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Abstract. The potential for the Higgs boson and Z-pole physics at the first muon collider is summarized, based on the discussions at the "Workshop on the Physics at the First Muon Collider and at the Front End of a Muon Collider".

INTRODUCTION

Muon colliders offer a wide range of opportunities for exploring the physics within and beyond the Standard Model (SM). Because the muon mass is about 200 times larger than the electron mass, s-channel production of the Higgs boson, and its associated advantages with regard to measurements of Higgs boson properties, is one of the unique features of a muon collider. Since the muons are produced in a decay channel by moving pions, the muons naturally carry a longitudinal polarization of about 20%. A collider allowing for adroit manipulation of the polarization and center of mass energy, combined with the prospect of luminosities in the range of $10^{32} < \mathcal{L} < 10^{33} \mathrm{cm}^{-2}\mathrm{s}^{-1}$ would make for a very powerful probe of the structure of the fundamental forces with unparalleled potential. In this report a summary of the Higgs and Z-pole physics, as well as some other aspects of the electroweak boson physics, as discussed at the workshop on the physics at the First Muon Collider (FMC), is presented [1].

EXPERIMENTAL CONSIDERATIONS

The experiments at the e^+e^--colliders LEP and SLC have shown that the calibration of the luminosity, beam energy and beam polarization is crucial for the physics results obtained. At LEP the luminosity is measured with small angle silicon based

calorimeters, counting Bhabha events to a precision of $\frac{\delta\mathcal{L}}{\mathcal{L}} = 10^{-3}$. The Bhabha cross section has been measured down to angles of about 30 mrad with respect to the beam direction. At the FMC, however, it is not clear if the muon Bhabha cross section can be measured down to small angles. The muons, with a lifetime of about 2 μs in their rest frame, can circulate in the machine for only about 800 turns at a center of mass energy $\sqrt{s} = 0.5$ TeV [2]. The electrons from the decay of the muon form a major problem. For $2 \cdot 10^{12}$ muons per bunch there are $3 \cdot 10^5/E_{\text{beam}}$ decays per meter, with E_{beam} in TeV. Because the final focus is tuned to the beam energy of the muon beams, the decay electrons will be sprayed out over the interaction region. Current detector designs [3] include an uninstrumented cone of $10° - 20°$ with respect to the beamline because of the large direct and induced backgrounds. It is thus unclear if a similar precision on the luminosity measurement can be obtained using muon Bhabha scattering or using an alternative method. For the discussions to follow it is assumed that at the FMC a precision on the luminosity measurement of $\delta\mathcal{L}/\mathcal{L} = 10^{-3}$ is achievable.

The beam energy at LEP is measured most accurately using the technique of resonant depolarization which has an ultimate accuracy of about 200 keV. This calibration, however, cannot be performed very often since it takes a long time for the polarization to build up in the beam. Moreover, it cannot be done during a physics run and has been performed with separated beams only. These and other limitations, resulted in a final uncertainty on $\sqrt{s}$ at LEP of about 1 MeV.

At the FMC the natural polarization of the muons [2] provides a mechanism to measure not only the beam energy but also the polarization itself. The precession of the polarization vector with respect to the momentum vector is governed by the muon spin tune, $\frac{g-2}{2}\gamma$, which corresponds to the number of precessions in one turn around the ring. Coincidentally, for muons at the Z-mass the spin tune is almost exactly 1/2. Thus, the spin flips each turn at $\sqrt{s} \approx M_Z$. The energy spectrum of the decay electrons depends on the muon polarization. By measuring the average energy of the decay electrons each turn at a fixed point along the circumference of the machine a measure of the beam energy and the polarization can be obtained. The measured energy spectrum will exhibit an oscillatory behavior as function of turn number. The frequency and amplitude of the oscillations measure the beam energy and the polarization, respectively. Initial studies for a perfect planar machine geometry show that an absolute energy calibration at the statistical level of $\delta E/E = 10^{-5}$ is easily feasible [4]. Systematic effects will be the dominant sources of uncertainty and are being studied. A clear advantage of this procedure is that the measurements are done concurrently with physics runs, directly sampling the interacting muon bunches.

The beam spread is controlled by the beam optics and a narrow band beam option at lower center of mass energies would provide a beam spread of $R = \sigma(p)/p = 3 \cdot 10^{-5}$.

HIGGS BOSONS

The SM as it stands is incomplete. Many fundamental questions of nature are left unanswered. Among them the question of electroweak symmetry breaking takes a prominent position. In the SM, the electroweak symmetry is broken spontaneously through a fundamental Higgs doublet field, giving rise to a single physical neutral Higgs boson (h^0). In the minimal supersymmetric standard model (MSSM), there are two neutral CP-even Higgs bosons (h^0, H^0), two charged Higgs bosons ($H^\pm$), and a CP-odd neutral Higgs boson (A^0). To understand the electroweak symmetry breaking and to explore new physics beyond the SM, the study of the Higgs sector is of highest priority for future collider experiments [5].

Although the Higgs boson masses are largely free parameters, theoretical arguments indicate that there exists an upper limit on the lightest Higgs boson mass, namely, $m_h < 150$ GeV/c^2 [5]. The FMC then is unique in studying light Higgs bosons since they can be produced through s-channel production at resonance, due to the sizeable coupling of a Higgs boson to muons [6–8].

The resonance cross section for Higgs boson production is given by

$$\sigma_h\left(\sqrt{\hat{s}}\right) = \frac{4\pi\Gamma(h \to \mu\mu)\,\Gamma(h \to X)}{(\hat{s} - m_h^2)^2 + m_h^2\Gamma_h^2}, \tag{1}$$

where $\hat{s}$ is the c.m. energy squared, Γ_h is the total width, and X denotes the final state. The Higgs coupling to fermions is proportional to the fermion mass so the corresponding s-channel process is highly suppressed at e^+e^- colliders with respect to muon colliders. The cross section must be convoluted with the machine energy spectrum, approximated by a Gaussian distribution of width $\sigma_{\sqrt{s}}$,

$$\bar{\sigma}_h\left(\sqrt{s}\right) = \int \sigma_h\left(\sqrt{\hat{s}}\right) \frac{\exp\left[-\dfrac{\left(\sqrt{\hat{s}} - \sqrt{s}\right)^2}{\left(2\sigma_{\sqrt{s}}^2\right)}\right]}{\sqrt{2\pi}\,\sigma_{\sqrt{s}}}\, d\sqrt{\hat{s}}. \tag{2}$$

The root mean square spread $\sigma_{\sqrt{s}}$ in c.m. energy is given in terms of the beam resolution R by

$$\sigma_{\sqrt{s}} = (7\text{ MeV})\left(\frac{R}{0.01\%}\right)\left(\frac{\sqrt{s}}{100\text{ GeV}}\right), \tag{3}$$

where a resolution down to $R = 0.003\%$ may be realized at the FMC. In comparison, a value of $R \sim 1\%$ is expected at the Next Linear e^+e^- Collider (NLC). To study a Higgs resonance one wants to be able to tune the machine energy to $\sqrt{s} = m_h$. For this purpose the monochromaticity of the beam energy is vital.

When the resolution is much larger than the Higgs width, $\sigma_{\sqrt{s}} \gg \Gamma_h$, the effective s-channel cross section is

$$\bar{\sigma}_h = \pi\sqrt{2\pi}\,\frac{\mathrm{BF}(h \to \mu\mu)\,\mathrm{BF}(h \to X)}{m_h^2}\cdot\frac{\Gamma_h}{\sigma_{\sqrt{s}}}\,. \tag{4}$$

It becomes clear that it would be desirable to get as good a beam energy resolution as possible because of the factor $\Gamma_h/\sigma_{\sqrt{s}}$. In the other extreme when the resolution is much smaller than the width, $\sigma_{\sqrt{s}} \ll \Gamma_h$, the effective cross section is

$$\bar{\sigma}_h = 4\pi\,\frac{\mathrm{BF}(h \to \mu\mu)\,\mathrm{BF}(h \to X)}{m_h^2}\,. \tag{5}$$
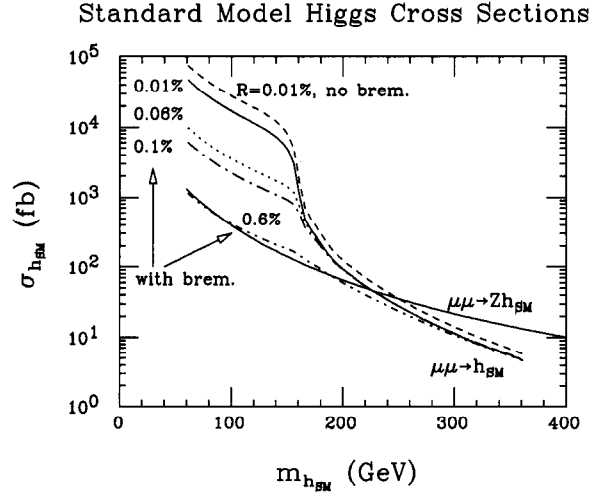
FIGURE 1. The s-channel cross section for $\mu^+\mu^- \to h$ for several choices of the beam resolution R. Also shown is the $\mu^+\mu^- \to Zh$ cross section at $\sqrt{s} = M_Z + \sqrt{2}\,m_h$, from Ref. [8].

Figure 1 illustrates the SM Higgs cross section for several choices of the machine resolution. For $m_h < 2M_W$, Γ_h is very narrow and a better beam resolution can significantly improve the signal rate. On the other hand, for $m_h > 2M_W$ the SM Higgs boson becomes increasingly broad and the effect of $\sigma_{\sqrt{s}}$ is negligible. To effectively explore Higgs physics, the resolution requirements for the machine thus depend on the Higgs width. Figure 2 gives both SM and SUSY Higgs width predictions versus the Higgs mass. A SM Higgs of mass $m_h \sim 100$ GeV/c^2 has a width of a few MeV. The width of the lightest SUSY Higgs may be comparable to that of the SM Higgs (if $\tan\beta \sim 1.8$) or much larger ($\Gamma_h \sim 0.5$ GeV for $\tan\beta \sim 20$). The width parameter characterizes the fundamental couplings of the Higgs boson to other particles. Figure 3 shows light Higgs resonance profiles versus the c.m. energy $\sqrt{s}$. With a resolution $\sigma_{\sqrt{s}}$ of order Γ_h the Breit-Wigner line shape can be measured and Γ_h determined. For the moment we must plan for a resolution $R \lesssim 0.01\%$ in order to be sensitive to Γ_h of a few MeV.

The prospects for observing the SM Higgs are evaluated in Fig. 4. The first two panels give the signal and background for a resolution $R = 0.003\%$. The third panel gives the luminosity needed for a 5σ detection in the dominant $b\bar{b}$ final state. The luminosity requirements are very reasonable, except for the Z-boson peak region.
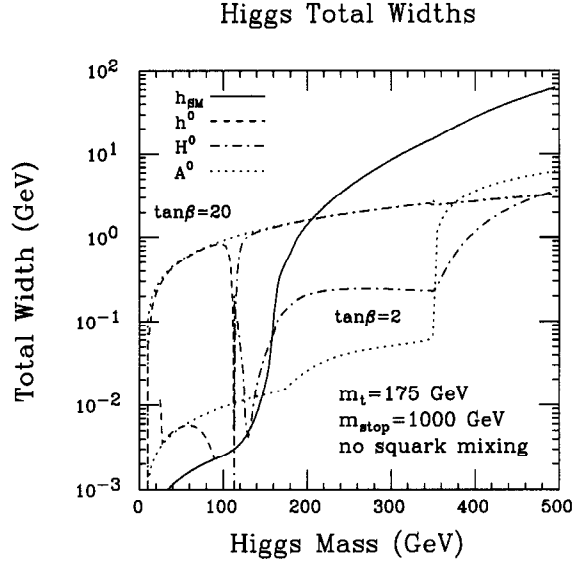
Higgs Total Widths

FIGURE 2. Total width of the SM and MSSM Higgs bosons with $\tan\beta = 2$ and 20, from Ref. [8].

It is likely that a SM-like Higgs boson will have been discovered at the LHC or NLC when the FMC starts its mission. The mass of a light Higgs boson will have been measured to an accuracy of approximately 200 MeV/c^2 [9]. From a rough scan for the s-channel h^0 signal over this 200 MeV range, the mass can be determined to an accuracy $\Delta m_h \sim \sigma_{\sqrt{s}}$. If $S/\sqrt{B} \gtrsim 3$ is required for detection or rejection of a Higgs signal and a resolution $R \sim 0.003\%$ ($\sigma_{\sqrt{s}} \sim 2$ MeV) is employed, then the necessary luminosity per scan point is 0.0015 fb^{-1} for $m_h \lesssim 2M_W$ and m_h not near M_Z. As an example, suppose that the LHC has measured $m_h = 110.0 \pm 0.1$ GeV/c^2. The number of scan points to cover a 200 MeV/c^2 region in $\sqrt{s}$ at the FMC is
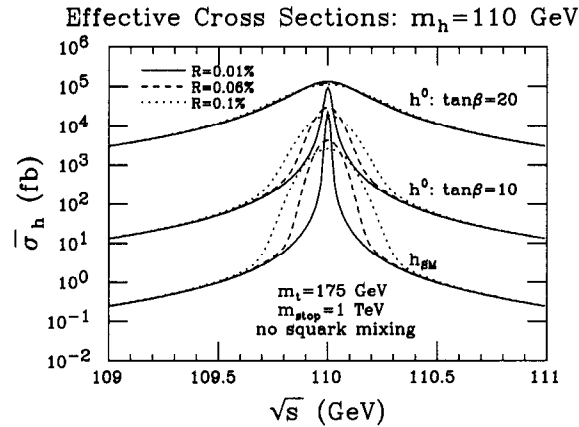
FIGURE 3. Effective s-channel Higgs cross section $\bar{\sigma}_h$ obtained by convoluting the Breit-Wigner resonance formula with a Gaussian distribution for resolution R, from Ref. [8].
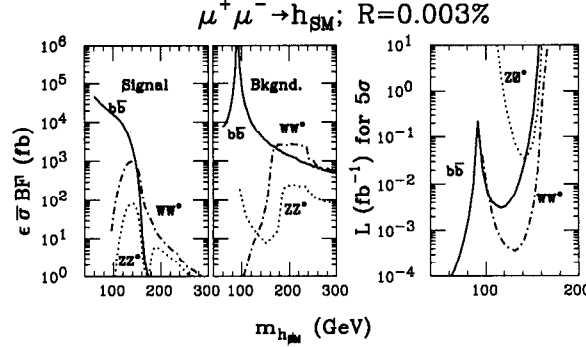
FIGURE 4. The SM Higgs cross sections and backgrounds in $b\bar{b}$, WW^* and ZZ^*. Also shown is the luminosity needed for a 5 standard deviation detection in the $b\bar{b}$ decay mode, from Refs. [7,8].

~ 200 MeV$/2$ MeV $= 100$, and a total luminosity of $100 \times (0.0015$ fb^{-1}/point$) = 0.15$ fb^{-1} is needed to discover the Higgs and reach an accuracy on its mass of

$$\Delta m_h \simeq \sigma_{\sqrt{s}} \sim 2 \text{ MeV}/c^2 . \tag{6}$$

Once m_h is determined to an accuracy $\Delta m_h \sim \mathcal{O}\left(\sigma_{\sqrt{s}}\right)$ a three point fine scan can be made with one setting at the apparent peak and two settings on the wings at $\pm\sigma_{\sqrt{s}}$ from the peak. The ratios of $\sigma(\text{wing}^i)/\sigma(\text{peak}^i)$ determine m_h and Γ_h. With a good energy resolution $R = 0.003\%$ and an integrated luminosity $L_{\text{total}} = 0.4$ fb^{-1}, for $m_h = 110$ GeV$/c^2$ and $\Gamma_h = 3$ MeV, accuracies of

$$\Delta\Gamma_h/\Gamma_h = 16\%, \quad \text{and} \quad \Delta m_h \sim 0.1 \text{ MeV}/c^2 \tag{7}$$

could be achieved. At the same time, branching ratios for the dominant decay channels can be also measured to a good precision, $i.e.$, 3% and 15% for $\sigma \cdot \text{BF}(b\bar{b})$ and $\sigma \cdot \text{BF}(W^+W^-)$, respectively. The high precision reached would have significant impact on the electroweak physics within and beyond the SM. For instance, with a measurement of an accuracy about 15% on the ratio $\text{BF}(W^+W^-)/\text{BF}(b\bar{b})$, one should be able to infer A^0 effects up to $M_{A^0} \gtrsim 400$ GeV$/c^2$ by comparing the predictions from the MSSM and the SM [7,8]. However, to reach the necessary precision within a sensible time scale, a machine luminosity of 10^{32} cm^{-2}s^{-1} (or 1 fb^{-1}/yr) and a good energy resolution $R = 0.003\%$ are highly desirable.

The heavier neutral MSSM Higgs bosons are also observable in the s-channel. Figure 5 give the cross sections and significance of the $\mathcal{CP}$-odd state A^0 versus the A^0 mass, assuming $R = 0.1\%$ and $L = 0.01$ fb^{-1}. Discovery and study of the A^0 is possible at all m_A if $\tan\beta > 2$ and at $m_A < 2m_t$ if $\tan\beta \lesssim 2$.

The possibility that A^0 and H^0 may be nearly mass degenerate is of particular interest for s-channel Higgs studies. In the large m_A limit, typical of many supergravity models, the masses of A^0, H^0 and $H^\pm$ are similar and h^0 is similar to h_{SM} in its properties. In this situation the A^0 and H^0 contributions can be separated by an s-channel scan; see Fig. 6.

$$\mu^+\mu^- \to A \to b\bar{b}, t\bar{t}: \quad R=0.01\%$$
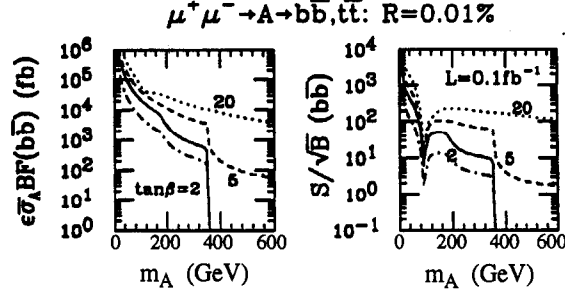
FIGURE 5. Cross sections and significance for detection of the A Higgs boson with an efficiency $\epsilon = 0.5$ and a luminosity $L = 0.1$ fb^{-1}, from Ref. [8].
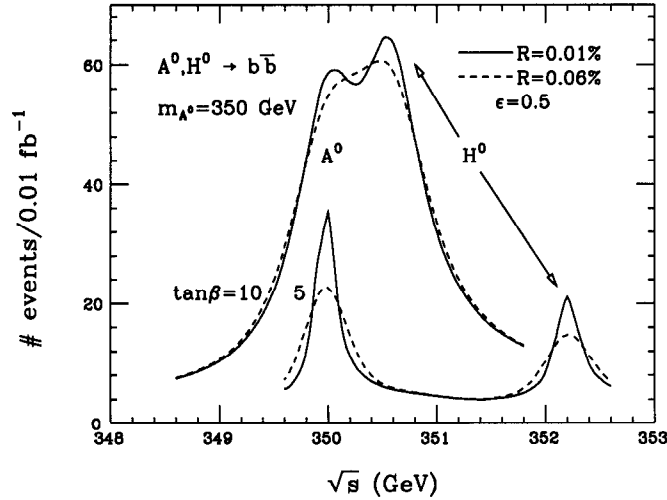
FIGURE 6. Separation of A^0 and H^0 Higgs signals for two values of $\tan\beta$, from Ref. [8].

It was reported during the workshop [10] that beam polarization is potentially useful for Higgs resonance studies, but only if the accompanying luminosity reduction is not significant. Large forward-backward asymmetries can also be used to enhance the Higgs "discovery" signal or improve precision measurements, particularly for the $\tau\bar{\tau}$ final state.

If the muon collider is running at energies above the narrow Higgs resonance, it may still be possible to pick up a signal sample through the photon radiation process $\mu^+\mu^- \to \gamma h$ with a cross section of the order of 0.1 fb. The authors in [11] studied this process and pointed out that the 1-loop contribution is comparable in size to the tree level result, and is especially sensitive to the Higgs coupling to the top quark and to anomalous Higgs couplings to W^+W^- and ZZ. It is also discussed [12] that $\mathcal{CP}$-odd kinematic variables may be constructed for processes like $h^0 \to W^+W^-, ZZ$ and $t\bar{t}$ so that one may be able to probe the $\mathcal{CP}$ properties of the Higgs boson couplings at muon colliders.

During the workshop, many other aspects of Higgs physics were discussed. They include Higgs boson searches at LEP [13,14]; Higgs physics at the Tevatron within

the SM [15] and within the MSSM [16], at the LHC [17,18], and at the NLC [19].

Z POLE ELECTROWEAK PHYSICS

The success of the SM has arguably been most beautifully demonstrated by the agreement of the very precise Z pole measurements at the e^+e^- colliders and the direct measurements. It has been unprecedented that an anticipated quark was discovered with a mass exactly within the range predicted from loop corrections within a theoretical framework. This is a remarkable feat for experimentalists and theorists alike and attests to the enormous success of the SM. Even though many measurements are now being carried out with excruciating precision, the SM shows no signs of giving up its claim of being the description of the fundamental interactions as we know them. Despite these enormous successes there are some discrepancies in the data which, given that the LEP Z pole era is over, will most likely stay with us for a long time. The most significant discrepancy from lepton colliders is the measurement of $\sin^2\theta_{\text{eff}}^{\text{lept}}$, defined as

$$\sin^2\theta_{\text{eff}}^{\text{lept}} \equiv \frac{1}{4}\left(1 - \frac{g_V^{f\,2}}{g_A^{f\,2}}\right) \quad ,$$

where $g_{V(A)}^f$ is the (axial-)vector coupling of the Z boson to fermion f. Currently SLD measures $\sin^2\theta_{\text{eff}}^{\text{lept}} = 0.23055 \pm 0.00041$ [20], derived from the left-right asymmetry of the total cross section and the leptonic forward-backward asymmetries, compared to the LEP average of $\sin^2\theta_{\text{eff}}^{\text{lept}} = 0.23196 \pm 0.00028$, determined from the Z partial decay widths and the forward-backward asymmetries [21]. The discrepancy between the measurements has a significance of 2.8σ. This discrepancy is rather significant. Considering only the A_{LR} measurement, combined with the direct determination of the W and top quark masses, the measurement implies a 95% CL upper bound on the Higgs mass of 77 GeV/c^2, while the direct searches at LEP yield a 95% CL lower limit on the Higgs mass also of 77 GeV/c^2 [22].

The FMC with its anticipated luminosity could contribute significantly to the physics in this sector [23]. Within a one year running period, due to a relaxed requirement on the beam energy resolution R for the broad Z-pole, a sample of 10^8 Z events could be recorded with both beams naturally polarized. To gauge the possible impact of such a large data sample it is instructive to look at the sensitivity of electroweak observables to SM parameters. Table 1 lists electroweak observable $\mathcal{O}$ together with its current measurement and sensitivity, $\Delta\mathcal{O}$, to the top quark mass, $m_t = 175.6 \pm 5.5$ GeV/c^2, the fine structure constant, $\alpha^{-1}(m_Z^2) = 128.896 \pm 0.090$, the strong coupling constant, $\alpha_s(m_Z^2) = 0.121 \pm 0.003$, and the Higgs boson mass, $60 < m_h < 1000$ GeV/c^2 [24]. The observables chosen are R_ℓ, the ratio of the hadronic over leptonic partial decay width of the Z, $R_\ell = \Gamma_{\ell\ell}/\Gamma_{\text{had}}$; $\Gamma_{\ell\ell}$, the Z partial decay width into leptons; $\sin^2\theta_{\text{eff}}^{\text{lept}}$; R_b, the fraction of hadronic Z decays coming from b quarks, $R_b = \Gamma_{b\bar{b}}/\Gamma_{\text{had}}$; and the mass of the W boson, m_W. The

choice of these observables is given by their experimental uncertainty compared to their sensitivity to the various parameters in the model.

Observable ($\mathcal{O}$)	Average Value	$\Delta\mathcal{O}(\delta M_t)$	$\Delta\mathcal{O}(\delta\alpha)$	$\Delta\mathcal{O}(\delta\alpha_s)$	$\Delta\mathcal{O}(\delta m_h)$
$R_\ell \cdot 10^3$	20755 ± 27	1.8	4.0	21	28
$\Gamma_{\ell\ell}$ (MeV)	83.91 ± 0.10	0.06	—	0.02	0.25
$\sin^2\theta_{\text{eff}}^{\text{lept}} \cdot 10^4$	2315.2 ± 2.3	2.0	2.3	0.05	15.4
$R_b \cdot 10^4$	2170 ± 9	2.0	0.2	0.05	0.4
m_W (MeV/c^2)	80430 ± 80	37	14	1	200

TABLE 1. Results for various electroweak observables and their sensitivity to the top quark mass, α, α_s and the Higgs boson mass.

From the table one should observe that $\Gamma_{\ell\ell}$ is insensitive to α, whereas R_ℓ is very sensitive to α_s. It is also clear that the constraint on the Higgs mass is dominated by the measurement of $\sin^2\theta_{\text{eff}}^{\text{lept}}$. The measurement of M_W will become as significant as the current measurement of $\sin^2\theta_{\text{eff}}^{\text{lept}}$ when the experimental accuracy reaches a level of 30 MeV/c^2. If at that point, however, the top mass uncertainty has not been reduced, the constraint from the M_W measurement would partially be spoiled by the top mass uncertainty.

Within the framework of the SM the value of $\alpha_s(m_Z^2)$ derived from an analysis of electroweak precision data depends essentially on R_ℓ, Γ_Z and σ_h^0, with σ_h^0 the peak hadronic Z pole cross section. Since R_ℓ is very sensitive to α_s, the strong coupling parameter can be determined from the parameter R_ℓ alone. For $m_Z = 91.1867$ GeV/c^2, and imposing $m_t = 175.6 \pm 5.5$ GeV/c^2 as a constraint, a value of $\alpha_s(m_Z^2) = 0.124 \pm 0.004 \pm 0.002$ is obtained, where the second uncertainty accounts for the change in the result when varying m_h in the range $60 < m_h < 1000$ GeV/c^2 [21]. The experimental uncertainty is dominated by the limited statistics of the leptonic Z decays. With an improvement in statistics of a factor of 10 over the current LEP statistics, an uncertainty on α_s of 0.001 could be obtained [23].

The quantity $\Gamma_{\ell\ell}$ is of particular interest since it is independent of α and is only mildly dependent on α_s. As such, it is a sensitive indicator of possible new physics beyond the SM. A precision measurement of $\Gamma_{\ell\ell}$, however, requires a very accurate absolute luminosity calibration. $\Gamma_{\ell\ell}$ can be determined with better precision indirectly from

$$\Gamma_Z = \Gamma_{\ell\ell}\left(3 + N_\nu \frac{\Gamma_{\nu\nu}}{\Gamma_{\ell\ell}} + R_\ell\right)$$

using the measurement of Γ_Z, the SM prediction for $\frac{\Gamma_{\nu\nu}}{\Gamma_{\ell\ell}}$ and the measurement of R_ℓ. The uncertainty on R_ℓ, as seen above, will be very small and independent of luminosity. The uncertainty is driven by the uncertainty on the Z width, which depends on the luminosity through the point to point errors in the energy scan and on the energy calibration. At the FMC, where a continuous energy calibration

should be feasible, the latter can be considerably reduced and an accuracy of $\frac{\delta\Gamma_{\ell\ell}}{\Gamma_{\ell\ell}} = 0.0003$ should be possible, compared to the current measurement of $\Gamma_{\ell\ell} = 83.91 \pm 0.10$.

Currently the most powerful way to determine $\sin^2\theta_{\text{eff}}^{\text{lept}}$ is to measure the left-right asymmetry, A_{LR}, defined as

$$A_{\text{LR}} = -\frac{1}{P}\frac{\sigma_{\text{L}} - \sigma_{\text{R}}}{\sigma_{\text{L}} + \sigma_{\text{R}}} \tag{8}$$

where $\sigma_{R(L)}$ is the total production cross section for a right (left) handed polarized electron beam with average polarization P. At the Z pole, ignoring photonic corrections, $A_{\text{LR}} = \mathcal{A}_e$, where the asymmetry of couplings, $\mathcal{A}_f$, is given by

$$\mathcal{A}_f \equiv \frac{g_L^{f\,2} - g_R^{f\,2}}{g_L^{f\,2} + g_R^{f\,2}} = \frac{2\,g_V^f\,g_A^f}{g_V^{f\,2} + g_A^{f\,2}}\ .$$

The measurement of A_{LR} at SLD, using a polarized electron beam with an average polarization during the last run of $\mathcal{P}_e = (76.5 \pm 0.8)\%$, yields a measurement of $\sin^2\theta_{\text{eff}}^{\text{lept}} = 0.23055 \pm 0.00041$ [20], of comparable precision to the combined LEP result, derived from the Z-pole and A_{FB} measurements. The power lies in the availability of polarized beams. The sensitivities of the two measurements are related as $\partial A_{\text{LR}}/\partial\sin^2\theta_{\text{eff}}^{\text{lept}} = \frac{1}{4}\partial A_{\text{FB}}/\partial\sin^2\theta_{\text{eff}}^{\text{lept}}$. Thus, compared to an A_{LR} measurement with fully polarized beams, a sixteen-fold larger data sample is required to achieve a similar accuracy in $\sin^2\theta_{\text{eff}}^{\text{lept}}$ from A_{FB}.

When both beams are polarized, the natural situation for a muon collider, equation 8 generalizes to

$$A_{\text{LR}} = \frac{1}{\mathcal{P}}\frac{\sigma_{\text{L}} - \sigma_{\text{R}}}{\sigma_{\text{L}} + \sigma_{\text{R}}} \qquad \text{with} \tag{9}$$

$$\mathcal{P} = \frac{P^+ - P^-}{1 - P^+ P^-} \tag{10}$$

where $P^{+(-)}$ refers to the average longitudinal polarization of the positively (negatively) charged incident fermion beam. The total cross section is given by

$$\sigma = \sigma_0\left\{(1 - P^+ P^-) + (P^+ - P^-)\,A_{\text{LR}}\right\}\ . \tag{11}$$

From equation 10 it can be seen that the quantity that controls the measurement of A_{LR} ($=\mathcal{A}_e$), and thus the precision of the $\sin^2\theta_{\text{eff}}^{\text{lept}}$ measurement, is $\mathcal{P}$, the effective polarization of the $\mu^+\mu^-$-system, which enhances the sensitivity of the measurement. For a 50% polarization of both beams, $P^{\pm} = \pm 50\%$, $\mathcal{P} = 80\%$. On top of that the cross section also increases by about 30%. Given the process under study a signal to background enhancement can be obtained by optimizing the polarization [10]. The most clear example is the production of the scalar Higgs boson,

where the same sign spin is the favored production mode with a strong suppression of the background.

In the current design of the FMC a big loss in luminosity is incurred for increased polarization [2]. For a polarization of 50% there is a loss in luminosity of about a factor of 4 per beam. The statistical precision with which A_{LR} can be measured is

$$\delta A_{\mathrm{LR}} = \frac{1}{\mathcal{P}} \frac{1}{\sqrt{N}} \, . \tag{12}$$

The relevant quantity therefore to collect the data to measure A_{LR} with a certain precision is $L_P \cdot \mathcal{P}^2$, where L_P is the loss in luminosity for beam polarization P [23]. Given the relation between the loss in luminosity and the beam polarization in the current design of the FMC, this quantity reaches a maximum for a loss in luminosity of about 0.7 corresponding to a beam polarization of about 30%. The advantage of fully polarized beams does not outweigh the loss in luminosity incurred in the current design. The uncertainties on A_{LR} and $\sin^2\theta_{\mathrm{eff}}^{\mathrm{lept}}$ are related through $\delta A_{\mathrm{LR}} = 7.9 \, \delta \sin^2\theta_{\mathrm{eff}}^{\mathrm{lept}}$. A data sample of 10^7 Z events with $\mathcal{P} = 0.5$, assuming equal statistical and systematic uncertainties, should make a measurement of $\delta \sin^2\theta_{\mathrm{eff}}^{\mathrm{lept}} <$ 10^{-4} easily feasible. The power of a measurement of $\sin^2\theta_{\mathrm{eff}}^{\mathrm{lept}}$ to such a precision can readily be seen from Fig. 7 which shows the dependence of $\sin^2\theta_{\mathrm{eff}}^{\mathrm{lept}}$ on m_h taking as input the current measured value for m_Z and assuming $m_t = 173.0 \pm 0.4$ GeV/c^2 and $\alpha^{-1}(m_Z^2) = 128.923 \pm 0.036$ [25]. Taking the current LEP central value for $\sin^2\theta_{\mathrm{eff}}^{\mathrm{lept}}$ a precision of 10^{-4} would constrain the Higgs mass to the range $170 < m_h < 370$ GeV/c^2. It should be noted that the uncertainty on the SM prediction is dominated by the uncertainty on α. An error on m_t of 2 GeV/c^2 is equivalent to an uncertainty on α of 0.04. With further improvements to the hadronic contribution of α to be anticipated, a precise measurement of $\sin^2\theta_{\mathrm{eff}}^{\mathrm{lept}}$ combined with a direct measurement of the Higgs mass, provides a very stringent consistency check of the SM.

It should be noted that relative total cross section measurements with different spin configurations also gives a measure of the polarization. This measures directly the polarization of the interacting muons and no corrections for beam transport to a polarimeter and sampled luminous region need to be applied.

Many more Z pole quantities can be studied at the FMC, notably in the b sector. This, however, has at present not been investigated and should be explored in future studies.

W MASS

Until very recently the mass of the W boson could only be measured directly in $\bar{p}p$ collisions. Precise measurements of the W mass have recently been obtained at LEP2 [21] using the enhanced statistical power of the rapidly varying total W^+W^- cross section at threshold, and the Breit-Wigner peaking behavior of the invariant
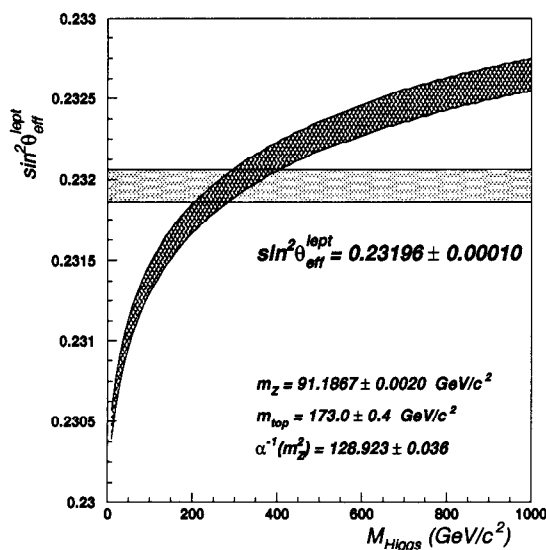
FIGURE 7. Dependence of $\sin^2\theta_{\text{eff}}^{\text{lept}}$ on m_h and the constraint on m_h a hypothetical measurement of $\sin^2\theta_{\text{eff}}^{\text{lept}}$ at the 10^{-4} level would yield, assuming an uncertainty on m_t of 2.0 GeV/c^2.

mass distribution of the W decay products. By measuring the WW threshold cross section at $\sqrt{s} = 161$ GeV, the four LEP experiments have obtained a combined W mass value of $m_W = 80.40^{+0.22}_{-0.21} \pm 0.03$ GeV/c^2. The second error is due to the LEP energy calibration. The direct reconstruction of the W mass from the decay products gives $m_W = 80.37 \pm 0.18 \pm 0.05 \pm 0.03$ GeV/c^2. The second uncertainty is due to effects of color reconnection and the last error is due to the LEP energy calibration. The achievable precision on m_W from LEP for an integrated luminosity of 500 pb^{-1} per experiment is estimated to be $\delta m_W = 35$ MeV/c^2 [26]. For an integrated luminosity of 10 fb^{-1} the Tevatron might be able to constrain m_W to about 20 MeV/c^2 [27]. The FMC is particularly well suited to a threshold measurement because of the very narrow beam spread, the ability to determine the beam energy during a physics run, and the reduced initial state radiation (ISR). Due to the large backgrounds, however, systematic errors arising from uncertainties in both the background level as well as the detection efficiencies will limit the ultimate precision [28,29]. The dominant physics background is mainly due to $Z\gamma$ production, which is almost independent of energy. When detection efficiencies and backgrounds, including the beam induced backgrounds, are largely energy independent, the best precision is obtained through a ratio of cross section measurements at an energy well below the WW threshold and at the threshold. The optimal threshold energy is also for the FMC [26]

$$\sqrt{s} \approx 2\,m_W + 0.5 \text{ GeV} . \tag{13}$$
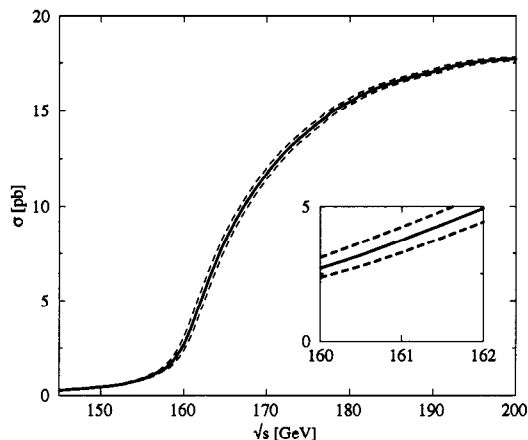
FIGURE 8. The cross section for $\mu^+\mu^- \to W^+W^-$ for $m_W = 80.3$ (± 0.2) GeV/c^2 indicated by the solid (dashed) lines. The inset shows the cross section in the region of maximum statistical sensitivity.

Figure 8 shows the cross section for $\mu^+\mu^- \to W^+W^-$ in the threshold region for three different W mass values. ISR effects have been included. With an integrated luminosity of $\mathcal{L} = 100$ pb^{-1} a precision of $\delta m_W = 6$ MeV/c^2 from the threshold measurement can be obtained when combining the three decay channels $\bar{q}q\bar{q}q$, $\bar{q}q\ell\nu$ and $\ell\nu\ell\nu$. A 10 MeV/c^2 uncertainty on m_W is equivalent to about a 0.5% uncertainty on the measured cross section.

TRIPLE AND QUARTIC GAUGE BOSON COUPLINGS

The non-Abelian $SU(2) \times U(1)$ gauge symmetry of the SM implies that the gauge bosons self-interact. These self-interactions give rise to very subtle interference effects in the SM. In fact, in the SM the couplings are uniquely determined by the gauge symmetry in order to preserve unitarity. An accurate measurement of the gauge boson self-interactions would constitute a stringent test of the gauge sector of the SM and any observed deviation of the couplings from their SM value would indicate new physics.

The formalism of effective Lagrangians is used to describe gauge boson interactions beyond the SM. The most general effective electroweak Lagrangian contains 2×7 free parameters [30]: $g_1^V, \kappa_V, \lambda_V, g_4^V, g_5^V, \tilde{\kappa}_V, \tilde{\lambda}_V$, with $V = \gamma, Z$. The parameter g_5^V, violates C and P but conserves CP; $g_4^V, \tilde{\kappa}_V$ and $\tilde{\lambda}_V$ violate CP. In the SM $g_1^V = 1, \kappa_V = 1$, and all other parameters vanish. For these two parameters one therefore introduces deviations from the SM values, $\Delta\kappa_V = \kappa_V - 1$ and $\Delta g_1^V = g_1^V - 1$.

Gauge boson self-interactions can be studied through di-boson production. The

cross sections for di-boson production are generally rather small and a study of the full fourteen-dimensional parameter space is impossible. In general, two approaches are followed to reduce the parameter space. The $\bar{p}p$ experiments generally set all parameters but two to their SM values and concentrate on $\Delta\kappa_V$, λ_V because they have a direct physical connection through the magnetic dipole and electric quadrupole moment of the W boson, $\mu_W = (e/2m_W)(1 + \kappa_\gamma + \lambda_\gamma)$ and $Q^e_W = (-e/m^2_W)(\kappa_\gamma - \lambda_\gamma)$ [31].

The second approach, followed mainly by the LEP experiments, constructs an effective Lagrangian with operators of higher dimension. By imposing some restriction, like retaining only the lowest dimension operators, respecting $\mathcal{C}, \mathcal{P}$ and $\mathcal{CP}$ invariance and requiring the Lagrangian to be invariant under $SU(2) \times U(1)$ and adding a Higgs doublet, the number of free parameters is reduced to just three [32]. With further, rather ad hoc, requirements the parameter space can be reduced to just two free parameters, with definite relations between the different parameters [33].

If in the processes of di-boson production the couplings deviate even modestly from their SM values, the gauge cancellations are destroyed and a large increase of the cross section is observed. Moreover, the differential distributions will be modified. A WWV interaction Lagrangian with constant anomalous couplings would thus violate unitarity at high energies and therefore the coupling parameters are modified to include form factors [34], that is, $\Delta\kappa(\hat{s}) = \Delta\kappa/(1 + \hat{s}/\Lambda^2)^2$ and $\lambda(\hat{s}) = \lambda/(1 + \hat{s}/\Lambda^2)^2$, where $\hat{s}$ is the square of the center of mass energy of the subprocess. Λ is a unitarity preserving form factor scale and indicates the scale at which new physics would manifest itself.

Currently the strongest limits come from the D0 experiment. From a combined fit to the results from the WW, WW/WZ and $W\gamma$ analyses based on the full Run I data, the limits obtained at 95% CL are ($\Lambda = 1.5$ TeV):

$$-0.33 < \Delta\kappa < 0.45 \quad (\lambda = 0)$$
$$-0.2 < \lambda < 0.2 \quad (\Delta\kappa = 0),$$

where it was assumed that the WWZ couplings and the $WW\gamma$ couplings were equal [35]. A relatively small improvement in the limits is anticipated for the combined Tevatron data. With $500\,pb^{-1}$ at $\sqrt{s} = 190$ GeV limits of the order of 0.05 to 0.1 on anomalous couplings are expected from LEP [26]. At the NLC, for a center of mass energy of $\sqrt{s} = 500$ GeV limits of $|\Delta\kappa_\gamma| < 2.4 \cdot 10^{-3}$ and $|\lambda_\gamma| < 1.8 \cdot 10^{-3}$ are expected [36]. The clear advantage of the FMC is the reduced initial state radiation (ISR), which will facilitate the event reconstruction and reduce the systematic uncertainties. No significant improvement in the limits over the constraints from an NLC are expected [37].

On the other hand, it was reported [38] at the workshop that both the NLC and a high energy muon collider with $\sqrt{s} = 0.5 - 1.5$ TeV may have significant sensitivity to perform direct tests of the quartic gauge boson couplings to a precision of theoretical interests. With an integrated luminosity of 200 fb^{-1} limits on the anomalous quartic couplings ℓ_4, ℓ_5, ℓ_6, ℓ_7 and ℓ_{10} of $\mathcal{O}(10^{-1})$ will be possible.

Also here, beam polarization significantly improves the sensitivity to anomalous couplings.

CONCLUSIONS

There is a rich physics program available with a muon collider. Among them, s-channel Higgs boson production may prove to be the "Crown jewel": precision measurements on the Higgs mass, width, decay branching fractions and couplings to other particles may provide invaluable information on physics within and beyond the SM. However, extremely good beam energy resolution ($R = 0.003\%$) and a high luminosity ($10^{32}\text{cm}^{-2}\text{s}^{-1}$) are highly desirable to fulfill the physics goal. Z pole physics will yield significant results if a data sample of 10^8 Z events (corresponding to an integrated luminosity of about 2 fb^{-1}) can be recorded with polarized beams. Spin manipulation is extremely powerful if it is built into the machine design from the start.

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