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Carl H. Albright, K.S. Babu and S.M. Barr

Fermi National Accelerator Laboratory
P.O. Box 500, Batavia, Illinois 60510

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A Minimality Condition and Atmospheric Neutrino Oscillations*†

Carl H. Albright[1]
Department of Physics, Northern Illinois University, DeKalb, IL 60115
and
Fermi National Accelerator Laboratory, P.O. Box 500, Batavia, IL 60510

K.S. Babu[2]
School of Natural Sciences, Institute for Advanced Study, Princeton, NJ 08540

S.M. Barr[3]
Bartol Research Institute, University of Delaware, Newark, DE 19716

Abstract

A structure is proposed for the mass matrices of the quarks and leptons that arises in a natural way from the assumption that the breaking of $SO(10)$ gauge symmetry is achieved by the smallest possible set of vacuum expectation values. This structure explains well many features of the observed spectrum of quarks and leptons. It reproduces the Georgi-Jarlskog mass relations and postdicts the charm quark mass in reasonable agreement with data. It also predicts a large mixing angle between ν_μ and ν_τ, as suggested by atmospheric neutrino data. The mixing angles of the electron neutrino are predicted to be small.

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†Email: [1]albright@fnal.gov, [2]babu@ias.edu, [3]smbarr@bartol.udel.edu

In this Letter we propose a structure for the quark and lepton mass matrices that arises naturally in supersymmetric $SO(10)$ from the simple assumption that $SO(10)$ *is broken to the Standard Model by the smallest possible set of vacuum expectation values* (VEVs). This structure reproduces many of the features of the known fermion mass spectrum. It also predicts a large value for the $\nu_\mu - \nu_\tau$ mixing angle, as is suggested by the atmospheric neutrino data [1]. Usually this angle is small (or not predicted) in grand unified theories, but in the present model its large value has a simple group-theoretical explanation.

The smallest set of vacuum expectation values that can break $SO(10)$ to the Standard Model consists of one adjoint (**45**) and one spinor (**16**) [2]. The spinor plays two necessary roles: it breaks the rank of the group from 5 to 4, and provides superlarge masses for the right-handed neutrinos. The adjoint also plays two roles: it completes the breaking of $SO(10)$ to the Standard Model (SM) group $SU(3)\times SU(2)\times U(1)$ and produces without fine-tuning the "doublet-triplet splitting" — that is, gives superlarge mass to the color-triplet partners of the SM Higgs doublets, while leaving those doublets light.

Our assumption of minimality requires that there is only **one** adjoint Higgs. It has recently been shown that this is enough to break $SO(10)$ with no fine-tuning, while preserving gauge-coupling unification [3]. Besides its economy, the postulate of having only one adjoint seems to be desirable in the context of perturbative heterotic string theory where there are limitations on multiple adjoints [4]. With only one adjoint, its VEV is fixed to be in the $B-L$ direction, as this is required by the Dimopoulos-Wilczek mechanism for doublet-triplet splitting [5,3]. The superlarge VEV of the spinor is, of course, also fixed: it must point in the $SU(5)$ singlet direction. With $SO(10)$ broken to the SM group by only these two definite VEVs, the possibilities for constructing realistic quark and lepton masses are quite constrained. This should be contrasted with other approaches that generate mass matrix textures in $SO(10)$ utilizing extended Higgs sector [6].

In "minimal $SO(10)$" the quark and lepton masses come from the operators $\mathbf{16}_i\mathbf{16}_j\mathbf{10}_H$, where i and j are family indices, and subscript H denotes a Higgs field. This leads to the "naive $SO(10)$ relations": $N = U \propto D = L$, with all these matrices being symmetric. (U,

D, L, and N denote, respectively, the mass matrices for the up quarks, down quarks, and charged leptons, and the Dirac mass matrix of the neutrinos.)

These relations, as is well known, lead to bad predictions: $U \propto D$ gives vanishing Cabibbo-Kobayashi-Maskawa angles and the relation $m_c^0/m_t^0 = m_s^0/m_b^0$, which is off by about an order of magnitude. (Superscript zero refers to parameters evaluated at the unification scale.) One way that $U \propto D$ can be avoided is by the quark and lepton mass matrices depending on $\langle \mathbf{16}_H \rangle$, which breaks $SO(10)$. ($\langle \mathbf{45}_H \rangle$ does not help here, as up and down quarks have the same $B-L$.) However, $\langle \mathbf{16}_H \rangle$ by itself leaves $SU(5)$ unbroken, which would still imply the "naive $SU(5)$ relation" $D = L^T$. This contains both the good prediction $m_b^0 = m_\tau^0$, and the bad predictions $m_s^0 = m_\mu^0$, and $m_d^0 = m_e^0$. Therefore, the quark and lepton mass matrices must also depend directly or indirectly on $\langle \mathbf{45}_H \rangle$, which is the only $SU(5)$-breaking VEV. Empirically, one finds the so-called Georgi-Jarlskog relations [7], $m_s^0 \cong m_\mu^0/3$ and $m_d^0 \cong 3m_e^0$. Since $\langle \mathbf{45}_H \rangle \propto B-L$, a natural explanation of the Georgi-Jarlskog factors of 3 and 1/3 is possible, as will be shown.

The assumption of minimal VEVs for $SO(10)$-breaking leads naturally, as will be seen, to the following forms for the quark and lepton mass matrices at the unification scale (with the convention that the left-handed fermions multiply them from the right, and the left-handed antifermions from the left):

$$U^0 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & \epsilon/3 \\ 0 & -\epsilon/3 & 1 \end{pmatrix} m_U, \qquad N^0 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -\epsilon \\ 0 & \epsilon & 1 \end{pmatrix} m_U,$$

$$D^0 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & \rho+\epsilon/3 \\ 0 & -\epsilon/3 & 1 \end{pmatrix} m_D, \quad L^0 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -\epsilon \\ 0 & \rho+\epsilon & 1 \end{pmatrix} m_D. \tag{1}$$

These matrices, since they leave u, d, and e^- massless, are obviously not the whole story. At the end of this Letter, we will discuss extending the model to include the first generation.

However, since $m_e \ll m_\mu$, $m_d \ll m_s$, and $m_u \ll m_c$, the effects of such first-generation physics should be quite small on the second and third generation parameters that we wish to fit. It turns out that with only two parameters, ϵ and ρ, one can get a good fit for five quantities that involve the second and third generations: m_c/m_t, m_s/m_b, m_μ/m_τ, m_b/m_τ, and V_{cb}. (The other mass ratio, m_b/m_t depends on an unknown ratio of VEVs.)

To give some insight into the structure of the matrices of Eq. (1), and why they arise naturally from the assumption of minimal VEVs, it will help to explain how they are built up logically, layer by layer, from the heaviest generation to the lightest. Because the third generation is by far the heaviest, and approximately satisfies the $SU(5)$ relation $m_b^0 = m_\tau^0$, we take the first layer to come from the simple term $\mathbf{16}_3\mathbf{16}_3\mathbf{10}_H$, giving the "1" entries in Eq. (1).

The second-generation masses, because of the Georgi-Jarlskog factors, must depend on $\langle \mathbf{45}_H \rangle$. The simplest choice is $\mathbf{16}_2\mathbf{16}_3\mathbf{10}_H\mathbf{45}_H$. This gives the "$\epsilon$" entries in Eq. (1), the factors of 1/3 just reflecting the fact that $\langle \mathbf{45}_H \rangle \propto B - L$ and that a quark has $B - L = 1/3$. It can be shown that $\langle \mathbf{45}_H \rangle \propto B - L$ also implies that this term contributes anti-symmetrically in flavor. (For this reason the terms $\mathbf{16}_2\mathbf{16}_2\mathbf{10}_H\mathbf{45}_H$ and $\mathbf{16}_3\mathbf{16}_3\mathbf{10}_H\mathbf{45}_H$ would not contribute.)

The matrices with only the "1" and "ϵ" entries, but without "ρ", would still not be realistic: the matrices U and D would be proportional, giving $V_{cb} = 0$ and $m_c^0/m_t^0 = m_s^0/m_b^0$, and the Georgi-Jarlskog factor would be 9 instead of 3. (The "see-saw" formula would give $m_\mu^0/m_\tau^0 \cong \epsilon^2$, and $m_s^0/m_b^0 \cong \epsilon^2/9$.)

It turns out that all three of these unrealistic features are cured in a single stroke by introducing a third layer that involves $\langle \mathbf{16}_H \rangle$. The simplest term, group theoretically, that can be written down is of the form $\mathbf{16}_2\mathbf{16}_3\mathbf{16}_H\mathbf{16}'_H$. $\mathbf{16}'_H$ is some spinor Higgs, distinct from $\mathbf{16}_H$, which breaks the electroweak symmetry but does *not* participate in the breaking of $SO(10)$ down to the Standard Model group [8]. (That is, the components that get VEVs are $\mathbf{1}(\mathbf{16}_H)$, and $\overline{\mathbf{5}}(\mathbf{16}'_H)$, where $\mathbf{p}(\mathbf{q})$ denotes a $\mathbf{p}$ of $SU(5)$ contained in a $\mathbf{q}$ of $SO(10)$.)

This term arises most naturally from "integrating out" $\mathbf{10}$'s of $SO(10)$, as shown in

Figure 1. The resulting operator is $\overline{\mathbf{5}}(\mathbf{16}_2)\mathbf{10}(\mathbf{16}_3)\langle\overline{\mathbf{5}}(\mathbf{16}'_H)\rangle\langle\mathbf{1}(\mathbf{16}_H)\rangle$. Note that this contributes to L and D, but not to U and N, and that it lopsidedly contributes to D_{23} and L_{32} but not to D_{32} and L_{23}. This is the origin of the "ρ" entries in Eq. (1). This lopsidedness, which has a group-theoretical origin, explains, as will be seen, why the 2-3 mixing is small for the quarks ($V_{cb} \ll 1$) but large for the leptons ($\sin^2 2\theta_{\mu\tau} \sim 1$).

There can be a relative phase, which we will call α, between the parameters ϵ and ρ. As is apparent from Eq. (1), this phase only enters at order ϵ/ρ, which will presently be seen to be a small parameter. (Henceforth the symbols ρ and ϵ will denote $|\rho|$ and $|\epsilon|$, and the phase will appear explicitly as α.) Diagonalizing the matrices in Eq. (1), one finds:

$$\begin{aligned}
m_b^0/m_\tau^0 &\cong 1 - \tfrac{2}{3}\tfrac{\rho}{\rho^2+1}(\epsilon\cos\alpha), \\
m_\mu^0/m_\tau^0 &\cong \epsilon\tfrac{\rho}{\rho^2+1}\left(1 - \tfrac{\rho^2-1}{\rho(\rho^2+1)}(\epsilon\cos\alpha)\right), \\
m_s^0/m_b^0 &\cong \tfrac{1}{3}\epsilon\tfrac{\rho}{\rho^2+1}\left(1 - \tfrac{1}{3}\tfrac{\rho^2-1}{\rho(\rho^2+1)}(\epsilon\cos\alpha)\right), \\
m_c^0/m_t^0 &\cong \epsilon^2/9, \\
V_{cb}^0 &\cong \tfrac{1}{3}\epsilon\tfrac{\rho^2}{\rho^2+1}\left(1 + \tfrac{2}{3}\tfrac{1}{\rho(\rho^2+1)}(\epsilon\cos\alpha)\right).
\end{aligned} \tag{2}$$

In these expressions terms that are down by order $O(\epsilon^2)$ have been dropped. (They affect the results at the fraction of a percent level.) Because ϵ is a small parameter, the following features of the observed masses and mixings have been reproduced by the model: the approximate equality of m_b^0 and m_τ^0; the fact that V_{cb}^0, m_μ^0/m_τ^0, and m_s^0/m_b^0 are all comparable, because $O(\epsilon)$, while m_c^0/m_t^0 is very much smaller, because $O(\epsilon^2)$; and the fact that m_μ^0/m_τ^0 is about 3 times m_s^0/m_b^0 (one of the the Georgi-Jarlskog relations). Also explained is the hierarchy among generations, which arises from the smallness of ϵ and from the rank-2 nature of the matrices.

Since there are five observables in terms of the two parameters ϵ and ρ in Eq. (2), the model predicts three relations among charged fermions. To study them we use the following input parameters: $m_\mu = 105.66$ MeV, $m_\tau = 1.777$ GeV, $m_s(1 \text{ GeV}) = (180 \pm 50)$ MeV, $m_b(m_b) = (4.26 \pm 0.11)$ GeV, $m_c(m_c) = (1.27 \pm 0.1)$ GeV [9], $M_t = 174.1 \pm 5.4$ GeV and $V_{cb} = 0.0395 \pm 0.0017$ [10]. The value of M_t quoted above corresponds to the running masses $m_t(m_t) = 165 \pm 5 \; GeV$.

To fit the data, various renormalization factors are needed. The factors, that will be denoted by η_i, that run the masses from the low scales up to the supersymmetry scale, M_{SUSY} (taken to be at m_t) are computed using 3-loop QCD and 1-loop QED or electroweak renormalization group equations (RGE), with inputs $\alpha_s(M_Z) = 0.118$, $\alpha(M_Z) = 1/127.9$ and $\sin^2\theta_W(M_Z) = 0.2315$. The relevant RGE can be found for instance in [11]. The results are $(\eta_\mu, \eta_\tau, \eta_s, \eta_b, \eta_c, \eta_t) = (0.982, 0.984, 0.426, 0.654, 0.473, 1.0)$.

The renormalization factors from M_{SUSY} up to the unification scale, M_G, are calculated using the 2-loop MSSM beta functions for all parameters [11], with $M_G = 2 \times 10^{16}$ GeV, and all SUSY thresholds taken to be at M_{SUSY}. These factors also depend on the value of tanβ, which is allowed *a priori* (by the perturbativity of the Yukawa couplings up to M_G) to be anywhere in the range $1.5 \leq \tan\beta \leq 65$. However, as will be seen, within our scheme the fits constrain tanβ to be between 10 and 40. Results will be presented for a "central" value of 30, and where significant the dependence on tanβ will be discussed. (In this model, since the light doublet, H' is a linear combination of $\overline{\mathbf{5}}(\mathbf{10})$ and $\overline{\mathbf{5}}(\mathbf{16'})$, $\tan\beta \neq m_t/m_b$. It is also not expected to be very small, since the same Yukawa coupling contributes to both the top and bottom quark masses.) The running factors for tan$\beta = 30$ are $(\eta_{\mu/\tau}, \eta_{s/b}, \eta_{c/t}, \eta_{b/\tau}, \eta_{cb}) = (0.956, 0.840, 0.691, 0.514, 0.873)$, where $\eta_{i/j} \equiv (m_i^0/m_j^0)/(m_i/m_j)_{M_{SUSY}}$, and $\eta_{cb} \equiv V_{cb}^0/(V_{cb})_{M_{SUSY}}$.

Aside from the running of the couplings described by the η's, there are finite corrections [12] to m_s, m_b and V_{cb} from gluino and chargino loops, which are proportional to tanβ and thus sizable for moderate to large tanβ. These will be denoted by the factors $(1+\Delta_s)$, $(1+\Delta_b)$, and $(1+\Delta_{cb})$, which depend on the supersymmetric spectrum:

$\Delta_b \simeq \tan\beta\{\frac{2\alpha_3}{3\pi}\frac{\mu M_{\tilde{g}}}{m^2_{\tilde{b}_L}-m^2_{\tilde{b}_R}}[f(m^2_{\tilde{b}_L}/M^2_{\tilde{g}}) - f(m^2_{\tilde{b}_R}/M^2_{\tilde{g}})] + \frac{\lambda_t^2}{16\pi^2}\frac{\mu A_t}{m^2_{\tilde{t}_L}-m^2_{\tilde{t}_R}}[f(m^2_{\tilde{t}_L}/\mu^2) - f(m^2_{\tilde{t}_R}/\mu^2)]\}$, where $f(x) \equiv \ln x/(1-x)$. Δ_s is given by the same expression but without the chargino contribution (the second term) and with $\tilde{b} \to \tilde{s}$. $\Delta_{cb} = -\Delta_b^{\rm chargino}$. One sees that even for $\tan\beta \approx 10$, these corrections are of order 10%. The analogous corrections to m_μ and m_τ arise only from Bino loops, while those to m_c and m_t lack the $\tan\beta$ enhancement, and so these are all negligible.

To fit for ρ and ϵ it is convenient to use the second and fifth relations of Eq. (2), since there is very little experimental uncertainty in m_μ, m_τ, and V_{cb}. This gives $\rho = [3V_{cb}/(m_\mu/m_\tau)](\frac{\eta_\tau\eta_{cb}}{\eta_\mu\eta_{\mu/\tau}})\ (1 - \frac{\epsilon\cos\alpha}{3}\frac{3\rho^2-1}{\rho(\rho^2+1)})(1-\Delta_{cb})$, and $\epsilon = [\frac{\rho^2+1}{\rho}(m_\mu/m_\tau)](\frac{\eta_\mu\eta_{\mu/\tau}}{\eta_\tau})$ $(1+\epsilon\cos\alpha\frac{\rho^2-1}{\rho(\rho^2+1)})$. One finds, for $\cos\alpha = 1$, that

$$\rho = 1.73\ (1-\Delta_{cb}), \qquad \epsilon = 0.136\ (1-0.5\Delta_{cb}). \tag{3}$$

The dependence on $\cos\alpha$, arising only at order ϵ/ρ, is rather weak: for $\cos\alpha = -1$, $\rho = 1.92\ (1-\Delta_{cb})$ and $\epsilon = 0.134\ (1-0.5\Delta_{cb})$. The dependence on $\tan\beta$, because it is only through the renormalization factors, is also fairly weak for $10 \le \tan\beta \le 40$. For example, increasing $\tan\beta$ to 40 increases ρ by 0.7% and decreases ϵ by 3%. Similarly, changing M_{SUSY} from m_t to 500 GeV only increases ρ by 3% and increases ϵ by 2%. Henceforth, all results will be stated for $\tan\beta = 30$, $M_{SUSY} = m_t$, and $\cos\alpha = 1$. Whenever results are very sensitive to these parameters, the dependence on them will be explicitly discussed.

Now that ρ and ϵ have been determined from V_{cb} and m_μ/m_τ, there are four other quantities that can be predicted, namely m_b, m_s, m_c, and $\sin^2 2\theta_{\mu\tau}$.

(i) m_b prediction:

The first relation of Eq. (2) implies $m_b(m_b) = m_\tau(m_\tau)(\frac{\eta_\tau}{\eta_b\eta_{b/\tau}})(1 - \frac{2}{3}\frac{\rho}{\rho^2+1}\epsilon\cos\alpha)\ (1+\Delta_b)$. For $\cos\alpha = 1$, this gives $m_b(m_b) = 5.0\ (1+\Delta_b)$ GeV. Comparing this with the experimental value 4.26 ± 0.11 GeV, one sees that $\Delta_b \cong -0.15$. This is quite a reasonable value if $\tan\beta \approx 30$. (With supergravity boundary conditions and a generic sparticle spectrum, the gluino loops contribute $\sim \pm 0.2$ to Δ_b, while the charginos contribute roughly a quarter as much and with the opposite sign [13]. We shall keep these numbers as a rough guide to estimate the corrections.) It should be noted that if tanβ is close to 1.6 or near 60, $m_b(m_b)$ will be in the acceptable range even with $\Delta_b = 0$. However, these extreme values of tanβ lead to wrong predictions of the charm mass ($m_c(m_c) \simeq 1.57\ GeV$ when tan$\beta \simeq 1.6$) and are thus disfavored within the model. An interesting consequence is that the model predicts the sign of μ (and A_t) to be such that it decreases the b–quark mass through the gluino and chargino graphs.

(ii) m_s prediction:

The first and third relation of Eq.(2) yield $m_s(1\ GeV) = m_\tau(m_\tau)\frac{1}{3}\epsilon\frac{\rho}{\rho^2+1}(\frac{\eta_\tau}{\eta_s\eta_{s/b}\eta_{b/\tau}})\ (1 - \frac{1}{3}\epsilon\cos\alpha\frac{3\rho^2-1}{\rho(\rho^2+1)})(1+\Delta_s)$. For $\cos\alpha = 1$ this gives $m_s(1\ GeV) = 176\ (1+\Delta_s)$ MeV. Taking $\Delta_s \simeq \Delta_b \cong -0.15$, which is justified if the gluino contribution dominates and $\tilde{s}$ and $\tilde{b}$ are nearly degenerate, we find $m_s(1\ GeV) = 150$ MeV, in excellent agreement with the experimental value of 180 ± 50 MeV.

(iii) m_c prediction:

The fourth relation of Eq. (2) implies $m_c(m_c) = m_t(m_t)\frac{1}{9}\epsilon^2(\frac{\eta_t}{\eta_c\eta_{c/t}})$. For $\cos\alpha = 1$, this gives $m_c = (1.05 \pm 0.11)(1 - \Delta_{cb})$ GeV. The error reflects the 1σ uncertainties in the experimental values of m_t, α_s ($= 0.118 \pm 0.004$), and V_{cb}. (These lead, respectively, to 6.5%, 7%, and 4% uncertainties for $m_c(m_c)$. It should also be noted that changing $\tan\beta$ by ± 10 changes the m_c prediction by $\mp 4\%$, changing M_{SUSY} to 500 GeV has less than a 2% effect, and changing $\cos\alpha$ to 0 reduces m_c by 3%.) Since $\Delta_{cb} \simeq -\Delta_b|^{\text{chargino}}$, it is reasonable to take $\Delta_{cb} \simeq -0.05$, using the supergravity–like spectrum as a guideline. This gives $m_c = 1.10 \pm 0.11$ GeV. This is in quite reasonable agreement with the experimental value $m_c(m_c) = (1.27 \pm 0.1)$ GeV. It is interesting that the sign of the correction term Δ_{cb} suggested by the supergravity spectrum is such that it improves the agreement of $m_c(m_c)$ with the experimental value.

It should also be emphasized that, whereas the predictions for m_b and m_s were, in a sense, group-theoretically built into the forms given in Eq. (1), it could not be known in advance that the prediction for m_c would come close.

(iv) $\sin^2 2\theta_{\mu\tau}$ **prediction:**

The neutrino-mixing matrix U_ν is defined by $\nu_f = \sum_m (U_\nu)_{fm}\nu_m$, where, ν_f and ν_m are the flavor and mass eigenstates, respectively. $f = e, \mu, \tau$, and $m = 1, 2, 3$. $U_\nu = U^{(L)\dagger} U^{(N)}$, where $U^{(L)}$ and $U^{(N)}$ are the unitary transformations of the left-handed fermions required to diagonalize, respectively, L and $M_\nu = -N^T M_R^{-1} N$. (M_R is the superheavy Majorana mass matrix of the right-handed neutrinos.)

The crucial point, easily seen from an inspection of Eq. (1), is that a large rotation in the 2-3 plane will be required to diagonalize the charged-lepton mass matrix, L. Calling this rotation angle $\theta_{23}^{(L)}$, one has that $\tan\theta_{23}^{(L)} \cong |L_{32}/L_{33}| \cong \rho + \epsilon\cos\alpha$. The actual $\nu_\mu - \nu_\tau$ mixing angle is the difference between $\theta_{23}^{(L)}$ and the corresponding rotation angle, $\theta_{23}^{(N)}$, required to diagonalize M_ν.

It might appear that one can know nothing about M_ν, and therefore about $\theta_{23}^{(N)}$, without knowing the precise form of M_R. However, this is not the case. From Eq. (1) it is apparent that in the limit $\epsilon \longrightarrow 0$ both N and $M_\nu = -N^T M_R^{-1} N$ are proportional to diag$(0, 0, 1)$, so that $\theta_{23}^{(N)} \longrightarrow 0$. Thus, formally, $\theta_{23}^{(N)} = O(\epsilon)$. If M_R^{-1} is parametrized by $(M_R^{-1})_{22} = (M_R^{-1})_{33} Y/\epsilon^2$, and $(M_R^{-1})_{23} = (M_R^{-1})_{32} = (M_R^{-1})_{33} X/\epsilon$, one finds (ignoring the first generation) that $\tan 2\theta_{23}^{(N)} \cong 2\epsilon\,|(1-X)/(1-2X+Y)|$. Unless X and Y are fine-tuned, this is indeed of order ϵ. Let κ be defined by $\mathrm{Re}(U_{23}^{(N)}) = \kappa\epsilon\cos\alpha$, in a phase convention where $U_{23}^{(L)}$ is real. If it is required that $m_{\nu_\mu}/m_{\nu_\tau} \approx 0.05$, as suggested by the atmospheric and solar neutrino data, then $|\kappa| \lesssim 2$. The $\mu - \tau$ mixing angle at the unification scale is then given by

$$\tan\theta_{\mu\tau} = \frac{\rho + (1-\kappa)\epsilon\cos\alpha}{1 + \kappa\rho\epsilon\cos\alpha}. \tag{4}$$

The one-loop renormalization group equation for this quantity [14] has the simple form $d(\ln\tan\theta_{\mu\tau})/d(\ln\mu) = -h_\tau^2/16\pi^2$. Integrating yields the result that (for $\tan\beta = 30$) $\tan\theta_{\mu\tau}$ is increased by a factor of 1.03 in running down to the weak scale from the unification scale.

Unlike the quark masses, the $\nu_\mu - \nu_\tau$ mixing angle is very sensitive to $\cos\alpha$, and therefore $\sin^2 2\theta_{\mu\tau}$ can be in a large range, from 1 down to about 1/4. Values > 0.7 obtain for most of the parameter range. For example, if $\cos\alpha = 0$, Eq. (4) simplifies to $\tan\theta_{\mu\tau} = \rho$, giving

$\sin^2 2\theta_{\mu\tau} = 0.78$, independent of κ. If $\kappa = 0$, then $\sin^2 2\theta_{\mu\tau} > 0.7$ for all $\cos\alpha$. $\sin^2 2\theta_{\mu\tau}$ reaches 1 for $\cos\alpha = 1$ and $\kappa = 2$, and reaches $\approx 1/4$ for $\cos\alpha = 1$ and $\kappa = -2$. (For recent attempts to generate large $(\nu_\mu - \nu_\tau)$ mixing in other ways see Ref. [15].)

There is not a unique way to extend this model to include the first generation. A simple possibility that gives a reasonable fit to the first-generation masses and mixings is to add (12) and (21) entries symmetrically to all the mass matrices. This would give several new predictions: (i) $\frac{m_d^0}{m_e^0} = 3(1 + \frac{2}{3\rho}\epsilon\cos\alpha)$ (one of the Georgi–Jarlskog relations), (ii) $|V_{us}^0| = |\sqrt{\frac{m_d^0}{m_s^0}}\frac{1}{(\rho^2+1)^{1/4}} - \sqrt{\frac{m_u^0}{m_c^0}}e^{i\phi}|$, (iii) $|V_{ub}^0| \simeq |\sqrt{\frac{m_d^0}{m_s^0}}\frac{m_s^0}{m_b^0}\frac{\rho}{(\rho^2+1)^{1/4}} - \sqrt{\frac{m_u^0}{m_c^0}}e^{i\phi}(\sqrt{\frac{m_c^0}{m_t^0}} - \frac{m_s^0}{m_b^0}\frac{1}{\rho})|$. If the phase parameter ϕ is near π, acceptable $|V_{us}|$ and $|V_{ub}|$ result. The leptonic mixing angles involving the electron are given by $|(U_\nu)_{e2}^0| \simeq |\sqrt{\frac{m_e^0}{m_\mu^0}}(\rho^2+1)^{1/4} + O(\epsilon)|$, and $|(U_\nu)_{e3}^0| \simeq |\sqrt{\frac{m_e^0}{m_\mu^0}}\frac{m_\mu^0}{m_\tau^0}\frac{(\rho^2+1)^{3/4}}{\rho} + O(\epsilon^2)|$ where the $O(\epsilon)$ and $O(\epsilon^2)$ terms represent corrections from the neutrino sector. Since these mixing angles are both small, their precise values are sensitive to the structure of M_R. These values are consistent with the small angle MSW oscillations for the solar neutrinos.

The model presented here can be tested in future experiments in several ways. (i) The prediction of $\tan\beta = 10 - 40$ can be tested once supersymmetric particles are discovered. (ii) The spectrum of the sparticles is predicted to be such that the gluino and the chargino corrections to m_b decrease its value by about 15 %. (iii) More precise determinations of $m_t, \alpha_3(M_Z)$ and V_{cb} and information about the sparticle spectrum will sharpen the model's prediction of $m_c(m_c)$. (iv) Large angle $(\nu_\mu - \nu_\tau)$ oscillations should be seen in long baseline experiments, but not in the ongoing accelerator experiments. The interpretation of the atmospheric neutrino anomaly in terms of $(\nu_\mu - \nu_\tau)$ oscillations should be confirmed. (v) There are also specific predictions in the model for proton decay branching ratios [16] and rare decays such as $\mu \to e\gamma$ [17].

In this Letter we have studied a simple form for the mass matrices that is motivated by general group-theoretical considerations, without examining a particular underlying unified model in great detail. That has been done in [18], where it is found that fermion mass matrices of the type discussed here can arise in realistic models.

REFERENCES

[1] K.S. Hirata *et al.*, *Phys. Lett.* **205B**, 416 (1988); K.S. Hirata *et al.*, *Phys. Lett.* **280B**, 146 (1992); Y. Fukuda *et al.*, *Phys. Lett.* **335B**, 237 (1994); D. Caspar *et al.*, *Phys. Rev. Lett.* **66**, 2561 (1991); R. Becker-Szendy *et al.*, *Phys. Rev.* **D46**, 3720 (1992); *Nucl. Phys.* **B** (Proc. Suppl.) **38**, 331 (1995); T. Kafka, *Nucl. Phys.* **B** (Proc. Suppl.) **35**, 427 (1994); M. Goodman, *ibid.* **38**, 337 (1995); W.W.M. Allison *et al.*, *Phys. Lett.* **391B**, 491 (1997).

[2] In order to break the gauge symmetry at the unification scale while preserving supersymmetry, it is generally necessary in $SO(10)$ that for every **16** that has a superlarge VEV, there be a corresponding $\overline{\mathbf{16}}$ with an equal VEV.

[3] S.M. Barr and S. Raby, *Phys. Rev. Lett.* **79**, 4748 (1997).

[4] K. Dienes, *Nucl. Phys.* **B488**, 141 (1997); K. Dienes and J. March-Russell, *Nucl. Phys.* **B479**, 113 (1996); Z. Kakushadze, G. Shiu and S.H. Tye, hep-th/9710149; J. Erler, *Nucl. Phys.* **B475**, 597 (1996).

[5] S. Dimopoulos and F. Wilczek, Report No. NSF-ITP-82-07 (1981), in *The unity of fundamental interactions*, Proceedings of the 19th Course of the International School of Subnuclear Physics, Erice, Italy, 1981, ed. A. Zichichi (Plenum Press, New York, 1983); K.S. Babu and S.M. Barr, *Phys. Rev.* **D48**, 5354 (1993).

[6] G. Anderson et al, *Phys. Rev.* **D49**, 3660 (1994); A. Kusenko and R. Shrock, *Phys. Rev.* **D49**, 4962 (1994); K.S. Babu and R.N. Mohapatra, *Phys. Rev. Lett.* **74**, 2418 (1995); L. Hall and S. Raby, *Phys. Rev.* **D51**, 6524 (1995); K.S. Babu and S.M. Barr, *Phys. Rev. Lett.* **75**, 2088 (1995); C. Albright and S. Nandi, *Phys. Rev. Lett.* **73**, 930 (1994) and *Phys. Rev.* **D 53**, 2699 (1996); Z. Berezhiani, *Phys. Lett.* **B409**, 220 (1997); K.C. Chou and Y.L. Yu, hep-ph/9708201.

[7] H. Georgi and C. Jarlskog, *Phys. Lett.* **86B**, 297 (1979).

[8] If $\mathbf{16}_H$ and $\mathbf{16}'_H$ were not distinct, the operator would be $\mathbf{16}_2\mathbf{16}_3(\mathbf{16}_H)^2$. This would

lead to flavor-symmetric contributions to the mass matrices, which would yield unrealistic predictions.

[9] J. Gasser and H. Leutwyler, *Phys. Rept.* **87**, 77 (1982).

[10] *Table of Partilce Properties*, Particle Data Group, 1998 [http://pdg.lbl.gov/].

[11] H. Arason *et al.*, *Phys. Rev.* **D46**, 3495 (1992); V. Barger, M. Berger and P. Ohmann, *Phys. Rev.* **D47**, 1093 (1993) and *Phys. Rev.* **D47**, 2038 (1993).

[12] L. Hall, R. Rattazzi and U. Sarid, *Phys. Rev.* **D50**,7048 (1994).

[13] T. Blazek, M. Carena, S. Raby, C.E.M. Wagner, *Phys. Rev.* **D56**, 6919 (1997).

[14] K.S. Babu, C.N. Leung, and J. Pantaleone, *Phys. Lett.* **319B**, 191 (1993).

[15] B. Brahmachari and R.N. Mohapatra, hep-ph/9710371; J. Sato and T. Yanagida, hep-ph/9710516; M. Drees, S. Pakvasa, X. Tata, and T. ter Veldhuis, hep-ph/9712392; M. Bando, T. Kugo, and K. Yoshioka hep-ph/9710417.

[16] K.S. Babu and S.M. Barr, Phys. Lett. **B381**, 137 (1996); P. Nath, Phys. Rev. Lett. **76**, 2218 (1996); K.S. Babu, J.C. Pati and F. Wilczek, hep-ph/9712307.

[17] R. Barbieri, L. Hall and A. Strumia, *Nucl. Phys.* **B445**, 219 (1995); M. Gomez, and H. Goldberg, *Phys. Rev.* **D53**, 5244 (1996).

[18] C. Albright and S.M. Barr, hep-ph/9712488.

FIGURES

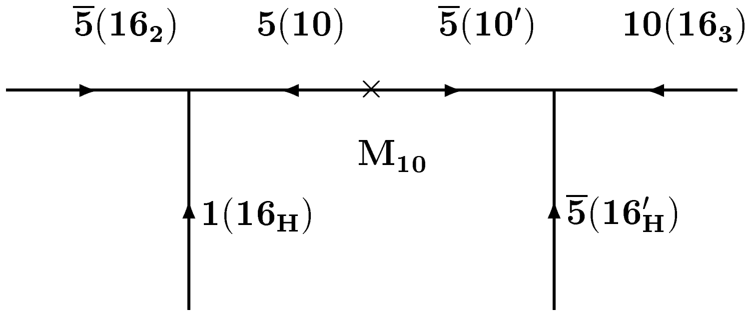

FIG. 1. A diagram that shows how vectors of fermions may be integrated out to produce the "ρ" terms in the mass matrices in (1). For group-theoretical reasons these produce lopsided contributions to the charged-lepton and down-quark mass matrices, that explain why V_{cb} is small while $\sin^2 2\theta_{\mu\tau}$ is large.