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Higher Twist Distribution Amplitudes of Vector Mesons in QCD: Formalism and Twist Three Distributions

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Abstract:

We present a systematic study of twist three light-cone distribution amplitudes of vector mesons in QCD, which is based on conformal expansion and takes into account meson and quark mass corrections. A complete set of distribution amplitudes is constructed for ρ, ω, K^* and ϕ mesons, which satisfies all (exact) equations of motion and constraints from conformal expansion. Nonperturbative input parameters are calculated from QCD sum rules, including an update of SU(3) breaking corrections in the leading twist distributions.

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1 Introduction

The success of QCD as fundamental theory of strong interactions is intimately tied to its ability to describe hard inclusive reactions, which has been tested in numerous experiments. Progress in the theory and phenomenology of hard exclusive processes [1] has been more modest for several reasons. First of all, exclusive processes are in general more difficult to study experimentally as they constitute only a small fraction of the inclusive rates at comparable momentum transfers. In addition, there is growing understanding that, although the quark counting rules appear to start working at small momentum transfers, QCD factorization in its standard form [2] may be valid only at very large momentum transfers. This is in contrast to inclusive processes like deep-inelastic scattering, where the leading twist factorization approximation is adequate already at $Q \sim 1$ GeV. Evidently, the situation calls for a systematic study of preasymtotic corrections to hard exclusive amplitudes.

Not much is known yet about these preasymptotic corrections. The task is complicated by the fact that it actually comprises two different problems. First, it is not known how to combine higher twist contributions to hadron form factors with the so-called "soft" or "end-point" contributions which are of the same order of magnitude. Second, in order to be able to calculate higher twist corrections, one needs to know *both* the dependence of the leading Fock state wave function – with a minimal number of (valence) partons – on transverse momentum *and* the distribution amplitudes with a non-minimal parton configuration with additional gluons and/or quark-antiquark pairs. These two effects are physically different, but related to each other by the (exact) QCD equations of motion; taking into account one effect and neglecting the other is inconsistent with QCD, unless kinematic suppression or enhancement of a particular mechanism can be established. One has to find a basis of independent distributions and to develop a meaningful approximation to describe them by a minimal number of parameters.

In this paper we address the second problem only, leaving aside the questions of "end-point" contributions and how to generalize the factorization formalism beyond leading twist. The main goal of our study is to find out whether higher twist components in hadrons have an economic description in QCD. Light-cone distributions beyond leading twist were previously addressed in Refs. [3, 4, 5, 6, 7, 8]. The existing results are, however, far from being complete and sometimes even contradictory. The aim of this paper is to develop a systematic formalism for constructing a basis of independent higher twist distributions in such a form that it is possible to model them while automatically including all QCD constraints. Our approach is an extension of earlier work [7] with the basic idea that the equations of motion can be solved order by order in the expansion in conformal spin. These relations are exact in perturbation theory although conformal symmetry is broken beyond one loop. Taking into account a few low-order terms in the conformal expansion, one obtains a consistent set of distribution amplitudes which involve a minimal number of independent nonperturbative parameters that can be estimated from QCD sum rules (or, eventually, calculated on the lattice). Although in principle this can be done for arbitrary twist, we concentrate on twist three distribution amplitudes of vector mesons in this work. A detailed treatment of twist four distributions will be presented elsewhere.

In Ref. [7], this program was realized for the pion for which a complete set of distributions of twist three and twist four is now available. Vector mesons bring in the complication of polarization and are more difficult to treat as the meson mass cannot be neglected. This requires a generalization of the techniques of [7], which we work out in the present paper. In addition, we take into account SU(3) flavour violation effects induced by quark masses and construct a complete set of twist three distribution amplitudes for ρ, ω, K^* and ϕ mesons, which is our main result. All necessary nonperturbative constants are calculated from QCD sum rules and the scale-dependence is worked out in leading logarithmic approximation.

Apart from providing the leading corrections to hard exclusive amplitudes, twist three distributions are of special interest as they are free from renormalon ambiguities (power divergences of the corresponding operators, in a different language) and their evolution with Q^2 is simple in the limit of a large number of colours, $N_c \to \infty$, as will be clarified in this work. One may hope that these distributions are accessible experimentally. Some immediate applications of our results, which we do not pursue in this paper, are to exclusive semileptonic and radiative B decays and to hard electroproduction of vector mesons at HERA.

The paper is organized as follows. Section 2 is mainly introductory. We collect necessary definitions and explain basic ideas. Section 3 contains a detailed study on chiral-odd distribution amplitudes, including solution of the equations of motion, conformal expansion and renormalization. A similar program is carried out for chiral-even distributions in Sec. 4. Section 5 contains explicit models for the ρ, K^* and ϕ meson distribution amplitudes up to twist three, which involve a minimal number of nonperturbative parameters and satisfy all QCD constraints. The final Sec. 6 is reserved for a summary and conclusions. The paper contains three appendices: in App. A we collect some useful formulae about orthogonal polynomials, in App. B we elaborate on the structure of conformal expansion for the so-called Wandzura-Wilczek contributions, and App. C contains QCD sum rules for the nonperturbative expansion coefficients used in Sec. 5.

2 General Framework

2.1 Kinematics and notations

Amplitudes of light-cone dominated processes involving vector mesons can be expressed in terms of matrix elements of gauge-invariant nonlocal operators sandwiched between the vacuum and the vector meson state,

$$\langle 0|\bar{u}(x)\Gamma[x,-x]d(-x)|\rho^-(P,\lambda)\rangle, \tag{2.1}$$

where Γ is a generic Dirac matrix structure and where we use the notation $[x, y]$ for the path-ordered gauge factor along the straight line connecting the points x and y:

$$[x,y] = \mathrm{P}\exp[ig\int_0^1 dt\,(x-y)_\mu A^\mu(tx+(1-t)y)]. \tag{2.2}$$

To simplify notations, we will explicitly consider charged ρ mesons; the distribution amplitudes of ρ^0 and of K^* and ϕ mesons can be obtained by choosing appropriate SU(3) currents. In order to be able to study SU(3) breaking effects, we keep all quark mass terms.

The asymptotic expansion of exclusive amplitudes in powers of large momentum transfer is governed by contributions from small transverse separations between the constituents, which are obtained by expanding amplitudes like (2.1) in powers of the deviation from the light-cone $x^2 = 0$. To implement the light-cone expansion in a systematic way, it is convenient to use light-like vectors. Let P_μ be the ρ meson momentum and $e^{(\lambda)}_\mu$ its polarization vector, so that

$$P^2 = m_\rho^2, \qquad e^{(\lambda)} \cdot e^{(\lambda)} = -1, \qquad P \cdot e^{(\lambda)} = 0. \tag{2.3}$$

We introduce light-like vectors p and z with

$$p^2 = 0, \qquad z^2 = 0,$$

such that $p \to P$ in the limit $m_\rho^2 \to 0$ and $z \to x$ for $x^2 \to 0$. From this it follows that

$$\begin{aligned}
z_\mu &= x_\mu - P_\mu \frac{1}{m_\rho^2}\left[xP - \sqrt{(xP)^2 - x^2 m_\rho^2}\right] \\
&= x_\mu - \frac{1}{2} P_\mu \frac{x^2}{zp} + \mathrm{O}(x^4) \\
&= x_\mu \left[1 - \frac{x^2 m_\rho^2}{4(zp)^2}\right] - \frac{1}{2} p_\mu \frac{x^2}{zp} + \mathrm{O}(x^4), \\
p_\mu &= P_\mu - \frac{1}{2} z_\mu \frac{m_\rho^2}{pz}.
\end{aligned} \tag{2.4}$$

Useful scalar products are

$$\begin{aligned}
zP = zp &= \sqrt{(xP)^2 - x^2 m_\rho^2}, \\
p \cdot e^{(\lambda)} &= -\frac{m_\rho^2}{2pz} z \cdot e^{(\lambda)}.
\end{aligned} \tag{2.5}$$

The polarizaton vector $e^{(\lambda)}$ can be decomposed into projections onto the two light-like vectors and the orthogonal plane:

$$\begin{aligned}
e^{(\lambda)}_\mu &= \frac{(e^{(\lambda)} \cdot z)}{pz} p_\mu + \frac{(e^{(\lambda)} \cdot p)}{pz} z_\mu + e^{(\lambda)}_{\perp\mu} \\
&= \frac{(e^{(\lambda)} \cdot z)}{pz}\left(p_\mu - \frac{m_\rho^2}{2pz} z_\mu\right) + e^{(\lambda)}_{\perp\mu}.
\end{aligned} \tag{2.6}$$

Note that

$$(e^{(\lambda)} \cdot z) = (e^{(\lambda)} \cdot x). \tag{2.7}$$

In terms of the original variables one has

$$e_\mu^{(\lambda)} = (e^{(\lambda)} \cdot x) \frac{P_\mu (xP) - x_\mu m_\rho^2}{(xP)^2 - x^2 m_\rho^2} + e_{\perp\mu}^{(\lambda)}. \tag{2.8}$$

We also need the projector onto the directions orthogonal to p and z,

$$g_{\mu\nu}^{\perp} = g_{\mu\nu} - \frac{1}{pz}(p_\mu z_\nu + p_\nu z_\mu), \tag{2.9}$$

and will often use the notations

$$a_{\cdot} \equiv a_\mu z^\mu, \qquad a_* \equiv a_\mu p^\mu/(pz), \tag{2.10}$$

for an arbitrary Lorentz vector a_μ.

We use the standard Bjorken-Drell convention [9] for the metric and the Dirac matrices; in particular $\gamma_5 = i\gamma^0\gamma^1\gamma^2\gamma^3$, and the Levi-Civita tensor $\epsilon_{\mu\nu\lambda\sigma}$ is defined as the totally antisymmetric tensor with $\epsilon_{0123} = 1$. The covariant derivative is defined as $D_\mu \equiv \overrightarrow{D}_\mu = \partial_\mu - igA_\mu$, which is consistent with the gauge phase factor (2.2), and we also use the notation $\overleftarrow{D}_\mu = \overleftarrow{\partial}_\mu + igA_\mu(x)$ in later sections. The dual gluon field strength tensor is defined as $\widetilde{G}_{\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu\rho\sigma}G^{\rho\sigma}$. Finally, we use a covariant normalization for one-particle states, i.e. $\langle\rho^-(P,\lambda)|\rho^-(P',\lambda')\rangle = (2\pi)^3 2P^0 \delta^{(3)}(P-P')\delta_{\lambda\lambda'}$.

2.2 Classification of two-particle distribution amplitudes

By definition, light-cone distribution amplitudes are obtained from Bethe-Salpeter wave functions at (almost) zero transverse separations of the constituents,

$$\phi(x) \sim \int^{k_\perp^2 < \mu^2} d^2k_\perp\, \phi(x, k_\perp)\,, \tag{2.11}$$

and are given by vacuum-to-meson matrix elements of nonlocal operators on the light-cone:

$$\langle 0|\bar{u}(z)\Gamma[z,-z]d(-z)|\rho^-(P,\lambda)\rangle. \tag{2.12}$$

Note that unlike in Eq. (2.1) the separation between the quark and the antiquark is strictly light-like. The expansion of (2.1) near the light-cone in terms of operators with light-like separations is the subject of the operator product expansion and considered at length in Ref. [10].

It turns out that all quark-antiquark distribution amplitudes of vector mesons can be classified in the same way as the more familiar nucleon structure functions (parton distributions) which correspond to the independent tensor structures in matrix elements of type

Twist	2	3	4
	$O(1)$	$O(1/Q)$	$O(1/Q^2)$
spin ave.	f_1	$\underline{e}$	f_4
$S_\parallel$	g_1	$\underline{h_L}$	g_3
$S_\perp$	$\underline{h_1}$	g_T	$\underline{h_3}$

Table 1: Spin, twist and chiral classification of the nucleon structure functions.

$\langle N(P,S)|\bar{\psi}(z)\Gamma[z,-z]\psi(-z)|N(P,S)\rangle$ over nucleon states $|N(P,S)\rangle$ with momentum P and spin S.

Jaffe and Ji [11] find nine independent quark distributions whose spin, twist and chiral classifications are shown in Tab. 1. The parton distributions in the first row are spin-independent, those in the second and third rows describe longitudinally ($S_\parallel$) and transversely ($S_\perp$) polarized nucleons. Each column refers to twist: a distribution of twist t contributes to inclusive cross sections with coefficients which contain $t-2$ or more powers of $1/Q$. The underlined distributions are referred to as chiral-odd, because they correspond to chirality-violating Dirac matrix structures $\Gamma = \{\sigma_{\mu\nu}i\gamma_5,\, 1\}$. The other distributions are termed chiral-even, because they are obtained for the chirality-conserving structures $\Gamma = \{\gamma_\mu,\, \gamma_\mu\gamma_5\}$.

The nucleon parton distributions are defined by

$$\begin{aligned}
&\langle N(P,S)|\bar{\psi}(z)\sigma_{\mu\nu}i\gamma_5[z,-z]\psi(-z)|N(P,S)\rangle \\
&\quad = 2\Bigg[(S_{\perp\mu}p_\nu - S_{\perp\nu}p_\mu)\int_{-1}^{1}dx\, e^{2ixp\cdot z}h_1(x,\mu^2) \\
&\quad\quad + (p_\mu z_\nu - p_\nu z_\mu)\frac{S\cdot z}{(p\cdot z)^2}M^2\int_{-1}^{1}dx\, e^{2ixp\cdot z}h_L(x,\mu^2) \\
&\quad\quad + (S_{\perp\mu}z_\nu - S_{\perp\nu}z_\mu)\frac{M^2}{p\cdot z}\int_{-1}^{1}dx\, e^{2ixp\cdot z}h_3(x,\mu^2)\Bigg],
\end{aligned} \tag{2.13}$$

$$\langle N(P,S)|\bar{\psi}(z)[z,-z]\psi(-z)|N(P,S)\rangle = 2M\int_{-1}^{1}dx\, e^{2ixp\cdot z}e(x,\mu^2), \tag{2.14}$$

$$\begin{aligned}
&\langle N(P,S)|\bar{\psi}(z)\gamma_\mu[z,-z]\psi(-z)|N(P,S)\rangle \\
&\quad = 2\left[p_\mu\int_{-1}^{1}dx\, e^{2ixp\cdot z}f_1(x,\mu^2) + z_\mu\frac{M^2}{p\cdot z}\int_{-1}^{1}dx\, e^{2ixp\cdot z}f_4(x,\mu^2)\right],
\end{aligned} \tag{2.15}$$

$$\langle N(P,S)|\bar{\psi}(z)\gamma_\mu\gamma_5[z,-z]\psi(-z)|N(P,S)\rangle$$

Twist	2	3	4
	$O(1)$	$O(1/Q)$	$O(1/Q^2)$
$e_\parallel$	$\phi_\parallel$	$\underline{h_\parallel^{(t)}}, \underline{h_\parallel^{(s)}}$	g_3
$e_\perp$	$\underline{\phi_\perp}$	$g_\perp^{(v)}, g_\perp^{(a)}$	$\underline{h_3}$

Table 2: Spin, twist and chiral classification of the ρ meson distribution amplitudes.

$$= 2M \left[p_\mu \frac{S \cdot z}{p \cdot z} \int_{-1}^{1} dx\, e^{2ixp\cdot z} g_1(x, \mu^2) + S_{\perp\mu} \int_{-1}^{1} du\, e^{2ixp\cdot z} g_T(x, \mu^2) \right.$$

$$\left. + z_\mu \frac{S \cdot z}{(p \cdot z)^2} M^2 \int_{-1}^{1} dx\, e^{2ixp\cdot z} g_3(x, \mu^2) \right], \tag{2.16}$$

where P_μ and S_μ are decomposed similarly to (2.4) and (2.6), respectively, with m_ρ replaced by the nucleon mass M; x is the Bjorken scaling variable. The nucleon spin vector is normalized as $S \cdot S = -1$, which differs from the definition used in [11] by a factor M^2.

The analysis of vector meson distribution amplitudes reveals an analogous pattern, which is no surprise as the operator structures are the same and the ρ meson polarization vector formally substitutes the nucleon spin vector in the Lorentz structures. We find eight independent two-particle distributions whose classification with respect to spin, twist and chirality is summarized in Tab. 2. One distribution amplitude is obtained for longitudinally ($e_\parallel$) and transversely ($e_\perp$) polarized ρ mesons of twist 2 and twist 4, respectively. On the other hand, the number of twist 3 distribution amplitudes is doubled for each polarization. Due to this analogous structure we take over the notations from Tab. 1 for some quantities. Again, the higher twist distribution amplitudes contribute to a hard exclusive amplitude with additional powers of $1/Q$ compared to the leading twist 2 ones. The underlined distribution amplitudes are chiral-odd, the others chiral-even. Because the matrix element $\langle 0|\bar{u}(z)\Gamma[z,-z]d(-z)|\rho^-(P,\lambda)\rangle$ depends on the polarization vector $e_\mu^{(\lambda)}$ linearly, there is no spin-independent distribution amplitude. This is in contrast to the nucleon parton distributions of Tab. 1, where the dependence on the polarization vector S_μ is determined by the density matrix $(1 + \gamma_5 \not{S})/2$ which contains a spin-independent part. One more difference is that $e_\mu^{(\lambda)}$ for the ρ meson is a vector, while S_μ for the nucleon is a pseudovector. Thus, an insertion of additional $i\gamma_5$ is necessary in order that matrix elements of relevant operators have the analogous Lorentz decomposition.

The explicit definitions of the chiral-odd ρ distributions are:

$$\langle 0|\bar{u}(z)\sigma_{\mu\nu}[z,-z]d(-z)|\rho^-(P,\lambda)\rangle =$$

$$= i f_\rho^T \left[(e_{\perp\mu}^{(\lambda)} p_\nu - e_{\perp\nu}^{(\lambda)} p_\mu) \int_0^1 du\, e^{i\xi p\cdot z} \phi_\perp(u, \mu^2) \right.$$

$$+ (p_\mu z_\nu - p_\nu z_\mu)\frac{e^{(\lambda)}\cdot z}{(p\cdot z)^2} m_\rho^2 \int_0^1 du\, e^{i\xi p\cdot z} h_\parallel^{(t)}(u,\mu^2)$$

$$+ \frac{1}{2}(e^{(\lambda)}_{\perp\mu} z_\nu - e^{(\lambda)}_{\perp\nu} z_\mu)\frac{m_\rho^2}{p\cdot z}\int_0^1 du\, e^{i\xi p\cdot z} h_3(u,\mu^2)\Bigg], \tag{2.17}$$

$$\langle 0|\bar u(z)[z,-z]d(-z)|\rho^-(P,\lambda)\rangle = -i\left(f_\rho^T - f_\rho\frac{m_u+m_d}{m_\rho}\right)(e^{(\lambda)}\cdot z) m_\rho^2\int_0^1 du\, e^{i\xi p\cdot z} h_\parallel^{(s)}(u,\mu^2), \tag{2.18}$$

while the chiral-even distributions are defined as (cf. [12])

$$\langle 0|\bar u(z)\gamma_\mu[z,-z]d(-z)|\rho^-(P,\lambda)\rangle =$$

$$= f_\rho m_\rho\left[p_\mu \frac{e^{(\lambda)}\cdot z}{p\cdot z}\int_0^1 du\, e^{i\xi p\cdot z}\phi_\parallel(u,\mu^2) + e^{(\lambda)}_{\perp\mu}\int_0^1 du\, e^{i\xi p\cdot z} g_\perp^{(v)}(u,\mu^2)\right.$$

$$\left. - \frac{1}{2} z_\mu \frac{e^{(\lambda)}\cdot z}{(p\cdot z)^2} m_\rho^2 \int_0^1 du\, e^{i\xi p\cdot z} g_3(u,\mu^2)\right] \tag{2.19}$$

and

$$\langle 0|\bar u(z)\gamma_\mu\gamma_5[z,-z]d(-z)|\rho^-(P,\lambda)\rangle =$$

$$= \frac{1}{2}\left(f_\rho - f_\rho^T\frac{m_u+m_d}{m_\rho}\right) m_\rho \epsilon_\mu{}^{\nu\alpha\beta} e^{(\lambda)}_{\perp\nu} p_\alpha z_\beta \int_0^1 du\, e^{i\xi p\cdot z} g_\perp^{(a)}(u,\mu^2). \tag{2.20}$$

Here and below we use the shorthand notation

$$\xi = u - (1-u) = 2u - 1.$$

The distribution amplitudes are dimensionless functions of u and describe the probability amplitudes to find the ρ in a state with minimal number of constituents — quark and antiquark — which carry momentum fractions u (quark) and $1-u$ (antiquark), respectively, and have a small transverse separation of order $1/\mu$. The nonlocal operators on the left-hand side are renormalized at scale μ, so that the distribution amplitudes depend on μ as well. This dependence can be calculated in perturbative QCD and will be considered below in Secs. 3 and 4.

The vector and tensor decay constants f_ρ and f_ρ^T are defined as usually as

$$\langle 0|\bar u(0)\gamma_\mu d(0)|\rho^-(P,\lambda)\rangle = f_\rho m_\rho e^{(\lambda)}_\mu, \tag{2.21}$$

$$\langle 0|\bar u(0)\sigma_{\mu\nu} d(0)|\rho^-(P,\lambda)\rangle = i f_\rho^T (e^{(\lambda)}_\mu P_\nu - e^{(\lambda)}_\nu P_\mu). \tag{2.22}$$

All eight distributions $\phi = \{\phi_\parallel, \phi_\perp, g_\perp^{(v)}, g_\perp^{(a)}, h_\parallel^{(t)}, h_\parallel^{(s)}, h_3, g_3\}$ are normalized as

$$\int_0^1 du\,\phi(u) = 1, \tag{2.23}$$

which can be checked by comparing both sides of the defining equations in the limit $z_\mu \to 0$ and using the equations of motion. The rationale for keeping the (tiny) corrections proportional to the u and d quark masses m_u and m_d is that it will allow us to calculate the SU(3) breaking corrections for K^* and ϕ mesons.

Note that the meson-to-vacuum matrix element vanishes for $\Gamma = i\gamma_5$, because it is not possible to construct a pseudoscalar quantity from p_μ, z_μ, and $e^{(\lambda)}_\mu$. On the other hand, (2.20) would correspond to (2.15) which defines the spin-averaged nucleon distributions and is the only exception to the complete analogy between the nucleon distribution functions and the ρ meson distribution amplitudes. In this case it is the difference in parity between $e^{(\lambda)}_\mu$ and S_μ, which leads to a completely different decomposition of the matrix elements.

Because of the analogous structure, the twist classification of the various distributions does not require a separate study and can be inferred directly from [11], see Tab. 2. Its physical interpretation, however, deserves a discussion.

One convenient way to understand the twist classification of distribution amplitudes directly from their definitions is to go over to the infinite momentum frame $p \cdot z \sim Q \to \infty$. From (2.6) it follows that in this frame $(e^{(\lambda)} \cdot z) \sim Q$ and $e^{(\lambda)}_\perp \sim 1$. This determines the power counting in Q of various terms on the right-hand side of (2.17) and (2.19), for example: The first, second, and third terms behave as $O(Q)$, $O(1)$ and $O(1/Q)$, respectively, and thus correspond to increasing twist.

A mathematically similar, but conceptually different approach to twist counting is based on the light-cone quantization formalism [13, 1, 11]. In this approach quark fields are decomposed into "good" and "bad" components, so that $\psi = \psi_+ + \psi_-$ with $\psi_+ = (1/2)\gamma_* \gamma_. \psi$ and $\psi_- = (1/2)\gamma_. \gamma_* \psi$. As discussed in [11], a "bad" component ψ_- introduces one unit of twist. Therefore, a quark-antiquark operator of type $\bar{u}d$ contains twist 2 $(\bar{u}_+ d_+)$, twist 3 $(\bar{u}_+ d_-,\ \bar{u}_- d_+)$, and twist 4 $(\bar{u}_- d_-)$ contributions. This explains why the number of twist 3 distribution amplitudes is doubled compared with the twist 2 and twist 4 ones (see Tab. 2).

The physical content of this classification is that a "good" component ψ_+ represents an independent degree of freedom corresponding to the particle content of the "Fock state". On the other hand, the "bad" components are not dynamically independent, but can be expressed in terms of the higher components in the Fock wave function with a larger number of constituents, in particular corresponding to a coherent quark-gluon pair. Only the twist 2 distribution amplitudes correspond to the valence quark-antiquark component in the ρ meson wave function, while the higher twist amplitudes involve contributions of multi-particle states. This point will be discussed in detail in Secs. 3 and 4.

One important comment is in order. The definition of twist based on power counting in the infinite momentum frame is convenient, because it is directly related to the power of $1/Q$, with which the corresponding distributions appear in the physical scattering amplitudes, and hence is frequently employed in recent works [11]. On the other hand, this definition is not Lorentz invariant and does not match the conventional and more consistent definition of twist as "dimension minus spin" of the relevant operators. For example, the nucleon structure function g_T which is identified as twist 3 by power counting in fact contains contributions of both operators of twist 2 and twist 3. Similarly, the distribution amplitudes of vector mesons that were identified as twist 3 above, actually contain contributions of twist 2 operators as well. In Secs. 3 and 4 we will study in detail the operator structure of twist 3 distribution amplitudes based on the operator product expansion. In this context the conventional "operator" definition of twist will be more adequate. The mismatch of different definitions of twist has to be kept in mind, but hopefully will not yield confusion.

To summarize, Eqs. (2.17)–(2.20) define a complete set of valence light-cone distribution amplitudes and provide full information on the quark-antiquark component of the Fock wave function of the ρ meson at zero transverse separation. As mentioned above, not all of these distributions are independent. In the following sections we will derive exact relations between the twist 3 quark-antiquark distribution amplitudes and those involving one additional gluon, which are introduced below.

2.3 Three-particle distribution amplitudes of twist three

Higher Fock components of the meson wave function are described by multi-particle distribution amplitudes. In this paper we will explicitly deal with three-particle twist 3 quark-antiquark-gluon distributions, defined as

$$\langle 0|\bar{u}(z)\gamma_\alpha[z,vz]gG_{\mu\nu}(vz)[vz,-z]d(-z)|\rho^-(P,\lambda)\rangle = ip_\alpha[p_\mu e^{(\lambda)}_{\perp\nu} - p_\nu e^{(\lambda)}_{\perp\mu}]f^V_{3\rho}\mathcal{V}(v,pz) + \dots \tag{2.24}$$

$$\langle 0|\bar{u}(z)\gamma_\alpha\gamma_5[z,vz]g\widetilde{G}_{\mu\nu}(vz)[vz,-z]d(-z)|\rho^-(P,\lambda)\rangle = p_\alpha[p_\nu e^{(\lambda)}_{\perp\mu} - p_\mu e^{(\lambda)}_{\perp\nu}]f^A_{3\rho}\mathcal{A}(v,pz) + \dots \tag{2.25}$$

$$\langle 0|\bar{u}(z)\sigma_{\alpha\beta}[z,vz]gG_{\mu\nu}(vz)[vz,-z]d(-z)|\rho^-(P,\lambda)\rangle =$$

$$= \frac{e^{(\lambda)}\cdot z}{2(p\cdot z)}[p_\alpha p_\mu g^\perp_{\beta\nu} - p_\beta p_\mu g^\perp_{\alpha\nu} - p_\alpha p_\nu g^\perp_{\beta\mu} + p_\beta p_\nu g^\perp_{\alpha\mu}]f^T_{3\rho}m_\rho\mathcal{T}(v,pz) + \dots, \tag{2.26}$$

where the ellipses stand for Lorentz structures of twist higher than three and where we used the following shorthand notation for the integrals defining three-particle distribution amplitudes:

$$\mathcal{F}(v,pz) \equiv \int \mathcal{D}\underline{\alpha}\, e^{-ipz(\alpha_u-\alpha_d+v\alpha_g)}\mathcal{F}(\alpha_d,\alpha_u,\alpha_g). \tag{2.27}$$

Here $\mathcal{F} = \{\mathcal{V},\mathcal{A},\mathcal{T}\}$ refers in an obvious way to the vector, axial-vector and tensor distributions, $\underline{\alpha}$ is the set of three momentum fractions: α_d (d quark), α_u (u quark) and α_g (gluon), and the integration measure is defined as

$$\int \mathcal{D}\underline{\alpha} \equiv \int_0^1 d\alpha_d \int_0^1 d\alpha_u \int_0^1 d\alpha_g\, \delta(1-\sum \alpha_i). \tag{2.28}$$

The normalization constants $f^V_{3\rho}$, $f^A_{3\rho}$, $f^T_{3\rho}$ are defined in such a way that

$$\int \mathcal{D}\underline{\alpha}\,(\alpha_d-\alpha_u)\,\mathcal{V}(\alpha_d,\alpha_u,\alpha_g) = 1,$$

$$\int \mathcal{D}\underline{\alpha}\, \mathcal{A}(\alpha_d, \alpha_u, \alpha_g) = 1,$$

$$\int \mathcal{D}\underline{\alpha}\, (\alpha_d - \alpha_u)\, \mathcal{T}(\alpha_d, \alpha_u, \alpha_g) = 1. \tag{2.29}$$

Choosing the normalization in this way we anticipate that the function $\mathcal{A}$ is symmetric and the functions $\mathcal{V}$ and $\mathcal{T}$ are antisymmetric under the interchange $\alpha_u \leftrightarrow \alpha_d$ in the SU(3) limit (cf. [3]), which follows from the behaviour of the corresponding matrix elements under G-parity transformations.

With these major definitions at hand, we now proceed to a systematic study of the twist 3 distribution amplitudes.

3 Chiral-odd Distribution Amplitudes

This section is devoted to the general discussion of chiral-odd distributions of twist 3. We demonstrate that the two-particle distribution amplitudes $h_{\parallel}^{(t)}(u, \mu^2)$ and $h_{\parallel}^{(s)}(u, \mu^2)$ can be eliminated in favour of independent dynamical degrees of freedom and expressed in terms of leading twist two– and three-particle distributions. The corresponding relations are worked out in detail and solved explicitly. A similar relation between the nucleon structure function g_1 and the twist 2 part of g_T is known as Wandzura-Wilczek relation [14]. We also investigate the expansion of twist 3 distributions in terms of matrix elements of conformal operators. We demonstrate that the equations of motion are satisfied order by order in the conformal expansion which provides, for this reason, a systematic approach to the construction of models of distribution amplitudes, consistent with QCD constraints. The renormalization of all distributions is worked out in the leading logarithmic approximation.

3.1 Equations of motion

The basis of twist 3 distributions defined in Sec. 2 is overcomplete. Due to the QCD equations of motion, the number of independent degrees of freedom is less than the number of independent Lorentz structures, and our first task will be to reveal the corresponding constraints.

The standard technique for this purpose is to derive relations between towers of local operators which arise in the Taylor expansion of the nonlocal operators in Eqs. (2.17), (2.18) and whose matrix elements are just moments of the distribution amplitudes. A more elegant and economic approach is to use exact operator identities between the nonlocal operators [5, 7] (see also [10]). In the present context, we need the identities

$$\frac{\partial}{\partial x_\mu} \{\bar{u}(x)\sigma_{\mu\nu}x^\nu[x,-x]d(-x)\} =$$

$$= i\int_{-1}^{1} dv\, v\; \bar{u}(x)x^\alpha\sigma_{\alpha\beta}[x,vx]x^\mu gG_{\mu\beta}(vx)[vx,-x]d(-x)$$

$$- ix^\beta\partial_\beta \{\bar u(x)[x,-x]d(-x)\} - (m_u-m_d)\bar u(x) \not x\, [x,-x]d(-x), \qquad (3.1)$$

$$\bar u(x)[x,-x]d(-x) - \bar u(0)d(0) =$$

$$= \int_0^1 dt \int_{-t}^t dv\, \bar u(tx)x^\alpha\sigma_{\alpha\beta}[tx,vx]x^\mu gG_{\mu\beta}(vx)[vx,-tx]d(-tx)$$

$$+ i\int_0^1 dt\, \partial^\alpha \left\{\bar u(tx)\sigma_{\alpha\beta}x^\beta[tx,-tx]d(-tx)\right\}$$

$$+i(m_u+m_d)\int_0^1 dt\, \bar u(tx) \not x\, [tx,-tx]d(-tx). \qquad (3.2)$$

Here we introduced a shorthand notation for the derivative over the total translation:

$$\partial_\alpha \{\bar u(tx)\Gamma[tx,-tx]d(-tx)\} \equiv \frac{\partial}{\partial y_\alpha}\{\bar u(tx+y)\Gamma[tx+y,-tx+y]d(-tx+y)\}\Big|_{y\to 0}, \qquad (3.3)$$

with the generic Dirac matrix structure Γ.

In the light-cone limit $x^2 \to 0$ matrix elements of the operators on both sides of Eqs. (3.1) and (3.2), sandwiched between the vacuum and the ρ meson state, can be expressed in terms of the distribution amplitudes defined in Sec. 2:

$$\langle 0|\bar u(x)\sigma_{\mu\nu}x^\nu[x,-x]d(-x)|\rho^-(P,\lambda)\rangle =$$

$$= if_\rho^T\left\{\left(e_\mu^{(\lambda)} - \frac{(e^{(\lambda)}\cdot x)}{Px}P_\mu\right)(Px)\int_0^1 du\, e^{i\xi P\cdot x}\Big[\phi_\perp(u,\mu^2) + O(x^2)\Big]\right.$$

$$\left. -m_\rho^2\frac{(e^{(\lambda)}\cdot x)}{Px}\left(x_\mu - \frac{x^2}{Px}P_\mu\right)\int_0^1 du\, e^{i\xi P\cdot x}\Big[h_\parallel^{(t)}(u,\mu^2) - \phi_\perp(u,\mu^2)\Big]\right\}, \qquad (3.4)$$

$$\langle 0|\bar u(x)[x,-x]d(-x)|\rho^-(P,\lambda)\rangle =$$

$$= -i\left(f_\rho^T - f_\rho\frac{m_u+m_d}{m_\rho}\right)(e^{(\lambda)}\cdot x)m_\rho^2\int_0^1 du\, e^{i\xi P\cdot x}\Big[h_\parallel^{(s)}(u,\mu^2) + O(x^2)\Big], \qquad (3.5)$$

and likewise for three-particle distributions.

The matrix elements of Eqs. (3.1) and (3.2) yield a system of integral equations between two- and three-particle light-cone distribution amplitudes:[1]

$$-ipz\int_0^1 du\, e^{i\xi pz}\xi\, h_\parallel^{(t)}(u) - 2\int_0^1 du\, e^{i\xi pz}\left(h_\parallel^{(t)}(u) - \phi_\perp(u)\right) =$$

$$= \zeta_{3\rho}^T(pz)^2\int_{-1}^1 dv\, v\mathcal{T}(v,pz) + (1-\delta_+)(pz)^2\int_0^1 e^{i\xi pz}h_\parallel^{(s)}(u) + i\delta_- pz\int_0^1 du\, e^{i\xi pz}\phi_\parallel(u), (3.6)$$

[1]The suppressed corrections $O(x^2)$ drop out; the $O(x^2)$ term in the Lorentz structure in the second line of (3.4), however, does give a contribution to the left-hand side of (3.1) after taking the derivative with respect to x_μ.

$$(1-\delta_+)\int_0^1 du\, e^{i\xi pz} h_\parallel^{(s)}(u) = i\zeta_{3\rho}^T pz \int_0^1 tdt \int_{-1}^1 dv\, \mathcal{T}(v, tpz) + \int_0^1 dt \int_0^1 du\, e^{i\xi tpz} h_\parallel^{(t)}(u)$$

$$- \delta_+ \int_0^1 dt \int_0^1 du\, e^{i\xi tpz} \phi_\parallel(u), \tag{3.7}$$

where we have discarded all corrections of order x^2, set $x_\mu = z_\mu$ and introduced the notations

$$\delta_\pm = \frac{f_\rho}{f_\rho^T} \frac{m_u \pm m_d}{m_\rho}, \qquad \zeta_{3\rho}^T = \frac{f_{3\rho}^T}{f_\rho^T m_\rho}. \tag{3.8}$$

Eqs. (3.6) and (3.7) are exact in QCD. Note the terms with total derivatives in (3.1) and (3.2), which induce mixing between $h_\parallel^{(s)}(u)$ and $h_\parallel^{(t)}(u)$. Such contributions are specific for exclusive processes and have no analogue in deep-inelastic scattering. Note also that quark mass corrections bring in the leading twist chiral-even distribution $\phi_\parallel(u)$.

We can solve Eqs. (3.6) and (3.7) for $h_\parallel^{(s)}(u)$ and $h_\parallel^{(t)}(u)$ in terms of the other distributions. To simplify the algebra, it is convenient to consider moments in an intermediate step. Defining

$$M_n^{\parallel,\perp} = \int_0^1 du\, \xi^n \phi_{\parallel,\perp}(u), \quad M_n^{(s),(t)} = \int_0^1 du\, \xi^n h_\parallel^{(s),(t)}(u) \tag{3.9}$$

and

$$\mathcal{T}_n(v) = (-i)^n \frac{\partial^n}{\partial \tau^n} \mathcal{T}(v,\tau)\bigg|_{\tau=0} = \int \mathcal{D}\underline{\alpha}\, (\alpha_d - \alpha_u - v\alpha_g)^n \mathcal{T}(\alpha_d, \alpha_u, \alpha_g) \tag{3.10}$$

and expanding (3.6) and (3.7) in powers of (pz), we obtain

$$M_n^{(t)} - \frac{2}{n+2} M_n^\perp - (1-\delta_+) \frac{(n-1)n}{n+2} M_{n-2}^{(s)} + \delta_- \frac{n}{n+2} M_{n-1}^\parallel - \zeta_{3\rho}^T \frac{(n-1)n}{n+2} \int_{-1}^1 dv\, v\mathcal{T}_{n-2}(v) = 0, \tag{3.11}$$

$$(1-\delta_+)\, M_n^{(s)} - \frac{1}{n+1} M_n^{(t)} + \delta_+ \frac{1}{n+1} M_n^\parallel - \zeta_{3\rho}^T \frac{n}{n+1} \int_{-1}^1 dv\, \mathcal{T}_{n-1}(v) = 0. \tag{3.12}$$

Combining these two equations, one gets the following recurrence relations for $h_\parallel^{(t)}$ and $h_\parallel^{(s)}$:

$$(n+2)\, M_n^{(t)} - n\, M_{n-2}^{(t)} = 2M_n^\perp + \zeta_{3\rho}^T \int_{-1}^1 dv\, \{(n-1)n\, v\mathcal{T}_{n-2}(v) + (n-2)n\, \mathcal{T}_{n-3}(v)\}$$

$$-\delta_+\, n\, M_{n-2}^\parallel - \delta_-\, n\, M_{n-1}^\parallel, \tag{3.13}$$

$$(1-\delta_+) \left\{(n+1)(n+2) M_n^{(s)} - (n-1)n M_{n-2}^{(s)}\right\} =$$

$$= 2M_n^\perp + \zeta_{3\rho}^T \int_{-1}^1 dv\, \{n(n+2)\, \mathcal{T}_{n-1}(v) + (n-1)n\, v\, \mathcal{T}_{n-2}(v)\}$$

$$- \delta_+\, (n+2)\, M_n^\parallel - \delta_-\, n\, M_{n-1}^\parallel. \tag{3.14}$$

Recurrence relations of this type are easily solved by transforming them into differential equations. For instance, for the distribution amplitude $h_\parallel^{(s)}(u)$ one finds a second order equation:

$$(1-\delta_+)\, u\bar{u}\, (h_\parallel^{(s)})''(u) = -\,\Phi(u) \tag{3.15}$$

with

$$\Phi(u) = 2\phi_\perp(u) - \delta_+ \left(\phi_\parallel(u) - \frac{1}{2}\,\xi\phi_\parallel'(u)\right) + \frac{1}{2}\,\delta_-\phi_\parallel'(u)$$

$$+\,\zeta_{3\rho}^T \frac{d}{du} \int_0^u d\alpha_d \int_0^{\bar{u}} d\alpha_u\, \frac{1}{1-\alpha_u-\alpha_d} \left(\alpha_d\, \frac{d}{d\alpha_d} + \alpha_u\, \frac{d}{d\alpha_u} - 1\right) \mathcal{T}(\underline{\alpha}). \tag{3.16}$$

Here and below we use the shorthand notation $\bar{u} = 1-u$. The solution of this equation with boundary conditions specified by the values of the first two moments reads

$$(1-\delta_+)\, h_\parallel^{(s)}(u) = \bar{u} \int_0^u dv\, \frac{1}{\bar{v}}\, \Phi(v) + u \int_u^1 dv\, \frac{1}{v}\, \Phi(v)\,. \tag{3.17}$$

The solution for $h_\parallel^{(t)}(u)$ can be obtained in a similar manner and reads:

$$h_\parallel^{(t)}(u) = \frac{1}{2}\,\xi \left(\int_0^u dv \frac{1}{\bar{v}}\, \Phi(v) - \int_u^1 dv \frac{1}{v}\, \Phi(v)\right) + \delta_+\phi_\parallel(u)$$

$$+\,\zeta_{3\rho}^T \frac{d}{du} \int_0^u d\alpha_d \int_0^{\bar{u}} d\alpha_u\, \frac{1}{1-\alpha_u-\alpha_d}\, \mathcal{T}(\underline{\alpha}). \tag{3.18}$$

According to the various "source" terms on the right-hand side of (3.13) and (3.14), one can decompose the solution in an obvious way into three pieces as

$$h_\parallel^{(t)}(u) = h_\parallel^{(t)WW}(u) + h_\parallel^{(t)g}(u) + h_\parallel^{(t)m}(u), \tag{3.19}$$

$$h_\parallel^{(s)}(u) = h_\parallel^{(s)WW}(u) + h_\parallel^{(s)g}(u) + h_\parallel^{(s)m}(u), \tag{3.20}$$

where $h_\parallel^{(t)WW}(u)$ and $h_\parallel^{(s)WW}(u)$ denote the "Wandzura-Wilczek" type contributions of twist 2 operators, $h_\parallel^{(t)g}(u)$ and $h_\parallel^{(s)g}(u)$ stand for contributions of three-particle distributions and $h_\parallel^{(t)m}(u)$ and $h_\parallel^{(s)m}(u)$ are due to the quark mass corrections. In particular, we get

$$h_\parallel^{(t)WW}(u) = \xi \left(\int_0^u dv \frac{\phi_\perp(v)}{\bar{v}} - \int_u^1 dv \frac{\phi_\perp(v)}{v}\right), \tag{3.21}$$

$$h_\parallel^{(s)WW}(u) = 2 \left(\bar{u} \int_0^u dv \frac{\phi_\perp(v)}{\bar{v}} + u \int_u^1 dv \frac{\phi_\perp(v)}{v}\right). \tag{3.22}$$

These are the analogues of the Wandzura-Wilczek contributions to the nucleon structure functions $g_T(x, Q^2)$ [14] and $h_L(x, Q^2)$ [11].

The relations Eqs. (3.17) and (3.18) are the principal results of this section: chiral-odd two-particle distribution amplitudes of twist 3 are expressed in terms of the leading twist amplitudes and the three-particle twist 3 distribution. In the next subsection we will discuss how to proceed further with this rather complicated formal solution, concentrating on the massless quark limit.

3.2 Conformal expansion

The conformal expansion of light-cone distribution amplitudes is analogous to the partial wave expansion of wave functions in standard quantum mechanics. In conformal expansion, the invariance of massless QCD under conformal transformations substitutes the rotational symmetry in quantum mechanics. In quantum mechanics, the purpose of partial wave decomposition is to separate angular degrees of freedom from radial ones (for spherically symmetric potentials). All dependence on the angular coordinates is included in spherical harmonics which form an irreducible representation of the group O(3), and the dependence on the single remaining radial coordinate is governed by a one-dimensional Schrödinger equation. Similarly, the conformal expansion of distribution amplitudes in QCD aims to separate longitudinal degrees of freedom from transverse ones. All dependence on the longitudinal momentum fractions is included in terms of functions (orthogonal polynomials) forming irreducible representations of the so-called collinear subgroup of the conformal group, SL(2,R), describing Möbius transformations on the light-cone. The transverse-momentum dependence (the scale-dependence) is governed by simple renormalization group equations: the different partial waves, labelled by different "conformal spins", behave independently and do not mix with each other. Since the conformal invariance of QCD is broken by quantum corrections, mixing of different terms of the conformal expansion is only absent to leading logarithmic accuracy. Still, conformal spin is a good quantum number in hard processes, up to small corrections of order α_s^2. Application of conformal symmetry to the studies of exclusive processes in leading twist have received a lot of attention in the literature, see e.g. [15, 16, 17].

Despite certain complications, the conformal expansion presents a natural approach to the study of higher twist distributions, which has even more power than in leading twist. The reason is that conformal transformations commute with the exact QCD equations of motion since the latter are not renormalized[2]. Thus, the equations of motion can be solved order by order in the conformal expansion. In this section, we use the approach of [18, 7] to work out the explicit form of the conformal expansion for the chiral-odd distributions $\phi_\perp(u), h_\parallel^{(t)}(u), h_\parallel^{(s)}(u)$ and $\mathcal{T}(\underline{\alpha})$, and solve the constraints (3.19) and (3.20) order by order in conformal spin.

Since quark mass terms break the conformal symmetry of the QCD Lagrangian explicitly, one might expect difficulties to incorporate SU(3) breaking corrections in the formalism. In

[2]More precisely, one can regularize the theory in such a way as to preserve the equations of motion.

fact, the inclusion of quark mass corrections turns out to be straightforward and produces two types of effects. First, matrix elements of conformal operators are modified and in general do not have the symmetry of the massless theory. This is not a "problem", since the conformal expansion is designed to simplify the transverse momentum dependence of the wave functions by relating it to the scale dependence of the relevant operators. This dependence is given by operator anomalous dimensions which are not affected by quark masses, provided they are smaller than the scales involved. Second, new higher twist operators arise, in which quark masses multiply operators of lower twist, see the previous section. These additional operators, again, do not pose a "problem" and can be expanded systematically in conformal partial waves, leaving the quark masses as multiplicative factors. Explicit examples are considered later in Sec. 5, while in this section we neglect operators proportional to the quark masses and set $\delta_\pm = 0$ in the formulae obtained in Sec. 3.1.

The conformal expansion of distribution amplitudes is especially simple when each constituent field has fixed (Lorentz) spin projection onto the light-cone. Such constituent fields correspond to the so-called primary fields in conformal field theories, and their conformal spin equals

$$j = \frac{1}{2}(l+s), \tag{3.23}$$

where l is the canonical dimension and s the (Lorentz) spin projection. Multi-particle states built of primary fields can be expanded in increasing conformal spin: the lowest possible spin equals to the sum of spins of constituents, and its "wave function" is given by the product of one-particle states. This state is nondegenerate and cannot mix with other states because of conformal symmetry. Its evolution is given, therefore, by a simple renormalization group equation and one can check (see Sec. 3.3) that the corresponding anomalous dimension is the lowest one in the whole spectrum. Therefore, this state is the only one which survives in the formal limit $Q^2 \to \infty$; following established tradition we will refer to the multi-particle state with the minimal conformal spin as "asymptotic distribution amplitude".

An explicit expression for the asymptotic distribution amplitude of a multi-particle state built of primary fields was obtained in Refs. [18, 7]:

$$\phi_{as}(\alpha_1, \alpha_2, \ldots, \alpha_m) = \frac{\Gamma[2j_1 + \ldots + 2j_m]}{\Gamma[2j_1]\ldots\Gamma[2j_m]} \alpha_1^{2j_1-1}\alpha_2^{2j_2-1}\ldots\alpha_m^{2j_m-1}. \tag{3.24}$$

Here the j_k are the conformal spins of the constituent fields (quark or gluons with fixed spin projections). This distribution has conformal spin $j = j_1 + \ldots + j_m$. Multi-particle irreducible representations with higher spin $j+n, n = 1,2,\ldots$ are given by orthogonal polynomials of m variables (with the constraint $\sum_{k=1}^m \alpha_k = 1$) with the weight function (3.24).

A classical example is the leading twist quark-antiquark distribution amplitude. The distribution amplitude $\phi_\perp(u)$, defined in (2.17), has the expansion

$$\phi_\perp(u) = 6u\bar{u}\sum_{n=0}^{\infty} a_n^\perp C_n^{3/2}(\xi), \tag{3.25}$$

where $C_n^{3/2}(\xi)$ are Gegenbauer polynomials (see e.g. [19]). The dimension of quark fields is $l = 3/2$ and the leading twist distribution corresponds to positive spin projection $s = +1/2$ for both the quark and the antiquark. Thus, according to (3.23), the conformal spin of each field is $j_q = j_{\bar{q}} = 1$; the asymptotic distribution amplitude (3.24) equals $\phi_{as}(\alpha_q, \alpha_{\bar{q}}) = 6\alpha_q\alpha_{\bar{q}}$ and has conformal spin $j = 2$. Taking into account $\alpha_q + \alpha_{\bar{q}} = 1$ and denoting $u = \alpha_q$ we arrive at the first term in the expansion (3.25). The Gegenbauer polynomials correspond to contributions with higher conformal spin $j + n$ and are orthogonal over the weight function $6u\bar{u}$.

Note that $a_0^\perp = 1$ due to the normalization condition (2.23). In the strict massless limit only the terms with even n survive in Eq. (3.25) because of G-parity invariance. The conformal expansion, however, can be performed at the operator level and is disconnected from particular symmetries of states such as G-parity. The following discussion is, therefore, valid for arbitrary n. We keep terms with $n = 2k+1$ for the later discussion of SU(3) breaking corrections.

The conformal expansion of the twist 3 two-particle distribution amplitudes $h_\parallel^{(t)}(u)$ and $h_\parallel^{(s)}(u)$ is less immediate. As a first step, one has to decompose them into components built of primary fields — with fixed spin projections. To this end, we define a set of auxiliary amplitudes $h^{\uparrow\downarrow}(u)$ and $h^{\downarrow\uparrow}(u)$ using the spin projection operators $P_+ = (1/2)\gamma_*\gamma_.$ and $P_- = (1/2)\gamma_.\gamma_*$ to single out quark states with $s = +1/2$ and $s = -1/2$, respectively, (see [7, 20]):

$$\langle 0|\bar{u}(z)\gamma_.\gamma_*[z,-z]d(-z)|\rho^-(P,\lambda)\rangle = f_\rho^T m_\rho^2 \frac{e^{(\lambda)}\cdot z}{p\cdot z}\int_0^1 du\, e^{i\xi pz} h^{\uparrow\downarrow}(u), \tag{3.26}$$

$$\langle 0|\bar{u}(z)\gamma_*\gamma_.[z,-z]d(-z)|\rho^-(P,\lambda)\rangle = f_\rho^T m_\rho^2 \frac{e^{(\lambda)}\cdot z}{p\cdot z}\int_0^1 du\, e^{i\xi pz} h^{\downarrow\uparrow}(u), \tag{3.27}$$

which are related to $h_\parallel^{(t)}(u)$ and $h_\parallel^{(s)}(u)$ by (see (2.17), (2.18))

$$h^{\uparrow\downarrow}(u) = h_\parallel^{(t)}(u) + \frac{1}{2}\frac{dh_\parallel^{(s)}(u)}{du}, \tag{3.28}$$

$$h^{\downarrow\uparrow}(u) = -h_\parallel^{(t)}(u) + \frac{1}{2}\frac{dh_\parallel^{(s)}(u)}{du}. \tag{3.29}$$

The conformal expansion of $h^{\uparrow\downarrow}(u)$ and $h^{\downarrow\uparrow}(u)$ is straightforward and is given by

$$h^{\uparrow\downarrow}(u) = 2\bar{u}\sum_{n=0}^\infty h_n^{\uparrow\downarrow} P_n^{(1,0)}(\xi), \tag{3.30}$$

$$h^{\downarrow\uparrow}(u) = 2u\sum_{n=0}^\infty h_n^{\downarrow\uparrow} P_n^{(0,1)}(\xi), \tag{3.31}$$

where $P_n^{(0,1)}(\xi)$ are Jacobi polynomials (see e.g. [19]) and the n-th term corresponds to conformal spin $j = n + 3/2$. Substituting these expansions in (3.28) and (3.29) and using the identities (A.1), (A.10) and (A.11) in App. A, we obtain

$$h_{\parallel}^{(t)}(u) = \sum_{n=0,2,4,\ldots} (H_n - H_{n-1})\, C_n^{1/2}(\xi) + \sum_{n=1,3,5,\ldots} (h_n - h_{n-1})\, C_n^{1/2}(\xi), \tag{3.32}$$

$$h_{\parallel}^{(s)}(u) = 4u\bar{u}\left(\sum_{n=0,2,4,\ldots} \frac{H_n - H_{n+1}}{(n+1)(n+2)} C_n^{3/2}(\xi) + \sum_{n=1,3,5,\ldots} \frac{h_n - h_{n+1}}{(n+1)(n+2)} C_n^{3/2}(\xi)\right), \tag{3.33}$$

where $H_{-1} = h_{-1} = 0$ is implied, and

$$\begin{aligned} H_n &\equiv \frac{h_n^{\uparrow\downarrow} - (-1)^n h_n^{\downarrow\uparrow}}{2}, \\ h_n &\equiv \frac{h_n^{\uparrow\downarrow} + (-1)^n h_n^{\downarrow\uparrow}}{2}, \end{aligned} \tag{3.34}$$

for $n = 0, 1, 2, \ldots$ correspond to G-parity conserving and G-parity violating contributions, respectively. Note that the coefficient in front of each orthogonal polynomial in (3.32) and (3.33) does not correspond to a definite conformal spin; in contrast to (3.25), (3.30), and (3.31), it is rather given by difference between the contributions of two successive conformal spins.

The conformal expansion of the twist 3 three-particle distribution amplitude gives yet another example for the general expression Eq. (3.24). The expansion reads

$$\mathcal{T}(\alpha_d, \alpha_u, 1 - \alpha_d - \alpha_u) = 360\alpha_d\alpha_u(1 - \alpha_d - \alpha_u)^2 \sum_{k,l=0}^{\infty} \omega_{k,l}^{T} J_{k,l}(\alpha_d, \alpha_u), \tag{3.35}$$

where $J_{k,l}(\alpha_d, \alpha_u) \equiv J_{k,l}(6, 2, 2, \alpha_d, \alpha_u)$ are particular Appell polynomials of two variables (see p269 of [19]). The conformal spin of a generic term in this expression equals $j = k + l + 7/2$ and is the same for all contributions with equal sum $n = k + l$. This illustrates that three-particle conformal representations are degenerate; the number of independent operators with the same spin in fact increases with the spin. Conformal symmetry does not allow mixing between contributions with different $j = n + 7/2$; it does allow, however, mixing with each other of different states with the same value of j. Therefore, the mixing matrix for higher twist operators becomes only block-diagonal in the conformal basis, instead of being diagonalized like in leading twist.

Next, we are going to demonstrate that conformal expansion is fully consistent with the equations of motion. To this end we need to show that the conformal expansion coefficients for two-particle twist 3 distributions can be expressed in terms of the expansion coefficients for three-particle distributions with the same conformal spin, and we need to separate the Wandzura-Wilczek contributions. The calculation is straightforward, although somewhat tedious.

We decompose

$$H_n = H_n^{WW} + H_n^g, \qquad h_n = h_n^{WW} + h_n^g, \tag{3.36}$$

and start with the Wandzura-Wilczek contributions (3.21) and (3.22) which give rise to auxiliary amplitudes $h^{\uparrow\downarrow WW}(u)$ with

$$h^{\uparrow\downarrow WW}(u) = 2\bar{u}\left(-\int_0^u dv\frac{\phi_\perp(v)}{\bar{v}} + \int_u^1 dv\frac{\phi_\perp(v)}{v}\right), \tag{3.37}$$

$$h^{\downarrow\uparrow WW}(u) = 2u\left(-\int_0^u dv\frac{\phi_\perp(v)}{\bar{v}} + \int_u^1 dv\frac{\phi_\perp(v)}{v}\right). \tag{3.38}$$

The integrals on the right-hand side of (3.37) and (3.38) can easily be done using (A.3) and (A.12):

$$-\int_0^u dv\frac{\phi_\perp(v)}{\bar{v}} + \int_u^1 dv\frac{\phi_\perp(v)}{v} = -3\sum_{n=0}^{\infty} a_n^\perp P_{n+1}^{(0,0)}(\xi). \tag{3.39}$$

Substituting the recurrence relations for Jacobi polynomials, (A.7), into this result, one immediately obtains

$$H_n^{WW} = \frac{3(n+1)}{2n+3}a_n^\perp; \qquad h_n^{WW} = -\frac{3(n+1)}{2n+1}a_{n-1}^\perp \qquad (n = 0, 2, 4, \ldots),$$

$$H_n^{WW} = -\frac{3(n+1)}{2n+1}a_{n-1}^\perp; \qquad h_n^{WW} = \frac{3(n+1)}{2n+3}a_n^\perp \qquad (n = 1, 3, 5, \ldots). \tag{3.40}$$

For even n, we find that $a_n^\perp$ which corresponds to the conformal spin $n+2$ in the expansion for the twist 2 distribution amplitude gives rise to H_n^{WW} and H_{n+1}^{WW} which corresponds to the conformal spin $n+3/2$ and $n+5/2$, respectively. Likewise for odd n, $a_n^\perp$ gives rise to h_n^{WW} and h_{n+1}^{WW}. These values of the conformal spin do not match the expansion in Eqs. (3.30) and (3.31). This is, however, not a contradiction since Wandzura-Wilczek terms are in fact not intrinsic twist 3 distributions, but correspond to matrix elements of twist 2 operators over ρ mesons with different (longitudinal) polarization. To relate matrix elements of conformal operators over longitudinal and transverse ρ mesons, one has to perform a spin rotation (in the ρ meson rest frame) which does not commute with the generators of collinear conformal group. As shown in App. B, this rotation gives rise to the shift in conformal spin and exactly explains the mismatch appearing in Eq. (3.40). Therefore, the conformal symmetry is realized in Wandzura-Wilczek contributions as well, but to see this one has to supplement the conformal classification of operators by conformal transformation properties of the meson states.

The three-particle contributions H_n^g and h_n^g of (3.36) can be treated similarly. From the solutions for $h_\parallel^{(t)g}(u)$ and $h_\parallel^{(s)g}(u)$ in Sec. 3.1 we obtain the corresponding auxiliary amplitudes:

$$h^{\uparrow\downarrow g} = \zeta_{3\rho}^T\left[\bar{u}\left(-\int_0^u dv\frac{K(v)}{\bar{v}} + \int_u^1 dv\frac{K(v)}{v}\right) + \frac{d}{du}\int_0^u d\alpha_d\int_0^{\bar{u}} d\alpha_u\frac{1}{\alpha_g}\mathcal{T}(\underline{\alpha})\right], \tag{3.41}$$

$$h^{\downarrow\uparrow g} = \zeta_{3\rho}^T \left[u \left(-\int_0^u dv \frac{K(v)}{\bar{v}} + \int_u^1 dv \frac{K(v)}{v} \right) - \frac{d}{du} \int_0^u d\alpha_d \int_0^{\bar{u}} d\alpha_u \frac{1}{\alpha_g} \mathcal{T}(\underline{\alpha}) \right], \quad (3.42)$$

where $\alpha_g = 1 - \alpha_d - \alpha_u$, and

$$K(u) = \frac{d}{du} \int_0^1 d\alpha_d \int_0^{\bar{u}} d\alpha_u \frac{1}{\alpha_g} \left(\alpha_d \frac{d}{d\alpha_d} + \alpha_u \frac{d}{d\alpha_u} - 1 \right) \mathcal{T}(\underline{\alpha}). \quad (3.43)$$

Substitution of the conformal expansion (3.35) into (3.43) yields

$$K(u) = 180\, u\bar{u} \sum_{k,l=0}^{\infty} \omega_{k,l}^T \frac{k!\, l!\, (-1)^k}{(k+l+2)!} \left(\frac{k-l}{(k+l+3)} P_{k+l+2}^{(1,1)}(\xi) + P_{k+l+1}^{(1,1)}(\xi) \right), \quad (3.44)$$

where we have used Eq. (A.15) to perform the integration. The final integration involving $K(v)$ on the right-hand side of (3.41) and (3.42) can be done similarly to (3.39) by using (A.3):

$$-\int_0^u dv \frac{K(v)}{\bar{v}} + \int_u^1 dv \frac{K(v)}{\bar{v}} = 180 \sum_{k,l=0}^{\infty} \omega_{k,l}^T \frac{k!\, l!\, (-1)^{k+1}}{(k+l+3)!} \left(\frac{k-l}{k+l+4} P_{k+l+3}^{(0,0)}(\xi) + P_{k+l+2}^{(0,0)}(\xi) \right), \quad (3.45)$$

and the last term of (3.41) and (3.42) can be integrated using (A.16):

$$\frac{d}{du} \int_0^u d\alpha_d \int_0^{\bar{u}} d\alpha_u \frac{1}{\alpha_g} \mathcal{T}(\underline{\alpha}) = 180 u\bar{u} \sum_{k,l=0}^{\infty} \omega_{k,l}^T \frac{k!\, l!(-1)^k}{(k+l+3)!} \left(\frac{k-l}{k+l+3} P_{k+l+2}^{(1,1)}(\xi) - P_{k+l+1}^{(1,1)}(\xi) \right). \quad (3.46)$$

Substituting (3.45) and (3.46) into (3.41), (3.42), and using the identities (A.5)–(A.7), we find

$$H_n^g = 180 \zeta_{3\rho}^T \sum_{k=0}^{n-2} \frac{k!(n-k-2)!(k+2)}{(n+2)!} (-1)^{n-k} \omega_{[k,n-k-2]}^T \qquad (n = 3, 4, 5, \ldots), \quad (3.47)$$

$$h_n^g = -180 \zeta_{3\rho}^T \sum_{k=0}^{n-2} \frac{k!(n-k-2)!(k+2)}{(n+2)!} (-1)^{n-k} \omega_{\{k,n-k-2\}}^T \qquad (n = 2, 3, 4, \ldots), \quad (3.48)$$

while $H_0^g = H_1^g = H_2^g = h_0^g = h_1^g = 0$. Here we introduced the following quantities:

$$\omega_{[k,l]}^T \equiv \frac{\omega_{k,l}^T - \omega_{l,k}^T}{2}, \quad (3.49)$$

$$\omega_{\{k,l\}}^T \equiv \frac{\omega_{k,l}^T + \omega_{l,k}^T}{2}. \quad (3.50)$$

We find that the coefficients $\omega_{k,l}^T$ with fixed $k+l = n-2$, which correspond to the conformal spin $j = n + 3/2$, all contribute to H_n^g and h_n^g corresponding to the same conformal spin

$j = n + 3/2$, as anticipated. For later convenience, we give the lowest order coefficients (3.47) and (3.48) explicitly:

$$H_3^g = \frac{15}{2}\zeta_{3\rho}^T\omega_{[1,0]}^T = -70\zeta_{3\rho}^T, \quad H_4^g = \zeta_{3\rho}^T\omega_{[2,0]}^T, \quad H_5^g = \zeta_{3\rho}^T\left(\frac{3}{2}\omega_{[3,0]}^T - \frac{1}{2}\omega_{[2,1]}^T\right), \ldots, \tag{3.51}$$

$$h_2^g = -15\zeta_{3\rho}^T\omega_{\{0,0\}}^T, \quad h_3^g = -\frac{3}{2}\zeta_{3\rho}^T\omega_{\{1,0\}}^T, \quad h_4^g = \zeta_{3\rho}^T\left(\frac{3}{4}\omega_{\{1,1\}}^T - 3\omega_{\{2,0\}}^T\right), \ldots, \tag{3.52}$$

where we substitute $\omega_{[0,1]}^T = 28/3$, which follows from the normalization condition (2.29).

From (3.36), (3.40), and (3.47) it follows that the two lowest coefficients H_0 and H_1 are completely determined by the value of $a_0^\perp = 1$, which results in $H_0 = 1$ and $H_1 = -2$. It is easy to see that these values for H_0 and H_1 ensure $\int_0^1 du\, h_\parallel^{(t)}(u) = \int_0^1 du\, h_\parallel^{(s)}(u) = 1$ and therefore, the normalization condition for $\phi_\perp(u)$ ensures correct normalization of $h_\parallel^{(t)}(u)$ and $h_\parallel^{(s)}(u)$.

To summarize, we have demonstrated that the equations of motion that relate different twist 3 operators can be solved order by order in the conformal expansion. In other words, equations of motion impose "horizontal" relations between operators of the same conformal spin and do not involve other spins. This picture is somewhat complicated by the Wandzura-Wilczek contributions of the operators of lower (leading) twist which have a more peculiar structure. The explicit relations derived above can be made somewhat more compact by assuming G-parity invariance. In this case $a_n^\perp = 0$ for odd n, $\omega_{k,l}^T = -\omega_{l,k}^T$, $H_n = h_n^{\uparrow\downarrow}$, and $h_n = 0$. As mentioned above, the G-parity violating terms are only relevant for SU(3) breaking corrections in the distribution amplitudes of K^* mesons.

3.3 Renormalization and scale-dependence

The scale-dependence of the chiral-odd distribution amplitudes $\phi_\perp(u,\mu^2)$, $h_\parallel^{(t)}(u,\mu^2)$ and $h_\parallel^{(s)}(u,\mu^2)$ is governed by the renormalization group (RG) equation for the relevant nonlocal light-cone operators appearing in the definitions (2.17), (2.18), and (2.26). Unlike inclusive processes, operators involving total derivatives have to be taken into account since they contribute to nonforward matrix elements, and this leads to additional operator mixing. For example, consider the set of local operators $\bar{u}(0)(\overleftarrow{D}.)^{n-k}\sigma_\perp.(\overrightarrow{D}.)^k d(0)$ $(k = 0, 1, 2, \ldots)$, which contribute to the n-th moment of the twist 2 distribution amplitude $\phi_\perp(u,\mu^2)$. These operators differ by total derivatives and all mix with each other under renormalization; to calculate the scale-dependence, one has to find the eigenvalues and eigenvectors of the corresponding anomalous dimension matrix.

As is well known [15, 16, 18], conformal expansion provides the solution to this problem. Analysis based on the anomalous Ward identities for the dilatation and special conformal transformation (which are members of the conformal group) shows [18] that, to leading logarithmic accuracy, the conformal operators with different conformal spin do not mix with each other and thus diagonalize the anomalous dimension matrix. As a result, by employing a conformal operator basis we do not encounter any additional operator mixing compared to inclusive processes. The relevant anomalous dimensions, which correspond to the eigenvalues of the anomalous dimension matrix, can be extracted directly from the results for renormalization of the corresponding parton distribution functions. In particular, the one-loop anomalous dimensions for $h_\parallel^{(t)}(u,\mu^2)$ and $h_\parallel^{(s)}(u,\mu^2)$ are the same as for the chiral-odd parton distribution functions [21, 22, 23, 24] as will be shown in the following.

Our main task in this section is to reveal the explicit operator content of the conformal operators, corresponding to particular coefficients in the conformal expansions (3.32), (3.33), (3.25), and (3.35). We give a one-to-one correspondence between the conformal basis and the basis used in the inclusive case. This allows us to determine the anomalous dimensions of the conformal operators and to find the evolution of the distribution amplitudes $h_\parallel^{(t)}(u,\mu^2)$ and $h_\parallel^{(s)}(u,\mu^2)$ through the conformal expansion. We will work out this program for arbitrary conformal spin.

One complication is that the conformal operator basis for the higher twist operators is degenerate (see (3.35)) and the mixing matrix becomes only block-diagonal instead of being fully diagonalized like in leading twist. Consequently, the conformal expansion for three-particle contributions to $h_\parallel^{(t)}(u,\mu^2)$ and $h_\parallel^{(s)}(u,\mu^2)$ does not resolve possible mixing between components with the same conformal spin. This is similar to mixing of the many quark-gluon correlation operators for the corresponding twist 3 parton distribution functions [21, 22, 23]. It has been shown recently [25, 26], however, that an important simplification occurs in the limit of a large number of colors or of large spin (moment of parton distribution function). In these limits all complicated mixing disappears and the twist 3 parton distribution functions obey simple DGLAP-type evolution. We will demonstrate that the twist 3 distribution amplitudes obey a similar pattern.

Let us start with the Wandzura-Wilczek terms $h_\parallel^{(t)WW}(u,\mu^2)$ and $h_\parallel^{(s)WW}(u,\mu^2)$ (see (3.32), (3.33), and (3.36)). The coefficients H_n^{WW} and h_n^{WW} in their conformal expansions have been expressed by $a_k^\perp$ of (3.25) as shown in (3.40). Therefore, the scale-dependence of $a_k^\perp$, i.e. of the twist 2 amplitude $\phi_\perp(u,\mu^2)$, completely determines that of $h_\parallel^{(t)WW}(u,\mu^2)$ and $h_\parallel^{(s)WW}(u,\mu^2)$. From (3.25) and orthogonality relations of Gegenbauer polynomials [19], we obtain

$$a_n^\perp(\mu^2) = \frac{2(2n+3)}{3(n+1)(n+2)} \int_0^1 du\, C_n^{3/2}(\xi)\phi_\perp(u,\mu^2). \tag{3.53}$$

Substituting (2.17) into the right-hand side of (3.53) gives

$$\left(f_\rho^T a_n^\perp\right)(\mu^2) = i\frac{2(2n+3)}{3(n+1)(n+2)}\frac{1}{(p\cdot z)^{n+1}}\langle 0|\Omega_n^\perp(0;\mu^2)|\rho^-(P,\lambda)\rangle \tag{3.54}$$

with

$$\Omega_n^\perp(x;\mu^2) = (i\partial.)^n\, \bar{u}(x) e_\perp^{(\lambda)\nu}\sigma_\nu . C_n^{3/2}\left(\frac{\overleftrightarrow{D}.}{\partial.}\right) d(x), \tag{3.55}$$

where the local operator on the right-hand side is renormalized at μ^2, $\overleftrightarrow{D}=\overrightarrow{D}-\overleftarrow{D}$, and ∂_μ is the total derivative (3.3). $\Omega_n^\perp(x,\mu^2)$ is the conformal operator of conformal spin $j=n+2$

[16, 18][3]. Therefore, it is RG covariant to leading logarithmic accuracy and satisfies the RG equation:

$$\left(\mu\frac{\partial}{\partial\mu}+\beta(g)\frac{\partial}{\partial g}+\frac{\alpha_s}{2\pi}\gamma_n^{\perp}\right)\Omega_n^{\perp}(x;\mu^2)=0, \tag{3.56}$$

where $\gamma_n^{\perp}$ is the one-loop anomalous dimension of the operator $\Omega_n^{\perp}$. To establish a formal connection with the results given in the literature, it is convenient to consider the case where $\Omega_n^{\perp}$ is diagonal in quark flavour corresponding to the flavour matrices λ_3, λ_8, and to take the forward matrix element of (3.56) over the nucleon state $|N(P,S)\rangle$. Because the total derivatives drop out in this matrix element, (3.56) reduces to the RG equation for $\langle N(P,S)|\overline{\psi}(0)\sigma_{\perp\cdot}(iD_\cdot)^n\psi(0)|N(P,S)\rangle$, which gives the n-th moment of the nucleon parton distribution function $h_1(x,\mu^2)$ of (2.13). By matching with the results for renormalization of $h_1(x,\mu^2)$ [24], we obtain $\gamma_n^{\perp}$ as

$$\gamma_n^{\perp}=4C_F\left(\psi(n+1)+\gamma_E-\frac{3}{4}+\frac{1}{n+1}\right), \tag{3.57}$$

where $\psi(n+1)=\sum_{k=1}^{n}1/k-\gamma_E$ is the digamma function, γ_E is the Euler constant, and $C_F=(N_c^2-1)/2N_c$. From (3.54)–(3.57), we obtain

$$\left(f_\rho^T a_n^{\perp}\right)(Q^2)=L^{\gamma_n^{\perp}/b}\left(f_\rho^T a_n^{\perp}\right)(\mu^2), \tag{3.58}$$

where $L\equiv\alpha_s(Q^2)/\alpha_s(\mu^2)$ and $b=(11N_c-2N_f)/3$. Combined with (3.32), (3.33) and (3.40), this result gives the μ^2-dependence of $h_{\parallel}^{(t)WW}(u,\mu^2)$ and $h_{\parallel}^{(s)WW}(u,\mu^2)$ and also determines evolution of the twist 2 distribution amplitude $\phi_{\perp}(u,\mu^2)$ of (3.25). We note that (3.58) for $n=0$ gives the scale-dependence of the tensor decay constant f_ρ^T as

$$f_\rho^T(Q^2)=L^{C_F/b}f_\rho^T(\mu^2), \tag{3.59}$$

because $a_0^{\perp}=1$.

The three-particle contributions $h_{\parallel}^{(t)g}(u,\mu^2)$ and $h_{\parallel}^{(s)g}(u,\mu^2)$ can be treated in a similar manner, although the discussion becomes more complicated because one has to deal with a degenerate representation of the conformal group (see (3.32), (3.33) and (3.36)). The relevant expansion coefficients H_n^g and h_n^g are expressed by $\omega_{k,l}^T$ in (3.35) and shown in (3.47) and (3.48). Thus the first step is to demonstrate that $\omega_{k,l}^T$ are given by matrix elements of the local conformal operators derived in [18]. Using (3.35) and the orthogonality relations (A.13) for the Appell polynomials, we obtain

$$\int \mathcal{D}\underline{\alpha}\, J_{k,n-k-2}(\alpha_d,\alpha_u)\mathcal{T}(\underline{\alpha})=\frac{360(-1)^n}{2^{n+1}(n+1)(2n+1)!!}\sum_{r=0}^{n-2}\omega_{r,n-r-2}^T W_{n-r-2,k}^{(n-1)} \tag{3.60}$$

[3]In principle, one can construct a tower of conformal operators $(\partial_\cdot)^m\Omega_n^{\perp}$ $(m=0,1,\ldots)$ with the same conformal spin, but with the different "third component" of it.

for $k = 0, 1, \ldots, n-2$, and $n = 2, 3, \ldots$ Making use of

$$\langle 0|\bar{u}(tz)\sigma^{\nu}.[tz, vz]gG_{\nu}.(vz)[vz, wz]d(wz)|\rho^-(P,\lambda)\rangle =$$

$$= (p\cdot z)(e^{(\lambda)}\cdot z) f_{3\rho}^T m_\rho \int \mathcal{D}\underline{\alpha}\, e^{-ip\cdot z(t\alpha_u + w\alpha_d + v\alpha_g)}\mathcal{T}(\underline{\alpha}), \tag{3.61}$$

which is equivalent to (2.26), the left-hand side of (3.60) gives

$$\frac{1}{f_{3\rho}^T m_\rho}\frac{1}{(e^{(\lambda)}\cdot z)(p\cdot z)^{n-1}}\langle 0|\Lambda^T_{k,n-k-2}(0)|\rho^-(P,\lambda)\rangle, \tag{3.62}$$

with

$$\Lambda^T_{k,l}(0) = (i\partial.)^{k+l}\, J_{k,l}\left(\frac{D^y_.}{\partial.}, \frac{D^x_.}{\partial.}\right)\bar{u}(x)\sigma^{\nu}.gG_{\nu}.(0)d(y)\Bigg|_{x=y=0}, \tag{3.63}$$

where the covariant derivatives D^x_μ and D^y_μ act on the coordinates x and y, respectively. $\Lambda^T_{k,n-k-2}$ ($k = 0, 1, \ldots, n-2$) are the twist 3 conformal operators of spin $j = n + 3/2$, forming a degenerate basis for three-particle representation [18].[4] Inverting the matrix $W^{(n-1)}_{k',k}$ in (3.60), we obtain

$$f_{3\rho}^T \omega^T_{k,n-k-2} = \frac{N^T_n(-1)^k}{90k!(n-k-2)!}\langle 0|\Theta^T_{k,n-k-2}(0)|\rho^-(P,\lambda)\rangle, \tag{3.64}$$

where N^T_n is the dimensionless and scale-independent constant:

$$N^T_n \equiv \frac{2^{n-1}(n+1)(2n+1)!!}{m_\rho(e^{(\lambda)}\cdot z)(p\cdot z)^{n-1}}. \tag{3.65}$$

The numerical factor in front of the matrix element (3.64) is put for later convenience. The operators $\Theta^T_{k,n-k-2}$ are given by linear combinations of $\Lambda^T_{r,n-r-2}$ ($r = 0, 1, \ldots, n-2$) and therefore have conformal spin $j = n + 3/2$.

The second step is to determine the explicit form of the relevant conformal operator $\Theta^T_{k,n-k-2}$. For the few lowest conformal spins, it is easy to express $\Theta^T_{k,n-k-2}$ as a linear combination of $\Lambda^T_{r,n-r-2}$ using (3.60)–(3.63), but the procedure becomes more complicated for higher conformal spins. Using orthogonality relations for the Appell polynomials, (A.14), it proves possible, however, to determine $\Theta^T_{k,n-k-2}$ for general n up to total derivatives.

$$\int \mathcal{D}\underline{\alpha}\, \alpha_d^k \alpha_u^{n-k-2}\mathcal{T}(\underline{\alpha}) =$$

$$= \omega^T_{k,n-k-2}\frac{360(-1)^n k!(n-k-2)!}{2^{n+1}(n+1)(2n+1)!!} + (\text{terms involving } \omega^T_{l,r-l-2}\Big|_{r<n})$$

$$= \frac{1}{f_{3\rho}^T m_\rho(e^{(\lambda)}\cdot z)(p\cdot z)^{n-1}}\langle 0|\bar{u}(0)(i\overleftarrow{D}.)^{n-k-2}\sigma^{\nu}.gG_{\nu}.(0)(i\overrightarrow{D}.)^k d(0)|\rho^-(P,\lambda)\rangle, \tag{3.66}$$

[4]One can generate a tower of conformal operators with a different "third component" of conformal spin by acting repeatedly with $\partial.$ on (3.63).

where the last line is obtained by substituting (3.61) into the left-hand side. From (3.60)–(3.63) it follows that $\omega^T_{k,n-k-2}$ are given by the matrix elements of the operators of dimension $n+3$. This implies that "(terms involving $\omega^T_{l,r-l-2}|_{r<n}$)" corresponds to matrix elements of operators involving total derivatives, which are given by linear combinations of terms $\sim (\partial.)^{n-r}\,\Lambda^T_{k,r-k-2}$ $(k=0,1,\ldots,r-2;\; r=2,3,\ldots,n-1)$. Thus we conclude, from (3.64) and (3.66), that

$$\Theta^T_{k,n-k-2}(0) = \bar{u}(0)(-i\overleftarrow{D}.)^{n-k-2}\sigma^{\nu}.gG_{\nu}.(0)(i\overrightarrow{D}.)^k d(0) + (\text{total derivatives}). \tag{3.67}$$

The operators $\Theta^T_{k,n-k-2}$ $(k=0,1,\ldots,n-2)$ have the same conformal spin and may mix with each other, although they do not mix with $\Theta^T_{l,r-l-2}$ for $n \neq r$. To write down the corresponding RG equation, it is convenient to introduce combinations that are even and odd under the substitution $k \to n-k-2$:

$$S^{\pm}_{n;k} \equiv \frac{\Theta^T_{k,n-k-2} \pm \Theta^T_{n-k-2,k}}{2} \qquad (k=0,1,\ldots,\kappa^{\pm}_n), \tag{3.68}$$

where

$$\kappa^+_n = \left[\frac{n}{2}\right] - 1, \qquad \kappa^-_n = \left[\frac{n-1}{2}\right] - 1. \tag{3.69}$$

It is straightforward to see that $S^+_{n;k}$ and $S^-_{n;k}$ possess opposite "parity" under the G-parity transformation. Therefore, these two sets of the operators do not mix with each other. The RG equations are given by

$$\left(\mu\frac{\partial}{\partial\mu} + \beta(g)\frac{\partial}{\partial g}\right) S^{\pm}_{n;k}(0;\mu^2) = -\frac{\alpha_s}{2\pi}\sum_{l=0}^{\kappa^{\pm}_n}\left(\Gamma^{T\pm}_n\right)_{k,l} S^{\pm}_{n;l}(0;\mu^2), \tag{3.70}$$

where the one-loop anomalous dimension Γ^{T+}_n (Γ^{T-}_n) is a $[n/2]\times[n/2]$ $([(n-1)/2]\times[(n-1)/2])$ matrix, describing the mixing. Note that the number of independent operators and thus the size of the mixing matrix increase with conformal spin.

In order to determine the anomalous dimension matrices $\Gamma^{T\pm}_n$ in (3.70) we make contact with inclusive processes. Similarly to the discussion of the Wandzura-Wilczek part, we consider the case where the operators $S^{\pm}_{n;k}$ are flavour-diagonal and take the forward matrix element of (3.70) in between nucleon states $|N(P,S)\rangle$. The total derivatives in (3.67) drop out, and (3.70) reduces to the RG equations for

$$\langle N(P,S)|\overline{\psi}(i\overleftarrow{D}.)^{n-k-2}\sigma^{\nu}.gG_{\nu}.(i\overrightarrow{D}.)^k\psi \pm (k \to n-k-2)|N(P,S)\rangle,$$

familiar from studies of the evolution of the n-th moment of the nucleon parton distributions $e(x,\mu^2)$ and $h_L(x,\mu^2)$ defined in (2.14) and (2.13). It is straightforward to see that these operators for the upper and lower sign, which are even and odd under $k \to n-k-2$, coincide exactly with the basis employed in renormalizing the parton distribution functions

$e(x,\mu^2)$ [22] and $h_L(x,\mu^2)$ [21], respectively.[5] Matching with these results, we obtain

$$\left(\Gamma_n^{T+}\right)_{k,l} = -Y_{k+2,l+2} \qquad (k,l=0,1,\ldots,\kappa_n^+), \tag{3.71}$$

$$\left(\Gamma_n^{T-}\right)_{k,l} = -X_{k+2,l+2} \qquad (k,l=0,1,\ldots,\kappa_n^-), \tag{3.72}$$

where $Y_{i,j}$ and $X_{i,j}$ are the mixing matrices in the notation of [22] and [21], i.e. are given by Eqs. (3.12)–(3.16) of [22] and Eqs. (3.14)–(3.16) of [21], respectively. By solving (3.70), we obtain

$$S_{n;k}^{\pm}(0;Q^2) = \sum_{l=0}^{\kappa_n^{\pm}} \left(L^{\Gamma_n^{T\pm}/b}\right)_{k,l} S_{n;l}^{\pm}(0;\mu^2), \tag{3.73}$$

and the matrix elements of $S_{n;k}^{\pm}$ are related to $\omega_{[k,n-k-2]}^T$ and $\omega_{\{k,n-k-2\}}^T$ of (3.49) and (3.50) as (see (3.64) and (3.68))

$$\left(f_{3\rho}^T\omega_{[k,n-k-2]}^T\right)(\mu^2) = \frac{N_n^T(-1)^{n-k+1}}{180k!(n-k-2)!}\langle 0|S_{n;k}^{\mp}(0;\mu^2)|\rho^-(P,\lambda)\rangle, \tag{3.74}$$

$$\left(f_{3\rho}^T\omega_{\{k,n-k-2\}}^T\right)(\mu^2) = \frac{N_n^T(-1)^{n-k}}{180k!(n-k-2)!}\langle 0|S_{n;k}^{\pm}(0;\mu^2)|\rho^-(P,\lambda)\rangle. \tag{3.75}$$

Here the upper (lower) superscript should be understood for $n=2,4,6,\ldots$ ($n=3,5,7,\ldots$) on the right-hand side. The results (3.71)–(3.75), combined with (3.32), (3.33), (3.47), (3.48) and (3.59) give the μ^2-dependence of the three-particle contributions $h_{\parallel}^{(t)g}(u,\mu^2)$ and $h_{\parallel}^{(s)g}(u,\mu^2)$ (recall that $\zeta_{3\rho}^T$ in (3.47) and (3.48) is given by (3.8)). The evolution of the three-particle twist 3 distribution amplitude $\mathcal{T}(\underline{\alpha})$ of (3.35) is thus specified completely. For the first few moments we obtain from (3.71)–(3.75):[6]

$$f_{3\rho}^T(Q^2) = L^{(23C_F+6C_G)/6b} f_{3\rho}^T(\mu^2), \tag{3.76}$$

$$\left(f_{3\rho}^T\omega_{[2,0]}^T\right)(Q^2) = L^{(106C_F+75C_G)/30b}\left(f_{3\rho}^T\omega_{[2,0]}^T\right)(\mu^2), \tag{3.77}$$

$$\begin{pmatrix} 3f_{3\rho}^T\omega_{[0,3]}^T \\ f_{3\rho}^T\omega_{[2,1]}^T \end{pmatrix}^{Q^2} = L^{\Gamma_5^{T+}/b}\begin{pmatrix} 3f_{3\rho}^T\omega_{[0,3]}^T \\ f_{3\rho}^T\omega_{[2,1]}^T \end{pmatrix}^{\mu^2}; \quad \Gamma_5^{T+} = \begin{pmatrix} \frac{41}{10}C_F+\frac{15}{6}C_G, & \frac{29}{15}C_F-\frac{11}{6}C_G \\ \frac{13}{15}C_F-\frac{13}{12}C_G, & \frac{47}{10}C_F+\frac{29}{12}C_G \end{pmatrix}, \tag{3.78}$$

corresponding to $n=3,4,5$. Here $C_G=N_c$, and $\omega_{[0,1]}^T=28/3$ is substituted in (3.76) (see (3.51)). Substitution of (3.76)–(3.78) in (3.51) determines the scale dependence of H_n^g ($n=3,4,5$).

[5]For the case of the parton distribution $h_L(x,\mu^2)$, the relevant quark-antiquark-gluon operator has an additional $i\gamma_5$ inbetween the quark fields. The evolution of the corresponding operator is not affected by the insertion of $i\gamma_5$ [25].

[6]It is worth noting that the evolution of $f_{3\rho}^T\omega_{\{k,n-k-2\}}^T$ coincides exactly with that of the coefficients, appearing in the conformal expansion of the twist 3 three-particle distribution amplitude of the pion [7].

The results in (3.71)–(3.78) illustrate a complicated mixing pattern characteristic for the higher twist operators. In particular, $H_n^g(Q^2)$ for $n \geq 5$ ($h_n^g(Q^2)$ for $n \geq 4$) are not directly related to $H_n^g(\mu^2)$ ($h_n^g(\mu^2)$), in contrast to (3.58) for the twist 2 operator.

There exist, however, two important limits, $N_c \to \infty$ and $n \to \infty$, where the three-particle coefficients H_n^g and h_n^g obey a simple evolution equation. The mathematical reason for this simplification is the same as for the similar simplification observed for the nucleon parton distributions $h_L(x, \mu^2)$ and $e(x, \mu^2)$ in [25, 22, 23]. To show this, it is convenient to express H_n^g and h_n^g directly by matrix elements of $S_{n;k}^{\pm}$, by substituting (3.74) and (3.75) into (3.47) and (3.48). We find

$$\left(f_\rho^T H_n^g\right)(\mu^2) = -\frac{N_n^T}{m_\rho(n+1)!}\sum_{k=0}^{\kappa_n^-}\left(1 - \frac{2(k+2)}{n+2}\right)\langle 0|S_{n;k}^-(0;\mu^2)|\rho^-(P,\lambda)\rangle, \qquad (3.79)$$

$$\left(f_\rho^T h_n^g\right)(\mu^2) = -\frac{N_n^T}{m_\rho(n+1)!}\sum_{k=0}^{\kappa_n^+}\left(1 - \frac{1}{2}\delta_{k,\kappa_n^+}\right)\langle 0|S_{n;k}^+(0;\mu^2)|\rho^-(P,\lambda)\rangle, \qquad (3.80)$$

for $n = 2, 4, 6, \ldots$, and

$$\left(f_\rho^T H_n^g\right)(\mu^2) = -\frac{N_n^T}{m_\rho(n+1)!}\sum_{k=0}^{\kappa_n^+}\langle 0|S_{n;k}^+(0;\mu^2)|\rho^-(P,\lambda)\rangle, \qquad (3.81)$$

$$\left(f_\rho^T h_n^g\right)(\mu^2) = -\frac{N_n^T}{m_\rho(n+1)!}\sum_{k=0}^{\kappa_n^-}\left(1 - \frac{2(k+2)}{n+2}\right)\langle 0|S_{n;k}^-(0;\mu^2)|\rho^-(P,\lambda)\rangle, \qquad (3.82)$$

for $n = 3, 5, 7, \ldots$ Setting $\mu^2 = Q^2$ in (3.79)–(3.82) and substituting (3.73) into them, we would reproduce the complicated mixing discussed above. However, in the large N_c limit, that is, neglecting terms $O(1/N_c^2)$ in the anomalous dimension matrices $\Gamma_n^{T\pm}$ of (3.71) and (3.72), the following exact relations have been derived [25, 22]:

$$\sum_{k=0}^{\kappa_n^+}\left(\Gamma_n^{T+}\right)_{k,l} = \gamma_n^{T+} \qquad (n = 3, 5, 7, \ldots), \qquad (3.83)$$

$$\sum_{k=0}^{\kappa_n^+}\left(1 - \frac{1}{2}\delta_{k,\kappa_n^+}\right)\left(\Gamma_n^{T+}\right)_{k,l} = \left(1 - \frac{1}{2}\delta_{\kappa_n^+,l}\right)\gamma_n^{T+} \qquad (n = 2, 4, 6, \ldots), \qquad (3.84)$$

and

$$\sum_{k=0}^{\kappa_n^-}\left(1 - \frac{2(k+2)}{n+2}\right)\left(\Gamma_n^{T-}\right)_{k,l} = \left(1 - \frac{2(l+2)}{n+2}\right)\gamma_n^{T-} \qquad (n = 3, 4, 5, 6, \ldots), \qquad (3.85)$$

where

$$\gamma_n^{T+} = 2N_c\left(\psi(n+1) + \gamma_E - \frac{1}{4} - \frac{1}{2(n+1)}\right), \qquad (3.86)$$

$$\gamma_n^{T-} = 2N_c \left(\psi(n+1) + \gamma_E - \frac{1}{4} + \frac{3}{2(n+1)} \right). \tag{3.87}$$

As a consequence of these relations, we obtain

$$\left(f_\rho^T H_n^g\right)(Q^2) = L^{\gamma_n^{T\mp}/b} \left(f_\rho^T H_n^g\right)(\mu^2), \tag{3.88}$$

$$\left(f_\rho^T h_n^g\right)(Q^2) = L^{\gamma_n^{T\pm}/b} \left(f_\rho^T h_n^g\right)(\mu^2), \tag{3.89}$$

where the upper (lower) superscript should be understood for $n = 2, 4, 6, \ldots$ ($n = 3, 5, 7, \ldots$) on the right-hand side. Therefore, in the large N_c limit, H_n^g and h_n^g obey simple DGLAP-type evolution equations similarly to the twist 2 case (3.58); they are governed by the anomalous dimensions given in analytic form in (3.86) and (3.87). We note that these anomalous dimensions correspond to the lowest eigenvalues of the mixing matrices $\Gamma_n^{T\pm}$ [25, 22].

The phenomenon leading to (3.88) and (3.89) can be stated as decoupling of the three-particle operators, which correspond to the higher eigenvalues of the mixing matrix, from the RG equation. The same decoupling is observed at large n for arbitrary values of N_c [25, 23]. In this case, we obtain (3.83)–(3.85) with the anomalous dimensions (3.86) and (3.87) shifted by

$$\gamma_n^{T\pm} \to \gamma_n^{T\pm} + (4C_F - 2N_c)\left(\ln n + \gamma_E - \frac{3}{4}\right). \tag{3.90}$$

With this modification of the anomalous dimensions, the results (3.88) and (3.89) are valid to $O\left((1/N_c^2)\ln(n)/n\right)$ accuracy.

These simplifications provide useful insight both into the model-building of the distribution amplitudes, and a convenient description of their scale-dependence: In the large N_c-limit, each conformal partial wave of the three-particle contributions is described by a single nonperturbative parameter, as demonstrated in (3.88) and (3.89). This is remarkable because in general each conformal spin involves many independent nonperturbative matrix elements (see (3.79)–(3.82)). This point can be made stronger with the full account for effects subleading in N_c but for large conformal spin j, as shown in (3.90). Furthermore, (3.90) proves the conjecture made in [10, 7] that the lowest anomalous dimension of the twist 3 three-particle operators is increasing as $\sim \ln j$, similarly to the twist 2 case (3.57). This ensures convergence of the series in the Appell polynomials at least for high energy scales. Combined with the fact that the distribution amplitudes can be resolved order by order in the conformal spin, the truncation of the conformal expansion at some low order provides a useful and consistent approximation of the full amplitude.

To summarize, we have worked out the scale dependence of chiral-odd twist 3 distribution amplitudes $h_\parallel^{(t)}(u,\mu^2)$ and $h_\parallel^{(s)}(u,\mu^2)$ in the leading logarithmic approximation. In the two limits, $N_c \to \infty$ and $n \to \infty$, the evolution of $h_\parallel^{(t)}(u,\mu^2)$ and $h_\parallel^{(s)}(u,\mu^2)$ is drastically simplified and reduces to a DGLAP-type equation. The discussion in this section completes the results for the chiral-odd distribution amplitudes up to twist 3, which can be predicted based on the QCD constraints from equations of motion, conformal invariance, and renormalization group invariance.

4 Chiral-even Distribution Amplitudes

The analysis in the previous section can directly be extended to the chiral-even distribution amplitudes. In Sec. 4.1 we derive the constraint equations for $g_\perp^{(v)}(u)$ and $g_\perp^{(a)}(u)$ imposed by the QCD equations of motion, and identify the contribution to $g_\perp^{(v)}(u)$ and $g_\perp^{(a)}(u)$ from the twist 2 distribution amplitudes $\phi_\parallel(u)$ and $\phi_\perp(u)$ and the three-particle distribution amplitudes $\mathcal{V}(\underline{\alpha})$ and $\mathcal{A}(\underline{\alpha})$. In Sec. 4.2, we study the conformal expansion for $\phi_\parallel(u)$, $g_\perp^{(v)}(u)$ and $g_\perp^{(a)}(u)$. In Sec. 4.3, we work out the renormalization of $g_\perp^{(v)}(u)$ and $g_\perp^{(a)}(u)$, utilizing the fact that the conformal symmetry is preserved at one-loop level. Our presentation in this section will be brief, since the methods and the results are in parallel with those in Sec. 3.

4.1 Equations of motion

To derive the constraint relations among the chiral-even distribution amplitudes we again use operator identities (to twist 3 accuracy) for the nonlocal operators in (2.19) and (2.20) [10]:

$$\begin{aligned}\overline{u}(x)\gamma_\mu[x,-x]d(-x) = & \int_0^1 dt\, \frac{\partial}{\partial x_\mu}\,\overline{u}(tx)\not{x}[tx,-tx]d(-tx)\\ & - \int_0^1 dt\, t\int_{-t}^{t} dv\, \overline{u}(tx)[tx,vx]g\tilde{G}_{\mu\nu}(vx)x^\nu\not{x}\gamma_5[vx,-tx]d(-tx)\\ & - i\int_0^1 dt\int_{-t}^{t} dv\, v\,\overline{u}(tx)[tx,vx]gG_{\mu\nu}(vx)x^\nu[vx,-tx]\not{x}d(-tx)\\ & - i\epsilon_{\mu\nu\alpha\beta}\int_0^1 dt\, t\, x^\nu\partial^\alpha\left[\overline{u}(tx)\gamma^\beta\gamma_5[tx,-tx]d(-tx)\right]\\ & + (m_u-m_d)x^\nu\int_0^1 dt\, t\,\overline{u}(tx)\sigma_{\nu\mu}[tx,-tx]d(-tx), \end{aligned} \tag{4.1}$$

and

$$\begin{aligned}\overline{u}(x)\gamma_\mu\gamma_5[x,-x]d(-x) = & \int_0^1 dt\, \frac{\partial}{\partial x_\mu}\,\overline{u}(tx)\not{x}\gamma_5[tx,-tx]d(-tx)\\ & - \int_0^1 dt\, t\int_{-t}^{t} dv\, \overline{u}(tx)[tx,vx]g\tilde{G}_{\mu\nu}(vx)x^\nu\not{x}[vx,-tx]d(-tx)\\ & - i\int_0^1 dt\int_{-t}^{t} dv\, v\,\overline{u}(tx)[tx,vx]gG_{\mu\nu}(vx)x^\nu[vx,-tx]\not{x}\gamma_5 d(-tx)\\ & - i\epsilon_{\mu\nu\alpha\beta}\int_0^1 dt\, t\, x^\nu\partial^\alpha\left[\overline{u}(tx)\gamma^\beta[tx,-tx]d(-tx)\right]\\ & + (m_u+m_d)x^\nu\int_0^1 dt\, t\,\overline{u}(tx)\sigma_{\nu\mu}\gamma_5[tx,-tx]d(-tx), \end{aligned} \tag{4.2}$$

where ∂_α is the total derivative defined in (3.3), and the terms proportional to quark masses originate from the use of QCD equation of motion. By sandwiching these equations between the vacuum and the ρ meson state, and taking the light-cone limit $x \to z$, one obtains the following relations among the distribution amplitudes:

$$\begin{aligned}
\int_0^1 du\, e^{i\xi pz} g_\perp^{(v)}(u) \;&=\; \int_0^1 dt \int_0^1 du\, e^{it\xi pz}\phi_\parallel(u) - \zeta_{3\rho}^A (pz)^2 \int t^2 dt \int_{-1}^1 dv\, \mathcal{A}(v, tpz) \\
&- \zeta_{3\rho}^V (pz)^2 \int_0^1 dt\, t^2 \int_{-1}^1 dv\, v\, \mathcal{V}(v, tpz) - \frac{1}{2}(pz)^2 \left(1 - \tilde{\delta}_+\right) \int_0^1 dt\, t^2 \int_0^1 du\, e^{it\xi pz} g_\perp^{(a)}(u) \\
&- i\tilde{\delta}_-(pz) \int_0^1 dt\, t \int_0^1 du\, e^{it\xi pz}\phi_\perp(u), \qquad (4.3)
\end{aligned}$$

and

$$\begin{aligned}
\frac{1}{2}\left(1 - \tilde{\delta}_+\right) \int_0^1 du\, e^{i\xi pz} g_\perp^{(a)}(u) \;&=\; \int_0^1 dt\, t \int_0^1 du\, e^{it\xi pz} g_\perp^{(v)}(u) \\
&+ i\zeta_{3\rho}^V (pz) \int_0^1 dt\, t^2 \int_{-1}^1 dv \mathcal{V}(v, tpz) + i\zeta_{3\rho}^A (pz) \int_0^1 dt\, t^2 \int_{-1}^1 dv\, v \mathcal{A}(v, tpz) \\
&- \tilde{\delta}_+ \int_0^1 dt\, t \int_0^1 du\, e^{it\xi pz}\phi_\perp(u), \qquad (4.4)
\end{aligned}$$

where $\mathcal{V}(v, tpz)$ and $\mathcal{A}(v, tpz)$ are defined in (2.27) and we introduced the notations

$$\tilde{\delta}_\pm \equiv \frac{f_\rho^{T^2}}{f_\rho^2}\delta_\pm = \frac{f_\rho^T}{f_\rho}\frac{m_u \pm m_d}{m_\rho}, \qquad \zeta_{3\rho}^{V,A} = \frac{f_{3\rho}^{V,A}}{f_\rho m_\rho}. \qquad (4.5)$$

In order to solve these equations, we expand them in powers of pz and transform them into relations among the moments of distribution amplitudes:

$$\begin{aligned}
(n+1) M_n^{(v)} = M_n^\parallel + \frac{n(n-1)}{2}\left(1 - \tilde{\delta}_+\right) M_{n-2}^{(a)} + \zeta_{3\rho}^A n(n-1) \int_{-1}^{1} dv\, \mathcal{A}_{n-2}(v) \\
+ \zeta_{3\rho}^V n(n-1) \int_{-1}^{1} dv\, v\, \mathcal{V}_{n-2}(v) - \tilde{\delta}_- n M_{n-1}^\perp, \qquad (4.6)
\end{aligned}$$

and

$$\frac{1}{2}(n+2)\left(1 - \tilde{\delta}_+\right) M_n^{(a)} = M_n^{(v)} + \zeta_{3\rho}^V n \int_{-1}^{1} dv\, \mathcal{V}_{n-1}(v) + \zeta_{3\rho}^A n \int_{-1}^{1} dv\, v\, \mathcal{A}_{n-1}(v) - \tilde{\delta}_+ M_n^\perp, \qquad (4.7)$$

where $\mathcal{V}_n(v)$ and $\mathcal{A}_n(v)$ are defined similarly to (3.10) from $\mathcal{V}(\underline{\alpha})$ and $\mathcal{A}(\underline{\alpha})$, and we introduced the shorthand notations $M_n^{(a),(v)} \equiv \int_0^1 du\, \xi^n\, g_\perp^{(a),(v)}$. From these equations, one obtains a recurrence relation for $M_n^{(a)}$ as

$$\begin{aligned}
\left(1-\widetilde{\delta}_+\right)\left((n+2)(n+1)M_n^{(a)} - n(n-1)M_{n-2}^{(a)}\right) &= \\
&= 2M_n^{\parallel} + 2\zeta_{3\rho}^V \int_{-1}^1 dv\, \left[n(n+1)\mathcal{V}_{n-1}(v) + n(n-1)v\mathcal{V}_{n-2}(v)\right] \\
&\quad + 2\zeta_{3\rho}^V \int_{-1}^1 dv\, \left[n(n+1)v\mathcal{A}_{n-1}(v) + n(n-1)\mathcal{A}_{n-2}(v)\right] \\
&\quad - 2(n+1)\widetilde{\delta}_+ M_n^\perp - 2n\widetilde{\delta}_- M_{n-1}^\perp.
\end{aligned} \tag{4.8}$$

This equation is similar to (3.14) for $h_\parallel^{(s)}(u)$, and can be cast into the form of a differential equation as

$$\left(1-\widetilde{\delta}_+\right) u\bar{u}(g_\perp^{(a)})''(u) = -\Psi(u), \tag{4.9}$$

where

$$\begin{aligned}
\Psi(u) &= 2\phi_\parallel(u) + \widetilde{\delta}_+\xi\phi'_\perp(u) + \widetilde{\delta}_-\phi'_\perp(u) \\
&\quad + 2\zeta_{3\rho}^V \frac{d}{du}\int_0^u d\alpha_d \int_0^{\bar{u}} d\alpha_u\, \frac{1}{1-\alpha_d-\alpha_u}\left(\alpha_d\frac{d}{d\alpha_d} + \alpha_u\frac{d}{d\alpha_u}\right)\mathcal{V}(\underline{\alpha}) \\
&\quad + 2\zeta_{3\rho}^A \frac{d}{du}\int_0^u d\alpha_d \int_0^{\bar{u}} d\alpha_u\, \frac{1}{1-\alpha_d-\alpha_u}\left(\alpha_d\frac{d}{d\alpha_d} - \alpha_u\frac{d}{d\alpha_u}\right)\mathcal{A}(\underline{\alpha}).
\end{aligned} \tag{4.10}$$

From this equation, one immediately obtains the solution for $g_\perp^{(a)}(u)$ as

$$\left(1-\widetilde{\delta}_+\right) g_\perp^{(a)}(u) = \bar{u}\int_0^u dv\, \frac{1}{\bar{v}}\,\Psi(v) + u\int_u^1 dv\, \frac{1}{v}\,\Psi(v). \tag{4.11}$$

Combining this result with (4.7), one obtains the solution for $g_\perp^{(v)}(u)$ as

$$\begin{aligned}
g_\perp^{(v)}(u) &= \frac{1}{4}\left[\int_0^u dv\, \frac{1}{\bar{v}}\,\Psi(v) + \int_u^1 dv\, \frac{1}{v}\,\Psi(v)\right] + \widetilde{\delta}_+\phi_\perp(u) \\
&\quad + \zeta_{3\rho}^A \int_0^u d\alpha_d \int_0^{\bar{u}} d\alpha_u\, \frac{1}{1-\alpha_d-\alpha_u}\left(\frac{d}{d\alpha_d} + \frac{d}{d\alpha_u}\right)\mathcal{A}(\underline{\alpha}) \\
&\quad + \zeta_{3\rho}^V \frac{d}{du}\int_0^u d\alpha_d \int_0^{\bar{u}} d\alpha_u\, \frac{\mathcal{V}(\underline{\alpha})}{1-\alpha_d-\alpha_u}.
\end{aligned} \tag{4.12}$$

Eqs. (4.11) and (4.12) again allow the decomposition of $g_\perp^{(v)}(u)$ and $g_\perp^{(a)}(u)$ into several terms according to the various source terms:

$$g_\perp^{(v)}(u) = g_\perp^{(v)WW}(u) + g_\perp^{(v)g}(u) + g_\perp^{(v)m}(u), \tag{4.13}$$

$$g_\perp^{(a)}(u) = g_\perp^{(a)WW}(u) + g_\perp^{(a)g}(u) + g_\perp^{(a)m}(u), \tag{4.14}$$

where $g_\perp^{(v)WW}(u)$ and $g_\perp^{(a)WW}(u)$ denotes the contribution from the twist 2 distribution amplitudes (Wandzura-Wilczek part), $g_\perp^{(v)g}(u)$ and $g_\perp^{(a)g}(u)$ are the contribution from the three-particle distribution amplitudes $\mathcal{V}$ and $\mathcal{A}$. In particular, we get

$$g_\perp^{(v)WW}(u) = \frac{1}{2}\left[\int_0^u dv\,\frac{1}{\bar{v}}\phi_\parallel(v) + \int_u^1 dv\,\frac{1}{v}\phi_\parallel(v)\right], \tag{4.15}$$

$$g_\perp^{(a)WW}(u) = 2\bar{u}\int_0^u dv\,\frac{1}{\bar{v}}\phi_\parallel(v) + 2u\int_u^1 dv\,\frac{1}{v}\phi_\parallel(v). \tag{4.16}$$

4.2 Conformal expansion

The conformal expansion for the chiral-even distribution amplitude can be performed similarly to the case for the chiral-odd ones in Sec. 3.2. In the following, we restrict ourselves to the massless case.

For completeness, we start with the expansion for the twist 2 distribution amplitude $\phi_\parallel(u)$. It reads

$$\phi_\parallel(u) = 6u\bar{u}\sum_{n=0}^{\infty} a_n^\parallel C_n^{3/2}(\xi), \tag{4.17}$$

where $C_n^{3/2}(\xi)$ is the Gegenbauer polynomial, and each term corresponds to the conformal spin $n+2$. We note $a_0^\parallel = 1$ due to the normalization condition (2.23). Because of the G-parity invariance of the ρ meson distribution amplitude (likewise for other mesons such as ω, ϕ), it follows that $a_n^\parallel = 0$ for $n = 1, 3, 5, \ldots$ in (4.17). In Sec. 5, however, we shall treat an application to the K^* distribution amplitude with explicit SU(3) breaking due to the quark masses. With this in mind, we keep all terms $a_n^\parallel$ in (4.17). For the same reason, we shall work out the conformal expansion for $g_\perp^{(v)}$, $g_\perp^{(a)}$, $\mathcal{V}(\underline{\alpha})$ and $\mathcal{A}(\underline{\alpha})$ by keeping both G-parity invariant and G-parity violating terms in the following.

To carry out the conformal expansion for $g_\perp^{(v)}(u)$ and $g_\perp^{(a)}(u)$, we again introduce a set of auxiliary amplitudes $g^{\uparrow\downarrow}(u)$ and $g^{\downarrow\uparrow}(u)$ [20] defined by

$$\langle 0|\overline{u}(z)\gamma.\gamma_\mu^\perp\gamma_*[z,-z]d(-z)|\rho^-(P,\lambda)\rangle = -f_\rho m_\rho e_{\perp\mu}^{(\lambda)}\int_0^1 du e^{i\xi pz} g^{\uparrow\downarrow}(u), \tag{4.18}$$

$$\langle 0|\overline{u}(z)\gamma_*\gamma_\mu^\perp\gamma.[z,-z]d(-z)|\rho^-(P,\lambda)\rangle = -f_\rho m_\rho e_{\perp\mu}^{(\lambda)}\int_0^1 du e^{i\xi pz} g^{\downarrow\uparrow}(u), \tag{4.19}$$

which are related to $g_\perp^{(v)}(u)$ and $g_\perp^{(a)}(u)$ as

$$g^{\uparrow\downarrow}(u) = g_\perp^{(v)}(u) + \frac{1}{4}\frac{d}{du}g_\perp^{(a)}(u), \tag{4.20}$$

$$g^{\downarrow\uparrow}(u) = g_\perp^{(v)}(u) - \frac{1}{4}\frac{d}{du}g_\perp^{(a)}(u). \tag{4.21}$$

The conformal expansion for $g^{\uparrow\downarrow}(u)$ and $g^{\downarrow\uparrow}(u)$ is given by

$$g^{\uparrow\downarrow}(u) = 2\overline{u}\sum_{n=0}^{\infty} g_n^{\uparrow\downarrow} P_n^{(1,0)}(\xi), \tag{4.22}$$

$$g^{\downarrow\uparrow}(u) = 2u\sum_{n=0}^{\infty} g_n^{\downarrow\uparrow} P_n^{(0,1)}(\xi). \tag{4.23}$$

Substituting these expansions in (4.20) and (4.21), one obtains

$$g_\perp^{(v)}(u) = \sum_{n=0,2,4,\ldots} \left(G_n - G_{n-1}\right) C_n^{1/2}(\xi) + \sum_{n=1,3,5,\ldots} \left(g_n - g_{n-1}\right) C_n^{1/2}(\xi), \tag{4.24}$$

$$g_\perp^{(a)}(u) = 8u\overline{u}\left(\sum_{n=0,2,4,\ldots} \frac{G_n - G_{n+1}}{(n+1)(n+2)} C_n^{3/2}(\xi) + \sum_{n=1,3,5,\ldots} \frac{g_n - g_{n+1}}{(n+1)(n+2)} C_n^{3/2}(\xi)\right) \tag{4.25}$$

where G_n and g_n represent, respectively, G-parity invariant and G-parity violating components in the expansion defined by

$$G_n \equiv \frac{g_n^{\uparrow\downarrow} + (-1)^n g_n^{\downarrow\uparrow}}{2},$$

$$g_n \equiv \frac{g_n^{\uparrow\downarrow} - (-1)^n g_n^{\downarrow\uparrow}}{2}, \tag{4.26}$$

for $n = 0, 1, 2, \ldots$, and $G_{-1} = g_{-1} = 0$ is implied. Note the difference between $\{G_n, g_n\}$ and $\{H_n, h_n\}$ (see (3.34)) owing to the chiral-even or -odd nature of the distribution amplitudes.

The conformal expansion for the three-particle distribution amplitudes $\mathcal{V}(\underline{\alpha})$ and $\mathcal{A}(\underline{\alpha})$ can be written down similarly to (3.35):

$$\mathcal{V}(\alpha_d, \alpha_u, 1-\alpha_d-\alpha_u) = 360\alpha_d\alpha_u(1-\alpha_d-\alpha_u)^2 \sum_{k,l=0}^{\infty} \omega_{k,l}^V J_{k,l}(\alpha_d, \alpha_u),$$

$$\mathcal{A}(\alpha_d, \alpha_u, 1-\alpha_d-\alpha_u) = 360\alpha_d\alpha_u(1-\alpha_d-\alpha_u)^2 \sum_{k,l=0}^{\infty} \omega_{k,l}^A J_{k,l}(\alpha_d, \alpha_u). \tag{4.27}$$

The G-parity invariance of the three-particle distribution amplitudes leads to $\omega_{k,l}^V = -\omega_{l,k}^V$ and $\omega_{k,l}^A = \omega_{l,k}^A$. As was stated at the begining of this subsection, we shall not assume this symmetry in the following. We also note the normalization condition in (2.29) gives $\omega^V_{[0,1]} = 28/3$ and $\omega^A_{0,0} = 1$. The conformal spin for each term in the above expansion is equal to $j = k + l + 7/2$, and the preservation of the conformal invariance at one-loop level prevents mixing among the contribution with different $n = k + l$.

Our next task is to identify the twist 2 (Wandzura-Wilczek) and the three-particle contibutions to G_n and g_n. We decompose

$$G_n = G_n^{WW} + G_n^g; \qquad g_n = g_n^{WW} + g_n^g, \tag{4.28}$$

and consider the Wandzura-Wilczek contribution first. Substituting (4.15) and (4.16) into (4.20) and (4.21) and using the formulae (A.12), (A.3), (A.8) and (A.9), we obtain the Wandzura-Wilczek contribution for $g^{\uparrow\downarrow}(u)$ and $g^{\downarrow\uparrow}(u)$ as

$$\begin{aligned} g^{\uparrow\downarrow WW}(u) &= \int_u^1 dv \frac{\phi_\parallel(v)}{v} \\ &= 2\bar{u} \sum_{n=0}^{\infty} a_n^\parallel \left[\frac{3(n+2)}{2(2n+3)} P_n^{(1,0)}(\xi) - \frac{3(n+1)}{2(2n+3)} P_{n+1}^{(1,0)}(\xi) \right], \end{aligned} \tag{4.29}$$

$$\begin{aligned} g^{\downarrow\uparrow WW}(u) &= \int_0^u dv \frac{\phi_\parallel(v)}{\bar{v}} \\ &= 2u \sum_{n=0}^{\infty} a_n^\parallel \left[\frac{3(n+2)}{2(2n+3)} P_n^{(0,1)}(\xi) + \frac{3(n+1)}{2(2n+3)} P_{n+1}^{(0,1)}(\xi) \right]. \end{aligned} \tag{4.30}$$

These give rise to G_n^{WW} and g_n^{WW} in (4.28) as

$$\begin{aligned} G_n^{WW} &= \frac{3(n+2)}{2(2n+3)} a_n^\parallel, \qquad g_n^{WW} = \frac{-3n}{2(2n+1)} a_{n-1}^\parallel, \qquad (n = 0, 2, 4 \ldots) \\ G_n^{WW} &= \frac{-3n}{2(2n+1)} a_{n-1}^\parallel, \qquad g_n^{WW} = \frac{3(n+2)}{2(2n+3)} a_n^\parallel. \qquad (n = 1, 3, 5 \ldots) \end{aligned} \tag{4.31}$$

For even n, we find that $a_n^\parallel$ which corresponds to the conformal spin $n+2$ in the expansion for the twist 2 distribution amplitude gives rise to G_n^{WW} and G_{n+1}^{WW} which corresponds to the conformal spin $n + 3/2$ and $n + 5/2$, respectively. Likewise for odd n, $a_n^\parallel$ gives rise to g_n^{WW} and g_{n+1}^{WW}. This is the same pattern as observed for the chiral-odd distribution amplitudes and is discussed in App. B.

From the solution for $g_\perp^{(v)}(u)$ and $g_\perp^{(a)}(u)$ in (4.11) and (4.12) we can identify the three-particle contribution to the auxiliary amplitude as

$$g^{\uparrow\downarrow g}(u) = \zeta_{3\rho}^V \left\{ \int_u^1 dv \, \frac{1}{v} H(v) + M(u) \right\} + \zeta_{3\rho}^A \left\{ \int_u^1 dv \, \frac{1}{v} L(v) + N(u) \right\}, \tag{4.32}$$

$$g^{\downarrow\uparrow g}(u) = \zeta_{3\rho}^V \left\{ \int_0^u dv \, \frac{1}{\bar{v}} H(v) + M(u) \right\} + \zeta_{3\rho}^A \left\{ \int_0^u dv \, \frac{1}{\bar{v}} L(v) + N(u) \right\}, \tag{4.33}$$

where the functions $H(u)$, $L(u)$, $M(u)$, $N(u)$ are defined as

$$H(u) = \frac{d}{du}\int_0^u d\alpha_d \int_0^{\overline{u}} d\alpha_u \frac{1}{\alpha_g}\left(\alpha_d \frac{d}{d\alpha_d} + \alpha_u \frac{d}{d\alpha_u}\right)\mathcal{V}(\underline{\alpha}), \tag{4.34}$$

$$L(u) = \frac{d}{du}\int_0^u d\alpha_d \int_0^{\overline{u}} d\alpha_u \frac{1}{\alpha_g}\left(\alpha_d \frac{d}{d\alpha_d} - \alpha_u \frac{d}{d\alpha_u}\right)\mathcal{A}(\underline{\alpha}), \tag{4.35}$$

$$M(u) = \frac{d}{du}\int_0^u d\alpha_d \int_0^{\overline{u}} d\alpha_u \frac{1}{\alpha_g}\mathcal{V}(\underline{\alpha}), \tag{4.36}$$

$$N(u) = \int_0^u d\alpha_d \int_0^{\overline{u}} d\alpha_u \frac{1}{\alpha_g}\left(\frac{d}{d\alpha_d} + \frac{d}{d\alpha_u}\right)\mathcal{A}(\underline{\alpha}), \tag{4.37}$$

with $\alpha_g = 1 - \alpha_d - \alpha_u$. The calculation of (4.32) and (4.33) can be carried out similarly to the chiral-odd distribution amplitudes in Sec. 3, using the explicit form for $\mathcal{V}(\underline{\alpha})$ and $\mathcal{A}(\underline{\alpha})$ in (4.27). First $H(u)$, $L(u)$, $M(u)$ and $N(u)$ in (4.34)–(4.37) can be easily obtained by using (A.15)–(A.18). The integration of $H(u)$ and $L(u)$ in (4.32) and (4.33) can be done by (A.3). Application of (A.8) and (A.9) to the results of those integrations and $N(u)$, and the use of (A.5) and (A.6) for $M(u)$ yields the anticipated form for the result. One eventually obtains

$$g^{\uparrow\downarrow g}(u) = 90(1-\xi)\sum_{n=2}^{\infty} P_n^{(1,0)}(\xi)\sum_{k=0}^{n-2}\frac{k!(n-k-2)!(-1)^k}{(n+1)!(n+1)} \times \{n - (2k+2-n)\}\left(-\zeta_{3\rho}^V \omega_{k,n-k-2}^V + \zeta_{3\rho}^A \omega_{k,n-k-2}^A\right), \tag{4.38}$$

$$g^{\downarrow\uparrow g}(u) = 90(1+\xi)\sum_{n=2}^{\infty} P_n^{(0,1)}(\xi)\sum_{k=0}^{n-2}\frac{k!(n-k-2)!(-1)^k}{(n+1)!(n+1)} \times \{n + (2k+2-n)\}\left(\zeta_{3\rho}^V \omega_{k,n-k-2}^V + \zeta_{3\rho}^A \omega_{k,n-k-2}^A\right), \tag{4.39}$$

where we kept the form $(2k+2-n)$ in $\{\ldots\}$ which is anti-symmetric under the interchange $k \leftrightarrow n-k-2$. From this equation we get $G_0^g = G_1^g = g_0^g = g_1^g = 0$, and

$$G_n^g = \begin{cases} 90\displaystyle\sum_{k=0}^{n-2}\frac{k!(n-k-2)!(-1)^{n+k}}{(n+1)!(n+1)}\left\{(2k+2-n)\zeta_{3\rho}^V \omega_{[k,n-k-2]}^V + n\zeta_{3\rho}^A \omega_{\{k,n-k-2\}}^A\right\}, \\ \qquad (n = 2,4,6\ldots) \\ 90\displaystyle\sum_{k=0}^{n-2}\frac{k!(n-k-2)!(-1)^{n+k}}{(n+1)!(n+1)}\left\{n\zeta_{3\rho}^V \omega_{[k,n-k-2]}^V + (2k+2-n)\zeta_{3\rho}^A \omega_{\{k,n-k-2\}}^A\right\}, \\ \qquad (n = 3,5,7\ldots) \end{cases} \tag{4.40}$$

$$g_n^g = \begin{cases} 90 \displaystyle\sum_{k=0}^{n-2} \frac{k!(n-k-2)!(-1)^{n+k+1}}{(n+1)!(n+1)} \left\{ n\zeta_{3\rho}^V \omega_{\{k,n-k-2\}}^V + (2k+2-n)\zeta_{3\rho}^A \omega_{[k,n-k-2]}^A \right\}, \\ \qquad (n = 2,4,6,\ldots) \\ 90 \displaystyle\sum_{k=0}^{n-2} \frac{k!(n-k-2)!(-1)^{n+k+1}}{(n+1)!(n+1)} \left\{ (2k+2-n)\zeta_{3\rho}^V \omega_{\{k,n-k-2\}}^V + n\zeta_{3\rho}^A \omega_{[k,n-k-2]}^A \right\}, \\ \qquad (n = 3,5,7,\ldots) \end{cases} \tag{4.41}$$

where we introduced the anti-symmetric and symmetric components of $\omega_{k,l}^{V,A}$ defined by

$$\omega_{[k,l]}^{V,A} \equiv \frac{\omega_{k,l}^{V,A} - \omega_{l,k}^{V,A}}{2}, \tag{4.42}$$

$$\omega_{\{k,l\}}^{V,A} \equiv \frac{\omega_{k,l}^{V,A} + \omega_{l,k}^{V,A}}{2}. \tag{4.43}$$

One finds here again that G_n and g_n which correspond to the conformal spin $j = 3/2 + n$ receives contributions from the coefficients $\omega_{k,l}^{V,A}$ with a fixed $k + l = n - 2$ which have the same conformal spin $j = 3/2 + n$. The first terms in (4.40) and (4.41) read

$$\begin{aligned} G_2^g &= 10\zeta_{3\rho}^A \omega_{00}^A = 10\zeta_{3\rho}^A, \\ G_3^g &= \frac{15}{8}\left(-3\zeta_{3\rho}^V \omega_{[0,1]}^V + \zeta_{3\rho}^A \omega_{\{0,1\}}^A\right) = \frac{15}{8}\left(-28\zeta_{3\rho}^V + \zeta_{3\rho}^A \omega_{\{0,1\}}^A\right), \\ G_4^g &= \frac{3}{5}\left(-2\zeta_{3\rho}^V \omega_{[0,2]}^V + 4\zeta_{3\rho}^A \omega_{\{0,2\}}^A - \zeta_{3\rho}^A \omega_{\{1,1\}}^A\right), \ldots, \\ g_2^g &= -10\zeta_{3\rho}^V \omega_{00}^V, \qquad g_3^g = \frac{15}{8}\left(3\zeta_{3\rho}^A \omega_{[0,1]}^A - \zeta_{3\rho}^V \omega_{\{0,1\}}^V\right), \\ g_4^g &= \frac{3}{5}\left(2\zeta_{3\rho}^A \omega_{[0,2]}^A - 4\zeta_{3\rho}^V \omega_{\{0,2\}}^V + \zeta_{3\rho}^V \omega_{\{1,1\}}^V\right), \ldots, \end{aligned} \tag{4.44}$$

where we have used the normalization condition for $\mathcal{A}(\underline{\alpha})$ and $\mathcal{V}(\underline{\alpha})$, $\omega_{00}^A = 1$ and $\omega_{[0,1]}^V = 28/3$. We also note that (4.31) for $n = 0, 1$ together with the conditions $a_0^\| = 1$, $G_0^g = G_1^g = 0$ determines the first two coefficients of G_n as $G_0 = 1$, $G_1 = -1/2$, which gives consistent normalization $\int_0^1 du\, g_\perp^{(v)}(u) = 1$ and $\int_0^1 du\, g_\perp^{(a)}(u) = 1$ through (4.24) and (4.25). We finally remind that the G-parity invariance of the three-particle distribution amplitudes imposes $\omega_{\{k,l\}}^V = 0$ and $\omega_{[k,l]}^A = 0$, leading to $g_n^g = 0$ and $G_n^g = g_n^{\uparrow\downarrow g}$.

4.3 Renormalization and scale-dependence

In this subsection, we discuss the renormalization of the chiral-even distribution amplitudes, $\phi_\|(u,\mu^2)$, $g_\perp^{(v)}(u,\mu^2)$ and $g_\perp^{(a)}(u,\mu^2)$, utilizing the conformal expansions derived in the previous subsection.

For completeness we start our discussion with the renormalization of the twist 2 distribution amplitude $\phi_{\parallel}(u)$. From (4.1) and the orthogonality relations of the Gegenbauer polynomials [19], one obtains

$$a_n^{\parallel}(\mu^2) = \frac{2(2n+3)}{3(n+1)(n+2)} \int_0^1 du\, C_n^{3/2}(\xi)\, \phi_{\parallel}(u,\mu^2). \tag{4.45}$$

Using (2.19) in the right-hand side of (4.45), we can express $a_n^{\parallel}$ in terms of the conformal operator:

$$a_n^{\parallel}(\mu^2) = \frac{2(2n+3)}{3 f_\rho m_\rho (n+1)(n+2)(e^{(\lambda)}\cdot z)(p\cdot z)^n} \langle 0|\Omega_n^{\parallel}(0;\mu^2)|\rho^-(P,\lambda)\rangle \tag{4.46}$$

with

$$\Omega_n^{\parallel}(x;\mu^2) = (i\partial.)^n\, \bar{u}(x)\gamma.C_n^{3/2}\left(\frac{\overleftrightarrow{D}.}{\partial.}\right) d(x), \tag{4.47}$$

where the local operator in the right-hand side is renormalized at μ^2, $\overleftrightarrow{D}\equiv \overrightarrow{D} - \overleftarrow{D}$, and ∂_μ is the total derivative (3.3). $\Omega_n^{\parallel}(x;\mu^2)$ is the conformal operator with conformal spin $j = n+2$ [18]. The scale dependence of $a_n^{\parallel}(\mu^2)$ is well known [1, 2]:

$$a_n^{\parallel}(Q^2) = L^{\gamma_n^{\parallel}/b} a_n^{\parallel}(\mu^2), \tag{4.48}$$

where $L \equiv \alpha_s(Q^2)/\alpha_s(\mu^2)$, $b = (11N_c - 2N_f)/3$ and the anomalous dimension $\gamma_n^{\parallel}$ for the conformal operator $\Omega_n^{\parallel}$ is given by

$$\gamma_n^{\parallel} = 4C_F\left(\psi(n+2) + \gamma_E - \frac{3}{4} - \frac{1}{2(n+1)(n+2)}\right). \tag{4.49}$$

For $n = 0$, $\Omega_0^{\parallel}$ is reduced to a conserved vector current, and hence its anomalous dimension vanishes to all orders. Combined with the normalization condition for $\phi_{\parallel}(u)$, $a_0^{\parallel} = 1$, this is consistent with the fact that f_ρ is scale independent. We thus omitted f_ρ in both sides of (4.48) (compare with (3.58) and (3.59)).

Next we proceed to discuss the renormalization of the twist 3 distribution amplitudes $g_\perp^{(v)}(u,\mu^2)$ and $g_\perp^{(a)}(u,\mu^2)$. As we saw in the previous subsections, they receive contributions from the twist 2 distribution amplitude (Wandzura-Wilczek parts), the three-particle distribution amplitudes and the terms proportional to the quark masses (see (4.13)−(4.16)). The scale-dependence of $\phi_{\parallel}(u,\mu^2)$ discussed above completely determines that of $g_\perp^{(v)WW}(u,\mu^2)$ and $g_\perp^{(a)WW}(u,\mu^2)$ through the relations (4.24), (4.25) and (4.31).

To understand the scale dependence of the three-particle distribution amplitudes $\mathcal{V}(\underline{\alpha})$ and $\mathcal{A}(\underline{\alpha})$, one needs to express $\omega_{k,l}^{V,A}$ in (4.27) in terms of the local conformal operators. Owing to the orthogonality relation (A.13) of the Appell polynomials with different $k+l$,

$\omega_{k,l}^{V,A}$ can be expressed in terms of the matrix elements of the conformal operators with a definite conformal spin $j = k + l + 7/2$. As we saw in Sec. 3, it suffices to know the form of the conformal operators up to total derivatives for the renormalization. For this purpose, we recall from (2.24) and (2.25)

$$\langle 0|\bar{u}(tz)\gamma.[tz,vz]g\widetilde{G}_{\perp}.(vz)[vz,wz]d(wz)|\rho^-(P,\lambda)\rangle$$
$$= i\epsilon_{\perp\cdot\mu\nu}p_\mu e_{\perp\nu}^{(\lambda)}(p\cdot z)f_{3\rho}^V\int\mathcal{D}\underline{\alpha}e^{-ip\cdot z(t\alpha_u+w\alpha_d+v\alpha_g)}\mathcal{V}(\underline{\alpha}), \tag{4.50}$$

$$\langle 0|\bar{u}(tz)\gamma.i\gamma_5[tz,vz]gG_{\perp}.(vz)[vz,wz]d(wz)|\rho^-(P,\lambda)\rangle$$
$$= i\epsilon_{\perp\cdot\mu\nu}p_\mu e_{\perp\nu}^{(\lambda)}(p\cdot z)f_{3\rho}^A\int\mathcal{D}\underline{\alpha}e^{-ip\cdot z(t\alpha_u+w\alpha_d+v\alpha_g)}\mathcal{A}(\underline{\alpha}). \tag{4.51}$$

To obtain the actual form of the conformal operators, we apply $\partial^{n-2}/\partial t^{n-k-2}\partial w^k$ on both sides of (4.50) and (4.51) and set $t = w = v = 0$. Using the integral formula for the Appell polynomial (A.14), one obtains

$$\langle 0|\bar{u}(0)(i\overleftarrow{D}.)^{n-k-2}\gamma.g\widetilde{G}_{\perp}.(0)(i\overrightarrow{D}.)^k d(0)|\rho(P,\lambda)\rangle$$
$$= if_{3\rho}^V\epsilon_{\perp\cdot\mu\nu}e_{\perp\mu}^{(\lambda)}p_\nu(p\cdot z)^{n-1}\left[\omega_{k,n-k-2}^V\frac{360(-1)^n k!(n-k-2)!}{2^{n+1}(n+1)(2n+1)!!}\right.$$
$$\left.+(\text{terms with }\omega_{l,r-l-2}^V|_{r<n})\right], \tag{4.52}$$

$$\langle 0|\bar{u}(0)(i\overleftarrow{D}.)^{n-k-2}\gamma.i\gamma_5 gG_{\perp}.(0)(i\overrightarrow{D}.)^k d(0)|\rho(P,\lambda)\rangle$$
$$= if_{3\rho}^A\epsilon_{\perp\cdot\mu\nu}e_{\perp\mu}^{(\lambda)}p_\nu(p\cdot z)^{n-1}\left[\omega_{k,n-k-2}^A\frac{360(-1)^n k!(n-k-2)!}{2^{n+1}(n+1)(2n+1)!!}\right.$$
$$\left.+(\text{terms with }\omega_{l,r-l-2}^A|_{r<n})\right]. \tag{4.53}$$

It is easy to see by induction that $\omega_{l,r-l-2}^{V,A}|_{r<n}$ in these equations are the matrix elements of the total derivatives of the lower conformal operators. Therefore we can identify the corresponding conformal operators for $\omega_{k,n-k-2}^{V,A}$ as

$$\left(f_{3\rho}^V\omega_{k,n-k-2}^V\right)(\mu^2) = \frac{(-1)^k N_n}{90k!(n-k-2)!}\langle 0|\Theta_{k,n-k-2}^V(0;\mu^2)|\rho^-(P,\lambda)\rangle, \tag{4.54}$$

$$\left(f_{3\rho}^A\omega_{k,n-k-2}^A\right)(\mu^2) = \frac{(-1)^k N_n}{90k!(n-k-2)!}\langle 0|\Theta_{k,n-k-2}^A(0;\mu^2)|\rho^-(P,\lambda)\rangle, \tag{4.55}$$

where the conformal operators are now obtained up to total derivatives as

$$\Theta_{k,n-k-2}^V(0) \equiv \bar{u}(0)(-i\overleftarrow{D}.)^{n-k-2}\gamma.\widetilde{G}_{\perp}.(0)(i\overrightarrow{D}.)^k d(0) + (\text{total derivatives}), \tag{4.56}$$

$$\Theta^A_{k,n-k-2}(0) \equiv \bar{u}(0)(-i\overleftarrow{D}.)^{n-k-2}\gamma.G_\perp.(0)(i\overrightarrow{D}.)^k i\gamma_5 d(0) + (\text{total derivatives}), \quad (4.57)$$

and we introduced for convenience a dimensionless and scale independent normalization constant N_n as

$$N_n \equiv \frac{2^{n-1}(2n+1)!!(n+1)}{i\epsilon_{\perp\cdot\mu\nu}p_\mu e^{(\lambda)}_{\perp\nu}(p\cdot z)^{n-1}}. \quad (4.58)$$

From Eqs. (4.54) and (4.55), we can obtain the scale-dependence of $(f^V_{3\rho}\omega^V_{k,n-k-2})(\mu^2)$ and $(f^A_{3\rho}\omega^A_{k,n-k-2})(\mu^2)$ by working out the renormalization of $\{\Theta^V_{k,n-k-2}, \Theta^A_{k,n-k-2}\}$ $(k=0,\ldots,n-2)$.[7] If we define $R^{V\pm}_{n,k} \equiv \Theta^V_{k,n-k-2} \pm \Theta^V_{n-k-2,k}$ $(k=0,1,\ldots,\kappa^\pm_n$ with $\kappa^\pm_n$ defined in (3.69)) and $R^{A\pm}_{n,k} \equiv \Theta^A_{k,n-k-2} \mp \Theta^A_{n-k-2,k}$ $(k=0,1,\ldots,\kappa^\mp_n)$, $\{R^{V+}_{n,k}, R^{A+}_{n,k}\}$ and $\{R^{V-}_{n,k}, R^{A-}_{n,k}\}$, respectively, have G-parity $(-1)^{n+1}$ and $(-1)^n$ and thus they do not mix with each other under renormalization.

By inserting (4.54) and (4.55) into (4.40) and (4.41) and recalling the definition of $\zeta^{V,A}_{3\rho}$ from (4.5), one can express the contribution from the three-particle distribution amplitude to the two-particle distribution amplitudes $g^{(v,a)}_\perp$ in terms of the conformal operators:

$$G^g_n(\mu^2) = \begin{cases} \dfrac{N_n}{f_\rho m_\rho (n+1)!(n+1)} \langle 0|\sum_{k=0}^{n-2}(k+1)R^-_{n,k}(0;\mu^2)|\rho^-(P,\lambda)\rangle, & (n=2,4,6,\ldots), \\ \dfrac{-N_n}{f_\rho m_\rho (n+1)!(n+1)} \langle 0|\sum_{k=0}^{n-2}(n-k-1)R^+_{n,k}(0;\mu^2)|\rho^-(P,\lambda)\rangle, & (n=3,5,7,\ldots), \end{cases} \quad (4.59)$$

$$g^g_n(\mu^2) = \begin{cases} \dfrac{-N_n}{f_\rho m_\rho (n+1)!(n+1)} \langle 0|\sum_{k=0}^{n-2}(n-k-1)R^+_{n,k}(0;\mu^2)|\rho^-(P,\lambda)\rangle, & (n=2,4,6,\ldots), \\ \dfrac{N_n}{f_\rho m_\rho (n+1)!(n+1)} \langle 0|\sum_{k=0}^{n-2}(k+1)R^-_{n,k}(0;\mu^2)|\rho^-(P,\lambda)\rangle, & (n=3,5,7,\ldots), \end{cases} \quad (4.60)$$

where the operators $R^\pm_{n,k}$ $(k=0,\ldots,n-2)$ are defined as

$$R^\pm_{n,k} = \Theta^V_{k,n-k-2} \pm \Theta^V_{n-k-2,k} \mp \Theta^A_{k,n-k-2} + \Theta^A_{n-k-2,k}. \qquad (k=0,1,\ldots,n-2) \quad (4.61)$$

Here we note that $\{R^+_{n,k}\}$ and $\{R^-_{n,k}\}$ have opposite parity under G-parity transformations and constitute another operator basis which is equivalent to $\{R^{V+}_{n,k}, R^{A+}_{n,k}\}$ and $\{R^{V-}_{n,k}, R^{A-}_{n,k}\}$, respectively, either of which has the same number of independent operators $\kappa^+_n + \kappa^-_n + 2 = n-1$. If we define the anomalous dimension matrix for $\{R^\pm_{n,k}\}$ as $\Gamma^\pm_n$, the scale dependence of $R^\pm_{n,k}$ is given by

$$R^\pm_{n,k}(0;Q^2) = \sum_{l=0}^{n-2}\left(L^{\Gamma^\pm_n/b}\right)_{k,l} R^\pm_{n,l}(0;\mu^2). \quad (4.62)$$

[7]Since $\Theta^V_{k,n-k-2}$ and $\Theta^A_{k,n-k-2}$ have the same conformal spin, they generally mix with each other under renormalization, even though they originate from different distribution amplitudes.

The renormalization for $\{R^{\pm}_{n,k}\}$ can be conveniently worked out by considering the forward matrix elements with respect to a spin 1/2 target (say a quark) as was done in Sec. 3. In this case, the contribution from the total derivative terms disappear and its matrix element $\langle R^{\pm}_{n,k}\rangle$ is reduced to $\langle\bar{\psi}(iD.)^{n-k-2}\gamma.\widetilde{G}_{\perp}.(iD.)^{k}\psi \pm (k \to n-k-2)\rangle + \langle\bar{\psi}(iD.)^{k}\gamma.G_{\perp}.(iD.)^{n-k-2}i\gamma_5\psi \mp (k \to n-k-2)\rangle$. By now, renormalization of $\{R^{\pm}_{n,k}\}$ has been solved by several different approaches in the context of the Q^2 evolution of the transverse spin structure function $g_2(x,Q^2)$ [10, 27, 28]. In particular, $\{R^{+}_{n,k}\}$ is exactly the operator basis used in [27] for the renormalization of $g_2(x,Q^2)$. From the result in [27], Γ^{+}_{n} in (4.62) is obtained as

$$\left(\Gamma^{+}_{n}\right)_{k,l} = -X^{n+1}_{k+1,l+1}, \qquad (k,l = 0,1,\ldots,n-2) \tag{4.63}$$

where $X^{n+1}_{k,l}$ is given in Eqs. (15)–(17) of [27] with $n \to n+1$. The explicit form of Γ^{-}_{n} is not available in the literature, but can be obtained from the kernel given in [10]. (Since $R^{-}_{n,k}$ does not contribute to the deep-inelastic scattering which is the charge conjugation even process, it has not been receiving attention up to now.)

By using the basis $\{R^{+}_{n,k}\}$ and $\{R^{-}_{n,k}\}$, (4.54) and (4.55) can be rewritten as

$$\left(f^{V}_{3\rho}\omega^{V}_{[k,n-k-2]} \pm f^{A}_{3\rho}\omega^{A}_{\{k,n-k-2\}}\right)(\mu^2) = \frac{(-1)^k N_n}{180k!(n-k-2)!}\langle 0|R^{\mp}_{n,k}(0;\mu^2)|\rho^-(P,\lambda)\rangle, \tag{4.64}$$

for the G-parity invariant components and

$$\left(f^{V}_{3\rho}\omega^{V}_{\{k,n-k-2\}} \mp f^{A}_{3\rho}\omega^{A}_{[k,n-k-2]}\right)(\mu^2) = \frac{(-1)^k N_n}{180k!(n-k-2)!}\langle 0|R^{\pm}_{n,k}(0;\mu^2)|\rho^-(P,\lambda)\rangle, \tag{4.65}$$

for the G-parity violating components. In (4.64) and (4.65), upper and lower signs correspond to $n = 2,4,6,\ldots$ and $n = 3,5,7,\ldots$, respectively. For illustration, we give here the explicit form of the scale dependence of the left hand side of (4.64) for $n = 2,3,4$ from (4.62):

$$f^{A}_{3\rho}(Q^2) = L^{\Gamma^{-}_{2}/b} f^{A}_{3\rho}(\mu^2), \qquad \Gamma^{-}_{2} = -\frac{1}{3}C_F + 3C_G, \tag{4.66}$$

$$\begin{pmatrix} \frac{28}{3}f^{V}_{3\rho} - f^{A}_{3\rho}\omega^{A}_{\{0,1\}} \\ \frac{28}{3}f^{V}_{3\rho} + f^{A}_{3\rho}\omega^{A}_{\{0,1\}} \end{pmatrix}^{Q^2} = L^{\Gamma^{+}_{3}/b}\begin{pmatrix} \frac{28}{3}f^{V}_{3\rho} - f^{A}_{3\rho}\omega^{A}_{\{0,1\}} \\ \frac{28}{3}f^{V}_{3\rho} + f^{A}_{3\rho}\omega^{A}_{\{0,1\}} \end{pmatrix}^{\mu^2},$$

$$\Gamma^{+}_{3} = \begin{pmatrix} \frac{4}{3}C_F + 3C_G, & \frac{2}{3}C_F - \frac{2}{3}C_G \\ \frac{5}{3}C_F - \frac{4}{3}C_G, & \frac{13}{6}C_F + 3C_G \end{pmatrix}, \tag{4.67}$$

$$\begin{pmatrix} 2f^{V}_{3\rho}\omega_{[0,2]} + 2f^{A}_{3\rho}\omega^{A}_{\{0,2\}} \\ -f^{A}_{3\rho}\omega^{A}_{\{1,1\}} \\ -2f^{V}_{3\rho}\omega_{[0,2]} + 2f^{A}_{3\rho}\omega^{A}_{\{0,2\}} \end{pmatrix}^{Q^2} = L^{\Gamma^{-}_{4}/b}\begin{pmatrix} 2f^{V}_{3\rho}\omega_{[0,2]} + 2f^{A}_{3\rho}\omega^{A}_{\{0,2\}} \\ -f^{A}_{3\rho}\omega^{A}_{\{1,1\}} \\ -2f^{V}_{3\rho}\omega_{[0,2]} + 2f^{A}_{3\rho}\omega^{A}_{\{0,2\}} \end{pmatrix}^{\mu^2}, \tag{4.68}$$

where we have used $\omega^A_{00} = 1$ and $\omega^V_{[0,1]} = 28/3$, and the anomalous dimensions Γ^-_2, Γ^-_4 can be obtained from the kernel given in [10].[8]

As one can see from this illustration, the μ^2 evolution of $\{\omega^V_{[k,n-k-2]}, \omega^A_{\{k,n-k-2\}}\}$ and $\{\omega^V_{\{k,n-k-2\}}, \omega^A_{[k,n-k-2]}\}$ becomes extremely complicated for general n. However, as was the case for the chiral-odd distribution amplitude in Sec. 3, the μ^2-dependence of the three-particle contribution to the twist 3 two-particle distribution amplitudes, i.e. $G^g_n(\mu^2)$ and $g^g_n(\mu^2)$, is greatly simplified in the large N_c limit. It has been shown in [26] that the combination of $R^{\pm}_{n,k}$ in (4.59) and (4.60) renormalize multiplicatively at $N_c \to \infty$ to $O(1/N_c^2)$ accuracy. We thus get in this limit

$$G^g_n(Q^2) = L^{\gamma_n/b} G^g_n(\mu^2),$$

$$g^g_n(Q^2) = L^{\gamma_n/b} g^g_n(\mu^2), \tag{4.69}$$

with a common anomalous dimension

$$\gamma_n = 2N_c \left(\psi(n+1) + \gamma_E - \frac{1}{4} + \frac{1}{2(n+1)} \right). \tag{4.70}$$

This γ_n has been shown to be the lowest eigenvalue of $\Gamma^{\pm}_n$ [26]. This reduction to the simple evolution equation (4.69) is equivalent to the fact that the coefficients of $R^{\pm}_{n,k}$ in (4.59) and (4.60) constitute the left eigenvector of $\Gamma^{\pm}_n$ with the eigenvalue γ_n in this limit:

$$\sum_{k=0}^{n-2} (k+1) \left(\Gamma^-_n\right)_{k,l} = (l+1)\gamma_n,$$

$$\sum_{k=0}^{n-2} (n-k-1) \left(\Gamma^+_n\right)_{k,l} = (n-l-1)\gamma_n, \tag{4.71}$$

which can be compared with (3.83)–(3.85) for the chiral-odd case. The renormalization of the flavour singlet part of $g_\perp^{(v,a)}(u,\mu^2)$ (for ω and ϕ mesons) is complicated by additional mixing with the purely gluonic twist 3 distribution amplitudes. For this mixing, no simplification occurs in the $N_c \to \infty$ limit.

As was discussed for the chiral-odd distribution amplitudes in Sec. 3, another simplification of the evolution equation for $g_\perp^{(v,a)}(u,\mu^2)$ occures at large n with arbitrary N_c. In this limit, the scale dependence of $G^g_n(\mu^2)$ and $g^g_n(\mu^2)$ is described by the same equation (4.69) with a slightly shifted anomalous dimension

$$\gamma_n \to \gamma_n + (4C_F - 2N_c) \left(\ln n + \gamma_E - \frac{3}{4} \right). \tag{4.72}$$

Combined with this simplification at $n \to \infty$, the result at large N_c in (4.69) is valid to $O((1/N_c^2) \ln(n)/n)$ accuracy as was the chiral-odd case in Sec. 3.

[8] Eq. (6.2) in [10] for the kernel contains a misprint: the delta function in the last line should be replaced by $\delta(\alpha - u)$.

To summarize this section, we have solved the renormalization of the nonsinglet chiral-even twist 3 distribution amplitudes, $g_\perp^{(v)}(u)$ and $g_\perp^{(a)}(u)$. We found that in the limits $N_c \to \infty$ and $n \to \infty$, the scale-dependence of the three-particle contribution to $g_\perp^{(v)}(u)$ and $g_\perp^{(a)}(u)$ is described by a simple DGLAP type evolution equation similar to that for the twist 2 distribution amplitude. Combined with the results of the previous section, this means that this simplification for the scale-dependence is universal for all twist 3 nonsinglet distribution amplitudes.

5 Models for Distribution Amplitudes

In this section we present explicit models for the two-particle distribution amplitudes of twist 3, taking into account contributions of the first few conformal partial waves. The main observation and important point is that the QCD equations of motion are satisfied order by order in the conformal expansion, which guarantees the consistency of the approximation. Our approximation thus introduces a minimum number of nonperturbative parameters describing matrix elements of certain local operators between the vacuum and the meson state, which we estimate using QCD sum rules [29, 3]. More sophisticated models can be constructed in a systematic way by adding contributions of higher conformal partial waves when estimates of the relevant nonperturbative matrix elements will become available.

Our approach involves the implicit assumption that the conformal partial wave expansion is well-convergent. This can be justified rigorously at large scales, since the anomalous dimensions of twist 2 and twist 3 operators increase logarithmically with the conformal spin j, but is nontrivial at relatively low scales of order $\mu \sim (1-2)\,\text{GeV}$ which we choose as reference scale. We believe that this assumption is natural and in fact necessary for any model of distribution amplitudes at scales where they evolve perturbatively; the last word, however, has to come from experiment.

Since orthogonal polynomials of high orders are rapidly oscillating functions, a truncated expansion in conformal partial waves almost necessarily is oscillatory as well. Such a behaviour is clearly unphysical, but does not constitute a real problem because physical observables are given by convolution integrals of distribution amplitudes with smooth coefficient functions. A classical example for this feature is the $\gamma\gamma^*$-meson form factor which is governed by the quantity

$$\int du\, \frac{1}{u}\, \phi(u) \sim \sum a_i,$$

where the coefficients a_i are exactly the "reduced matrix elements" in the conformal expansion. The oscillating terms are averaged over and strongly suppressed. Stated otherwise: models of distribution amplitudes should generally be understood as distributions (in the mathematical sense).

V	$\rho^\pm$	$K^{*\pm}$	ϕ
f_V[MeV]	198 ± 7	226 ± 28	254 ± 3

Table 3: Experimental values of couplings to the vector current [30].

V	$\rho^\pm$	$K^{*\pm}$	ϕ
$f_V^T(1\,\mathrm{GeV})$[MeV]	160 ± 10	185 ± 10	215 ± 15
$a_1^\parallel(1\,\mathrm{GeV})$	0	0.19 ± 0.05	0
$a_2^\parallel(1\,\mathrm{GeV})$	0.18 ± 0.10	0.06 ± 0.06	0 ± 0.1
$a_1^\perp(1\,\mathrm{GeV})$	0	0.20 ± 0.05	0
$a_2^\perp(1\,\mathrm{GeV})$	0.2 ± 0.1	0.04 ± 0.04	0 ± 0.1

Table 4: The tensor couplings and lowest Gegenbauer moments of vector mesons from QCD sum rules, see App. C.

5.1 Leading twist distributions

The twist 2 distribution amplitudes of vector mesons have received much attention in the literature. Their study was pioneered by Chernyak and Zhitnitsky (see [3] for a comprehensive review). More recently, the results for ρ mesons were critically examined in [12]. In this paper we complete the update [12] by the reanalysis of SU(3) breaking corrections, see App. C.

A simple model of twist 2 distributions includes contributions of the three lowest conformal spins ("S, P and D waves"):

$$\phi_\parallel(u) = 6u\bar{u}\left[1 + 3a_1^\parallel\,\xi + a_2^\parallel\,\frac{3}{2}(5\xi^2 - 1)\right], \tag{5.1}$$

$$\phi_\perp(u) = 6u\bar{u}\left[1 + 3a_1^\perp\,\xi + a_2^\perp\,\frac{3}{2}(5\xi^2 - 1)\right]. \tag{5.2}$$

We recall that $\bar{u} = 1 - u$ and $\xi = 2u - 1$. The values of the decay constants f_V, f_V^T and the Gegenbauer moments a_1, a_2 are collected in Tabs. 3 and 4 for $V = \rho$, K^* and ϕ mesons. The scaling laws for the coefficients $a_n^\parallel$ and $a_n^\perp$ are given in Eqs. (4.48) and (3.58), respectively.

The corresponding distributions, evaluated at $\mu = 1\,\mathrm{GeV}$, are shown in Fig. 1. Note that the leading twist distributions for longitudinally and transversely polarized vector mesons are very similar to each other.

In the following we neglect the masses of u and d quarks and do not account for ρ–ω mixing. In this approximation the couplings and distribution amplitudes of $\rho^\pm$, ρ^0 and ω

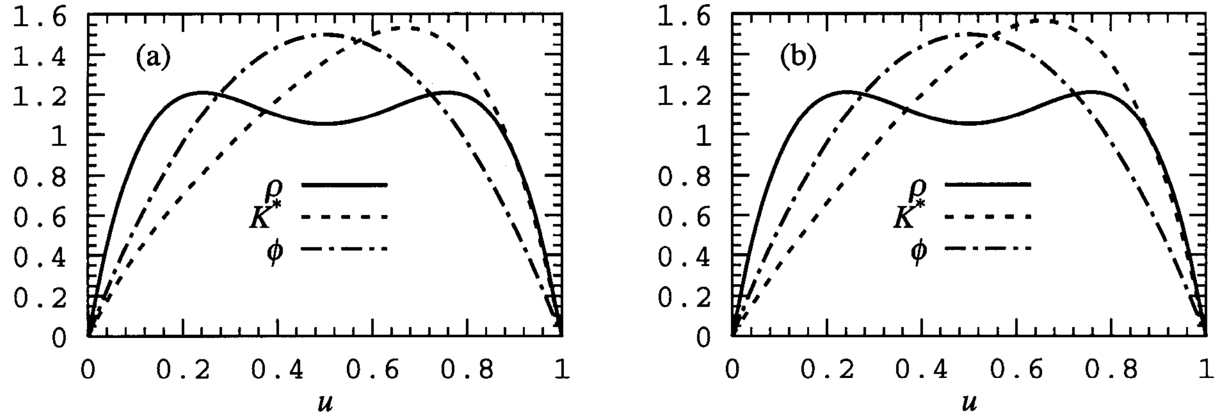

Figure 1: (a) Leading twist 2 distribution amplitude $\phi_{\parallel}(u)$ for a longitudinally polarized vector meson, (b) $\phi_{\perp}(u)$ for a transversely polarized one. Renormalization point is $\mu = 1\,\text{GeV}$.

mesons are equal if one chooses properly normalized currents, i.e. $(\bar{u}u \pm \bar{d}d)/\sqrt{2}$ for the ω and ρ^0 meson, respectively, in Eqs. (2.17), (2.19), (2.21) and (2.22).

The model distribution amplitudes for the K^* meson given by Chernyak and Zhitnitsky [31, 3] involve an additional contribution $\sim a_3 C_3^{3/2}(\xi)$. We have not included this term since estimates of high partial waves from QCD sum rules are not reliable. Our estimates for $a_1^{\perp}$ and $a_2^{\perp}$ differ significantly from the results of [31, 3], see Ref. [12] for details.

5.2 Three-particle distributions of twist three

Three-particle twist 3 distribution amplitudes were defined in Sec. 2.3 and their conformal expansion is considered in detail in Secs. 3 and 4. Assuming SU(3) flavour symmetry, the lowest order contributions to the conformal expansion are

$$\mathcal{V}(\alpha_d, \alpha_u, \alpha_g) = 5040\,(\alpha_d - \alpha_u)\alpha_d\alpha_u\alpha_g^2, \tag{5.3}$$

$$\mathcal{A}(\alpha_d, \alpha_u, \alpha_g) = 360\,\alpha_d\alpha_u\alpha_g^2\big[1 + \omega_{1,0}^{A}\frac{1}{2}(7\alpha_g - 3)\big], \tag{5.4}$$

$$\mathcal{T}(\alpha_d, \alpha_u, \alpha_g) = 5040\,(\alpha_d - \alpha_u)\alpha_d\alpha_u\alpha_g^2. \tag{5.5}$$

Our expressions for $\mathcal{V}$ and $\mathcal{A}$ agree with the corresponding "asymptotic distributions" in [3, 32]. The result for $\mathcal{T}$ is new. An important point to note is that the contribution of leading conformal spin $j = 7/2$ to the distribution amplitudes $\mathcal{V}$ and $\mathcal{T}$ vanishes by virtue of G-parity invariance (in the SU(3) limit). Hence, if one takes into account the leading $j = 7/2$ contribution to the distribution $\mathcal{A}$ only, it is consistent to put $\mathcal{V}$ and $\mathcal{T}$ to zero; the expressions given in (5.3) and (5.5) correspond to contributions of the next-to-leading conformal spin $j = 9/2$ and have to be taken into account together with the *correction* to $\mathcal{A}$ proportional to $\omega^A_{1,0}$, which has the same spin.

The decay constants $f^V_{3\rho}$, $f^A_{3\rho}$ and the few first coefficients $\omega^A_{i,k}$, $\omega^V_{i,k}$ were estimated from QCD sum rules in Ref. [4], in particular[9]

$$f^A_{3\rho} = (0.5 - 0.6) \cdot 10^{-2}\,\mathrm{GeV}^2, \qquad f^V_{3\rho} = 0.2 \cdot 10^{-2}\,\mathrm{GeV}^2, \qquad \omega^A_{1,0} = -2.1. \tag{5.6}$$

We have derived a new sum rule for $f^T_{3\rho}$ (see App. C) from which we obtain the estimate

$$f^T_{3\rho} = (0.3 \pm 0.3) \cdot 10^{-2}\,\mathrm{GeV}^2. \tag{5.7}$$

Again, the renormalization scale is $\mu = 1\,\mathrm{GeV}$. The anomalous dimensions of the couplings can be found in Eqs. (4.66), (4.67) and (3.76).

Estimates of quark mass corrections are difficult to obtain from such complicated sum rules. A detailed study of this point goes beyond the scope of this paper. For the present purpose we neglect SU(3) corrections to three-particle distributions and assume that the dimensionless couplings $\zeta_3^{V,A,T}$ defined in (3.8) and (4.5) are the same for all vector mesons. The most interesting effect which we miss in this "poor-man's" approximation is that the leading conformal spin contribution $\sim 360\alpha_d\alpha_u\alpha_g^2$ reappears in the distribution amplitudes $\mathcal{V}$ and $\mathcal{T}$ for K^* mesons. These terms deserve a further study. Our preferred values for the parameters determining three-particle distributions are collected in Tab. 5.

5.3 Two-particle distributions of twist three

As repeatedly emphasized above, the equations of motion allow the elimination of two-particle distribution amplitudes of higher twist in favour of leading twist and three-particle distribution amplitudes as independent dynamical degress of freedom. With leading twist and three-particle distributions as specified above, we get the following *exact* expressions (including terms up to conformal spin 9/2):

$$\begin{aligned} h_\parallel^{(s)}(u) &= 6u\bar{u}\left[1 + a_1^\perp\xi + \left(\frac{1}{4}a_2^\perp + \frac{35}{6}\zeta_3^T\right)(5\xi^2 - 1)\right] \\ &\quad + 3\,\delta_+\,(3u\bar{u} + \bar{u}\ln\bar{u} + u\ln u) + 3\,\delta_-\,(\bar{u}\ln\bar{u} - u\ln u), \end{aligned} \tag{5.8}$$

$$\begin{aligned} h_\parallel^{(t)}(u) &= 3\xi^2 + \frac{3}{2}\,a_1^\perp\,\xi(3\xi^2 - 1) + \frac{3}{2}a_2^\perp\,\xi^2\,(5\xi^2 - 3) + \frac{35}{4}\zeta_3^T(3 - 30\xi^2 + 35\xi^4) \\ &\quad + \frac{3}{2}\,\delta_+\,(1 + \xi\,\ln\bar{u}/u) + \frac{3}{2}\,\delta_-\,\xi\,(2 + \ln u + \ln\bar{u}), \end{aligned} \tag{5.9}$$

$$g_\perp^{(a)}(u) = 6u\bar{u}\left[1 + a_1^\parallel\xi + \left\{\frac{1}{4}a_2^\parallel + \frac{5}{3}\zeta_3^A\left(1 - \frac{3}{16}\,\omega^A_{1,0}\right) + \frac{35}{4}\zeta_3^V\right\}(5\xi^2 - 1)\right]$$

[9] The model distributions proposed in [4] include in addition the $j = 11/2$ terms with coefficients $\omega^A_{1,1} = 11.7$, $\omega^A_{2,0} = 7$ and $\omega^V_{2,0} = -1.9$. We do not include these contributions for simplicity and because the corresponding QCD sum rules are less reliable.

V	$\rho^\pm$	$K^{*\pm}$	ϕ
ζ_3^A	0.032	0.032	0.032
ζ_3^V	0.013	0.013	0.013
ζ_3^T	0.024	0.024	0.024
$\omega_{1,0}^A$	−2.1	−2.1	−2.1
δ_+	0	0.24	0.46
δ_-	0	−0.24	0
$\tilde{\delta}_+$	0	0.16	0.33
$\tilde{\delta}_-$	0	−0.16	0

Table 5: Masses and couplings entering Eqs. (5.8)–(5.11). Renormalization point is $\mu = 1\,\mathrm{GeV}$. We use $m_s(1\,\mathrm{GeV}) = 150\,\mathrm{MeV}$ and put the u and d quark mass zero.

$$+\,6\,\tilde{\delta}_+\,(3u\bar{u} + \bar{u}\ln\bar{u} + u\ln u) + 6\,\tilde{\delta}_-\,(\bar{u}\ln\bar{u} - u\ln u), \tag{5.10}$$

$$\begin{aligned} g_\perp^{(v)}(u) &= \frac{3}{4}(1+\xi^2) + a_1^\parallel\,\frac{3}{2}\,\xi^3 + \left(\frac{3}{7}\,a_2^\parallel + 5\zeta_3^A\right)\left(3\xi^2 - 1\right) \\ &+ \left(\frac{9}{112}\,a_2^\parallel + \frac{105}{16}\,\zeta_3^V - \frac{15}{64}\,\zeta_3^A\omega_{1,0}^A\right)\left(3 - 30\xi^2 + 35\xi^4\right) \\ &+ \frac{3}{2}\,\tilde{\delta}_+\,(2 + \ln u + \ln\bar{u}) + \frac{3}{2}\,\tilde{\delta}_-\,(2\xi + \ln\bar{u} - \ln u), \end{aligned} \tag{5.11}$$

where, for simplicity, we used asymptotic leading twist distribution amplitudes in the correction terms proportional to quark masses $\sim \delta_\pm$.[10]

The resulting ρ meson distributions are plotted in Fig. 2 together with the corresponding asymptotic distributions and with the distributions calculated in the Wandzura-Wilczek approximation. It is seen that gluon corrections of twist 3 are generally important and tend to broaden the distributions. For the second moments we get (at the scale 1 GeV):

$$\int_0^1 du\,(2u-1)^2\,h_\parallel^{(s)}(u) = 0.24\ (0.20), \tag{5.12}$$

$$\int_0^1 du\,(2u-1)^2\,h_\parallel^{(t)}(u) = 0.63\ (0.60), \tag{5.13}$$

$$\int_0^1 du\,(2u-1)^2\,g_\perp^{(a)}(u) = 0.25\ (0.20), \tag{5.14}$$

[10] For realistic parameter values the correction in a_2 is small and can safely be neglected.

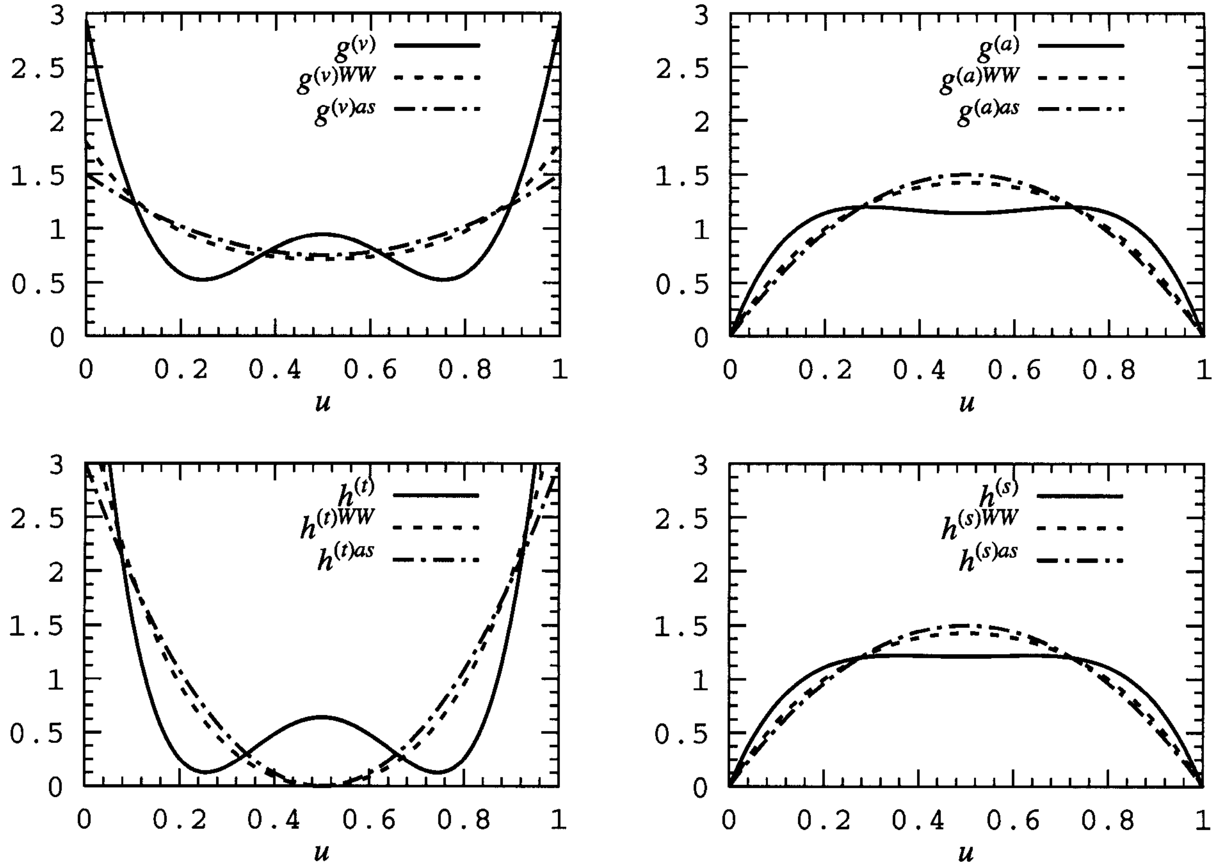

Figure 2: Two-particle twist 3 distribution amplitudes for the ρ meson.

$$\int_0^1 du\,(2u-1)^2\, g_\perp^{(v)}(u) = 0.47\ (0.40), \tag{5.15}$$

where the numbers in parenthesis give the asymptotic values. As already mentioned, the oscillatory behavior of the distributions depicted in Fig. 2 is an artifact of the expansion in orthogonal polynomials and will be smoothened by contributions of higher-order partial waves. We expect, nevertheless, that our approximation is sufficient for calculating most overlap integrals that appear in physical applications.

In Fig. 3 we compare the distribution amplitudes of ρ, K^* and ϕ mesons, which differ due to the nonzero strange quark mass. Note that quark mass corrections to two-particle distributions in general involve logarithms of the momentum fraction and are not reduced to polynomials. This means that in this case the expansion in conformal partial waves does not correspond to an expansion in local operators, which is similar to what was observed in [33, 7] for bilinear twist 4 operators. The quark mass effects are not large, but can result in a logarithmic enhancement of distributions close to the end-points $u \to 0$ and $u \to 1$, see Eqs. (5.8)–(5.11) and Fig. 3. Because of that, the calculation of SU(3) breaking effects

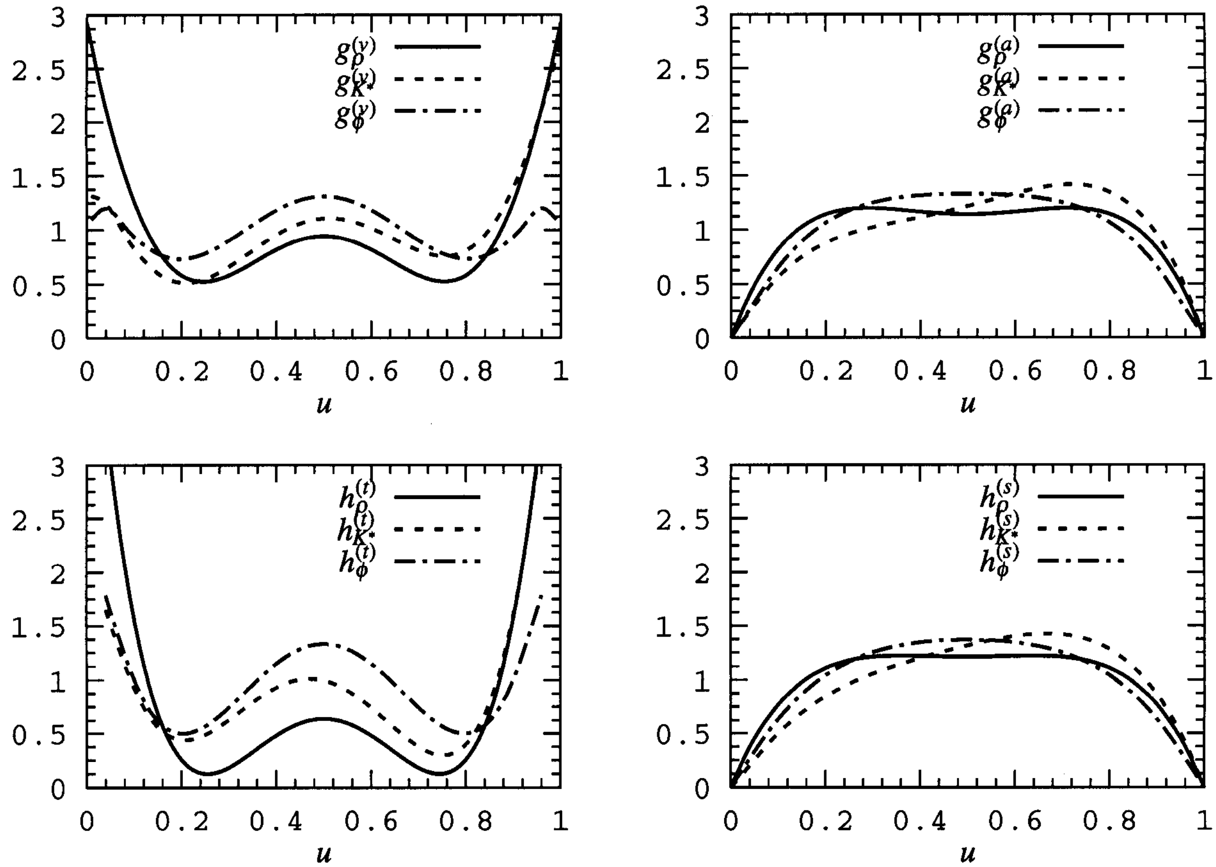

Figure 3: Two-particle twist 3 distribution amplitudes for the ρ, K^* and ϕ meson.

requires caution for physical observables which are sensitive to the end-point region where the linear approximation in m_s breaks down.[11]

6 Summary and Conclusions

In the present paper, we have studied the twist three distribution amplitudes of vector mesons in QCD and expressed them in a model-independent way by a minimal number of nonperturbative parameters. The one key ingredient in our approach was the use of the QCD equations of motion which allowed us to reveal the interrelations between the

[11] For finite quark masses, renormalization also gets complicated due to the absence of conformal symmetry. For example, $\Theta^T_{k,n-k-2}$ could receive additional mixing not only with $m_q\Omega^{\|}_{n-1}$, but also with operators involving total derivatives (see Eqs. (3.67) and (4.47)). Therefore, the scale dependence of $f_3^T\omega^T_{k,n-k-2}$ may not be described by a simple extension of the anomalous dimension matrix involving the mixing with $m_q f_V a^{\|}_{n-1}$, which is in contrast to the renormalization of twist 3 parton distributions. The complete clarification of this point is beyond the scope of this work.

different distribution amplitudes of a given twist and to obtain exact integral representations for distribution amplitudes that are not dynamically independent. The other ingredient was the use of conformal expansion which, analogously to partial wave decomposition in quantum mechanics, allows one to separate transverse and longitudinal variables in the wave function: The dependence on transverse coordinates is represented as scale-dependence of the relevant operators and is governed by renormalization-group equations, the dependence on the longitudinal momentum fraction is described in terms of irreducible presentations of the corresponding symmetry group, the collinear conformal group SL(2,R). The conformal partial wave expansion is explicitly consistent with the equations of motion since the latter are not renormalized. The expansion thus makes maximum use of the symmetry of the theory in order to simplify the dynamics, which is related, in the perturbative domain, to renormalization properties of twist three operators.

As it was known for some time [26, 25], anomalous dimensions of twist three operators increase logarithmically with the spin. Like in the leading twist case, this property ensures convergence of the conformal expansion at sufficiently large scales and suggests that only the few lowest "harmonics" are important in calculations of physical observables.

Based on this assumption, we have derived explicit and consistent models for all two- and three-particle distribution amplitudes of ρ, ω, K^* and ϕ mesons of twist two and three including contributions up to conformal spin $j = 9/2$. The relevant nonperturbative parameters ("reduced matrix elements") were estimated from QCD sum rules. The results are immediately applicable to a range of phenomenologically interesting processes like exclusive semileptonic or radiative B decays and hard electroproduction of vector mesons at HERA.

Our formalism is — in principle — applicable to arbitrary twist, although its realization will become technically more involved. The application to twist four distribution amplitudes of vector mesons will be presented elsewhere.

Acknowledgments

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A Formulae for Orthogonal Polynomials

In this appendix, we collect useful formulae for the orthogonal polynomials which appear in the conformal expansion.

Differentiation formula for Gegenbauer polynomials:

$$\frac{d}{d\xi}(1-\xi^2)C_n^{3/2}(\xi) = -(n+1)(n+2)C_n^{1/2}(\xi). \tag{A.1}$$

Differentiation formulae for Jacobi polynomials:

$$\frac{d}{d\xi}P_n^{(\nu_1,\nu_2)}(\xi) = \frac{n+\nu_1+\nu_2+1}{2}P_{n-1}^{(\nu_1+1,\nu_2+1)}(\xi), \tag{A.2}$$

$$(1+\xi)P_n^{(1,1)}(\xi) = 2\frac{d}{d\xi}\left[\frac{(n+1)}{(n+2)(2n+3)}P_{n+2}^{(0,0)}(\xi) + \frac{1}{n+2}P_{n+1}^{(0,0)}(\xi) + \frac{1}{2n+3}P_n^{(0,0)}(\xi)\right]. \tag{A.3}$$

The equation (A.3) is obtained from (A.2) combined with (A.4) below.
Recurrence formulae for Jacobi polynomials:

$$(1+\xi)P_n^{(1,1)}(\xi) = \frac{(n+1)(n+3)}{(n+2)(2n+3)}P_{n+1}^{(1,1)}(\xi) + P_n^{(1,1)}(\xi) + \frac{n+1}{2n+3}P_{n-1}^{(1,1)}(\xi) \tag{A.4}$$

$$= \frac{2(n+1)}{2n+3}\left(P_n^{(1,0)}(\xi) + P_{n+1}^{(1,0)}(\xi)\right), \tag{A.5}$$

$$(1-\xi)P_n^{(1,1)}(\xi) = \frac{2(n+1)}{2n+3}\left(P_n^{(0,1)}(\xi) - P_{n+1}^{(0,1)}(\xi)\right), \tag{A.6}$$

$$P_n^{(0,0)}(\xi) = \frac{n+1}{2n+1}P_n^{(1,0)}(\xi) - \frac{n}{2n+1}P_{n-1}^{(1,0)}(\xi) = \frac{n+1}{2n+1}P_n^{(0,1)}(\xi) + \frac{n}{2n+1}P_{n-1}^{(0,1)}(\xi), \tag{A.7}$$

$$P_n^{(0,0)}(\xi) + P_{n+1}^{(0,0)}(\xi) = (1+\xi)P_n^{(0,1)}(\xi), \tag{A.8}$$

$$P_n^{(0,0)}(\xi) - P_{n+1}^{(0,0)}(\xi) = (1-\xi)P_n^{(1,0)}(\xi). \tag{A.9}$$

Relations between Jacobi and Gegenbauer polynomials:

$$(1+\xi)P_n^{(0,1)}(\xi) + (1-\xi)P_n^{(1,0)}(\xi) = 2C_n^{1/2}(\xi), \tag{A.10}$$

$$(1+\xi)P_n^{(0,1)}(\xi) - (1-\xi)P_n^{(1,0)}(\xi) = 2C_{n+1}^{1/2}(\xi), \tag{A.11}$$

$$(n+2)P_n^{(1,1)}(\xi) = 2C_n^{3/2}(\xi). \tag{A.12}$$

Orthogonality relations for Appell polynomials [19]:

$$\int \mathcal{D}\underline{\alpha}\, \alpha_d\alpha_u\alpha_g^2 J_{k,l}(\alpha_d,\alpha_u)J_{m,n}(\alpha_d,\alpha_u) = \delta_{k+l,m+n}\frac{(-1)^{k+l}}{2^{k+l+3}(k+l+3)(2k+2l+5)!!}W_{k,m}^{(k+l+1)}, \tag{A.13}$$

where $W^{(k+l+1)}_{k,m} \equiv \partial^{m+n} J_{k,l}(\alpha_d, \alpha_u)/\partial\alpha_d^m \partial\alpha_u^n$ is a $(k+l+1)\times(k+l+1)$ symmetric matrix. This result can be obtained from the following relations:

$$\int \mathcal{D}\underline{\alpha}\, \alpha_d^{m+1}\alpha_u^{n+1}\alpha_g^2 J_{k,l}(\alpha_d,\alpha_u) =$$

$$= \begin{cases} 0 & (m+n < k+l) \\ \delta_{m,k}\dfrac{(-1)^{k+l}k!l!}{2^{k+l+3}(k+l+3)(2k+2l+5)!!} & (m+n = k+l), \end{cases} \tag{A.14}$$

while the integral is in general nonzero for $m+n > k+l$.
Integral formulae for Appell polynomials:

$$\frac{d}{du}\int_0^u d\alpha_d \int_0^{\overline{u}} d\alpha_u \frac{1}{1-\alpha_d-\alpha_u}\left(\alpha_d\frac{\partial}{\partial\alpha_d}+\alpha_u\frac{\partial}{\partial\alpha_u}-1\right)\alpha_d\alpha_u(1-\alpha_d-\alpha_u)^2 J_{k,l}(\alpha_d,\alpha_u) =$$

$$= \frac{u\overline{u}}{2}\frac{k!l!(-1)^k}{(k+l+2)!}\left(\frac{k-l}{k+l+3}P^{(1,1)}_{k+l+2}(\xi)+P^{(1,1)}_{k+l+1}(\xi)\right), \tag{A.15}$$

$$\frac{d}{du}\int_0^u d\alpha_d \int_0^{\overline{u}} d\alpha_u \alpha_d\alpha_u(1-\alpha_d-\alpha_u)J_{k,l}(\alpha_d,\alpha_u) =$$

$$= \frac{u\overline{u}}{2}\frac{k!l!(-1)^k}{(k+l+3)!}\left(\frac{k-l}{k+l+3}P^{(1,1)}_{k+l+2}(\xi)-P^{(1,1)}_{k+l+1}(\xi)\right), \tag{A.16}$$

$$\frac{d}{du}\int_0^u d\alpha_d \int_0^{\bar{u}} d\alpha_u \frac{1}{1-\alpha_d-\alpha_u}\left(\alpha_d\frac{\partial}{\partial\alpha_d}-\alpha_u\frac{\partial}{\partial\alpha_u}\right)\alpha_d\alpha_u(1-\alpha_d-\alpha_u)^2 J_{k,l}(\alpha_d,\alpha_u) =$$

$$= \frac{u\bar{u}}{2}\frac{k!l!(-1)^k}{(k+l+3)!}\left[\frac{(k+l+2)(k+l+4)}{k+l+3}P^{(1,1)}_{k+l+2}(\xi)+(k-l)P^{(1,1)}_{k+l+1}(\xi)\right]. \tag{A.17}$$

$$\int_0^u d\alpha_d \int_0^{\bar{u}} d\alpha_u \frac{1}{1-\alpha_d-\alpha_u}\left(\frac{\partial}{\partial\alpha_d}+\frac{\partial}{\partial\alpha_u}\right)\alpha_d\alpha_u(1-\alpha_d-\alpha_u)^2 J_{k,l}(\alpha_d,\alpha_u) =$$

$$= \frac{k!l!(-1)^k}{(k+l+3)!4}\left[\frac{-k+l}{2k+2l+5}\left(P^{(0,0)}_{k+l+1}(\xi)-P^{(0,0)}_{k+l+3}(\xi)\right)\right.$$

$$\left.+\frac{k+l+2}{2k+2l+7}\left(P^{(0,0)}_{k+l+2}(\xi)-P^{(0,0)}_{k+l+4}(\xi)\right)\right]. \tag{A.18}$$

The results (A.15)−(A.17) can be obtained by differentiating and/or integrating the Appell polynomials $J_{k,l}(\alpha_d,\alpha_u)$ term by term. To obtain (A.18), it is convenient to calculate its derivative first, which can be done similarly to (A.15), (A.16) and (A.17), and then integrate the result with the condition that it vanishes at $u=0$.

B Conformal Expansion of Wandzura-Wilczek Contributions

In this appendix we explain the structure of the conformal expansion of the Wandzura-Wilczek contributions to twist 3 chiral-odd two-particle distribution amplitudes, Eq. (3.40). As mentioned in Sec. 3.2, the conformal spin assignment for these terms seemingly does not match the expansion in Eqs. (3.30) and (3.31), which calls for an explanation.

The basic idea is the following: the expansion derived in Eqs. (3.30) and (3.31) is based on the conformal expansion of the corresponding operators. The distribution amplitudes $h_{\parallel}^{(t)}$ and $h_{\parallel}^{(s)}$ are obtained as matrix elements of these operators between the vacuum and the longitudinally polarized ρ meson state. We call the state conformal and assign a corresponding conformal spin, if it is annihilated by a conformal operator. The difficulty with the Wandzura-Wilczek terms is due to the fact that these contributions involve matrix elements over the ρ meson with a different (transverse) polarization. Working out the Wandzura-Wilczek contributions to $h_{\parallel}^{(t)}$, $h_{\parallel}^{(s)}$ essentially corresponds to reexpressing these matrix elements in terms of similar matrix elements over the longitudinally polarized meson, using Lorentz symmetry. In our context it is important that the transversely polarized state is related (in the ρ meson rest frame) to the longitudinally polarized state by a spin rotation which does not commute with the generators of the collinear conformal group. Working out the necessary commutation relations, we reproduce the particular spin structure appearing in (3.40).

It is convenient to work in the helicity basis: ρ meson states with $\lambda = \pm 1$ correspond to transverse polarization, $\lambda = 0$ denotes the longitudinal polarization. Equation (3.54) reads

$$f_\rho^T a_n^\perp \propto \langle 0|\Omega_n^\perp|\rho(P,\lambda=\pm 1)\rangle, \tag{B.1}$$

where the proportionality factor is irrelevant for what follows. Following Ohrndorf [18], we define a set of eigenstates $|j,m\rangle$ with conformal spin j and the "third projection" of the spin $m = j, j+1, j+2, \ldots$, such that

$$\begin{aligned} J_3|j,m\rangle &= m|j,m\rangle, & J^2|j,m\rangle &= j(j-1)|j,m\rangle, \\ J_-|j,m\rangle &= -(j-m)|j,m-1\rangle, & J_+|j,m\rangle &= (j+m)|j,m+1\rangle. \end{aligned} \tag{B.2}$$

The generators $J_\pm$, J_3 satisfy the canonical commutation relations of the algebra of the group of hyperbolic rotations, O(2,1), and are related to the generators of the collinear conformal group by

$$J_+ = \frac{i}{\sqrt{2}}P_., \quad J_- = \frac{i}{\sqrt{2}}K_*, \quad J_3 = \frac{i}{2}(D + M_{*.}), \tag{B.3}$$

where P_μ and $M_{\mu\nu}$ are the usual generators of Poincare group, D is the generator of dilatations and K_μ generates conformal transformations. Note that J_+ and J_- are just "step-up" and "step-down" operators in this basis; the state with the *lowest* value of m, $m_{\min} = j$, is annihilated by J_-.

Following the discussion in Ref. [18], it is easy to show that

$$J_-\Omega_n^\perp|0\rangle = [J_-,\Omega_n^\perp]|0\rangle = 0, \quad J_3\Omega_n^\perp|0\rangle = (n+2)\Omega_n^\perp|0\rangle, \quad J^2\Omega_n^\perp|0\rangle = (n+2)(n+1)\Omega_n^\perp|0\rangle, \tag{B.4}$$

so that we can identify

$$|n+2,n+2\rangle \equiv \Omega_n^\perp|0\rangle, \tag{B.5}$$

and rewrite Eq. (B.1) as

$$f_\rho^T a_n^\perp \propto \langle n+2,n+2|\rho(P,\lambda=\pm1)\rangle. \tag{B.6}$$

This confirms that $a_n^\perp$ corresponds to conformal spin $j=n+2$, as stated in the main text.

To determine the corresponding contribution to $h_\parallel^{(t)}$, $h_\parallel^{(s)}$, we have to recast Eq. (B.6) in a different form corresponding to a matrix element over the longitudinally polarized meson. For definiteness, take $\lambda=+1$. In the ρ meson rest frame the $\lambda=+1$ state is related to the $\lambda=0$ state by the spin rotation

$$|\rho(P=0,\lambda=+1)\rangle \propto (M_{23}+iM_{31})|\rho(P=0,\lambda=0)\rangle, \tag{B.7}$$

where $(M_{23}+iM_{31})$ is the step-up operator of ordinary angular momentum. $|\rho(P,\lambda)\rangle$ is then obtained by a Lorentz boost in P^3 direction:

$$|\rho(P,\lambda)\rangle = \mathcal{U}(\omega)|\rho(P=0,\lambda)\rangle, \tag{B.8}$$

where

$$\mathcal{U}(\omega) = e^{-i\omega M^{03}} = e^{-i\omega M_{*\cdot}}, \quad \mathrm{th}(\omega) = P^3/P^0. \tag{B.9}$$

We can thus write

$$f_\rho^T a_n^\perp \propto \langle\Psi|\rho(P,\lambda=0)\rangle, \tag{B.10}$$

where

$$|\Psi\rangle = \mathcal{U}(\omega)(M_{23}+iM_{13})\,\mathcal{U}^{-1}(\omega)|n+2,n+2\rangle. \tag{B.11}$$

In the following we demonstrate that $|\Psi\rangle$ is given by a superposition of three conformal states,

$$|\Psi\rangle = C_1|n+\frac{3}{2},n+\frac{3}{2}\rangle + C_2|n+\frac{3}{2},n+\frac{5}{2}\rangle + C_3|n+\frac{5}{2},n+\frac{5}{2}\rangle, \tag{B.12}$$

where the C_k are C-numbers. If established, Eq. (B.12) shows that contributions of $a_n^\perp$ to the matrix element over a longitudinal ρ meson correspond to conformal spins $j=n+\frac{3}{2}$ and $j=n+\frac{5}{2}$, which explains the pattern appearing in Eq. (3.40).

To prove (B.12), we first note that

$$\mathcal{U}(\omega)\,(M_{23}+iM_{13})\,\mathcal{U}^{-1}(\omega) = M_{23}\,\mathrm{ch}(\omega) + M_{20}\,\mathrm{sh}(\omega) + i\{M_{13}\,\mathrm{ch}(\omega)+M_{10}\,\mathrm{sh}(\omega)\}. \tag{B.13}$$

From this and Eq. (B.11), it follows

$$|\Psi\rangle = [\exp(\omega)/2\,(M_{2\cdot}+iM_{1\cdot}) - \exp(-\omega)\,(M_{2*}+iM_{1*})]\,|n+2,n+2\rangle \tag{B.14}$$

with $z^\mu = (1,0,0,-1)$ and $p^\mu = (1,0,0,1)$ (note that $p\cdot z = 2$). Applying to (B.14) the commutation relations

$$[J_3, M_{2\cdot} + iM_{1\cdot}] = -\frac{1}{2}\,(M_{2\cdot} + iM_{1\cdot}), \qquad [J_3, M_{2*} + iM_{1*}] = \frac{1}{2}\,(M_{2*} + iM_{1*}) \tag{B.15}$$

and

$$[J_-, M_{2\cdot} + iM_{1\cdot}] = 0, \qquad [J_-, M_{2*} + iM_{1*}] \propto K_2 + iK_1, \qquad [J_-, K_2 + iK_1] = 0, \tag{B.16}$$

it immediately follows that

$$\begin{aligned} (M_{2\cdot} + iM_{1\cdot})|n+2, n+2\rangle &\propto |n + \frac{3}{2}, n + \frac{3}{2}\rangle, \\ (M_{2*} + iM_{1*})|n+2, n+2\rangle &\propto a_1|n + \frac{3}{2}, n + \frac{5}{2}\rangle + a_2|n + \frac{5}{2}, n + \frac{5}{2}\rangle, \end{aligned} \tag{B.17}$$

with C-number coefficients a_i, which proves Eq. (B.12).

A similar discussion also explains the mismatch observed in Eq. (4.31) for chiral-even distribution amplitudes.

C QCD Sum Rules for Expansion Coefficients of Distribution Amplitudes

The method of QCD sum rules in its application to distribution amplitudes of light mesons was pioneered by Chernyak and Zhitnitsky and is comprehensively discussed in [3]. In this appendix we collect QCD sum rules and results for the twist 2 distribution amplitudes of the vector mesons ρ, K^* and ϕ as well as for the twist 3 distribution amplitudes of the ρ meson. Numerical results presented below are obtained using the following input parameters:

$$\begin{aligned} \overline{m}_s(1\,\mathrm{GeV}) &= (150 \pm 50)\,\mathrm{MeV}, & \left\langle \tfrac{\alpha_s}{\pi} G^2 \right\rangle &= (0.012 \pm 0.006)\,\mathrm{GeV}^4, \\ \langle \bar{q}q\rangle(1\,\mathrm{GeV}) &= (-240 \pm 20)\,\mathrm{MeV}^3, & \langle \bar{s}s\rangle(1\,\mathrm{GeV}) &= 0.8\langle \bar{q}q\rangle(1\,\mathrm{GeV}), \\ \langle \bar{q}\sigma gGq\rangle(1\,\mathrm{GeV}) &= 0.8\langle \bar{q}q\rangle(1\,\mathrm{GeV}), & \langle \bar{s}\sigma gGs\rangle(1\,\mathrm{GeV}) &= 0.8\langle \bar{q}\sigma gGq\rangle(1\,\mathrm{GeV}), \\ \Lambda^{(3)}_{\mathrm{QCD}} &= 400\,\mathrm{MeV} \Longrightarrow \alpha_s(1\,\mathrm{GeV}) = 0.56 \end{aligned} \tag{C.1}$$

and assuming factorization of the vacuum expectation values of four-fermion operators. The SU(3) breaking effects in the sum rules are due to explicit corrections proportional to the quark masses, the difference in values of the condensates of strange and nonstrange quarks, and differences in the values of the continuum thresholds s_0 and Borel parameters M^2. Instead of fitting the continuum thresholds separately for each meson and for each sum rule, in this paper we prefer to determine $s_{0,\rho}$ from the simplest sum rules for vector (tensor) couplings and use the relations

$$s_{0,K^*} - s_{0,\rho} = m^2_{K^*} - m^2_\rho, \qquad s_{0,\phi} - s_{0,\rho} = m^2_\phi - m^2_\rho, \tag{C.2}$$

which are known to hold with reasonable accuracy for the spectra of resonances in the respective channels. Similar relations are assumed between the "working windows" in the Borel parameter.

C.1 Distribution amplitudes of twist two

Most of the relevant formulas were previously obtained in [31, 3, 12]. For the ρ meson, we quote the results from [12]. For the other mesons we present a new analysis which includes the radiative corrections calculated in [12] and the SU(3) breaking terms calculated in [31]. Unlike in Ref. [3, 31], where sum rules for the moments of distribution amplitudes were derived, we prefer to consider the sum rules directly for the coefficients a_n (Gegenbauer moments) in the conformal expansion, see [12] for a discussion.

QCD sum rules for even Gegenbauer moments can be derived from the *diagonal* correlation functions of the conformal operators introduced in Sec. 3.3 and Sec. 4.3:

$$\mathrm{D}_n^{\parallel\{\perp\}} = i\int d^4y\, e^{ipy}\langle 0|T\Omega_n^{\parallel\{\perp\}}(y)\Omega_0^{\dagger\parallel\{\perp\}}(0)|0\rangle, \tag{C.3}$$

see Eqs. (3.53)–(3.55) and (4.45)–(4.47) for precise definitions.

One finds the following sum rules for vector and tensor couplings [29, 31] of the K^* meson:

$$\begin{aligned} f_{K^*}^2\, e^{-m_{K^*}^2/M^2} &= \frac{M^2}{4\pi^2}\left(1-e^{-s_0^\parallel/M^2}\right)\left(1+\frac{\alpha_s}{\pi}\right)+\frac{1}{12M^2}\left\langle\frac{\alpha_s}{\pi}G^2\right\rangle+\frac{\overline{m}_s\langle\bar{s}s\rangle}{M^2} \\ &+\frac{16\pi\alpha_s}{81M^4}\left(\langle\bar{q}q\rangle^2+\langle\bar{s}s\rangle^2\right)-\frac{16\pi\alpha_s}{9M^4}\langle\bar{q}q\rangle\langle\bar{s}s\rangle, \end{aligned} \tag{C.4}$$

$$\begin{aligned} (f_{K^*}^T(\mu))^2 e^{-m_{K^*}^2/M^2} &= \left(\frac{\alpha_s(\mu^2)}{\alpha_s(M^2)}\right)^{\frac{2\gamma_0^\perp}{\beta_0}}\left\{1+\frac{\alpha_s(\mu^2)-\alpha_s(M^2)}{2\pi}\,\frac{\gamma_0^\perp}{\beta_0}\left(\frac{\gamma_0^{\perp(1)}}{\gamma_0^\perp}-\frac{\beta_1}{\beta_0}\right)\right\}\times \\ \times\ &\left[\frac{1}{4\pi^2}\int_0^{s_0^\perp}ds\, e^{-s/M^2}\left\{1+\frac{\alpha_s}{\pi}\left(\frac{7}{9}+\frac{2}{3}\ln\frac{s}{M^2}\right)\right\}-\frac{1}{12M^2}\left\langle\frac{\alpha_s}{\pi}G^2\right\rangle+\frac{\overline{m}_s\langle\bar{s}s\rangle}{M^2}\right. \\ &\left.-\frac{1}{3M^4}\overline{m}_s\langle\bar{s}\sigma gGs\rangle-\frac{32\pi\alpha_s}{81M^4}\left(\langle\bar{q}q\rangle^2+\langle\bar{s}s\rangle^2\right)\right]. \end{aligned} \tag{C.5}$$

In the sum rule for $f_{K^*}^T$, $\beta_0=9, \beta_1=64$, and $\gamma_0^{\perp(1)}=310/9$ (for three running flavours) is the two-loop anomalous dimension calculated in [34].

For arbitrary (even) Gegenbauer moments one obtains:

$$\frac{3(n+1)(n+2)}{2(2n+3)}\, f_{K^*}^2 a_n^\parallel(\mu) e^{-m_{K^*}^2/M^2} \;=$$

$$
= \frac{1}{2\pi^2}\frac{\alpha_s}{\pi}M^2\left(1-e^{-s_0^{\parallel}/M^2}\right)\int_0^1 du\, u\bar{u}\, C_n^{3/2}(2u-1)\,\ln^2\frac{u}{\bar{u}} + \frac{1}{2M^2}\,\overline{m}_s\langle\bar{s}s\rangle(n+1)(n+2)
$$

$$
+\frac{1}{24M^2}\left\langle\frac{\alpha_s}{\pi}G^2\right\rangle(n+1)(n+2) - \frac{1}{48M^4}\,\overline{m}_s\langle\bar{s}\sigma gGs\rangle n(n+1)(n+2)(n+3)
$$

$$
-\frac{8\pi\alpha_s(\mu)}{9M^4}\langle\bar{q}q\rangle\langle\bar{s}s\rangle(n+1)(n+2) + \frac{4\pi\alpha_s}{81M^4}(\langle\bar{q}q\rangle^2+\langle\bar{s}s\rangle^2)(n+1)^2(n+2)^2, \qquad \text{(C.6)}
$$

$$
\frac{3(n+1)(n+2)}{2(2n+3)}\,(f_{K^*}^T(\mu))^2 a_n^{\perp}(\mu)e^{-m_{K^*}^2/M^2} =
$$

$$
= \frac{1}{2\pi^2}\frac{\alpha_s}{\pi}M^2\left(1-e^{-s_0^{\perp}/M^2}\right)\int_0^1 du\, u\bar{u}\, C_n^{3/2}(2u-1)\left(\ln u + \ln\bar{u} + \ln^2\frac{u}{\bar{u}}\right)
$$

$$
+\frac{1}{24M^2}\left\langle\frac{\alpha_s}{\pi}G^2\right\rangle(n^2+3n-2) - \frac{1}{48M^4}\,\overline{m}_s\langle\bar{s}\sigma gGs\rangle(n+1)(n+2)(n^2+3n+8)
$$

$$
+\frac{4\pi\alpha_s}{81M^4}(\langle\bar{q}q\rangle^2+\langle\bar{s}s\rangle^2)(n-1)(n+1)(n+2)(n+4) + \frac{1}{2M^2}\,\overline{m}_s\langle\bar{s}s\rangle(n+1)(n+2). \qquad \text{(C.7)}
$$

The sum rules for ρ are obtained by setting m_s zero and s to q in the condensates. For ϕ, one has to replace q by s in the condensates and to double the terms linear in m_s. On the left-hand sides one has to insert the proper meson masses. All renormalization scale dependent quantities are evaluated at the scale $\mu \sim 1$ GeV.

Gegenbauer moments with odd n are nonvanishing for the K^* meson only. They can be determined most conveniently from the *nondiagonal* correlation functions[12]

$$
\mathrm{ND}_n^{\parallel\{\perp\}} = i\int d^4y\, e^{ipy}\langle 0|T\Omega_n^{\parallel\{\perp\}}(y)\Omega_0^{\dagger\perp\{\parallel\}}(0)|0\rangle. \qquad \text{(C.8)}
$$

The sum rules read [31]:

$$
\frac{3(n+1)(n+2)}{2(2n+3)}\,f_{K^*}^T(\mu)f_{K^*}m_{K^*}a_n^{\parallel}(\mu)e^{-m_{K^*}^2/M^2} =
$$

$$
= \frac{3}{8\pi^2}\,\overline{m}_s M^2\left(1-e^{-s_0^{\parallel}/M^2}\right) + \frac{1}{2}(n+1)(n+2)(\langle\bar{s}s\rangle - \langle\bar{q}q\rangle)
$$

$$
-\frac{1}{24M^2}(n+1)^2(n+2)^2(\langle\bar{s}\sigma gGs\rangle - \langle\bar{q}\sigma gGq\rangle), \qquad \text{(C.9)}
$$

$$
\frac{3(n+1)(n+2)}{2(2n+3)}\,f_{K^*}^T(\mu)f_{K^*}m_{K^*}a_n^{\perp}(\mu)e^{-m_{K^*}^2/M^2} =
$$

[12]Since perturbative contributions to $\mathrm{D}^{(n)}$ vanish for odd n. Note also that from $\mathrm{ND}^{(0)}$ one obtains the relative sign between f_V and f_V^T.

$$= \frac{3}{8\pi^2}\,\overline{m}_s M^2 \left(1 - e^{-s_0^\perp/M^2}\right) + \frac{1}{2}(n+1)(n+2)(\langle\bar{s}s\rangle - \langle\bar{q}q\rangle)$$

$$- \frac{1}{48M^2}(n+1)(n+2)(n^2+3n+4)(\langle\bar{s}\sigma gGs\rangle - \langle\bar{q}\sigma gGq\rangle). \qquad \text{(C.10)}$$

In these sum rules again the right-hand sides are to be evaluated at fixed scale μ of order 1 GeV.

The results are collected in Tab. 4, where the quoted errors are due to uncertainties in input parameters and to the variation of the Borel parameter within the range $M^2 \approx (1-2)\,\mathrm{GeV}^2$ (for the ρ meson), with the value of the continuum threshold $s_{0,\rho}^{\|} = 1.5\ \mathrm{GeV}^2$ fitted to reproduce the experimental value of the vector coupling. As discussed in detail in [12], the sum rule for the tensor couplings contains contaminating contributions of states with the opposite parity 1^+, which can be effectively taken into account by using a lower value of the continuum threshold $s_{0,\rho}^{\perp} = 1.2\ \mathrm{GeV}^2$. For other mesons, we assume validity of the relations (C.2). Note that in the numerical analysis of Gegenbauer moments we substitute the couplings on the left-hand sides by their sum rules (C.4) and (C.5) rather than using the values given in the table.

C.2 Distribution amplitudes of twist three

The vector $f_{3\rho}^V$ and axial $f_{3\rho}^A$ twist 3 couplings are defined as the local matrix elements

$$\langle 0|\bar{d}\gamma_\mu \left[gG_{\rho\lambda}(i\overrightarrow{D}.) - (i\overleftarrow{D}.)gG_{\rho\lambda}\right] u|\rho^+\rangle \equiv \langle 0|\bar{u}\gamma_\mu \left[gG_{\rho\lambda}(i\overrightarrow{D}.) - (i\overleftarrow{D}.)gG_{\rho\lambda}\right] d|\rho^-\rangle$$

$$= i(pz)\,p_\mu(p_\lambda e_\rho^\perp - p_\rho e_\lambda^\perp) f_{3\rho}^V + \ldots, \qquad \text{(C.11)}$$

$$\langle 0|\bar{d}\gamma_\mu\gamma_5 g\widetilde{G}_{\lambda\rho}u|\rho^+\rangle = \langle 0|\bar{u}\gamma_\mu\gamma_5 g\widetilde{G}_{\lambda\rho}d|\rho^-\rangle = p_\mu(p_\rho e_\lambda^\perp - p_\lambda e_\rho^\perp) f_{3\rho}^A + \ldots, \qquad \text{(C.12)}$$

and have been estimated using the sum rule approach in Ref. [35] together with a few matrix elements of higher dimension (and conformal spin). The results are given in the text[13].

The tensor coupling is defined as

$$\langle 0|\bar{d}\sigma_{\mu\cdot} \left[gG^{\mu\cdot}(i\overrightarrow{D}.) - (i\overleftarrow{D}.)gG^{\mu\cdot}\right] u|\rho\rangle = (ez)(pz)^2 m_\rho f_{3\rho}^T \qquad \text{(C.13)}$$

and can be estimated from correlation functions of this operator with the vector and/or tensor current. The correlation function with the vector current is chirality-violating and is expanded in operators with odd dimension. The leading contribution of the mixed quark-gluon condensate, however, vanishes, and the first corrections comes from dimension 7 operators whose vacuum expectation values are known only very poorly. For this reason

[13] Apparently the corresponding sum rules have never been published and are not available. We thank V. Chernyak for correspondence on this point.

this sum rule is essentially useless. From the "diagonal" correlation function with the tensor current,

$$i \int d^4y\, e^{iqy} \langle 0|T\bar{d}(y)\sigma_{\mu\cdot} \left[gG^{\mu\cdot}(i\overrightarrow{D}.) - (i\overleftarrow{D}.)gG^{\mu\cdot} \right] u(y)\bar{u}(0)\sigma_{\mu\cdot}d(0)|0\rangle, \tag{C.14}$$

we obtain the sum rule

$$e^{-m_\rho^2/M^2} m_\rho f_\rho^T(\mu) f_{3\rho}^T(\mu) = \frac{\alpha_s}{720\pi^3} \int_0^{s_0} ds\, e^{-s/M^2} + \frac{1}{36} \left\langle \frac{\alpha_s}{\pi} G^2 \right\rangle + \frac{32}{27M^2} \pi\alpha_s \langle \bar{q}q \rangle^2, \tag{C.15}$$

which we have studied numerically. As a general feature of the sum rules for matrix elements of operators with high dimension, this sum rule is dominated at small $M^2 \sim$(1–2) GeV^2 by the condensates of high dimension (four-quark operators, for the case at hand) and is not stable. At larger values of the Borel parameter the stability of the sum rule is very much improved, suggesting the value $f_{3\rho}^T(1\,\mathrm{GeV}) \approx 0.3 \cdot 10^{-2}\,\mathrm{GeV}^2$. This number has to be considered as a rough estimate, however, since at large values of the Borel parameter the contributions of higher mass resonances and of the continuum are out of control. Note that, similar to the case of the twist 2 tensor coupling considered above, the sum rule (C.15) includes contributions of states with opposite (positive) parity. Ascribing a 100% error to this result, we arrive at the range given in (5.7) as our best estimate.

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