}_V = i\int d^4y e^{iqy}\langle P(p)|T[\bar{q}\gamma_\mu b](y) j_B^\dagger(0)|0\rangle \;=\; \Pi_+^P(q+2p)_\mu + \Pi_-^P q_\mu, \tag{3}$$

$$\mathrm{CF}_T = i\int d^4y e^{iqy}\langle P(p)|T[\bar{q}\sigma_{\mu\nu}q^\nu b](y) j_B^\dagger(x)|0\rangle \;=\; 2iF_T^P(p_\mu q^2 - (pq)q_\mu), \tag{4}$$

which are calculated in an expansion around the light-cone $x^2 = 0$. The expansion goes in inverse powers of the b quark virtuality, which, in order for the light-cone expansion to be applicable, must be of order m_b. This restricts the accessible range in q^2 to $m_b^2 - q^2 \lesssim O(m_b)$ parametrically. For physical B mesons, I choose $m_b^2 - q^2 \leq 18\,\mathrm{GeV}^2$. Note also that for very large q^2 the influence of the next nearby pole (B^* for f_+^π) becomes more prominent.

It proves convenient to perform the calculation for an arbitrary weak vertex $\Gamma = \{\gamma_\mu, \sigma_{\mu\nu}q^\nu\}$, which, neglecting for the moment radiative corrections, yields:

$$\begin{aligned}
\mathrm{CF}_\Gamma = {} & \frac{f_\pi}{4}\int_0^1 du \Bigg[-\phi_\pi(u) Tr(\Gamma S_b(Q)\not{p}) \\
& + \frac{m_\pi^2}{m_u + m_d}\left\{ -\phi_P(u) Tr(\Gamma S_b(Q)) + \frac{i}{6}\,\phi_\sigma(u)\,\frac{\partial}{\partial Q_\alpha} Tr(\Gamma S_b(Q)\sigma_{\alpha\beta})p^\beta \right\} \\
& + \left\{ g_1(u) - \int_0^u dv\, g_2(v) \right\} \frac{\partial^2}{\partial Q_\alpha \partial Q^\alpha} Tr(\Gamma S_b(Q)\not{p}) - g_2(u)\,\frac{\partial}{\partial Q_\alpha} Tr(\Gamma S_b(Q)\gamma_\alpha) \Bigg] \\
& + \frac{f_\pi}{4}\int_0^1 dv \int_0^1 \mathcal{D}\underline{\alpha}\, \frac{1}{\bar{s}^2} \Bigg[\frac{4 f_{3\pi}}{f_\pi} v(pq)\phi_{3\pi}(\underline{\alpha}) Tr(\Gamma\not{p}) + (2\phi_\perp(\underline{\alpha}) - \phi_\parallel(\underline{\alpha})) Tr(\Gamma(\not{q} + m_b)\not{p}) \\
& + 2v\left\{ \phi_\parallel(\underline{\alpha}) Tr(\Gamma\not{p}\not{q}) - 2(pq)\phi_\perp(\underline{\alpha}) Tr(\Gamma) \right\} + \left\{ 2\tilde{\phi}_\perp(\underline{\alpha}) - \tilde{\phi}_\parallel(\underline{\alpha}) \right\} Tr(\Gamma(\not{q} + m_b)\not{p}) \\
& + 4iv\tilde{\phi}_\perp(\underline{\alpha}) Tr(\Gamma\sigma_{\alpha\beta}) q^\alpha p^\beta \Bigg] .
\end{aligned} \tag{5}$$

[2] Note also that not much is known about ϕ_B, whereas the analysis of light meson distribution amplitudes is facilitated by the fact that it can be organized in an expansion in conformal spin, much like the partial wave expansion of scattering amplitudes in quantum mechanics in rotational spin.

Explicit expressions for $\Pi_\pm$ and F_T were already obtained in [15, 16]. Here $Q = q + \bar{u}p$, $s = m_b^2 - Q^2 = m_b^2 - up_B^2 - \bar{u}q^2$, $\mathcal{D}\underline{\alpha} = d\alpha_1 d\alpha_2 d\alpha_3 \delta(1-\alpha_1-\alpha_2-\alpha_3)$ and $\widetilde{s} = m_b^2 - (q+(\alpha_1 + v\alpha_3)p)^2$. $S_b(Q) = (\widehat{Q} + m_b)/(-s)$ is the b quark propagator. In the above expression, $\phi_{\pi,K}$ is the leading twist 2 distribution amplitude, ϕ_P and ϕ_σ are the two-particle distribution amplitudes of twist 3, g_1 and g_2 of twist 4, all of which are defined in [19]. The twist 3 and 4 two-particle distribution amplitudes are determined completely in terms of the twist 3 and 4 three-particle distributions amplitudes $\phi_{3\pi}$, $\phi_{\|,\perp}$ and $\widetilde{\phi}_{\|,\perp}$ [19]. Note that in the above expression corrections in the light meson mass are neglected ($m_\pi^2/(m_u + m_d)$, however, is expressed in terms of the quark condensate and taken into account). Their inclusion, of potential relevance in $B \to K$ transitions, is not straightforward and requires an extension of the method developed in Ref. [19] to include meson and quark mass corrections in the twist 4 distribution amplitudes. According to [20], the numerical impact on the form factors is small, around 5% and most pronounced at large q^2.

3. It is convenient to calculate also the radiative corrections for arbitrary weak vertex. To twist 2 accuracy, the light quarks are massless and carry only longitudinal momentum. The one-loop calculation does not pose any particular technical complications, but results in bulky expressions which I refrain from quoting here. The general structure is, as to be expected, similar to that for the form factor f_+^π obtained in [17]. Full formulas will be presented in [21]. The separation of perturbative and nonperturbative contributions introduces an arbitrary logarithmic (infra-red) factorization scale. The condition that the correlation function be independent of that scale leads to an evolution equation for the distribution amplitude, which was first derived and solved in [6] to leading logarithmic accuracy. In the present context, with full $O(\alpha_s)$ corrections to the perturbative part included, one has to use the next-to-leading order evolution of the distribution amplitude, which was derived in closed form in [22]. A natural choice for the factorization scale is the virtuality of the b quark, $\mu_{\rm IR}^2 \sim u\, m_b$. For technical reasons it is, however, more convenient to choose a fixed scale like $\mu_{\rm IR}^2 = m_B^2 - m_b^2$, which is of the same order. The numerical impact of changing the scale is minimal.[3] The penguin form factor depends also on an ultra-violet scale, the renormalization-scale of the local operator $\bar{q}\sigma_{\mu\nu}q^\nu b$ appearing in the effective weak Lagrangian. A natural choice for this ultra-violet scale is $\mu_{\rm UV} = m_b$.

As for the size of radiative corrections, it turns out that they are dominated by the correction to the pseudoscalar B vertex, which, as discussed below, yields large cancellations against the corresponding corrections to the leptonic B decay constant f_B.

4. Let me now derive the light-cone sum rules. The correlation functions CF_Γ, calculated for unphysical p_B^2, can also be written as dispersion relations over the physical cut. Singling out the contribution of the B meson, one has e.g. for Π_+:

$$\mathrm{CF}_{\Pi_+} = \frac{m_B^2 f_B}{m_b}\, f_+(q^2)\, \frac{1}{m_B^2 - p_B^2} + \text{higher poles and cuts,} \tag{6}$$

[3]This is in contrast to the π electromagnetic form factor which is rather sensitive to the shape of the distribution amplitude near the end-points.

where f_B is the leptonic decay constant of the B meson, $f_B m_B^2 = m_b\langle B|j_B^\dagger|0\rangle$. In order to enhance the ground-state B contribution to the right-hand side, one performs a Borel-transformation,

$$\widehat{B}\,\frac{1}{s-p_B^2} = \frac{1}{M^2}\exp(-s/M^2) \tag{7}$$

with Borel parameter M^2. The next step is to invoke quark-hadron duality to approximate the contributions of hadrons other than the ground-state B meson, so that finally

$$\widehat{B}\,\mathrm{CF}_{\Pi_+} = \frac{1}{M^2}\,\frac{m_B^2 f_B}{m_b}\,f_+(q^2)\,e^{-m_B^2/M^2} + \frac{1}{M^2}\,\frac{1}{\pi}\int_{s_0}^{\infty} ds\,\mathrm{ImCF}_{\Pi_+}(s)\,\exp(-s/M^2). \tag{8}$$

This equation is the light-cone sum rule for f_+, and those for F_0 and f_T look similar. Here s_0 is the so-called continuum threshold, which separates the ground-state from the continuum contributions. s_0 and M^2 are in principle free parameters of the light-cone sum rules, but can be fixed by requiring stability of the sum rule under their change. In the present context, one can decrease their influence considerably by also writing f_B as QCD sum rule, depending on the same parameters s_0 and M^2. From the analysis of the latter sum rule, one finds $s_0 \approx 35\,\mathrm{GeV}^2$ and $M^2 \approx$(4–8) GeV^2. The resulting value for f_B is (150–200) MeV, in perfect agreement with the results from lattice simulations. This procedure makes the form factors largely independent of m_b, s_0 and M^2; the remaining dependence will be included in the error estimate. Note also that subtraction of the continuum contribution from both sides of (8) introduces a lower limit of integration $u \geq (m_b^2 - q^2)/(s_0 - q^2)$ in (5), which behaves as $1 - 1/m_b$ for large m_b and thus corresponds to the dynamical configuration of the Feynman mechanism.

Let me now specify the nonperturbative input. For the b quark I use the one-loop pole mass $m_b = (4.7 \pm 0.1)\,\mathrm{GeV}$, which is consistent with a recent determination from the Υ mesons [23]. For the light mesons, I need to specify the distribution amplitudes. Fortunately, conformal symmetry of massless QCD combined with the nonlocal string operators technique developed in [24], provides a very powerful tool to describe higher twist distribution amplitudes in a mutually consistent and most economic way (see [25] for a detailed discussion). The determination of the relevant nonperturbative parameters from QCD sum rules was pioneered in [26]. In [19], the twist 3 and 4 π distribution amplitudes were obtained including contributions up to conformal spin 11/2 in terms of 6 independent nonperturbative parameters whose values were determined from QCD sum rules. The leading twist 2 distribution amplitude, on the other hand, can be expanded in Gegenbauer polynomials $C_i^{3/2}$:

$$\phi_{\pi,K} = 6u(1-u)\left(1 + \sum_{i=1}^{\infty} a_i(\mu) C_i^{3/2}(2u-1)\right). \tag{9}$$

The Gegenbauer moments a_i renormalize multiplicatively. For π, all odd moments vanish because of the π's definite G-parity. Although a fixed order expansion

$$\phi_{\pi,K(n)} = 6u(1-u)\left(1 + \sum_{i=1}^{n} a_i(\mu) C_i^{3/2}(2u-1)\right)$$

in the heavily oscillatory Gegenbauer polynomials does yield oscillatory expressions, they should be understood in the sense of mathematical distributions which are to be convoluted with smooth functions. This effectively smoothes out the oscillations, so that e.g. in B decay form factors the contributions of higher Gegenbauer moments die off rapidly.

To be specific, I use the π distribution amplitude as obtained in [27] (see also [28]),

$$a_2^\pi(1\,\mathrm{GeV}) = 0.44, \quad a_4^\pi(1\,\mathrm{GeV}) = 0.25. \tag{10}$$

For the K, on the other hand, the nonzero value of the strange quark mass induces nonvanishing values of the odd moments. I use

$$a_1^K(1\,\mathrm{GeV}) = 0.17, \quad a_2^K(1\,\mathrm{GeV}) = 0.2, \tag{11}$$

where the first value was obtained in [26] and the second one comes from an analysis of the sum rule for the π in [27] with account for SU(3) breaking effects.

The results are displayed in Fig. 1. The form factor f_+^π coincides with the one obtained in [17]. I plot each form factor using the twist 2 distribution amplitudes as specified above and with and without $O(\alpha_s)$ corrections, and also using the asymptotic distribution amplitude $\phi_{\pi,(0)}$ and $\phi_{K,(1)}$ to illustrate the impact of nonasymptotic contributions. The plotted curves were obtained with $m_b = 4.7\,\mathrm{GeV}$, $s_0 = 35\,\mathrm{GeV}^2$ and $M^2 = 6\,\mathrm{GeV}^2$. The distribution amplitudes are evaluated at the scale $\mu^2 = m_B^2 - m_b^2$. Apparently, the net effect of radiative corrections on the form factors is rather small. This is due to an effect already observed in [17]: the radiative corrections to the QCD sum rule for f_B are rather large, which is due to the large vertex corrections to the pseudoscalar B vertex. In the radiative corrections to the light-cone sum rules, the same vertex appears with corrections of similar size, so that they cancel between left and right-hand side of (8), leaving a net effect of around 10%.

It is also interesting to note that the effect of nonasymptotic twist 2 distribution amplitudes is small to moderate in all cases and most pronounced at large q^2. For all form factors, the effect of three-particle twist 3 and 4 quark-gluon contributions (and their induced effects in the two-particle distribution amplitudes) are small ($\sim 5\%$), so that the considerable theoretical uncertainty of these terms does not play. This also shows that the expansion in contributions of increasing twist is under good control.

As is expected from the definition of F_0, which refers to a scalar current, it increases less sharply in q^2 than the other form factors. A good parametrization for the q^2 dependence can be given in terms of three parameters as

$$F(q^2) = \frac{F(0)}{1 - a_F\,(q^2/m_B^2) + b_F\,(q^2/m_B^2)^2}\,. \tag{12}$$

The parameters are given in Tab. 1 together with errors obtained from varying all input parameters within their respective allowed range. For comparison, I also give the results for f_+^π quoted in [12] and f_+^K obtained in [15], the latter one obtained in leading-logarithmic accuracy. The table confirms what can also be inferred from the figure, namely that f_+^π and f_T^π nearly coincide. Comparison with the K form factors shows that the main SU(3)

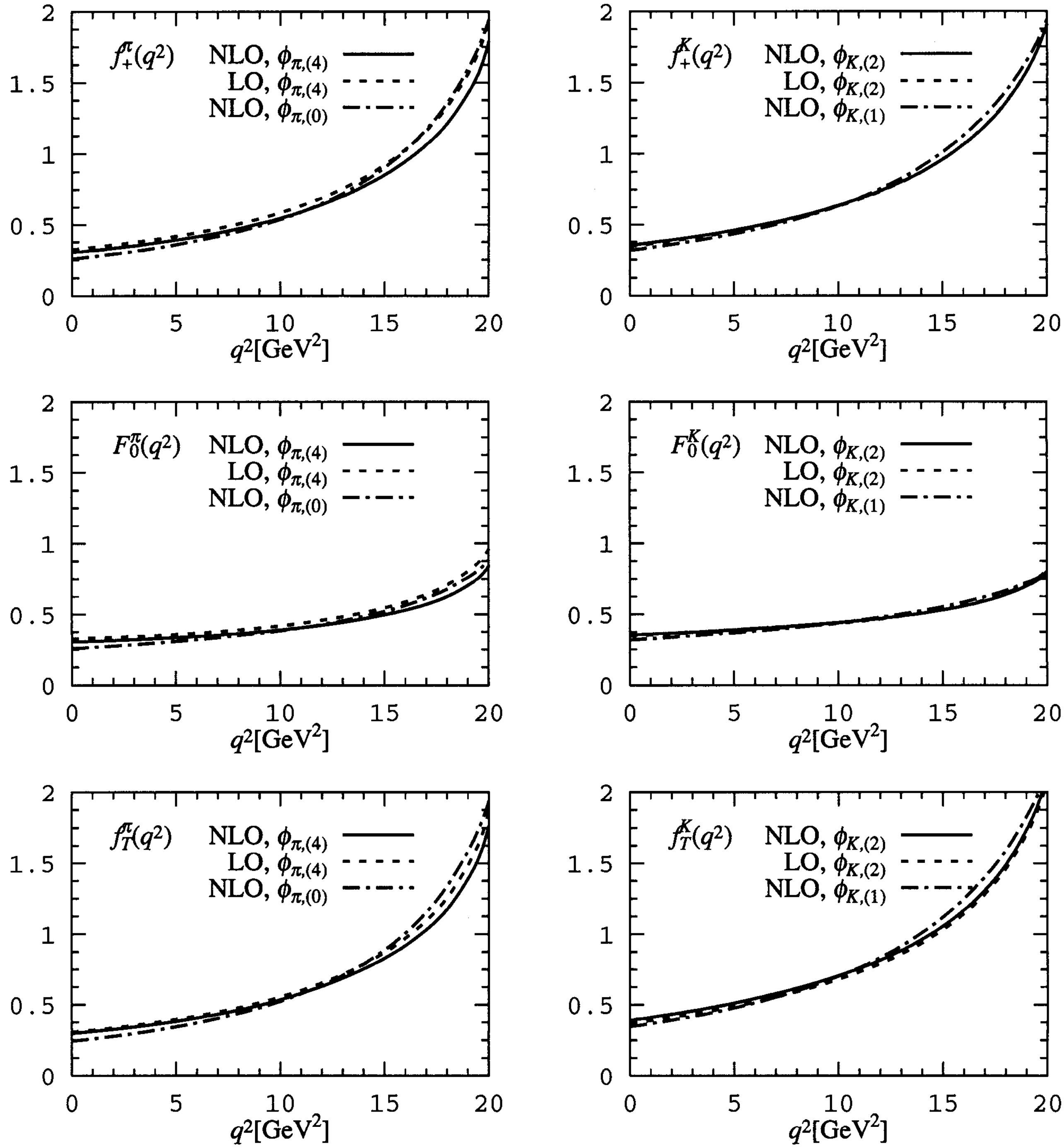

Figure 1: Form factors from light-cone sum rules in various approximations.

	f_+^π	f_+^K	F_0^π	F_0^K	$f_T^\pi(m_b)$	$f_T^K(m_b)$	f_+^π [12]	$f_+^{K,LO}$ [15]
$F(0)$	0.30	0.35	$\equiv f_+^\pi(0)$	$\equiv f_+^K(0)$	0.30	0.39	0.27	0.33
	± 0.04	± 0.05			± 0.04	± 0.05		
a_F	1.35	1.37	0.39	0.40	1.34	1.37	1.50	1.14
b_F	0.27	0.35	0.62	0.41	0.26	0.37	0.52	0.05

Table 1: Results for form factors in the parametrization of Eq. (12). Renormalization scale for f_T is $\mu = m_b$.

q^2	$f^\pi_{+,\rm latt}(q^2)$[4, 2]	$f^\pi_{+,\rm LCSR}(q^2)$	$F^\pi_{0,\rm latt}(q^2)$[4, 2]	$F^\pi_{0,\rm LCSR}(q^2)$
$14.9\,{\rm GeV}^2$	0.85 ± 0.20	0.85 ± 0.15	0.46 ± 0.10	0.5 ± 0.1
$17.2\,{\rm GeV}^2$	1.10 ± 0.27	1.1 ± 0.2	0.49 ± 0.10	0.60 ± 0.15
$20.0\,{\rm GeV}^2$	1.72 ± 0.50	1.8	0.56 ± 0.12	0.8

Table 2: Comparison of lattice results for $B \to \pi$ form factors with results from light-cone sum rules. The errors for lattice results are those quoted in [2].

breaking effect is in the normalization $F(0)$, whereas the q^2 dependence is only slightly modified. This can be understood from the fact that the formation of a π or K meson is proportional to their respective decay constants $f_{\pi,K}$, so that one would naively expect and enhancement $\sim f_K/f_\pi = 1.2$ of the K form factors (at least if the three-parton states are not important), which is essentially what I find.

A comparison with lattice results from the UKQCD collaboration is given in Tab. 2. The agreement with the lattice data is excellent as it was already found for $B \to \rho$ form factors in [14, 29]. The LCSR point at $q^2 = 20\,{\rm GeV}^2$ is just for illustration, because of which I also refrain from assigning it an error.

5. Summarizing, I have calculated the semileptonic and penguin form factors of $B \to \pi$ and $B \to K$ transitions from light-cone sum rules. A new feature was the inclusion of one-loop radiative corrections to the leading twist contributions. The results are summarized in Fig. 1 and Tab. 1. The impact of radiative corrections and higher twist contributions is small, so that the achievable accuracy is limited by the inherent systematic uncertainty of light-cone sum rules, which is associated with the extraction of the B meson ground-state contribution out of the continuum of states coupling to the same current. This uncertainty is estimated to be $\sim 10\%$ and of the same size as the uncertainty induced by the input parameters in the sum rule. Hence, further refinement of the calculation by including higher order terms or two-loop radiative corrections is not expected to yield higher accuracy of the result. It would, however, be useful to have an independent determination of the few lowest moments of the twist 2 π and K meson distribution amplitudes from lattice simulations. The existing results [30] have large uncertainties, and in view of the recent improvements of the methods of lattice QCD and the availability of much more powerful computers, it seems feasible to obtain more accurate results. The application of these results would not be restricted to B meson decays, but also be of direct relevance for the description of other hard exclusive processes, for instance single meson production at HERA.

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