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Lattice QCD Calculations of Leptonic and Semileptonic Decays

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Abstract. In lattice QCD, obtaining properties of heavy-light mesons has been easier said than done. Focusing on the B meson's decay constant, it is argued that towards the end of 1997 the last obstacles were removed, at least in the quenched approximation. These developments, which resulted from a fuller understanding and implementation of ideas in effective field theory, bode well for current studies of neutral meson mixing and of semileptonic decays.

INTRODUCTION

Eleven years ago at *Lattice '87*, three talks gave birth to a new field of research, the study of heavy quarks in lattice QCD. Estia Eichten [1] emphasized that lattice QCD should provide reliable information on hadrons with heavy quarks, which would help determine the Cabibbo-Kobayashi-Maskawa (CKM) matrix. He suggested starting with the *static* approximation and adding corrections in $1/m_Q$. One of the developments to grow from this suggestion is the heavy-quark effective theory (HQET), an enormous subject in its own right. Peter Lepage [2] introduced non-relativistic QCD (NRQCD) as a tool for studying spectroscopy and matrix elements of systems with one or more heavy quarks. This effective theory also has enjoyed widespread application. Finally, Luciano Maiani [3] presented, among other things, the first well-known calculation from lattice QCD of the leptonic decay constant of the B meson. Maiani and his collaborators took Wilson's action for light quarks and bravely applied it to the b quark, even though their lattice's ultraviolet cutoff was below the mass m_b .

The idea that lattice QCD could play a role in interpreting future experiments was exciting, and it attracted much attention. In addition to the spectra of quarkonium and "heavy-light" hadrons, considerable effort has been devoted to the hadronic matrix elements for leptonic and semileptonic decays of heavy-light mesons (B , B_s , D , and D_s) and neutral meson mixing. The decay constant of the

¹⁾ Fermilab is operated by Universities Research Association Inc., under contract with the United States' Department of Energy.

generic heavy-light pseudoscalar meson, denoted f_P , parameterizes the hadronic amplitude for leptonic decays. It has received the most attention, especially f_B . It was expected to be the most straightforward of matrix elements and, thus, a bellwether. Unfortunately, a reliable calculation could not be done quickly. Fortunately, the technical and conceptual difficulties have been largely overcome, and the calculation of f_B in lattice QCD has grown up at last.

Computationally f_B is indeed just as easy as f_π , but the interpretation of the results has not been obvious. Consequently, the literature contains a wide range of estimates, some of which should not be taken seriously. For example, several early calculations in the static limit contained too much contamination from radial excitations B [4], and the results are misleadingly high.

More recently, a fuller understanding of the interplay between the effective theories and the lattice has helped to reduce the effects of lattice artifacts. The effective theories NRQCD and HQET are derived from QCD by lowering the renormalization point, or cutoff, μ until $|\mathbf{p}| \ll \mu \approx m_Q$, where $|\mathbf{p}|$ is the heavy quark's typical momentum and m_Q its mass. Because $|\mathbf{p}|/m_Q \ll 1$ the interactions in the effective Lagrangian can be organized in powers of $|\mathbf{p}|/m_Q$. (In quarkonium $|\mathbf{p}| \sim \alpha_s m_Q$; in heavy-light systems $|\mathbf{p}| \sim \Lambda_{\text{QCD}} \sim 200$ MeV.) The two effective theories share the same Lagrangian, although the power of $|\mathbf{p}|/m_Q$ assigned to operators of higher dimension can differ. One can take the effective theories' Lagrangian and introduce the lattice as an ultraviolet regulator, choosing the lattice spacing a so that $m_Q \approx \mu \sim \pi/a$. This lattice theory is often called lattice NRQCD [5]. There are discretization effects, which are the higher-dimension operators multiplied by calculable coefficients. Although some of these operators are new, many are the same as in the $1/m_Q$ expansion. Consequently, physical $1/m_Q$ effects and artificial a effects have become intertwined. Lattice practitioners must disentangle them and remove the lattice artifacts, at least to the desired accuracy.

Alternatively, one can start with an action derived for $m_q a \ll 1$, such as Wilson's, and ask what happens when one applies it for $m_Q a \approx 1$. This, essentially, is the way of Ref. [3]. Since then, many experts have said (and still do, out of habit), that a heavy quark cannot be put directly on the lattice because $m_Q a \approx 1$. On the other hand, numerous calculations have been published with $m_Q a \approx 1$, so there must be more to the story. Indeed, the lattice theory does not break down. Instead, the lattice artifacts are again intertwined with the $1/m_Q$ effects, as in lattice NRQCD. The same operators appear, but the coefficients are different, though still calculable. For a class of actions based on the Wilson action, it has been shown, to all orders in perturbation theory, that the coefficients remain small, for all $m_Q a$ [6].

In preparing a brief review of a subject one is faced with the choice between a catalog of all recent results or a synthesis of developments over a longer period of time. The proceedings of the Lattice conferences provide excellent examples of the former [7–9]. By contrast, this paper gives a view of the (theoretical and computational) progress, focusing on f_B . Owing to space limitations the material presented on the allied subject of neutral meson mixing and on phenomenologically promising form factors of semileptonic decays is brief.

NUMERICAL LATTICE CALCULATIONS

When one thinks of lattice QCD, one usually thinks of large-scale numerical calculations. This approach computes the functional integrals of quantum field theory by applying a Monte Carlo method with importance sampling. Statistical errors arise here, and with more and more computer time these errors can be made arbitrarily small. Over the years various clever techniques have been devised to enhance the “signal-to-noise” ratio, that is, to reduce the statistical error for fixed computing resources.

To use Monte Carlo methods, three modifications are introduced. First, spacetime becomes a finite box, usually with periodic boundary conditions. Second, the spacetime continuum becomes a discrete lattice. Last, the so-called quenched approximation is applied. The first two are common to many kinds of numerical analysis, but the last requires a short explanation. The quenched approximation treats a hadron’s valence quarks and all the exchanged gluons fully, including retardation, but the back-reaction of closed quark loops on the gluons is omitted. For particle physics a more descriptive name (and one of the original names) would be the valence approximation, but the term “quenched,” taken from statistical mechanics, is more commonly used. The back-reaction of closed quark loops is computationally burdensome. Because its omission saves computer time, the quenched approximation is a useful way to control the other errors and, thus, to teach theorists how to analyze uncertainties.

Let us return to the first two approximations. A volume larger than a few fm on a side should be good enough. After all, one does not expect the true size and boundary of the universe to effect the physics of hadrons. It is with the lattice itself that the subject becomes a craft. The continuum limit can be reached by taking the lattice spacing $a \rightarrow 0$ with brute force, or by improving the action to reduce discretization effects, or by a combination of the two. The crudest form of brute force, namely to take $m_Q a \ll 1$, would require, for the bottom quark, a fantastically small lattice spacing. This has never been done.

B, D, K & CKM

Table 1 contains a list of specific reactions with conventional parameterizations of hadronic matrix elements. Together each pair can determine the listed element of the CKM matrix. Every element of the CKM matrix appears except V_{tb} and the entire (3×3 unitary) matrix can be constrained.

In the Standard Model, the leptonic partial width of a generic pseudoscalar meson P , containing quarks of flavors p and q , is given by

$$\Gamma_{P \rightarrow l\nu} = \left(\frac{\text{known}}{\text{factors}} \right) f_P^2 |V_{pq}|^2. \quad (1)$$

If one could compute f_P (i.e., f_K or f_D or f_B) with a reliably estimated uncertainty, a measurement of the partial width is tantamount to a measurement of $|V_{pq}|$.

TABLE 1. How to combine exclusive experimental measurements with calculations in (lattice) QCD to obtain the Cabibbo-Kobayashi-Maskawa matrix.

measure	compute	determine	measure	compute	determine
$\pi \rightarrow \mu\nu$	f_π	$ V_{ud} $			
$K \rightarrow \mu\nu$	f_K	$ V_{us} $	$K \rightarrow \pi e\nu$	$f_\pm^{K\pi}(q^2)$	$ V_{us} $
$B \rightarrow \tau\nu$	f_B	$ V_{ub} $	$B \rightarrow \pi e\nu$	$f_\pm^{B\pi}(q^2)$	$ V_{ub} $
			$\rightarrow \rho e\nu$	$A_1^{B\rho}(q^2)$	
$D \rightarrow \mu\nu$	f_D	$ V_{cd} $	$D \rightarrow \pi e\nu$	$f_\pm^{D\pi}(q^2)$	$ V_{cd} $
			$\rightarrow \rho e\nu$	$A_1^{D\rho}(q^2)$	
$D_s \rightarrow \mu\nu$	f_{D_s}	$ V_{cs} $	$D \rightarrow K e\nu$	$f_\pm^{DK}(q^2)$	$ V_{cs} $
			$\rightarrow K^* e\nu$	$A_1^{DK^*}(q^2)$	
$B_c \rightarrow \mu\nu$	f_{B_c}	$ V_{cb} $	$B \rightarrow D e\nu$	$h_\pm(w)$	$ V_{cb} $
			$\rightarrow D^* e\nu$	$h_{A_1}(w)$	
Neutral meson mixing					
$K^0 \leftrightarrow \bar{K}^0$	$\frac{8}{3}m_K^2 f_K^2 B_K$	$\varepsilon_K(\rho, \eta)$			
$B_d^0 \leftrightarrow \bar{B}_d^0$	$\frac{8}{3}m_{B_d}^2 f_{B_d}^2 B_{B_d}$	$ V_{td} $	$B_s^0 \leftrightarrow \bar{B}_s^0$	$\frac{8}{3}m_{B_s}^2 f_{B_s}^2 B_{B_s}$	$ V_{ts} $

Similarly, the differential decay rate of a semileptonic decay is given by

$$\frac{d\Gamma_{P \rightarrow H l \nu}}{dq^2} = \left(\frac{\text{known}}{\text{factors}} \right) \mathcal{F}^2(q^2) |V_{pq}|^2, \quad (2)$$

where q is momentum carried off by the leptons, and \mathcal{F} is the appropriate form factor. These processes do not suffer the helicity suppression of the leptonic decays, so the statistical error of the experimental measurements is smaller. Furthermore, lattice QCD can provide the q^2 dependence, at least when the momentum of the daughter hadron is not too large. If theory and experiment exhibit the same shape as a function of q^2 , one's confidence in the systematics increases qualitatively.

Neutral meson mixing reveals a glimpse of the third row of the CKM matrix. For example, the mass difference of neutral B mesons is given by

$$x_q = \frac{\Delta M_{B_q^0}}{\Gamma_{B_q^0}} = \left(\frac{\text{known}}{\text{factors}} \right) \frac{8}{3} m_{B_q}^2 f_{B_q}^2 B_{B_q} |V_{tq}^* V_{tb}|^2, \quad (3)$$

where the flavor q can be either down or strange. The notation employing $\frac{8}{3}m_{B_q}^2$, $f_{B_q}^2$, and the ‘‘bag parameter’’ B_{B_q} is historical and is taken from the kaon system. (In the so-called vacuum saturation approximation, $B_B = 1$.) Nevertheless, this formula, more so than leptonic B decay, motivates the interest in f_{B_d} and f_{B_s} .

Figure 1 shows a time-line of calculations of f_B with lattice QCD [10–30]. The results included in Fig. 1 have been selected with two criteria: Conference proceedings, which are almost always followed (eventually) by papers in refereed journals, have been omitted. Otherwise, I have taken papers whose authors were self-confident enough to quote a result in the abstract. The second criterion is not

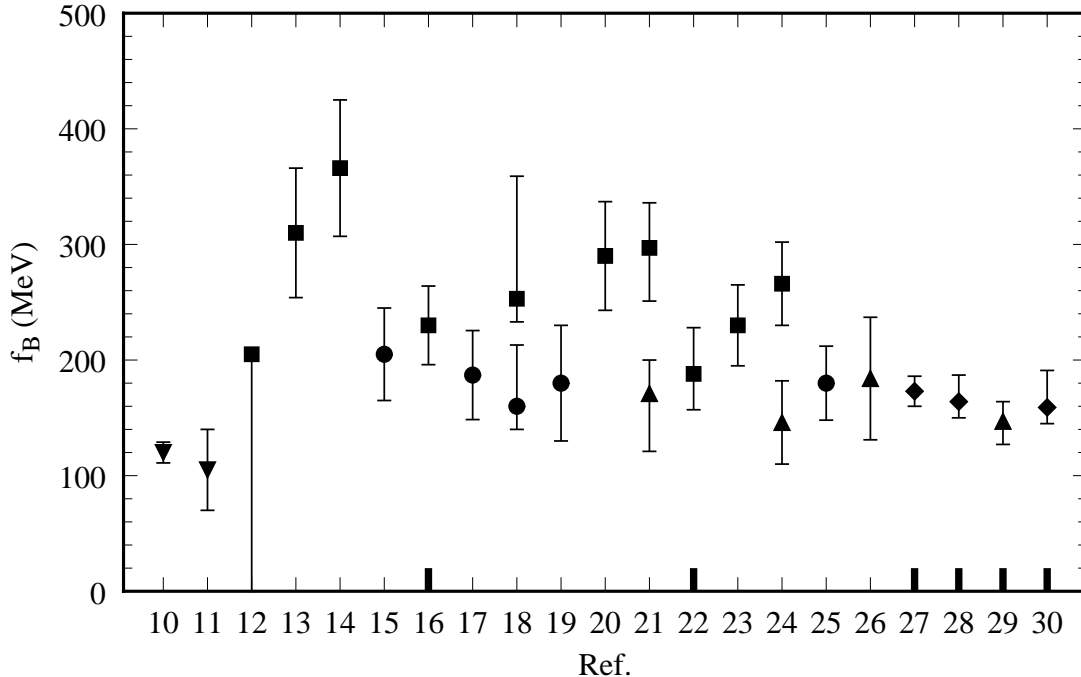


FIGURE 1. Time-line of calculations of f_B with lattice QCD. Methods shown are extrapolation from $m_Q \leq m_c$ to m_b (inverted triangles), the static limit $m_Q \rightarrow \infty$ (squares), interpolation between m_c and ∞ (circles), NRQCD (triangles), and that of Ref. [6]. Entries with a thick tick-mark on the horizontal axis control lattice spacing effects, as explained in the text.

necessarily fair to cautious innovators, but, on the other hand, it generates the picture that is seen by outsiders. Figure 1 does contain a few exceptions to the second criterion, to include calculations that offered new technical developments.

One can divide the time-line into infancy [10–14], childhood [15–22], youth [23–26], and adulthood [27–30]. (A similar, more discriminating classification has been made by Bernard [31].) In adulthood, with refereed publications starting in November 1997, the scatter that characterizes the field through its youth has settled down. The results with a thick tick-mark on the horizontal axis use several lattice spacings [16,22,27,28,30] or a fully consistent implementation of lattice NRQCD [29]. Thus, one could say that Refs. [16,22] are mature results of the static approximation but, because the contribution of order $1/m_b$ is not negligible, not of f_B .

HEAVY QUARKS AND LATTICE FIELD THEORY

In the introduction, I explained that physical $1/m_Q$ effects and artificial a effects are intertwined and that a better appreciation of the intertwining was required before calculations of f_B could mature. In particular, a theoretical analysis [6] of Wilson quarks away from the small-mass limit was needed to obtain the results in

Refs. [27,28,30]. This section summarizes some of the main ideas by comparing and contrasting the effective theories in the continuum and on the lattice.

The effective Hamiltonian of QCD for heavy quarks can be written

$$H = M_1 \bar{\Psi}\Psi + \bar{\Psi}\gamma_0 A_0\Psi + \bar{\Psi}h\Psi, \quad (4)$$

where Ψ is the fermion field and h is given by an expansion in $1/m_Q$, namely

$$h = -\frac{\mathbf{D}^2}{2M_2} - z_B \frac{i\boldsymbol{\Sigma} \cdot \mathbf{B}}{2M_2} + z_{\text{s.o.}} \frac{\{\boldsymbol{\gamma} \cdot \mathbf{D}, \boldsymbol{\alpha} \cdot \mathbf{E}\}}{8M_2^2} - z_4 \frac{(\mathbf{D}^2)^2}{8M_2^3} + \dots, \quad (5)$$

corresponding to the kinetic energy, hyperfine splitting, spin-orbit splitting, relativistic corrections, etc.² This result follows from a series of Foldy-Wouthuysen-Tani transformations, or from noticing the heavy-quark symmetries of QCD, writing down allowed operators with arbitrary coefficients, and matching physical observables to standard QCD. With radiative corrections, one finds $z = 1 + O(g^2)$.

The rest mass of a quark is M_1 and the kinetic energy of a quark is $\mathbf{p}^2/2M_2$. It is convenient to call M_2 the kinetic mass, even though Lorentz invariance implies $M_2 = M_1$. Let us write $H = M_1 \bar{\Psi}\Psi + H_Q$. The rest-mass term and H_Q commute, even in the interacting theory, so eigenstates of H are simultaneously eigenstates of H_Q . Thus, one can drop the rest-mass term or readjust M_1 according to convenience, without changing the dynamics of heavy-quark QCD. The physically relevant parameter is the kinetic mass, which one adjusts so that $M_2 = m_Q$.

The lattice Hamiltonian takes the same structure as in Eqs. (4) and (5), but with changes to the operators' coefficients. One can express this by replacing h in Eq. (4) with $h_{\text{lat}} = h + \delta h$ and writing

$$\delta h = ab_B i\boldsymbol{\Sigma} \cdot \mathbf{B} + a^2 b_{\text{s.o.}} \{\boldsymbol{\gamma} \cdot \mathbf{D}, \boldsymbol{\alpha} \cdot \mathbf{E}\} + a^3 b_4 (\mathbf{D}^2)^2 + a^3 w_4 \sum_i D_i^4 + \dots, \quad (6)$$

where the coefficients $b = b(m_Q a, g^2)$ and $w = w(m_Q a, g^2)$ are functions of the (lattice) quark mass $m_Q a$ and the gauge coupling g^2 . The same operators appear, as well as others, such as the last one, that break rotational symmetry. Part of the craft of the numerical work is to adjust the underlying lattice action so that these artifact coefficients b and w vanish, at least to some accuracy.

In lattice NRQCD and HQET this pattern arises by construction, in particular one sets $M_1 = 0$. It is fairly straightforward to adjust the b s and w s to vanish at the tree level of perturbation theory in g^2 . Beyond the tree level it is still possible, but arduous perturbative calculations are needed. Power-law divergences appear in loop diagrams, so one cannot take $a \rightarrow 0$ by brute force [2]. That means that the lattice artifacts can be removed only by further refinements of the NRQCD action. For heavy-light mesons, the action of the light quarks must be improved to a consistent level [32], as in Ref. [29].

²⁾ Analogous expansions are introduced for the operators mediating electroweak transitions.

With actions for Wilson quarks, such as the clover action³ or the Wilson action itself, the pattern sketched here is less immediate. It is guaranteed, however, by the heavy-quark symmetry of the lattice action. Because the lattice violates Lorentz invariance, the rest and kinetic masses differ; in practice $M_1 < M_2$. The bs and ws are, however, *bounded* for all masses. As $m_Q a \rightarrow 0$, these coefficient functions go to a constant⁴ or vanish like $(m_Q a)^{s_0}$, for some integer s_0 , by Symanzik's standard analysis of cutoff effects. On the other hand, as $m_Q a \rightarrow \infty$, they vanish like $(1/m_Q a)^{s_\infty}$, for some s_∞ , by heavy-quark symmetry [6]. The full functional dependence on $m_Q a$ is *calculable* in perturbation theory and, sometimes, nonperturbatively. The coefficient functions bs and ws depend on the details of the lattice action. For example, with the Wilson action $b_B \neq 0$, but with the clover action one can adjust an unphysical coupling until b_B vanishes.

LEPTONIC DECAYS

Let us now return to Fig. 1. The plotting symbols distinguish methods. Squares denote calculations in the static approximation, and triangles denote calculations with lattice NRQCD. Inverted triangles [10,11], circles, and diamonds denote treatments of numerical data from the Wilson or clover action.

Because the latter connects naturally with the effective theories, Ref. [6] suggested treating the bottom quark on the lattice by adjusting the bare mass until $M_2 = m_b$. In addition, a suitably normalized operator, essentially one built from the heavy-quark field Ψ , must be used for the current. Results with this approach are denoted with diamonds. The discretization errors are of order $\Lambda_{\text{QCD}} a$ to some power, from matrix elements of the operators in Eq. (6). (These effects are then multiplied by a coefficient b or w , which is a number of order 1 or, in limiting cases, smaller still.) With this interpretation of the numerical data from the clover action, the lattice results for f_B are nearly independent of the lattice spacing [27,28].

In earlier work with Wilson quarks, the tuning of the mass and the normalization of the current introduced discretization effects of order $m_Q a$ or $(m_Q a)^2$. The authors minimized these lattice artifacts by reducing the quark mass of the lattice calculations below that of the charmed quark. This is still large: $m_c a \sim 5\Lambda_{\text{QCD}} a$. Then the results were extrapolated up to m_b with fits to $1/m_Q$ expansions, either with (circles) or without (inverted triangles) the help of the static value. This intertwines lattice artifacts and $1/m_Q$ effects in ways that depend on details of the fits, not least because one cannot verify whether the $1/m_Q$ expansion converges for the low quark masses, on which the fits are based.

Table 2 tabulates the mature results [27–30] for f_B , f_{B_s} , f_D , and f_{D_s} , along with an average [34] of results from experimental measurements of $\Gamma_{D_s \rightarrow \mu\nu}$, combined with $|V_{cs}|$. While it is tempting to average the results, it is subtle to do so properly,

³⁾ The clover action adds a term to the Wilson action to eliminate the leading lattice artifact [33].

⁴⁾ If the limit is a constant, the lattice artifact still vanishes, owing to the explicit powers of a .

TABLE 2. Compendium of recent results for decay constants of heavy-light mesons. The experimental average comes, in fact, from measurements of $|V_{cs}|f_{D_s}$, which then take $|V_{cs}|$ from unitarity or from neutrino production of charm

method	Ref.	f_B				f_{B_s}			
		MeV	stat	syst	quench	MeV	stat	syst	quench
[6]	[27]	173	04	09	09	199	03	10	10
[6]	[28]	164	$^{+14}_{-11}$	08	$^{+10\%}_{-00\%}$	185	$^{+13}_{-08}$	09	$^{+10\%}_{-00\%}$
NRQCD	[29]	147	11	11	$^{+08}_{-12}$	175	08	13	10
[6]	[30]	159	11	$^{+22}_{-09}$	$^{+21}_{-00}$	175	10	$^{+28}_{-10}$	$^{+25}_{-01}$
method	Ref.	f_D				f_{D_s}			
		MeV	stat	syst	quench	MeV	stat	syst	quench
[6]	[27]	197	02	14	10	224	02	16	12
[6]	[28]	194	$^{+14}_{-10}$	10	$^{+10\%}_{-00\%}$	213	$^{+14}_{-11}$	11	$^{+10\%}_{-00\%}$
[6]	[30]	195	11	$^{+15}_{-08}$	$^{+15}_{-00}$	213	09	$^{+23}_{-09}$	$^{+17}_{-00}$
experiment	[34]					243		36 (total)	

because the systematic errors are largely, but not entirely, common. If you must have an average, consult Draper’s review [9], or use your own eye.

The central values in Table 2 (except the experiment!) are in the quenched approximation. The *error bars*, however, reflect estimates of the associated uncertainty. The best estimation is that of the MILC Collaboration [30] who have some lattice results including up and down quark loops. (The strange and heavier quarks are still quenched.) The partially unquenched data sometimes lie higher than the quenched data, sometimes not, leading to the very asymmetric error estimate. These results are encouraging, not least because they suggest that the wait for a fully unquenched calculation will not be too much longer.

NEUTRAL MESON MIXING

Because the calculation is technically more demanding, the literature contains fewer calculations of the bag parameters of B_d^0 and B_s^0 meson mixing than of the decay constants. Recent publications report $B_{B_s}/B_{B_d} \approx 1$ and

$$\hat{B}_{B_d} = \begin{cases} 1.03 \pm 0.06 \pm 0.18 & [35] \\ 1.40 \pm 0.06 \pm 0.26^{+0.04} & [36] \\ 1.23 \pm 0.05 \pm 0.15 & [36, 35] \\ 1.17 \pm 0.09 \pm 0.05 & [37] \end{cases}, \quad (7)$$

where \hat{B}_{B_d} is the renormalization-scheme independent combination. The first three entries use the static approximation. The third entry comes from an analysis in Ref. [36] of the data in Ref. [35]. The last entry uses lattice NRQCD and finds, additionally, that the dependence on $1/m_Q$ is not large. See Ref. [9] for more results. There are also results for the ratio of matrix elements:

$$\frac{\frac{8}{3}m_{B_s}^2 f_{B_s}^2 B_{B_s}}{\frac{8}{3}m_{B_d}^2 f_{B_d}^2 B_{B_d}} = \begin{cases} 1.38 \pm 0.07 & [35] \\ 1.76 \pm 0.10^{+57}_{-42} & [38] \end{cases} . \quad (8)$$

On the whole, the impression is that more work needs to be done to gain control over the systematic errors; calculations of B_B are not as mature as those of f_B .

The kaon's bag parameter B_K is needed to predict ε_K , a measure of indirect CP violation in $K \rightarrow \pi\pi$. Given B_K from (lattice) QCD, a measurement of ε_K traces a hyperbola in the complex V_{td} plane. The history of B_K has similarities to that of f_B : numerical work was a greater challenge than initially hoped, and theoretical insight is needed as a guide. For example, the lattice-spacing dependence (with staggered fermions) is surprisingly steep, but one now knows to extrapolate to the continuum limit in a^2 (rather than a) [39]. Two recent calculations find

$$B_K(\text{NDR}, 2 \text{ GeV}) = \begin{cases} 0.62 \pm 0.02 \pm 0.02 & [40] \\ 0.628 \pm 0.042 & [41] \end{cases} , \quad (9)$$

where the main uncertainty comes from the continuum extrapolation. Quenching and degenerate quark-mass effects may each lead to underestimates of order 5% [42].

SEMILEPTONIC DECAYS

Semileptonic decays are wonderful for learning about the first two rows of the CKM matrix. Because a hadron is in the final state, lattice calculations of the form factors are more difficult than decay constants, but not much more difficult.

In these decays there is an additional kinematic variable, the momentum $|\mathbf{p}'|$ of the daughter hadron (in the parent's rest frame). The Lorentz invariant q^2 is linearly related to $|\mathbf{p}'|$. Until recently, calculations of these form factors were done with $m_Q \lesssim m_c$ and extrapolated up to m_c or m_b with $1/m_Q$ expansions. Since the kinematically allowed range of q^2 depends on the heavy-quark mass, another extrapolation is made. It is clear that the extrapolations once again intertwine artificial a effects with physical $1/m_Q$ and q^2 dependence. The details of the intertwining are not transparent (to me, anyway), so I reserve comment and direct the reader to reviews by Onogi [7] and by Draper [9].

More recently, calculations have been done with lattice NRQCD or with Wilson quarks interpreted [6] as suggested by Eqs. (4)–(6). These techniques are especially powerful here, because the ability to compute directly at the B mass decouples extrapolations in q^2 from the mass dependence. Calculations are underway for form factors of heavy-to-light transitions, such as $D \rightarrow \pi l\nu$ and $D \rightarrow K l\nu$, which together yield $|V_{cd}/V_{cs}|$ [43,44], and $B \rightarrow \pi l\nu$, which yields $|V_{ub}|$ [43,45,46].

I would like to conclude with a preliminary result [47] on the zero-recoil form factor for the decay $B \rightarrow D l\nu$, which will improve the determination of $|V_{cb}|$. A similar study of $B \rightarrow D^* l\nu$ is in progress. Until now there have been calculations of the shape of the form factors for $B \rightarrow D^{(*)} l\nu$, but the normalization has been

computed only poorly, in perturbation theory. It is possible, however, to handle almost all of the normalization nonperturbatively. For example [48],

$$\frac{\langle D|\mathcal{V}_0|B\rangle\langle B|\mathcal{V}_0|D\rangle}{\langle D|\mathcal{V}_0|D\rangle\langle B|\mathcal{V}_0|B\rangle} = \frac{h_+^{B\rightarrow D}(1)h_+^{D\rightarrow B}(1)}{h_+^{D\rightarrow D}(1)h_+^{B\rightarrow B}(1)} = |h_+^{B\rightarrow D}(1)|^2, \quad (10)$$

and here at Fermilab we have found analogous ratios for $h_-^{B\rightarrow D}(1)$ and $h_{A_1}^{B\rightarrow D^*}(1)$. For the class of actions considered in Ref. [6], the remaining radiative corrections have been computed [49], and they are small.

In Eq. (2) for $B \rightarrow D l \nu$ one requires $\mathcal{F} = h_+ - (m_B - m_D)h_-/(m_B + m_D)$. The advantage of the new method is that, in effect, it calculates not $\mathcal{F}(1)$ but the deviation of $\mathcal{F}(1)$ from 1. We find

$$\mathcal{F}(1) = 1.069 \pm 0.008 \pm 0.002 \pm 0.025, \quad (11)$$

where the uncertainties are Monte Carlo statistics, tuning of the quark masses, and a certain parametrically small contribution omitted from h_- . Uncertainties from lattice artifacts and from quenching have not yet been taken into account.

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