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and the Heavy Quark Limit**

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# Radiative Corrections to the Decay $B \rightarrow \pi e \nu$ and the Heavy Quark Limit

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## Abstract:

We calculate radiative corrections to the light-cone sum rule for the semileptonic form factor in  $B \rightarrow \pi e \nu$  decays and thus remove the major uncertainty in determining the CKM mixing angle  $|V_{ub}|$  by this method. We discuss the remaining uncertainties and perspectives for further studies. The structure of the radiative corrections suggests factorization of soft (end-point) and hard rescattering contributions in heavy-to-light decays in the heavy quark limit.

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**1.** Precise determination of the CKM mixing angle  $|V_{ub}|$  will most likely come from semileptonic  $B$  decays. Two competing strategies exist, the study of the end-point region in inclusive decays and the analysis of various exclusive decay channels, most notably  $B \rightarrow \pi e \nu$  which is the easiest one to access experimentally. The theoretical challenge in the latter case is to calculate the  $B \rightarrow \pi$  transition form factor  $f_+(q^2)$  induced by the weak vector current:

$$\langle \pi(p_\pi) | \bar{u} \gamma_\mu b | B(p_B) \rangle = (p_B + p_\pi)_\mu f_+(q^2) + (p_B - p_\pi)_\mu f_-(q^2). \quad (1)$$

Here  $q^2 = (p_B - p_\pi)^2$  is the square of the momentum transferred to the lepton pair; the second form factor  $f_-$  does not contribute to the decay rate for zero mass leptons and will not be considered in this letter. The decay rate is dominated by contributions from small values of  $q^2$ , for which the  $u$  quark produced in the weak decay has large energy of order  $m_b/2$  in the  $B$  meson rest frame ( $m_b$  is the  $b$  quark pole mass). A consistent QCD description of such processes exists up to now only in the Sudakov limit [1], in which case contributions from large transverse quark-antiquark separations are suppressed and the form factor is dominated by hard gluon exchange [2]. This limit is theoretically interesting, but not relevant for realistic  $b$  quark masses. In fact, we will argue that “soft” contributions related to large transverse distances exceed the “hard” contributions by an order of magnitude (and have opposite sign). Any theoretical approach to heavy-to-light decay form factors aiming at quantitative predictions has to deal with these “soft” terms explicitly, as do e.g. light-cone sum rules and various models [3].

The consistent separation of “hard” and “soft” contributions in heavy-to-light decays presents an unsolved problem which, to our knowledge, has never been tackled in a systematic way<sup>1</sup>. In this letter we examine a possibility for such a separation, based on our calculation of the radiative corrections to the corresponding light-cone sum rules.

The basic idea of the light-cone sum rule approach is to consider a two-point correlation function replacing the  $B$  meson state by a suitable interpolating current:

$$\begin{aligned} \Pi_\mu(p_B^2, q^2) &= i \int d^4x e^{-ip_B x} \langle \pi(p_\pi) | T \{ \bar{u}(0) \gamma_\mu b(0) \bar{b}(x) i \gamma_5 d(x) \} | 0 \rangle \\ &= (p_B + p_\pi)_\mu \Pi_+(p_B^2, q^2) + (p_B - p_\pi)_\mu \Pi_-(p_B^2, q^2). \end{aligned} \quad (2)$$

The Lorentz-invariant function  $\Pi_+$  has a pole at  $p_B^2 = m_B^2$  corresponding to the contribution of the  $B$  meson:

$$\Pi_+(p_B^2, q^2) = \frac{f_B m_B^2}{m_b} \frac{f_+(q^2)}{m_B^2 - p_B^2} + \dots, \quad (3)$$

where  $m_B$  is the  $B$  meson (pole) mass and the dots stand for contributions from higher-mass resonances and the continuum. The  $B$  meson decay constant  $f_B$  is usually defined as

$$m_b \langle 0 | \bar{d} i \gamma_5 b | B \rangle = m_B^2 f_B. \quad (4)$$

On the other hand, the correlation functions in (2) can be calculated in the Euclidean region,  $p_B^2 - m_B^2$  large and negative, using the light-cone expansion. Up to higher twist

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<sup>1</sup>One reason being that in the classical application, the pion form factor, the “soft” terms are suppressed by one power of the momentum transfer.

corrections the product of  $b$  quark fields can be substituted by a perturbative propagator and the tree-level result is

$$\Pi_+(p_B^2, q^2) = \frac{1}{2} f_\pi m_b \int_0^1 du \frac{\phi_\pi(u, \mu)}{m_b^2 - up_B^2 - \bar{u}q^2}. \quad (5)$$

Here and below  $\bar{u} = 1 - u$ ,  $\phi_\pi(u, \mu)$  is the leading twist pion distribution amplitude defined by

$$\langle \pi(p_\pi) | \bar{u}(0) \gamma_\nu \gamma_5 d(x) | 0 \rangle \stackrel{x^2=0}{=} -i f_\pi p_{\pi\nu} \int_0^1 du e^{i\bar{u}p_\pi x} \phi_\pi(u, \mu) \quad (6)$$

in Fock-Schwinger gauge and  $\mu$  is the factorization (renormalization) scale. The variable  $u$  has the physical meaning of the momentum fraction carried by the  $u$  quark in the infinite momentum frame.

Next, we use the concept of duality, assuming that the contribution of the  $B$  meson to (3) corresponds to an integral over the spectral density calculated within the light-cone expansion, Eq. (5), in a certain duality interval:

$$\Pi_+^{\text{meson}}(p_B^2, q^2) = \int_{m_b^2}^{s_0} \frac{ds}{s - p_B^2} \rho(s, q^2). \quad (7)$$

The width of the duality interval is characterized by the parameter  $s_0$  (continuum threshold) and in general is of order  $s_0 - m_b^2 \sim O(m_b)$ . The spectral density  $\rho(s, q^2)$  immediately follows from Eq. (5) after a simple change of variables  $s \rightarrow (m_b^2 - q^2)/u + q^2$ . Thus, in this particular case the duality restriction on the maximum invariant energy  $s_0$  translates into a restriction on the *minimum* momentum fraction  $u_0$  carried by the  $u$  quark:

$$u_0 = \frac{m_b^2 - q^2}{s_0 - q^2}. \quad (8)$$

Equating the representations (3) and (5) and isolating the contribution of the  $B$  meson according to (7), we obtain the simplest version of the light-cone sum rule for  $f_+(q^2)$ , which neglects higher twist and radiative corrections. In order to suppress the contributions of higher order states, it is written in a Borel transformed form, which amounts to replacing the factor  $1/(s - p_B^2)$  by  $\exp(-s/M^2)$  and  $1/(m_B^2 - p_B^2)$  by  $\exp(-m_B^2/M^2)$ :

$$\frac{f_B m_B^2}{f_\pi m_b^2} e^{(m_b^2 - m_B^2)/M^2} f_+(q^2) = \frac{1}{2} \int_{u_0}^1 du \frac{\phi_\pi(u, \mu)}{u} e^{-\bar{u}(m_b^2 - q^2)/(uM^2)}. \quad (9)$$

Here  $M^2$  is the Borel parameter.

**2.** The accuracy of the sum rule (9) can be improved by including higher twist and radiative corrections. The former ones have been calculated earlier [4, 5]; Eq. (79) in [5] gives the complete result to twist 4 accuracy, which we will use in the numerical analysis. We have calculated first order radiative corrections to the leading twist term (9) The calculation is straightforward and similar to earlier calculations of the radiative correction

to the  $\gamma^* \rightarrow \pi\gamma$  form factor [6], but more cumbersome because of the nonzero  $b$  quark mass. In the  $\overline{\text{MS}}$ -scheme, the result reads:

$$\begin{aligned}
\frac{f_B m_B^2}{f_\pi m_b^2} e^{(m_b^2 - m_B^2)/M^2} f_+(q^2) &= \frac{1}{2} \int_{u_0}^1 du \frac{\phi_\pi(u, \mu)}{u} e^{-\bar{u}(m_b^2 - q^2)/(uM^2)} \\
&- C_F \frac{\alpha_s}{4\pi} e^{m_b^2/M^2} \left\{ \int_0^1 du \frac{\phi_\pi(u, \mu)}{\bar{u}} \int_{m_b^2}^{s_0} dt \frac{m_b^2 - t}{t - q^2} \left( \frac{2}{t} + L_1 \right) e^{-t/M^2} \right. \\
&+ \int_0^{u_0} du \frac{\phi_\pi(u, \mu)}{\bar{u}} e^{-\frac{m_b^2 - \bar{u}q^2}{uM^2}} \int_{m_b^2}^{s_0} dt (m_b^2 - t) \frac{e^{-\frac{ut + \bar{u}q^2 - m_b^2}{uM^2}}}{m_b^2 - ut - \bar{u}q^2} L_2 \\
&+ \int_{u_0}^1 du \frac{\phi_\pi(u, \mu)}{\bar{u}} e^{-\frac{m_b^2 - \bar{u}q^2}{uM^2}} \left[ \int_{m_b^2}^{s_0} dt (m_b^2 - t) \frac{e^{-\frac{ut + \bar{u}q^2 - m_b^2}{uM^2}} - 1}{m_b^2 - ut - \bar{u}q^2} L_2 + \int_{m_b^2}^{s_0} dt L_2 \right. \\
&\quad \left. \left. - \int_{m_b^2}^{t_0} dt \frac{m_b^2 - q^2}{t - q^2} \left[ \frac{m_b^2 - t}{t(m_b^2 - q^2)} + L_1 \right] e^{-\frac{t - m_b^2}{uM^2}} \right] \right. \\
&+ \int_{u_0}^1 du \frac{\phi_\pi(u, \mu)}{u} e^{-\frac{m_b^2 - \bar{u}q^2}{uM^2}} \left[ \frac{5}{2} - \frac{\gamma_E}{2} + 2 \ln \frac{uM^2}{m_b^2} - \frac{3}{2} \ln \frac{uM^2}{\mu^2} + \frac{1}{2} \text{Ei} \left( \frac{m_b^2 - t_0}{uM^2} \right) \right. \\
&\quad + \int_{m_b^2}^{s_0} dt L_2 + \int_{s_0}^\infty dt \frac{(m_b^2 - q^2) L_2}{m_b^2 - ut - \bar{u}q^2} + \int_{m_b^2}^{t_0} dt \left( \frac{1}{t} - L_2 \right) e^{-\frac{t - m_b^2}{uM^2}} \\
&\quad + \int_{m_b^2}^{t_0} dt \left[ \frac{1}{2} - \frac{(t - m_b^2)^2}{2t^2} + (m_b^2 - q^2) (L_1 + L_2) \right] \frac{e^{-\frac{t - m_b^2}{uM^2}} - 1}{m_b^2 - t} \\
&\quad \left. \left. - \int_{t_0}^\infty \frac{dt}{m_b^2 - t} \left[ \frac{1}{2} - \frac{(t - m_b^2)^2}{2t^2} + (m_b^2 - q^2) (L_1 + L_2) \right] \right] \right\} \quad (10)
\end{aligned}$$

where  $m_b$  is the one-loop pole mass,  $\text{Ei}(x)$  the exponential-integral function, defined as  $\text{Ei}(x) = -\int_{-x}^\infty dy e^{-y}/y$ ,  $t_0 \equiv us_0 + \bar{u}q^2$  and

$$L_1 = \frac{1}{t - q^2} \left[ -1 + \ln \frac{(t - m_b^2)^2}{t\mu^2} \right]; \quad L_2 = \frac{1}{t - q^2} \left[ -\frac{m_b^2}{t} + \ln \frac{(t - m_b^2)^2}{t\mu^2} \right]. \quad (11)$$

Details of the calculation will be published elsewhere. We have checked that the  $\mu$  dependent terms cancel the  $\mu$  dependence of the distribution amplitude  $\phi_\pi(u, \mu)$  to leading logarithmic accuracy [7]. We have also checked that our results agree with the recent results of Ref. [8], where a similar calculation is reported, both analytically for the amplitude

in momentum space and numerically for the final sum rule after Borel transformation and continuum subtraction. Our representation for the final answer is simpler.

**3.** In order to gain a better understanding of the structure of radiative corrections, it is convenient to go to the heavy quark limit  $m_b \rightarrow \infty$ . In this limit the dimensionful parameters  $f_B$ ,  $s_0$  and  $M^2$  have to be rescaled as

$$\begin{aligned} f_B &= 1/\sqrt{m_b} f^{\text{stat}}(\mu = m_b), & m_B - m_b &= \bar{\Lambda}, \\ s_0 &= m_b^2 + 2m_b\omega_0, & M^2 &= 2m_b\tau, \end{aligned} \quad (12)$$

where  $\omega_0$  and  $\tau$  are the nonrelativistic continuum threshold and the Borel parameter, respectively. The heavy quark limit of the form factor depends crucially on the value of  $q^2$ . As we are only interested in small values  $q^2 \ll m_b^2$ , we will set  $q^2 = 0$  in this discussion for simplicity.

In this limit the momentum fraction cut-off (8) becomes  $u_0 = 1 - 2\omega_0/m_b$ , so that the integration region in (9) shrinks to a narrow interval near the end-point, corresponding to all the pion momentum being carried by the  $u$  quark. Since close to the end-point  $\phi_\pi(u, \mu) \sim (1 - u)$ , we obtain

$$\frac{f^{\text{stat}}(\mu = m_b)}{\sqrt{m_b} f_\pi} e^{-\bar{\Lambda}/\tau} f_+(0) = -\frac{2}{m_b^2} \phi'_\pi(1) \int_0^{\omega_0} d\omega \omega e^{-\omega/\tau} \quad (13)$$

where  $\phi'_\pi(u) = (d/du)\phi_\pi(u)$ . Note that  $\phi'_\pi(1) < 0$ . One thus finds  $f_+(0) \sim m_b^{-3/2}$ , at least at tree level [9, 10].

The full expression for the radiative corrections looks rather complicated; to make its structure more transparent we take the limit  $\tau \rightarrow \infty$ , corresponding to the so-called local duality approximation:

$$\begin{aligned} &\frac{f^{\text{stat}}(m_b)}{f_\pi} [m_b^{3/2} f_+(0)] = \\ &= -\omega_0^2 \phi'_\pi(1) \left[ 1 + \frac{\alpha_s}{\pi} C_f \left( \frac{1 + \pi^2}{4} + \ln \frac{m_b}{2\omega_0} - \frac{1}{2} \ln^2 \frac{m_b}{2\omega_0} + \frac{1}{2} \ln \frac{2\omega_0}{\mu} \right) \right] \\ &\quad - \omega_0^2 \frac{\alpha_s}{\pi} C_f \left[ \left( 1 - \ln \frac{2\omega_0}{\mu} \right) \int_0^1 du \left( \frac{\phi_\pi(u)}{\bar{u}^2} + \frac{\phi'_\pi(1)}{\bar{u}} \right) - \ln \frac{2\omega_0}{\mu} \int_0^1 du \frac{\phi_\pi(u)}{\bar{u}} \right]. \end{aligned} \quad (14)$$

This expression deserves to be studied in some detail. Let us interpret the two pieces: the first term on the right-hand side must be identified with the soft (end-point) contribution including the Born-term and its radiative correction, while the second term corresponds to the usual mechanism of hard gluon exchange.

The dependence on the collinear factorization scale  $\mu$  must cancel the scale dependence of the pion distribution amplitude. This implies that the structure of terms in  $\ln \mu$  in the hard contribution is fixed by the structure of the leading order soft term which is

proportional to  $\phi'_\pi(1, \mu)$ . Indeed, we find

$$\begin{aligned} \frac{d}{d \ln \mu} \phi'_\pi(1, \mu) &= \frac{\alpha_s}{\pi} C_f \frac{d}{dx} \left[ \int_0^1 dy V_0(x, y) \phi_\pi(y, \mu) \right]_{x \rightarrow 1} \\ &= -\frac{\alpha_s}{\pi} C_f \left\{ \int_0^1 du \left[ \frac{\phi_\pi(u) + \bar{u} \phi'_\pi(1)}{\bar{u}^2} + \frac{\phi_\pi(u)}{\bar{u}} \right] - \frac{1}{2} \phi'_\pi(1) \right\}, \end{aligned} \quad (15)$$

where  $V_0(x, y)$  is the usual Brodsky-Lepage kernel, so that the structure of  $\ln \mu$  terms in (14) is reproduced. Note the subtraction term accompanying the naively divergent expression  $\int du \phi_\pi(u, \mu)/\bar{u}^2$  [2], which is similar to the usual “plus” prescription in the evolution kernel.

Local duality means that we identify the  $B$  meson with a  $b$  quark accompanied by an arbitrary number of light quarks and gluons with total energy less than  $\omega_0$  (in the  $b$  quark rest frame). Consider the “deep inelastic” cross section of neutrino scattering off a pion, in which one selects the contribution of the charged weak current with a  $b$  quark in the final state:  $d\sigma/dM_X^2(\nu_e + \pi \rightarrow X_b + e)$ , where  $M_X^2$  is the invariant mass of the hadronic final state. In the approximation adopted in this letter we identify the integral of this cross section over the region of small invariant masses  $M_X^2 < m_b^2 + 2m_b\omega_0$  with the square of the form factor for the inverse process  $F(\nu\pi \rightarrow eB)$  (up to kinematical factors). This interpretation is useful in several aspects. It is easy to check that the “soft” contribution in (14) corresponds to the would-be leading twist contribution to the deep inelastic cross section, while the “hard” contribution (involving interaction with the quark-spectator) is of higher twist. These two terms are of the same order (in the  $b$  quark mass) since the leading twist contribution is additionally suppressed by a factor  $1/m_b^2$  for small values of the invariant mass of the hadronic system, because the corresponding parton distribution (in the pion, for the case at hand) vanishes at the end-point  $x \rightarrow 1$ . With a low collinear factorization scale  $\mu \sim \omega_0$  all quark mass dependence in (14) is due to the “soft” contribution and thus to the leading twist part of the cross section. This suggests that resummation of heavy quark mass logarithms can be done using the same techniques as for the end-point spectrum in inclusive  $b \rightarrow u$  decays [11]. A detailed discussion goes beyond the scope of this letter.

A final remark concerns the size of the radiative correction. With the natural factorization scale  $\mu = 2\omega_0 \simeq 2 \text{ GeV}$  and with  $m_b \simeq 5 \text{ GeV}$  the quark mass logarithms are of order unity and the large constant term dominates. It has to be compared, however, to the large radiative correction which was found in QCD sum rules for the decay constant  $f^{\text{stat}}$ . In the same (local duality) approximation and neglecting contributions of condensates, one finds [12]

$$f^{\text{stat}}(m_b) = \frac{\omega_0^{3/2}}{\pi} \left[ 1 + \frac{\alpha_s}{\pi} C_f \left( \frac{15}{8} + \frac{1}{6} \pi^2 + \frac{3}{4} \ln \frac{m_b}{2\omega_0} \right) \right]. \quad (16)$$

We see that the large radiative corrections almost cancel each other between  $f^{\text{stat}}$  and the right-hand side of (14). This shows that the form factor itself is free from large radiative corrections in the light-cone sum rule approach. The same cancellation takes place in the complete expressions with finite  $b$  quark mass.

**4.** We have carried out a detailed numerical analysis of the complete sum rules with finite  $b$  quark mass and including radiative and higher twist corrections. To this end we



substitute the value of  $f_B$  in (10) by the corresponding sum rule [13] including radiative corrections with the same value of the continuum threshold and the same Borel parameter. In particular, we use the range  $5 \text{ GeV}^2 < M^2 < 8 \text{ GeV}^2$  and  $m_b = \{4.6, 4.7, 4.8\} \text{ GeV}$  with the continuum thresholds  $s_0 = \{35, 34, 33.5\} \text{ GeV}^2$ , respectively. The resulting value for  $f_B$  is  $(175 \pm 25) \text{ MeV}$ , which is in agreement with the current lattice average [14]. We do not attempt a renormalization group improvement (resummation of  $\ln m_b$ ) of the sum rules and set the factorization scale to  $\mu = \sqrt{m_B^2 - m_b^2} \simeq 2.4 \text{ GeV}$ . The scale dependence of our results is in fact negligible. A remark is in order about the actual choice of  $M^2$  in Eq. (10). The expansion turns out to be essentially in inverse powers of  $uM^2$  rather than  $M^2$  itself. In order to avoid small values of this “effective” parameter  $uM^2$  for large  $q^2$  (where  $u$  can become small), we use the rescaled value  $M^2 = M_{2pt}^2/\langle u \rangle$  in the Borelized expression for the correlation function  $\Pi_+$  and  $M_{2pt}^2$  in the above mentioned range in the sum rule for  $f_B$ .  $\langle u \rangle$  is the average value of  $u$  in the integral in (9) with  $\langle u \rangle \approx 0.87$  for  $q^2 = 0$  and  $\langle u \rangle \approx 0.72$  for  $q^2 = 15 \text{ GeV}^2$ . Variations of the Borel parameter in the chosen window result in a minimal uncertainty in the prediction for the form factor of about 10%, independent of  $q^2$ . We are going to discuss the other remaining uncertainties in detail.

*Radiative corrections.* Radiative corrections to the light-cone sum rule have been previously suspected to be significant. It was expected, however, that large corrections to the correlation function  $\Pi_+$  partially cancel with large corrections to the coupling  $f_B$  in the ratio  $f_+ \sim \Pi_+/f_B$ . To quantify this effect, we write, schematically,

$$f_+ \sim \frac{\Pi_+^0 \left(1 + \Pi_+^1 \frac{\alpha_s}{\pi}\right)}{f_B^0 \left(1 + f_B^1 \frac{\alpha_s}{\pi}\right)}, \quad (17)$$

where  $\Pi_+^0$  and  $f_B^0$  are the tree-level contributions and  $\Pi_+^1$  and  $f_B^1$  specify the corrections. For central values of the input parameters, we find  $f_B^1 \simeq 3.0$  and  $\Pi_+^1 = \{2.4, 2.3, 2.2, 2.1\}$  for momentum transfers  $q^2 = \{0, 5, 10, 15\} \text{ GeV}^2$ , respectively. Thus the corrections indeed cancel each other to a large extent. The “hard” contribution to  $\Pi_+^1$ , defined by the terms in (10) involving an integral over  $u$  in the interval  $0 < u < u_0$ , is  $-0.5$  (at  $q^2 = 0$ ), whereas the radiative correction to the “soft” contribution  $u_0 < u < 1$  is  $+2.9$ . The resulting net effect of radiative corrections is shown in Fig. 1, where we plot  $f_+$  as a function of  $q^2$  for two different choices of the pion distribution amplitude (DA), see below. The curves marked LO (NLO) are obtained by neglecting (including) radiative corrections in *both* the numerator and the denominator in (17). The size of the correction is at most -7%, thus providing an *a posteriori* justification of the procedure of Refs. [4, 10, 5] to use a low value of  $f_B$  in leading order light-cone sum rules.

*Pion distribution amplitude.* The main input in the sum rules is the pion distribution amplitude  $\phi_\pi$  defined in (6), which can conveniently be expanded in a series of Gegenbauer polynomials with multiplicatively renormalizable coefficients  $a_n$  (to leading logarithmic accuracy):<sup>2</sup>

$$\phi_\pi(u, \mu) = 6u(1-u) \left[ 1 + a_2(\mu)C_2^{3/2}(2u-1) + a_4(\mu)C_4^{3/2}(2u-1) + \dots \right]. \quad (18)$$

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<sup>2</sup>We neglect two-loop anomalous dimensions and also the mixing of the  $a_n$ , which occurs at two-loop accuracy. The corresponding expressions are available in the literature and can easily be incorporated in the analysis. Their effect is, however, negligible compared to uncertainties in the numerical values of the  $a_n$ .

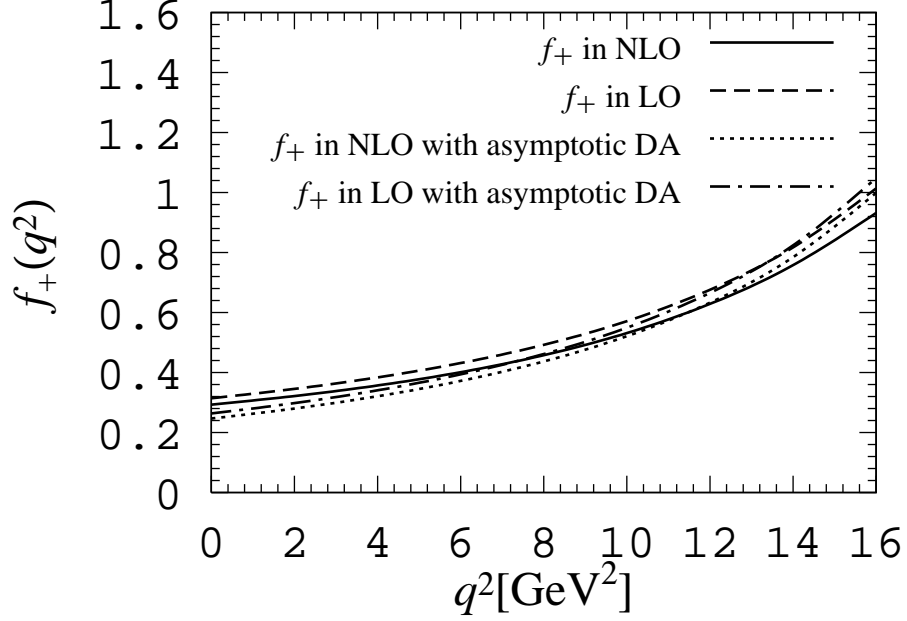


Figure 1:  $f_+(q^2)$  as a function of  $q^2$  for two different sets of the leading twist pion distribution amplitude. The effect of including radiative corrections is a reduction of the form factor by about (4–7)%.

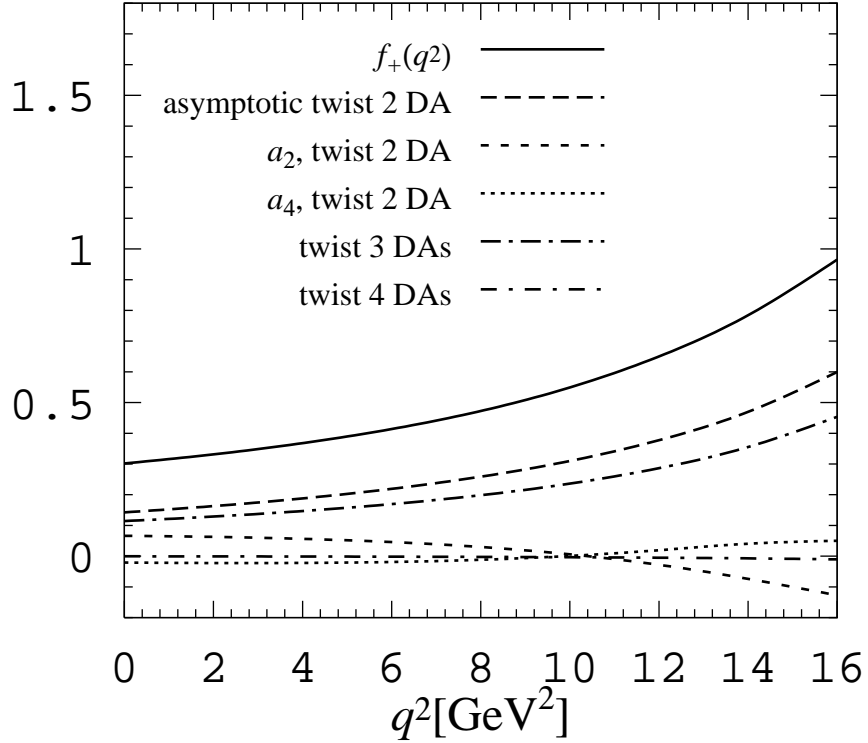


Figure 2: The several contributions to the light-cone sum rule for the form factor as a functions of  $q^2$ , using the leading twist distribution amplitude of [19].

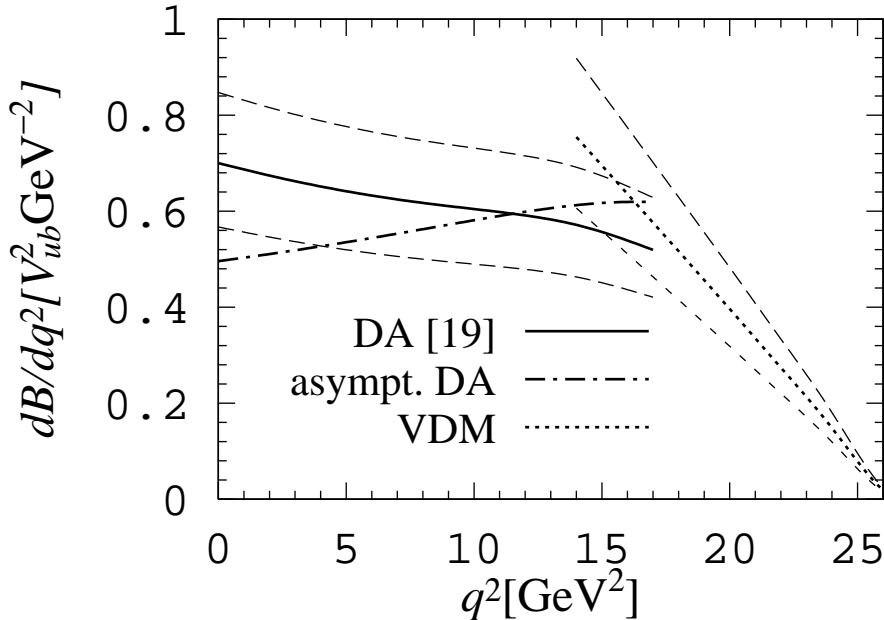


Figure 3: The spectrum  $dB(B \rightarrow \pi e \nu)/dq^2$  as a function of  $q^2$ . Solid line: central values of the input parameters, using the distribution amplitude (DA) [19]; dashed-dotted line: the same using the asymptotic DA; dotted line: the vector dominance approximation (VDM). The dashed lines indicate the range of the theoretical uncertainties. See also text.

The values of the nonperturbative coefficients  $a_n$  can be restricted from experimental data, most notably from the  $\gamma\gamma^*\pi$  form factor. As argued in [15], the recent measurements by CLEO [16] are consistent with the distribution being close to its asymptotic form. On the other hand, the old QCD sum rule results [17] indicate sizable corrections which, in view of the criticism raised in [18], are probably overestimated. In our analysis we use the QCD sum rule estimates  $a_2 = 0.35$  and  $a_4 = 0.18$  at the scale  $\mu = 2.4$  GeV [19, 5] as upper bounds for possible corrections to the asymptotic distributions. The resulting  $q^2$  dependence of the form factor is displayed in Fig. 1 for the two choices — asymptotic and QCD sum rule motivated — of the pion distribution amplitude. In Fig. 2 we show contributions of the asymptotic distribution and the corrections separately. It is seen that (a) the contribution of the second Gegenbauer polynomial  $\sim a_2$  can reach 20% and significantly affects the shape of the form factor as a function of  $q^2$  and that (b) with current estimates of their magnitude, the contributions of higher polynomials ( $\sim a_4$ ) are negligible. We stress that the size of these corrections will eventually be determined from experimental data.

*Higher twist corrections.* From Fig. 2 it is apparent that the twist three contribution to the sum rule is large and of the same order as the leading twist contribution. An inspection shows that this large contribution is entirely due to the asymptotic two-particle distribution amplitudes of twist three, whose normalization is fixed by the quark condensate [20]. A variation of  $\langle \bar{q}q \rangle$  within the conservative limits  $-(230 - 250 \text{ MeV})^3$  yields an uncertainty in  $f_+(0)$  of at most  $\pm 0.012$ , i.e. 4%. Corrections to the asymptotic two-particle twist 3 distribution amplitudes are related to the three-particle distribution of twist 3 and

proportional to the coupling  $f_{3\pi}$  defined by the matrix element

$$\langle 0 | \bar{d} \sigma_{\mu\alpha} \gamma_5 G^\alpha_{\nu} u | \pi^+(p) \rangle = 2i f_{3\pi} p_\mu p_\nu + \dots \quad (19)$$

The constant  $f_{3\pi}$  was estimated from QCD sum rules to be  $f_{3\pi} = 0.0026 \text{ GeV}^2 \pm 30\%$  (at the scale  $2.4 \text{ GeV}$ ) [17]. The corresponding contribution to the form factor is within 2%. Finally, the twist 4 contributions also turn out to be unimportant numerically, see Fig. 2, so that we use the full set as specified in [5] without detailed error analysis. The only essential uncertainty in the higher twist effects thus comes from the yet uncalculated radiative corrections to the twist 3 contribution: a +30% correction would increase the form factor by about 10%.

Our final result for the spectrum  $dB(B \rightarrow \pi e \nu)/dq^2$  as a function of  $q^2$  using the QCD sum rule motivated pion distribution amplitude and taking central values of the input parameters is shown in Fig. 3 (solid curve). We evaluate the light-cone sum rules for  $q^2 < 17 \text{ GeV}^2$ , where both the twist-expansion and the contribution of higher states are well under control. At higher values of  $q^2$  the decay rate is strongly suppressed by the phase space factor; for comparison, we show (dotted line) the spectrum calculated in the vector dominance (VDM) approximation in this region, with the coupling  $g_{BB^*\pi} = (29 \pm 3)$  [5]. The uncertainties are illustrated by dashed lines. The spectrum decreases monotonically with  $q^2$ , which is a consequence of the broad pion distribution amplitude of [19]. The asymptotic pion distribution produces a different shape, see the dash-dotted curve, with a maximum around  $q^2 \sim 16 - 18 \text{ GeV}^2$ . We repeat that these two distributions present two extreme possibilities and the ambiguity will eventually be removed. And vice versa, measuring the spectrum in  $B \rightarrow \pi e \nu$  decays can distinguish between different shapes of the pion distribution amplitude.

To summarize, in this letter we have calculated the radiative correction to the light-cone sum rule for the semileptonic  $B \rightarrow \pi e \nu$  form factor. We have studied its behaviour in the heavy quark limit, which, as we believe, teaches us that factorization of “hard” and “soft” subprocesses is generally valid in the heavy quark limit. From the numerical analysis of the sum rule, we obtain the form factor at  $q^2 = 0$ :

$$f_+(0) = 0.25 \pm 0.03 \pm 0.01 \pm 0.01 \quad (20)$$

assuming the asymptotic pion distribution amplitude and

$$f_+(0) = 0.30 \pm 0.03 \pm 0.01 \pm 0.01 \quad (21)$$

using the QCD sum rule motivated distribution [19]. The first error comes from the variation of the Borel parameter within  $5 \text{ GeV}^2 < M^2 < 8 \text{ GeV}^2$ , the second one from the error in the quark condensate and the third one from the combined uncertainties in  $m_b$  and  $f_{3\pi}$ . Combining all errors in quadrature and averaging over the two different leading twist distributions, we get  $f_+(0) = 0.28 \pm 0.05$  as our final result.

We also give a simple parametrization of the  $q^2$  dependence. For the asymptotic distribution we find:

$$f_+(q^2) = \frac{0.25 \pm 0.03}{1 - 1.72 q^2/m_B^2 + 0.716 q^4/m_B^4}, \quad (22)$$

and for the QCD sum rule distribution [19]:

$$f_+(q^2) = \frac{0.30 \pm 0.03}{1 - 1.32 q^2/m_B^2 + 0.208 q^4/m_B^4}. \quad (23)$$

Both representations reproduce the exact light-cone sum rule results to within 1% for  $0 < q^2 < 17 \text{ GeV}^2$ . The influence of the other input parameters on the  $q^2$ -dependence is negligible, so we do not give errors in the denominators in (22) and (23).

The total combined uncertainty of  $\pm 0.05$  can be reduced to  $\pm 0.03$  by calculating radiative corrections to the asymptotic two-particle distributions of twist 3 and from more detailed information on the pion distribution amplitude when it becomes available.

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