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November 1998

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$B \to Dl\nu$ form factors and the determination of $|V_{cb}|$

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The zero recoil limit of the $B \to Dl\nu$ form factors is calculated on the lattice, which provides a model-independent determination of $|V_{cb}|$. Considering a ratio of form factors, in which the bulk of statistical and systematic errors cancel, we obtain a precise result both for $h_+(1)$ and for $h_-(1)$.

1. Introduction

For the determination of $|V_{cb}|$ through the exclusive decay $B \to D^{(*)}l\nu$, the theoretical calculation of the form factor $\mathcal{F}_{B\to D^{(*)}}(w)$, especially in the zero recoil limit, is necessary. Previously this has been done using the zero-recoil sum rule [1] or the heavy quark expansion of the form factor [2,3]. Both of these calculations, however, need to introduce an assumption or a theoretical model to deal with hadronic effects away from the infinite quark mass limit. In this talk, we present a lattice calculation of the form factor $\mathcal{F}_{B\to D}(1)$, which can be used for a model independent determination of $|V_{cb}|$, including deviations from the heavy quark limit.

2. $B \to Dl\nu$ Form Factors

The differential decay rate of $B \to Dl\nu$ is proportional to the square of

$$\mathcal{F}_{B\to D}(w) = h_+(w) - \frac{m_B - m_D}{m_B + m_D} h_-(w). \qquad (1)$$

$h_+(w)$ and $h_-(w)$ are form factors defined through

$$\langle D(\mathbf{p}')|\mathcal{V}^\mu|\bar{B}(\mathbf{p})\rangle = \sqrt{m_B m_D}$$
$$\times [h_+(w)(v + v')^\mu + h_-(w)(v - v')^\mu], \qquad (2)$$

where $v_\mu = p_\mu/m_B$, $v'_\mu = p'_\mu/m_D$ and $w = v \cdot v'$. In the heavy quark mass limit, $h_-(w)$ vanishes, and $h_+(w)$ agrees with the universal form factor $\xi(w)$ (the Isgur-Wise function), which is normalized in the zero recoil limit $\xi(1) = 1$ [4].

The $1/m_Q$ expansion may be used to describe the deviation from the heavy quark limit. At zero recoil, the expansion becomes

$$h_+(1) = 1 - c_+^{(2)}\left(\frac{1}{m_c} - \frac{1}{m_b}\right)^2 + O(1/m_Q^3), (3)$$

$$h_-(1) = 0 - c_-^{(1)}\left(\frac{1}{m_c} - \frac{1}{m_b}\right) + O(1/m_Q^2). \quad (4)$$

The absence of the $O(1/m_Q)$ term in the expansion of $h_+(1)$, which is a part of the Luke's theorem [5], is a consequence of the symmetry under the exchange of m_c and m_b (see eq.(2)), and this particular form of the expansion is also restricted by the symmetry (anti-symmetry for $h_-(1)$). Our task is to determine the coefficients $c_+^{(2)}$ and $c_-^{(1)}$, for which there has been no model independent calculation.

3. $h_+(1)$

The vector current $\mathcal{V}_\mu = \bar{c}\gamma_\mu b$ appearing in eq.(2) must be related to the lattice counterpart V_μ^{latt} using the perturbative relation $\mathcal{V}_\mu = Z_V V_\mu^{latt}$. This is true even for the equal mass ($m_c = m_b$) case, because the lattice (local) vector current is not conserved. Without a two-loop calculation, this perturbative matching could in principle be a source of large systematic uncertainty of $O(\alpha_s^2) \sim 5\%$, which is too large to obtain the precision we seek for the form factor ($< 5\%$). The statistical error in the lattice calculation would also be a problem, if we employed the usual method to extract the matrix element from three-point correlator.
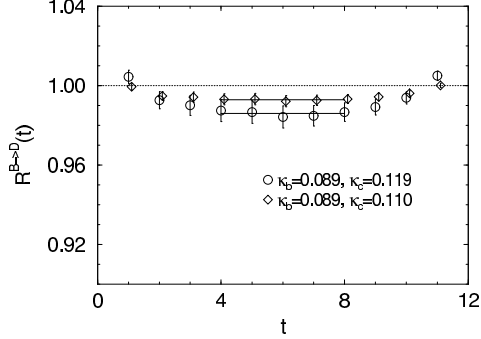
Figure 1. $R^{B \to D}(t)$ as a function of t.

To reduce these errors, we define a ratio at zero recoil

$$R^{B \to D} = \left[\frac{\langle D|\mathcal{V}_0|B\rangle\langle B|\mathcal{V}_0|D\rangle}{\langle D|\mathcal{V}_0|D\rangle\langle B|\mathcal{V}_0|B\rangle} \right]^{cont}$$

$$= \frac{h_+^{B \to D}(1)h_+^{D \to B}(1)}{h_+^{D \to D}(1)h_+^{B \to B}(1)} = |h_+^{B \to D}(1)|^2, (5)$$

where we used the property $h_+^{D \to D}(1) = h_+^{B \to B}(1) = 1$ derived from current conservation. $R^{B \to D}$ may be related to the lattice counterpart

$$\frac{Z_{V^{cb}} Z_{V^{bc}}}{Z_{V^{cc}} Z_{V^{bb}}} \times \left[\frac{\langle D|V_0|B\rangle\langle B|V_0|D\rangle}{\langle D|V_0|D\rangle\langle B|V_0|B\rangle} \right]^{latt}. \quad (6)$$

The ratio of matching factor can be safely evaluated with perturbation theory, since a large cancellation of perturbative coefficients takes place in the ratio. The one-loop calculation is discussed in a separate talk [6].

To calculate the hadronic amplitude, we define

$$R^{B \to D}(t) = \frac{C^{DV_0B}(t)C^{BV_0D}(t)}{C^{DV_0D}(t)C^{BV_0B}(t)} \to R^{B \to D}, \quad (7)$$

where $C^{DV_0B}(t)$ is a three-point correlator, whose initial and final states are fixed at $t = 0$ and $t = N_t/2$ respectively and the vector current is moved to find a plateau. Our calculation is done on a $12^3 \times 24$ lattice at $\beta = 5.7$. The Fermilab action [7] is used for heavy quark with $c_{sw} = 1/u_0^3$.

Figure 1 shows a nice plateau for the ratio $R^{B \to D}(t)$. Even with only 200 configurations the statistical error is remarkably small ($< 1\%$), because of the cancellation of statistical fluctuations in the ratio.

Fitting the plot with a constant we obtain $|h_+(1)|^2$ for each combination of initial and final heavy quark masses. To fix the parameter in
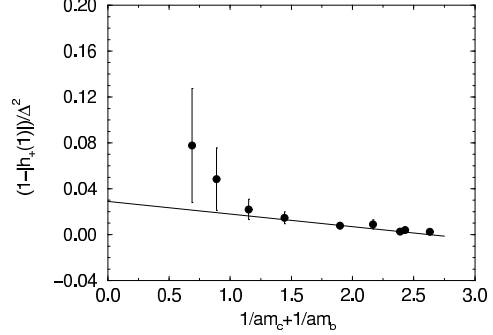

Figure 2. $(1 - |h_+(1)|)/(1/m_c - 1/m_b)^2$ versus $1/am_c + 1/am_b$.

the $1/m_Q$ expansion, we choose six values of the heavy quark mass covering the physical m_b and m_c, and fit the results with the form

$$h_+(1) = 1 - c_+^{(2)} \left(\frac{1}{m_c} - \frac{1}{m_b} \right)^2$$

$$+ c_+^{(3)} \left(\frac{1}{m_c} + \frac{1}{m_b} \right) \left(\frac{1}{m_c} - \frac{1}{m_b} \right)^2, \quad (8)$$

where the $O(1/m_Q^3)$ term is required to explain the data. Figure 2 shows the quantity $(1 - h_+(1))/(1/m_c - 1/m_b)^2 = c_+^{(2)} - c_+^{(3)}(1/m_c + 1/m_b)$, from which we extract the coefficients $c_+^{(2)}$ and $c_+^{(3)}$. Our result is $c_+^{(2)} = 0.029(11)$ and $c_+^{(3)} = 0.011(4)$. In physical units we obtain $c_+^{(2)} = (0.20(4)\text{GeV})^2$ and $c_+^{(3)} = (0.26(3)\text{GeV})^3$. Shifman et al. [1] derived a bound $c_+^{(2)} > (\mu_\pi^2 - \mu_G^2)/2 = (0.26^{+0.09}_{-0.12}\text{GeV})^2$ using the zero-recoil sum rule. [1] Our result is consistent with this bound within errors.

4. $h_-(1)$

$h_-(1)$ cannot be obtained from the matrix element at zero recoil. We introduce a finite (but small) D meson momentum $\mathbf{p}'$ and define a ratio

$$R^{B \to D}_{V_i/V_0} = \frac{\langle D(\mathbf{p}')|V_i|B(\mathbf{0})\rangle}{\langle D(\mathbf{p}')|V_0|B(\mathbf{0})\rangle}$$

$$= \frac{1}{2}v_i' \left(1 - \frac{h_-(w)}{h_+(w)} \right)$$

$$\times \left[1 - \frac{1}{4} \left(1 - \frac{h_-(w)}{h_+(w)} \right) \mathbf{v}'^2 + \cdots \right], \quad (9)$$

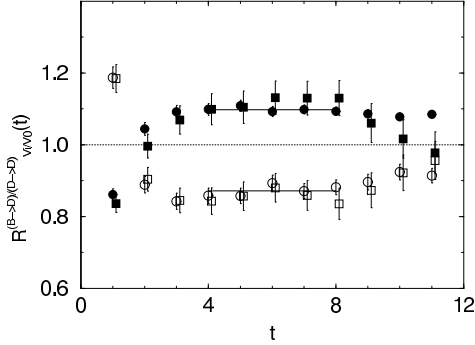[1] For the hadronic parameters we used $\mu_\pi^2 = 0.5(1)\text{GeV}^2$ and $\mu_G^2 = 0.36\text{GeV}^2$.

Figure 3. $R_{V^i/V^0}^{(B\to D)/(D\to D)}(t)$ for the final state momentum $(1,0,0)$ (circles) and $(2,0,0)$ (squares). $B \to D$ (filled symbols) and $D \to B$ (open symbols) are shown.

where we use the definition of the form factors and expand in small $\mathbf{v'}^2$. It is more convenient to define a double ratio

$$R_{V_i/V_0}^{(B\to D)/(D\to D)} = R_{V_i/V_0}^{B\to D}/R_{V_i/V_0}^{D\to D}$$
$$= \left(1 - \frac{h_-(w)}{h_+(w)}\right)\left[1 - \frac{1}{4}\frac{h_-(w)}{h_+(w)}\mathbf{v'}^2 + \cdots\right] \quad (10)$$

The property of elastic scattering $h_-^{D\to D}(w) = 0$ is used for the denominator.

Figure 3 shows the corresponding ratio $R_{V_i/V_0}^{(B\to D)/(D\to D)}(t)$ on the lattice for two different values of the D meson momentum. As in the calculation of $h_+(1)$, the plateau is very clear and we can extract $(1 - h_-(w)/h_+(w))$ from this plot. The small correction proportional to $\mathbf{v'}^2$ may be eliminated by extrapolating to the $\mathbf{v'}^2 \to 0$ limit. We also observe an antisymmetric property $h_-^{D\to B}(w) = -h_-^{B\to D}(w)$.

The heavy quark mass dependence can be obtained with a similar strategy. We fit our data with

$$h_-(1) = -c_-^{(1)}\left(\frac{1}{m_c} - \frac{1}{m_b}\right)$$
$$+ c_-^{(2)}\left(\frac{1}{m_c} + \frac{1}{m_b}\right)\left(\frac{1}{m_c} - \frac{1}{m_b}\right), \quad (11)$$

and obtain $c_-^{(1)} = 0.23(3)$ and $c_-^{(2)} = 0.06(1)$, which correspond to $c_-^{(1)} = 0.26(4)\mathrm{GeV}$ and $c_-^{(2)} = (0.29(3)\mathrm{GeV})^2$ in physical units.

5. $\mathcal{F}_{B\to D}(1)$

The results with physical mass parameter are

$$h_+(1) = 1.016 \pm 0.003 \pm 0.002 \pm 0.006, \quad (12)$$
$$h_-(1) = -0.112 \pm 0.014 \pm 0.011 \pm 0.025, \quad (13)$$

where $h_+(1)$ includes the one-loop correction $+0.025(6)$ [6]. Errors arise from statistics, mass parameter determination, and our estimate for higher order perturbative correction in the order given. Using (1) we obtain

$$\mathcal{F}_{B\to D}(1) = 1.069 \pm 0.008 \pm 0.002 \pm 0.025. \quad (14)$$

6. Conclusions

Using a ratio in which a large cancellation of statistical and systematic errors takes place, we have calculated the $B \to Dl\nu$ form factors very precisely, which may lead to a better determination of $|V_{cb}|$. In the error bar given above, however, we have not yet included the discretization error and the effect of quenching. It is our hope that the bulk of these errors also cancels in the ratio. We leave these important issues for future study.

Our method can also be applied to the $B \to D^*l\nu$ form factor, which currently yields an experimentally more precise determination of $|V_{cb}|$.

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