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Ola Törnkvist

*Fermi National Accelerator Laboratory
P.O. Box 500, Batavia, Illinois 60510*

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Magnetic Fields from the Electroweak Phase Transition*

Ola Törnkvist[‡]

*NASA/Fermilab Astrophysics Center
MS-209, P.O. Box 500, Batavia, IL 60510-0500, U.S.A.*

and

*Department of Applied Mathematics and Theoretical Physics
University of Cambridge, Cambridge CB3 9EW, United Kingdom[§]*

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Abstract

I review some of the mechanisms through which primordial magnetic fields may be created in the electroweak phase transition. I show that no magnetic fields are produced initially from two-bubble collisions in a first-order transition. The initial field produced in a three-bubble collision is computed. The evolution of fields at later times is discussed.

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[‡]Electronic address: O.Tornkvist@damtp.cam.ac.uk

[§]Present address.

1 Introduction

Observations, of e.g. the Faraday rotation of the polarisation of light from distant sources as it passes through galaxies, reveal that galaxies typically possess a magnetic field with strength of the order of 10^{-6} Gauss. The direction of the field appears to be correlated with each galaxy's axis of rotation, which suggests that it was generated during the epoch of galaxy formation through magnetohydrodynamic (MHD) processes. Such processes include dynamo mechanisms, which may amplify the magnetic field by many orders of magnitude, and nonlinear inverse-cascade mechanisms, which increase the correlation length by transferring power to long-wavelength modes [1].

Because the magnetic field \mathbf{B} enters homogeneously in the MHD equations, an initial seed field is needed, which must be larger than about 10^{-20} Gauss. The most natural origin of such a seed field is the electroweak phase transition (EWPT), as that is the earliest time a magnetic field can come into existence as a gauge-invariant physical quantity. The initial field is very strong, $B_0 \sim M_W^2 \sim 10^{24}$ Gauss, and has a correlation length $\xi \sim M_W^{-1}$. As the universe expands, the average magnetic field decreases as the inverse square of the scale factor, $B(t) \propto [a(t)]^{-2}$, since in the conducting plasma the magnetic flux through surfaces bounded by comoving closed curves is conserved. Silk damping, the diffusion of photons in the charged plasma, may further reduce the field.

A seed field can arise in the EWPT in several interesting ways [1]. If the transition is first-order, the field may result from the turbulent motion of charged plasma layers on expanding bubble walls, or in the collision of bubbles that contain different phases of the Higgs field [2, 3]. Also in a second-order transition the seed field may emerge spontaneously in the EWPT, as proposed by Vachaspati [4], through the process of symmetry breaking itself.

In this talk, I will concentrate on magnetic-field generation in collisions of electroweak bubbles. For this purpose, it is necessary first to reexamine the definition of the electromagnetic field in the broken-symmetry phase.

2 Gauge-invariant definition of the electromagnetic field

In the unitary gauge, $\Phi = (0, \rho)^\top$ with $\rho \equiv \text{constant}$, the electromagnetic field tensor takes the familiar form $\mathcal{F}_{\mu\nu}^{\text{em}} = \partial_\mu A_\nu - \partial_\nu A_\mu$, where $A_\mu = \sin\theta_w W_\mu^3 + \cos\theta_w Y_\mu$ and W_μ^3, Y_μ are defined by their occurrence in the covariant derivative of the Higgs field, $\mathcal{D}_\mu \Phi = (\partial_\mu - igW_\mu^a \tau^a / 2 - ig'Y_\mu / 2)\Phi$. Here A_μ , by construction, does not couple to the Higgs field Φ ; it represents the massless photon.

For a general Higgs field $\Phi = (\phi_1(x), \phi_2(x))^\top$, the field A_μ as defined above couples to

$\phi_1(x)$ and becomes massive. Evidently, at the position x , A_μ is no longer the photon field, and the curl of A_μ cannot be interpreted as the electromagnetic field tensor. Instead, one must find a gauge-invariant definition of $\mathcal{F}_{\mu\nu}^{\text{em}}$ whose value in any gauge coincides with that in the unitary gauge. Such a definition was proposed in Ref. [5]. In terms of the three-component unit isovector $\hat{\phi}^a = (\Phi^\dagger \tau^a \Phi)/(\Phi^\dagger \Phi)$ it is given by

$$\mathcal{F}_{\mu\nu}^{\text{em}} := -\sin\theta_w \hat{\phi}^a F_{\mu\nu}^a + \cos\theta_w F_{\mu\nu}^Y + \frac{\sin\theta_w}{g} \epsilon^{abc} \hat{\phi}^a (D_\mu \hat{\phi})^b (D_\nu \hat{\phi})^c, \quad (1)$$

where $(D_\mu \hat{\phi})^a = \partial_\mu \hat{\phi}^a + g\epsilon^{abc} W_\mu^b \hat{\phi}^c$. The last term in Eq. (1) correctly takes into account electromagnetic fields associated with the phases of the Higgs field. Unlike previous definitions, Eq. (1) eliminates contributions from neutral currents and subtracts out the W -boson magnetic dipole-moment density also when the magnitude $(\Phi^\dagger \Phi)^{1/2}$ has a spatial dependence [5]. In addition, it satisfies the Bianchi identity $\epsilon^{\mu\nu\alpha\beta} \partial_\nu \mathcal{F}_{\alpha\beta}^{\text{em}} = 0$, which ensures that there is no magnetic charge or magnetic current in the absence of magnetic monopoles.

3 Magnetic fields from bubble collisions

In a first-order EWPT magnetic fields can be created in the collision of expanding bubbles that contain different phases of the Higgs field. This was first investigated in the abelian U(1) model [2]. Because of the phase gradient, a gauge-invariant current $j_k = iq[\phi^\dagger D_k \phi - (D_k \phi)^\dagger \phi]$ develops across the surface of intersection of the two bubbles, where $D_k = \partial_k - iqV_k$. The current gives rise to a ring-like flux of the field strength $\partial_i V_j - \partial_j V_i$ which takes the shape of a girdle encircling the bubble intersection region.

This mechanism can be generalised to the electroweak SU(2)×U(1) theory, where the initial Higgs field in the two bubbles is of the form

$$\Phi_1 = \begin{pmatrix} 0 \\ \rho_1(x) \end{pmatrix}, \quad \Phi_2 = \exp(i\frac{\theta}{2} n^a \tau^a) \begin{pmatrix} 0 \\ \rho_2(x) \end{pmatrix} \quad (2)$$

and the Higgs phases have equilibrated to constant values inside each bubble. Saffin and Copeland [3] found that such an initial configuration can be written globally in the same form as Φ_2 , provided that $\theta \rightarrow \theta(x)$ and $\rho_2(x)$ is replaced by a different function $\rho_3(x)$. One may also assume that the initial configuration has zero field strength, $F_{\mu\nu}^a = F_{\mu\nu}^Y = 0$. Then, except in the pathological case of singular vacuum configurations, one may choose also the gauge potentials to be zero, $W_\mu^a = Y_\mu = 0$.

For $\Phi = \Phi_1$ one obtains $\hat{\phi} = \hat{\phi}_0 \equiv (0, 0, -1)$. It is easy to show that, as Φ interpolates from Φ_1 to Φ_2 , the isovector $\hat{\phi}$ is rotated by an angle θ about the axis \mathbf{n} . Saffin and Copeland discovered that, in the cases where \mathbf{n} is parallel or perpendicular to $\hat{\phi}_0$, the

dynamics reduces to the U(1) problem already solved [3]. The ring-like flux in these two cases corresponds to the gauge field of an electroweak Z -string or W -string, respectively.

The issue of magnetic-field creation in bubble collisions has so far not been properly addressed in the electroweak theory. Because the field tensors and gauge potentials are initially zero, the only contribution to the magnetic field comes from the last term in Eq. (1), which reduces to $\sin \theta_w \hat{\phi} \cdot (\partial_\mu \hat{\phi} \times \partial_\nu \hat{\phi})/g$. Because $\hat{\phi}$ is obtained by rotating $\hat{\phi}_0$ by an angle θ about \mathbf{n} , one finds that $\partial_\mu \hat{\phi} = \partial_\mu \theta (\mathbf{n} \times \hat{\phi})$. It then follows that neither electromagnetic fields nor electric currents are present initially in the collision [5]. I emphasise that this result is very different from the U(1) case, where a field strength was present from the instant of collision of the bubbles.

Let us now consider the subsequent field evolution. Because $\mathcal{F}_{\mu\nu}^{\text{em}}$ satisfies the Bianchi identity, $\partial \mathbf{B}/\partial t = -\nabla \times \mathbf{E} = 0$ initially. Likewise, since the electric current vanishes initially, it follows that $\partial \mathbf{E}/\partial t = \nabla \times \mathbf{B} = 0$ initially. Non-zero \mathbf{E} and \mathbf{B} can thus arise only in ‘‘second order’’ through later evolution of the Higgs field that renders the last term in Eq. (1) nonzero.

As we have seen, no magnetic field is produced initially in the collision of two bubbles in the EWPT. One can show that the reason is that the unit vector \mathbf{n} is a constant. In contrast, in a three-bubble collision \mathbf{n} will have a spatial dependence, and a magnetic field will appear already at the instant of collision. If three bubbles are needed, however, the importance of bubble collisions for magnetic-field generation is diminished, as the third bubble must impinge before the phases of the first two have equilibrated. The initial configuration of three bubbles can be written globally as $\Phi = \exp[i(f(x)m_a + g(x)n_a)\tau^a] (0, \rho(x))^\top$, where the constant unit vectors \mathbf{m} and \mathbf{n} are not collinear. Defining $R \equiv \sqrt{f^2 + g^2}$ and taking, for simplicity, $\mathbf{m} \perp \mathbf{n}$, we obtain

$$\mathcal{F}_{\mu\nu}^{\text{em}} = \frac{4 \sin \theta_w}{g} f_{[\mu, \nu]} g_{\nu]} \left(-\frac{\sin 2R}{2R} (m_1 n_2 - m_2 n_1) + \frac{\sin^2 R}{R^2} (f n_3 - g m_3) \right), \quad (3)$$

which yields a nonzero magnetic field as long as $\nabla f \times \nabla g \neq 0$.

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