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Particle Physics Models of Inflation and the Cosmological Density Perturbation

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Abstract

This is a review of particle-theory models of inflation, and of their predictions for the primordial density perturbation that is thought to be the origin of structure in the Universe. It contains mini-reviews of the relevant observational cosmology, of elementary field theory and of supersymmetry, that may be of interest in their own right. The spectral index $n(k)$, specifying the scale-dependence of the spectrum of the curvature perturbation, will be a powerful discriminator between models, when it is measured by Planck with accuracy $\Delta n \sim 0.01$. The usual formula for n is derived, as well as its less familiar extension to the case of a multi-component inflaton; in both cases the key ingredient is the separate evolution of causally disconnected regions of the Universe. Primordial gravitational waves will be an even more powerful discriminator if they are observed, since most models of inflation predict that they are completely negligible. We treat in detail the new wave of models, which are firmly rooted in modern particle theory and have supersymmetry as a crucial ingredient. The review is addressed to both astrophysicists and particle physicists, and each section is fairly homogeneous regarding the assumed background knowledge.

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1 Introduction

We do not know the history of the observable Universe before the epoch of nucleosynthesis, but it is widely believed that there was an early era of cosmological inflation [202, 176, 194, 195]. During this era, the Universe was filled with a homogeneous scalar field ϕ , called the inflaton field, and essentially nothing else. The potential $V(\phi)$ dominated the energy density of the Universe, decreasing slowly with time as ϕ rolled slowly down the slope of V .

The attraction of this paradigm is that it can set the initial conditions for the subsequent hot big bang, which otherwise have to be imposed by hand. One of these is that there be no unwanted relics (particles or topological defects which survive to the present and contradict observation). Another is that the initial density parameter should have the value $\Omega = 1$ to very high accuracy, to ensure that its present value has at least roughly this value. There is also the requirement that the Universe be homogeneous and isotropic to high accuracy.

All of these virtues of inflation were noted when it was first proposed by Guth in 1981 [127],² and very soon a more dramatic one was also noticed [132, 278, 128]. Starting with a Universe which is absolutely homogeneous and isotropic at the classical level, the inflationary expansion of the Universe will ‘freeze in’ the vacuum fluctuation of the inflaton field so that it becomes an essentially classical quantity. On each comoving scale, this happens soon after horizon exit.³ Associated with this vacuum fluctuation is a primordial energy density perturbation, which survives after inflation and may be the origin of all structure in the Universe. In particular, it may be responsible for the observed cosmic microwave background (cmb) anisotropy and for the large-scale distribution of galaxies and dark matter. Inflation also generates primordial gravitational waves as a vacuum fluctuation, which may contribute to the low multipoles of the cmb anisotropy.

When it was first proposed in 1982, this remarkable paradigm received comparatively little attention. For one thing observational tests were weak, and for another the inflationary density perturbation was not the only candidate for the origin of structure. In particular, it seemed as if cosmic strings or other topological defects might do the job instead. This situation changed dramatically in 1992, when COBE measured the cmb anisotropy on large angular scales [276], and another dramatic change is now in progress with the advent of smaller scale measurement. Subject to confirmation of the latter, it seems that the paradigm of slow-roll inflation is the only one not in conflict with observation.

The inflaton field perturbation, except in contrived models, has practically zero mass and negligible interaction. As a result, the primordial density perturbation is gaussian; in other words, its fourier components $\delta_{\mathbf{k}}$ are uncorrelated and have random phases. Its spectrum $\mathcal{P}_{\mathcal{R}}(k)$, defined roughly as the expectation value of $|\delta_{\mathbf{k}}|^2$ at the epoch of horizon exit, defines all of its stochastic properties.⁴ The shape of the spectrum is conveniently

²Guth’s paper gave inflation its name, and for the first time spelled out its virtues in setting initial conditions. Earlier authors had contemplated the possibility of inflation, as reviewed comprehensively in Reference [247].

³A comoving scale a/k is said to leave the horizon when $k = aH$, where $a(t)$ is the scale factor of the Universe and $H = \dot{a}/a$ is the Hubble parameter.

⁴To be precise, $\mathcal{P}_{\mathcal{R}}$ is the spectrum of a quantity \mathcal{R} to be defined later, which is a measure of the spatial curvature seen by comoving observers.

defined by the spectral index $n(k)$, defined as

$$n(k) - 1 \equiv d \ln \mathcal{P}_{\mathcal{R}} / d \ln k, \quad (1)$$

Slow-roll inflation predicts a slowly-varying spectrum, corresponding to $|n - 1|$ significantly below 1. In some models of inflation, $n(k)$ is practically constant on cosmological scales, leading to the alternative definition

$$\mathcal{P}_{\mathcal{R}}(k) \propto k^{n-1}, \quad (2)$$

$$\mathcal{P}_{\text{grav}}(k) \propto k^{n_{\text{grav}}}. \quad (3)$$

The gravitational wave amplitude is also predicted to be gaussian, again with a primordial spectrum $\mathcal{P}_{\text{grav}}(k)$ which is slowly varying.

The spectra $\mathcal{P}_{\mathcal{R}}(k)$ and $\mathcal{P}_{\text{grav}}(k)$ provide the contact between theory and observation. The latter is negligible except in a very special class of inflationary models, and we shall learn a lot if it turns out to be detectable. For the moment, observation gives only the magnitude of $\mathcal{P}_{\mathcal{R}}(k)$ at the scale $k^{-1} \sim 10^3$ Mpc (the COBE normalization $\frac{2}{5} \mathcal{P}_{\mathcal{R}}^{1/2} = 1.91 \times 10^{-5}$) plus a bound on its scale dependence corresponding to $n = 1.0 \pm 0.2$.

The observational constraint $\mathcal{P}_{\mathcal{R}}^{1/2} \sim 10^{-5}$ was already known when inflation was proposed, and was soon seen to rule out an otherwise viable model. Since then, practically all models have been constructed with the constraint in mind, so that its power has not always been recognized; the huge class of models which it rules out have simply never been exhibited.

The situation regarding the spectral index is quite different. The present result $n(k) = 1.0 \pm 0.2$ is only mildly constraining for inflationary models, its most notable consequence being to rule out ‘extended’ inflation in all except very contrived versions. But this situation is going to improve in the foreseeable future, and after Planck flies in about ten years we shall probably know $n(k)$ to an accuracy $\Delta n \sim 0.01$. As this article demonstrates, such an accurate number will consign to the rubbish bin of history most of the proposed models of inflation.

What do we mean by a model of inflation? Before addressing the question we should be very clear about one thing. Observation, notably the COBE measurement of the cmb anisotropy, tells us that *when our Universe leaves the horizon*⁵ the potential $V(\phi)$ is far below the Planck scale. To be precise, $V^{1/4}$ is no more than a few times 10^{16} GeV, and it may be many orders of magnitude smaller. Subsequently, there are at most 60 e -folds of inflation, and only these have a directly observable effect. On the other hand, the history of our Universe begins with V presumably at the Planck scale, and to avoid fine tuning inflation should also begin then. This ‘primary inflation’, which may or may not join smoothly to the the last 60 e -folds, cannot be investigated by observation and is of comparatively little interest. It will not be treated in this review. So for us, a ‘model of inflation’ is a model of inflation that applies after the observable Universe leaves the horizon. It is a model of ‘observable’, as opposed to ‘primary’, inflation

⁵A comoving scale a/k is said to be outside the horizon when it is bigger than H^{-1} . Each scale of interest leaves the horizon at some epoch during inflation and enters it afterwards. The density perturbation on a given scale is essentially generated when it leaves the horizon. The comoving scale corresponding to the whole observable Universe (‘our Universe’) is entering the horizon at roughly the present epoch.

So what *is* meant by a ‘model of inflation’? The phrase is actually used by the community in two rather different ways. At the simplest level a ‘model of inflation’ is taken to mean a form for the potential, as a function of the fields giving a significant contribution to it. In single-field models there is just the inflaton field ϕ (defined as the one which is varying with time) whereas in hybrid inflation models most of the potential comes from a second field ψ which is fixed until the end of inflation. In both cases, one ends up by knowing $V(\phi)$, and the field value ϕ_{end} at the end of inflation. This allows one to calculate the spectrum $\mathcal{P}_{\mathcal{R}}(k)$, and in particular the spectral index $n(k)$. In some cases the prediction for $n(k)$ depends only on the shape of V , as is illustrated in the table on page 80. One can also calculate the spectrum $\mathcal{P}_{\text{grav}}$ of gravitational waves, but in most models they are too small to ever be detectable.

At a deeper level, one thinks of a ‘model of inflation’ as something analogous to the Standard Model of particle interactions. One imagines that Nature has chosen some extension of the Standard Model, and that the relevant scalar fields are part of that model. In this sense a ‘model of inflation’ is more than merely a specification of the the potential of the relevant fields. It will provide answers to at least some of the following questions. Have the relevant fields and interactions already been invoked in some other context, and if so are the parameters required for inflation compatible with what is already known? Do the relevant fields have gauge interactions? If so, are we dealing with the Standard Model interactions, GUT interactions, or interactions in a hidden sector? Is the potential the classical one, or are quantum effects important? In the latter case, are we dealing with perturbative or non-perturbative effects?

Of course, it would have been wonderful if inflation already dropped out of the Standard Model, but sadly that is not the case. Perhaps more significantly, it is not the case either for minimal supersymmetric extensions of the Standard Model.⁶

Taken in either sense, inflation model building has seen a recent renaissance. In this article, we review the present status of the subject, taking seriously present thinking about what is likely to lie beyond the Standard Model. In particular, we take seriously the idea that supersymmetry (susy) is relevant. At the fundamental level, susy is supposed to be local, corresponding to supergravity. When considering particle interactions in the vacuum, in particular predictions for what is seen at colliders and underground detectors, global susy usually provides a good approximation to supergravity. But, as we shall discuss in detail, global susy is *not* in general a valid approximation during inflation. This remains true no matter how low the energy scale, and no matter how small the field values, a fact ignored over the years by many authors.

Being only a symmetry, supersymmetry does not completely define the form of the field theory. In fact, in a supergravity theory the number of couplings that need to be specified is in principle infinite (a non-renormalizable theory). For guidance about the form of field theory, one may look to string theory. Taking it to denote the whole class of theories that give field theory as an approximation, string theory comes in many versions, but the two most widely studied are weakly coupled heterotic string theory [123, 140, 102, 66, 100, 67, 13, 141, 164] and Horava-Witten M-theory [303, 139, 244, 211]. In our present state of

⁶By ‘minimal’ we mean in this context extensions invoking only the supersymmetric partners of fermions and gauge bosons, with a reasonably simple supersymmetric extension of the Higgs sector. The simplest possible extension is called the Minimal Supersymmetric Standard Model (MSSM).

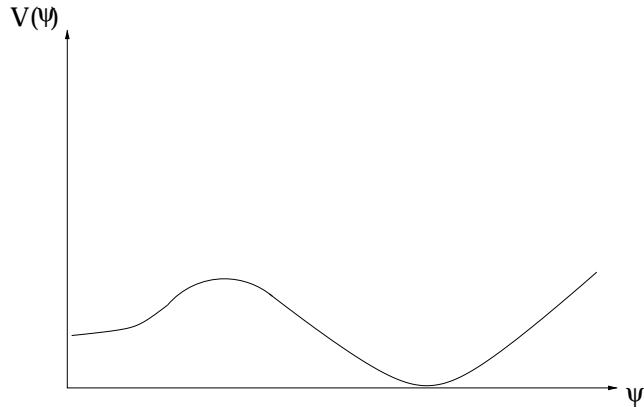


Figure 1: The Old Inflation potential

knowledge, this gives reasonably detailed information in the regime where all field values are $\ll M_{\text{P}}$, but almost nothing about the regime where some field value is $\gg M_{\text{P}}$.⁷ Accordingly, ‘models’ of inflation in the sense of forms for the potential can at present be promoted to particle-physics models only in this regime.

Let us briefly review the history. In Guth’s model of 1981 [127], some field that we shall call ψ is trapped at the origin, in a local minimum of the potential as illustrated in Figure 1. Inflation ends when it tunnels through the barrier, and descends quickly to the minimum of the potential which represents the vacuum. It was soon noted that this ‘old inflation’ is not viable because bubbles of the new phase never coalesce.

In 1982, Linde [198] and Albrecht and Steinhardt [6] proposed the first viable model of inflation, which has been the archetype for all subsequent models. Some field ϕ , which we shall call the inflaton, is slowly rolling down a rather flat potential $V(\phi)$. In the ‘new inflation’ model proposed in the above references, the potential has a maximum at the origin as in the full line of Figure 3, and inflation takes place near the maximum. It ends when ϕ starts to oscillate around the minimum, which again represents the vacuum.

The ‘new inflation’ model was a model in both senses of the word, specifying both the form of the potential and its possible origin in a Grand Unified Theory (GUT) theory of particle physics. The vacuum expectation value (vev) of the inflaton, was originally taken to be at the GUT scale. Later it was raised to the Planck scale (‘primordial inflation’) which weakened the connection with the GUT. Most of the models proposed in this first phase of particle-theory model building were very complicated, and are not usually mentioned nowadays. They were complicated because they worked under two restrictions, which have since been abandoned. First, the inflaton was required start out in thermal equilibrium (though Linde pointed out at an early stage that this is not mandatory [200]). Secondly, they worked almost exclusively with the paradigm of single-field inflation, as opposed to the hybrid inflation paradigm that we shall encounter in a moment.

While this phase of complicated model-building was getting under way, Linde proposed [199] in 1983 that instead the field might be rolling towards the origin, with a field value much bigger than M_{P} . He proposed a monomial potential, say $V \propto \phi^2$ or ϕ^4 , which was

⁷ M_{P} is the Planck mass, defined in units $\hbar = c = 1$ by $M_{\text{P}} = (8\pi G)^{-1/2} = 2.4 \times 10^{18}$ GeV.

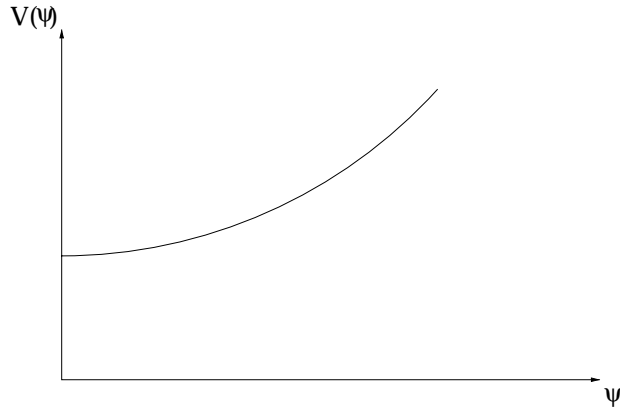


Figure 2: During hybrid inflation, the potential $V(\psi)$ is minimized at $\psi = 0$.

supposed to hold right back to the Planck epoch when $V \sim M_{\text{P}}^4$. At that epoch, ϕ was supposed to be a chaotically varying function of position.⁸ Working out the field dynamics, one finds that the observable Universe leaves the horizon when $\phi \sim 10M_{\text{P}}$, and inflation ends when $\phi \sim M_{\text{P}}$. Such big field values make it practically impossible to make a connection with particle physics. After some years, the monomial paradigm became the favoured one, and the search for a connection with particle physics was largely abandoned.

The seeds for the present renaissance of model-building were laid around 1990. First, in 1989, La and Steinhardt proposed what they called ‘extended inflation’ [182]. Its objective was to implement ‘old’ inflation by providing a mechanism for making the bubbles coalesce at the end of inflation. This mechanism was simply to add a slowly-rolling inflaton field ϕ to the original new inflation model, which makes the Hubble parameter decrease significantly with time. It invoked the extension of gravity known as Brans-Dicke theory, and for this reason it was called extended inflation. The original version conflicted with present-day tests of General Relativity, but more complicated versions were soon constructed that avoided this problem. This paradigm was practically killed in 1992, by the COBE detection [276] the cmb anisotropy. There was no sign of the bubbles formed at the end of inflation, yet all except very contrived versions of the paradigm required that there should be.

Going back to the historical development, it was known that like many extended gravity theories, extended inflation can be re-formulated as an Einstein gravity theory. Working from the beginning with Einstein gravity, Linde [203] and Adams and Freese [1] proposed in 1991 a crucial change in the idea behind extended inflation; until the end of inflation, tunneling is completely impossible (not just relatively unlikely) because the trapped field ψ has a coupling to the slowly-rolling inflaton field ϕ . During inflation, this changes the potential of the trapped field so that it becomes like the one shown in Figure 2. Only at the end of inflation is the final form of Figure 1 achieved, and only then does tunnelling take place. In this model, the bubbles can coalesce very quickly, and be completely invisible in the microwave background as required by observation.

⁸For this reason, inflation with such a potential is usually called ‘chaotic inflation’. We shall use the phrase ‘monomial inflation’, because the hypothesis of chaotic initial conditions has no necessary connection with the form of the potential during observable inflation. A wide variety of other monotonically increasing functions will also inflate at $\phi \gg M_{\text{P}}$, but they are seldom considered because there is too much freedom.

The logical end is perhaps not hard to guess; in 1991, Linde [204] dispensed with the bubbles altogether, by eliminating the dip of the potential at the origin. At the end of inflation, the field ψ now reverts to its vacuum value without any bubble formation, so that there is a second-order phase transition, instead of the first-order one of the original model. This final paradigm is known as hybrid inflation. It has led to the renaissance of inflation model building, firmly rooted in the concepts of modern particle theory, which is the focus of the present review.

The actual beginning of the renaissance can be traced to a paper in 1994 [60]. It contained the crucial observation that during hybrid inflation, the inflaton field ϕ is typically much less than M_{P} . As a result, contact with particle theory again becomes a realistic possibility. At the same time though, the above paper emphasized that a generic supergravity theory will fail to inflate no matter how small are the field values, because the inflaton mass is too big. Much effort has since been devoted to finding ways around this problem.

In addition to the small field value, hybrid inflation has another good feature. In single-field models the curve $V(\phi)$ must first support inflation, and then cease to support it so that inflation ends. There are only a few simple functions that achieve this, if one excludes field values much bigger than M_{P} . In the hybrid case, the job of ending inflation is done by the other field ψ , which greatly increases the range of simple possibilities.

We end this introduction with an overview of the present article, and a list of its omissions. The article is addressed to a wide audience, including both cosmologists and particle physicists. To cope with this problem, we have tried to make each section reasonably homogeneous regarding the background knowledge that is taken for granted, while at the same time allowing considerable variation from one section to another. Section 2 focusses on the cosmological quantities, that form a link between a model of inflation and observation. Section 3 gives the basics of the slow-roll paradigm of inflation, showing how the cosmological quantities are calculated. Section 4 is a specialized one, explaining how to derive the usual prediction of slow-roll inflation, and how to generalize it to the case of a multi-component inflaton. Section 5 summarizes some of the basic ideas of modern particle theory, which have been used in inflation model-building. Those with a background in particle theory will skip through it fairly quickly. Using these ideas, Section 6 reviews ‘models’ of inflation, taken to mean forms for the potential that have the general form suggested by particle theory.

Section 7 summarizes those aspects of supersymmetry which are most relevant for inflation model-building. It is addressed mainly to those who already have some understanding of that subject. As we explain there, the tree-level potential in a supersymmetric theory is the sum of an ‘ F -term’ and a ‘ D -term’. The terms have very different properties and in all models of inflation so far proposed one or other dominates. Section 8 deals with models of inflation where the F -term dominates, and Section 9 with those where the D -term dominates. We conclude in Section 10.

The above list of topics is formidable, but still not exhaustive. Let us mention the main omissions.

While the paradigm of slow-roll inflation is broadly necessary, in order to account for the near scale-independence of the primordial spectrum $\mathcal{P}_{\mathcal{R}}(k)$, brief interruptions of slow-roll are sometimes contemplated. So are sharp changes in the direction of slow-roll, of the kind described in Section 4. In both cases, the effect is to generate a sharp feature, in the

otherwise smooth primordial spectrum. At the time of writing there is no firm observational evidence for such a feature, and we mention only briefly the models that would predict one.

We shall not discuss the pre-big-bang idea, that a bounce at the Planck scale can do the job of inflation. In contrast with inflation, this paradigm provides no natural explanation of the near scale-independence of the spectrum of the primordial curvature perturbation, encoded by the result $n \simeq 1$. In slow-roll inflation this result is an automatic consequence of the near time-independence of the Hubble parameter, but no analogous quantity appears in the pre-big-bang paradigm.

Globally supersymmetric models using complicated particle physics, in particular a Grand Unified Theory (GUT) are not mentioned much. Like some simpler models that we do mention, these models usually lack any specific mechanism for controlling the supergravity corrections. Except for a brief mention of monomial potentials, models invoking field values much bigger than M_{P} are not mentioned. (All known models involving non-Einstein gravity are of this type.)

We do not consider the rather special inflationary potentials that can give an open Universe (negative spatial curvature) through bubble formation [118, 46, 206, 209], since little attention has so far been paid to these in the context of particle theory. We are basically focussing on the usual case, that Ω has been driven to 1 long before our Universe leaves the horizon during inflation, making its present value (including the contribution of any cosmological constant) also 1. However, most of what we do continues to apply if that is not the case, which arguably might happen for any form of the inflationary potential.⁹

We assume that the primordial density perturbation generated by the vacuum fluctuation of the inflaton is solely responsible for large scale structure, except possibly for a gravitational wave signal in the cmb anisotropy. This means that we ignore anything coming from topological defects, as well as the isocurvature density perturbation that could in principle be generated by the vacuum fluctuation of a non-inflaton field like the axion. Subject to confirmation from further observations, it looks as though such things cannot be entirely responsible for large scale structure, so indeed the simplest thing is to assume that they are entirely absent.

Finally, we are considering only models of *inflation*, not of the subsequent cosmology. In particular, we are not considering the reheating process by which the scalar field is converted into hot radiation. We are not considering the preheating process that might exchange energy between scalar fields before reheating [171, 172, 153, 11, 177, 154, 155, 156, 173, 174]. And we are definitely not considering baryogenesis, dark matter, or unwanted relics such as moduli. All of these phenomena are likely to involve fields, and interactions, that play no role during inflation. We generally set $\hbar = c = 1$, and we define the Planck mass by $M_{\text{P}} = (8\pi G)^{-1/2} = 2.4 \times 10^{18} \text{ GeV}$.

2 Observing the density perturbation (and gravitational waves?)

The vacuum fluctuation of the inflaton field generates a primordial energy density perturbation, and the vacuum fluctuation in the transverse traceless part of the metric generates

⁹With any potential, one can assume that Ω is fine-tuned to be small at the Planck scale [221], or else that the Universe is created at a finely-tuned point in field space [133, 207, 295, 42]. As usual, one can consider eliminating such fine-tuning by the anthropic principle.

gravitational waves. In this section we explain briefly how the primordial density perturbation, and the gravitational waves, are related to what is actually observed.

2.1 The primordial quantities

In the unperturbed Universe, the separation of comoving observers¹⁰ is proportional to the scale factor of the Universe $a(t)$, and we normalize it to 1 at the present epoch. The Hubble parameter is $H = \dot{a}/a$, and its present value $H_0 = 100h \text{ km s}^{-1} \text{ Mpc}^{-1}$ with h probably in the range 0.5 to 0.7. The corresponding Hubble distance is $cH_0^{-1} = 3000h^{-1} \text{ Mpc}$, which is roughly the size of the observable Universe.¹¹

Instead of the physical Cartesian coordinates \mathbf{r} it is more convenient to use coordinates \mathbf{x} such that $\mathbf{r} = a(t)\mathbf{x}$. Then the coordinate position of a comoving observer is time-independent, in the unperturbed Universe. The Fourier expansion of a perturbation $g(\mathbf{x}, t)$ is made inside a large comoving box, whose coordinate size L should be a few orders of magnitude bigger than that of the observable Universe. (On bigger scales it would not be justified to assume a homogeneous, isotropic universe.) The Fourier expansion is

$$g(\mathbf{x}, t) = \sum_{\mathbf{k}} g_{\mathbf{k}} e^{i\mathbf{k}\cdot\mathbf{x}}. \quad (4)$$

For mathematical purposes it is convenient to consider the limit of an infinite box,

$$g(\mathbf{x}, t) = \int d^3\mathbf{k} g(\mathbf{k}, t) e^{i\mathbf{k}\cdot\mathbf{x}}. \quad (5)$$

where $(L/2\pi)^3 g_{\mathbf{k}} \rightarrow (2\pi)^{-3/2} g(\mathbf{k})$. A useful way of specifying the physical wavenumber k/a is to give its present value k .

During inflation, aH increases with time, and a comoving scale a/k is said to leave the horizon when $aH/k = 1$. After inflation aH decreases, and the comoving scale is said to enter the horizon when $aH/k = 1$. For cosmologically interesting scales, horizon entry occurs long after nucleosynthesis. We shall occasionally refer to the long era between horizon exit and horizon entry as the *primordial* era. As we shall see, the evolution of the perturbations during the primordial era is simple, because causal processes cannot operate.

Leaving aside gravitational waves, there is only one independent primordial perturbation, because everything is generated from the vacuum fluctuation of the inflaton field. (We are considering the usual case of a single-component inflaton field.) Instead of the inflaton

¹⁰Both in the perturbed and unperturbed Universe, a comoving observer is defined as one moving with the flow of energy. Such observers measure zero momentum density at their own positions.

¹¹In the following we shorten ‘observable Universe’ to ‘Universe’. The unknown regions outside it are referred to as the ‘universe’ with capitalization.

field perturbation, it is actually more conveniently to consider a quantity $\mathcal{R}(\mathbf{k})$, defined by¹²

$$\mathcal{R}(\mathbf{k}) = \frac{1}{4}(a/k)^2 R^{(3)}(\mathbf{k}), \quad (6)$$

where $R^{(3)}$ is the spatial curvature scalar seen by comoving observers. Unlike the inflaton field perturbation, it is time-independent during the primeval era, and it continues to be well-defined after the inflaton field disappears.

A gravitational wave corresponds to a spatial metric perturbation h_{ij} which is traceless, $\delta^{ij}h_{ij} = 0$, and transverse, $\partial_i h_{ij} = 0$. This means that each Fourier component is of the form

$$h_{ij} = h_+ e_{ij}^+ + h_\times e_{ij}^\times. \quad (7)$$

In a coordinate system where \mathbf{k} points along the z -axis, the nonzero components of the polarization tensors are defined by $e_{xx}^+ = -e_{yy}^+ = 1$ and $e_{xy}^\times = e_{yx}^\times = 1$. The two independent quantities $h_{+,\times}$ are time-independent well outside the horizon.

Inflation generates gaussian fluctuations.¹³ This means that for each perturbation $g(\mathbf{x})$, at fixed t , the Fourier components are uncorrelated except for the expectation values

$$\langle g^*(\mathbf{k})g(\mathbf{k}') \rangle = \delta^3(\mathbf{k} - \mathbf{k}') \frac{2\pi^2}{k^3} \mathcal{P}_g(k). \quad (8)$$

The quantity $\mathcal{P}_g(k)$ is called the spectrum of $g(\mathbf{x})$, and it determines all of its stochastic properties.

The primordial perturbations consist of the three independent quantities \mathcal{R} , h_+ and h_\times , and from rotational invariance the last two have the same spectrum,

$$\mathcal{P}_{h_+} = \mathcal{P}_{h_\times} \equiv \mathcal{P}_{\text{grav}}/2. \quad (9)$$

We therefore have two independent spectra $\mathcal{P}_{\mathcal{R}}$ and $\mathcal{P}_{\text{grav}}$, determined in a slow-roll model of inflation by the formulas described in the next section. We shall see that they have at most mild scale dependence, and this is consistent with observation.

The spectral index $n(k)$ of the curvature perturbation (Eq. (1)) is a crucial point of comparison between theory and observation, and the same will be true of $n_{\text{grav}}(k)$ if the gravitational waves are detectable.

¹²The quantity we are calling \mathcal{R} was defined first by Bardeen Ref. [22], who called it ϕ_m . It was called \mathcal{R}_m by Kodama and Sasaki Ref. [165], and we drop the subscript following [221, 193, 287]. Later it was called ζ by Mukhanov et al [236], which is the other commonly used notation at present. It is a factor $\frac{3}{2}k^{-2}$ times the quantity δK of Ref. [212, 213]. On the scales far outside the horizon where it is constant (the only regime where it is of interest) it coincides with the quantity $\xi/3$ of Ref. [23] and the quantity ξ of Ref. [268]. On these scales, it is also equal to $-C\Phi$, where Φ is the commonly used 'gauge invariant potential' [22], and C is a factor of order unity which is constant both during radiation domination and during matter domination. During the latter epoch, $C = 5/3$.

¹³To be more precise, the gravitational wave amplitude is certainly gaussian, and so is the curvature perturbation if the inflaton field fluctuation $\delta\phi$ is gaussian. The latter is true if $\delta\phi$ is a practically free field, which is the case in practically all models of inflation. The gaussianity is inherited by all of the perturbations as long as they remain small.

2.2 The observable quantities

From these primordial quantities, one can calculate the observable quantities, provided that one knows enough about the nature and evolution of the unperturbed Universe after relevant scales enter the horizon. Since cosmological scales enter the horizon well after nucleosynthesis, one indeed has the necessary information, up to uncertainties in the Hubble parameter, the nature and amount of dark matter, the epoch of reionization and the magnitude of the cosmological constant.¹⁴ The observed quantities can be taken to be the matter density contrast $\delta \equiv \delta\rho/\rho$ (observed through the distribution and motion of the galaxies), and the cmb anisotropy. The latter consists of the temperature anisotropy $\Delta T/T$, which is already being observed, and two Stokes parameters describing the polarization which will be observed by the MAP [226] and Planck [253] satellites. It is convenient to make multipole expansions so that one is dealing with the temperature anisotropy $a_{\ell m}$, and the polarization anisotropies $E_{\ell m}$ and $B_{\ell m}$.¹⁵

Except for the density perturbation on scales where gravitational collapse has taken place, the observable quantities are related to the primordial ones through linear, rotationally invariant, transfer functions. For the density perturbation,

$$\delta(\mathbf{k}, t) = \mathcal{T}(k, t)\mathcal{R}(\mathbf{k}). \quad (10)$$

It can be observed both at the present, and (by looking out to large distances) at earlier times. The corresponding spectrum is

$$\mathcal{P}_\delta(k, t) = \mathcal{T}^2(k, t)\mathcal{P}_\mathcal{R}(k). \quad (11)$$

For the cmb anisotropy, ignoring the gravitational waves, one has

$$a_{\ell m} = \frac{4\pi}{(2\pi)^{3/2}} \int \mathcal{T}_\Theta(k, \ell) \mathcal{R}_{\ell m}(k) k dk, \quad (12)$$

$$E_{\ell m} = \frac{4\pi}{(2\pi)^{3/2}} \int \mathcal{T}_E(k, \ell) \mathcal{R}_{\ell m}(k) k dk, \quad (13)$$

$$B_{\ell m} = 0. \quad (14)$$

Here, the multipoles of \mathcal{R} are related to its Fourier components by

$$\mathcal{R}_{\ell m}(k) = k i^\ell \int \mathcal{R}(k, \hat{\mathbf{k}}) Y_{\ell m}(\hat{\mathbf{k}}) d\Omega_{\mathbf{k}}. \quad (15)$$

which is equivalent to the usual spherical expansion. They are uncorrelated except for the expectation values

$$\langle g_{\ell m}^*(k) g_{\ell' m'}(k') \rangle = \frac{2\pi^2}{k^3} \mathcal{P}_g(k) \delta(k - k') \delta_{\ell\ell'} \delta_{mm'}. \quad (16)$$

¹⁴In principle the reionization epoch can be calculated in terms of the other parameters, through the abundance of early rare objects, but present estimates are fairly crude.

¹⁵The polarization multipoles are defined with respect to spin-weighted spherical harmonics, to ensure the correct transformation of the Stokes parameters under rotation about the line of sight.

As a result, the multipoles of the cmb anisotropy are uncorrelated, except for the expectation values

$$\langle a_{\ell m}^* a_{\ell' m'} \rangle = C(\ell) \delta_{\ell\ell'} \delta_{mm'}, \quad (17)$$

$$\langle a_{\ell m}^* E_{\ell' m'} \rangle = C_{\text{cross}}(\ell) \delta_{\ell\ell'} \delta_{mm'}, \quad (18)$$

$$\langle E_{\ell m}^* E_{\ell' m'} \rangle = C_E(\ell) \delta_{\ell\ell'} \delta_{mm'}. \quad (19)$$

where

$$C(\ell) = 4\pi \int_0^\infty \mathcal{T}_\Theta^2(k, \ell) \mathcal{P}_\mathcal{R}(k) \frac{dk}{k}, \quad (20)$$

$$C_{\text{cross}}(\ell) = 4\pi \int_0^\infty \mathcal{T}_\Theta(k, \ell) \mathcal{T}_E(k, \ell) \mathcal{P}_\mathcal{R}(k) \frac{dk}{k}, \quad (21)$$

$$C_E(\ell) = 4\pi \int_0^\infty \mathcal{T}_E^2(k, \ell) \mathcal{P}_\mathcal{R}(k) \frac{dk}{k}. \quad (22)$$

The gravitational waves give contributions to the C 's which have a similar form, now with a nonzero C_B defined analogously to C_E .¹⁶ We shall not give their precise form, but note for future reference that they fall off rapidly above $\ell \sim 100$. The reason is that larger ℓ correspond to scales smaller than the horizon at photon decoupling; on such scales the amplitude of the gravitational waves has been reduced from its primordial value by the redshift.

We should comment on the meaning of the ‘expectation value’, denoted by $\langle \dots \rangle$. At the fundamental level, it denotes the quantum expectation value, in the state that corresponds to the vacuum during inflation. This state does not correspond to a definite perturbation $g(\mathbf{x})$ (because it does not correspond to definite $g(\mathbf{k})$), so it is a superposition of possible universes. As usual, this Schrödinger’s cat paradox does not prevent us from comparing with observation. We simply make the hypothesis that our Universe is a typical one, of the superposition defined by the quantum state. Except for the low multipoles of the cmb anisotropy, this makes observational quantities sharply defined, since they involve a sum over the practically continuous variables \mathbf{k} and ℓ . For the low multipoles the expected difference between the observed $|a_{\ell m}|^2$ and $\langle |a_{\ell m}|^2 \rangle$ (called cosmic variance) needs to be taken into account, but the hypothesis that we live in a typical universe is still a very powerful one.

For the density perturbation, the comparison of the above prediction with observation has been a major industry for many years. Since 1992 the same has been true of the cmb temperature anisotropy. Perhaps surprisingly, the result of all this effort is easy to summarize.

Observation is consistent with the inflationary prediction that the curvature perturbation is gaussian, with a smooth spectrum. The spectrum is accurately measured by COBE at the scale $k \simeq 7.5H_0$ (more or less the center of the range explored by COBE). Assuming that gravitational waves are negligible, it is [47]

$$\delta_H \equiv (2/5) \mathcal{P}_\mathcal{R}^{1/2} = 1.91 \times 10^{-5}. \quad (23)$$

¹⁶There is no cross term involving $B_{\ell m}$ because it would be odd under the parity transformation. (The vacuum state is parity invariant, and so is the Thompson scattering process responsible for the polarization.)

with an estimated 9% uncertainty at the $1\text{-}\sigma$ level. In writing this expression, we introduced the quantity δ_H which is normally used by observers.

Assuming that the spectral index is roughly constant over cosmological scales, observation constrains it to something like the range [195]

$$n = 1.0 \pm 0.2. \quad (24)$$

Gravitational waves have not so far been seen in the cmb anisotropy (or anywhere else). Observation is consistent with the hypothesis that they account for a significant fraction (less than 50% or so) of the mean-square cmb multipoles at $\ell \lesssim 100$. In quantifying their effect, it is useful to consider the quantity r defined in the next section. Up to a numerical factor it is $\mathcal{P}_{\text{grav}}/\mathcal{P}_{\mathcal{R}}$, and the factor is chosen so that in an analytic approximation due to Starobinsky [280],

$$r = C_{\text{grav}}(\ell)/C_{\mathcal{R}}(\ell) \quad (25)$$

for ℓ in the central COBE range.¹⁷ (Here $C_{\mathcal{R}}$ is the contribution of the curvature perturbation given by Eq. (20), and C_{grav} the contribution of gravitational waves.) We are saying that present observations require $r \lesssim 1$ or so. According to an accurate calculation [47], the relative contribution of gravitational waves to the COBE anisotropy is actually $0.75r$, reducing the deduced value of δ_H by a factor $\simeq (1 + 0.75r)^{-1/2}$ compared with Eq. (23).

What about the future? The magnificent COBE normalization will perhaps never to be improved, but this hardly matters since at present an understanding of even its order of magnitude is a major theoretical challenge. Much more interesting is the situation with the spectral index. The Planck satellite will probably measure $n(k)$ with an accuracy of order $\Delta n \sim 0.01$, which as already mentioned will be a powerful discriminator between models of inflation. The same satellite will also either tighten the limit on gravitational waves to $r \lesssim 0.1$, or detect them. This last figure is unlikely to be improved by more than an order of magnitude in the foreseeable future.

The Planck satellite probes a range $\Delta \ln k \simeq 6$, and will measure the scale-dependence $dn/d \ln k$ if it is bigger than a few times 10^{-3} .

We have emphasized the cmb anisotropy because of the promised high accuracy, but it will never be the whole story. It can directly probe only the scales $10 \text{ Mpc} \lesssim k^{-1} \lesssim 10^4 \text{ Mpc}$, where the upper limit is the size of the observable Universe, and the lower limit is the thickness of the last-scattering ‘surface’. At present it probes only the upper half of this range, $100 \text{ Mpc} \lesssim k^{-1} \lesssim 10^4 \text{ Mpc}$.¹⁸ Galaxy surveys probe the range $1 \text{ Mpc} \lesssim k^{-1} \lesssim 100 \text{ Mpc}$, providing a useful overlap in the future. The range $1 \text{ Mpc} \lesssim k^{-1} \lesssim 10^4 \text{ Mpc}$ is usually taken to be the range of ‘cosmological’ scales. If a signal of early reionization is seen in the cmb anisotropy, it will provide an estimate of the spectrum on a significantly smaller scale, $k^{-1} \sim 10^{-2} \text{ Mpc}$. Alternatively, the absence of a signal will provide a rough upper limit on this scale.

On smaller scales still, information on the spectrum of the primordial density perturbation is sparse, and consists entirely of upper limits. The most useful limit, from the

¹⁷A common alternative is to define r by setting $\ell = 2$ in Starobinsky’s calculation. This increases r by a factor 1.118 compared with the above definition.

¹⁸A very limited constraint is provided on much bigger scales through the Grishchuk-Zeldovich [126, 110] effect, which we shall not discuss.

viewpoint of constraining models of inflation, is the one on the smallest relevant scale which is the one leaving the horizon just before the end of inflation. It has been considered in Refs. [49, 262, 122], and for a scale-independent spectral index corresponds to $n \lesssim 1.3$.

3 The slow-roll paradigm

Inflation is defined as an era of repulsive gravity, $\ddot{a} > 0$, which is equivalent to $3P < -\rho$ where ρ is the energy density and P is the pressure. As noted earlier, we are concerned only with the era of ‘observable inflation’, which begins when the observable Universe leaves the horizon, since memory of any earlier epochs has been wiped out.

During inflation the density parameter Ω is driven towards 1. Subsequently it moves away from 1, and its present value is equal to its value at the beginning of observable inflation. We are taking that value to be close to 1, which means that Ω is close to 1 during observable inflation. This gives the energy density ρ in terms of the Hubble parameter,

$$3M_{\text{P}}^2 H^2 = \rho. \quad (26)$$

During observable inflation, the energy density and pressure are supposed to be dominated by scalar fields. Of the fields that contribute significantly to the potential, the inflaton field ϕ is by definition the only one with significant time-dependence, leading to

$$\rho = \frac{1}{2}\dot{\phi}^2 + V \quad (27)$$

$$P = \frac{1}{2}\dot{\phi}^2 - V. \quad (28)$$

(We make the usual assumption that ϕ has only one component, deferring the general case to Section 4.)

The evolution of ϕ is given by

$$\ddot{\phi} + 3H\dot{\phi} = -V', \quad (29)$$

where an overdot denotes d/dt and a prime denotes $d/d\phi$. This is equivalent to the continuity equation $\dot{\rho} = -3H(\rho + P)$, which with Eq. (26) is equivalent to

$$\dot{H} = -\frac{1}{2}\dot{\phi}^2/M_{\text{P}}^2. \quad (30)$$

3.1 The slowly rolling inflaton field

While cosmological scales are leaving the horizon, the slow-roll paradigm of inflation [176, 202, 194, 195] is practically mandatory in order to account for the near scale-invariance of spectrum of the primordial curvature perturbation.

The inflaton field ϕ is supposed to be on a region of the potential which satisfies the flatness conditions

$$\epsilon \ll 1 \quad (31)$$

$$|\eta| \ll 1, \quad (32)$$

where

$$\epsilon \equiv \frac{1}{2}M_{\text{P}}^2(V'/V)^2, \quad (33)$$

$$\eta \equiv M_{\text{P}}^2V''/V. \quad (34)$$

Also, it is supposed that the exact evolution Eq. (29) can be replaced by the slow-roll approximation

$$\dot{\phi} = -\frac{V'}{3H}. \quad (35)$$

The flatness conditions and the slow-roll approximation are the basic equations, needed to derive the standard prediction for the density perturbation and the spectral index. For potentials satisfying the flatness conditions, the slow-roll approximation is typically valid for a wide range of initial conditions (values of ϕ and $\dot{\phi}$ at an early time).

The first flatness condition $\epsilon \ll 1$ ensures that ρ is close to V and is slowly varying.¹⁹ As a result H is slowly varying, which implies that one can write $a \propto e^{Ht}$ at least over a Hubble time or so.

The second flatness condition $|\eta| \ll 1$ is actually a consequence of the first flatness condition plus the slow-roll approximation $3H\dot{\phi} = -V'$. Indeed, differentiating the latter one finds

$$\frac{\ddot{\phi}}{H\dot{\phi}} = \epsilon - \eta, \quad (36)$$

and from Eq. (29) the slow-roll approximation is equivalent to $|\ddot{\phi}| \ll H|\dot{\phi}|$.

A crucial role is played by the number of Hubble times $N(\phi)$ of inflation, still remaining when ϕ has a given value. From some time t to a fixed later time t_2 , the number of Hubble times is

$$N(t) \equiv \int_t^{t_2} H(t) dt. \quad (37)$$

The small change satisfies

$$dN \equiv -H dt (= -d \ln a). \quad (38)$$

During slow-roll inflation,

$$\frac{dN}{d\phi} = -\frac{H}{\dot{\phi}} = \frac{V}{M_{\text{P}}^2V'} \left(= \pm \left(\sqrt{2\epsilon} M_{\text{P}} \right)^{-1} \right). \quad (39)$$

The number of e -folds of slow-roll inflation, remaining at a given epoch, is

$$N(\phi) = \int_{\phi_{\text{end}}}^{\phi} M_{\text{P}}^{-2} \frac{V}{V'} d\phi, \quad (40)$$

where ϕ_{end} marks the end of slow-roll inflation.

¹⁹In what follows, we say that a function of time satisfying $|d \ln f / d \ln a| \ll 1$ is ‘slowly varying’. For a function of wavenumber k , ‘slowly varying’ will mean the same thing with a replaced by k .

3.2 The slow-roll predictions

In this subsection and the next, as well as in Section 4, we discuss predictions for $\mathcal{P}_{\mathcal{R}}$, n and $dn/d\ln k$. More material can be found in references [193, 194, 287, 195].

Two basic assumptions are made. One is that the inflaton field perturbation $\delta\phi$ has negligible interaction with other fields. This is equivalent to the validity during inflation of linear cosmological perturbation theory, in other words to the procedure of keeping only terms that are linear in the perturbations [195].

The other essential assumption is that well before horizon exit, when the particle concept makes sense, the relevant Fourier modes of $\delta\phi$ have zero occupation number. This vacuum assumption is more or less mandatory, since too many particles would give significant pressure and spoil inflation [193].

As a result of these assumptions, the primordial curvature perturbation is gaussian, with stochastic properties that are completely defined by its spectrum $\mathcal{P}_{\mathcal{R}}(k)$.

In this subsection, we make the usual assumption that the slow-roll paradigm is valid.

3.2.1 The spectrum

The perturbation $\delta\phi$ is best defined on spatially flat hypersurfaces. Then, in the slow-roll limit $\dot{H} \rightarrow 0$, one can ignore the effect of the metric perturbation [194, 195], and $\delta\phi$ satisfies

$$(\delta\phi)'' + 3H(\delta\phi)' + \left[V'' + \left(\frac{k}{a} \right)^2 \right] \delta\phi = 0. \quad (41)$$

The flatness condition (32) ensures that the mass-squared $2V''$ is negligible until at least a few Hubble times after horizon exit. This means that $\delta\phi$ can be treated as a massless free field. A few Hubble times after horizon exit, its vacuum fluctuation can be regarded as a classical quantity, and its spectrum is then

$$\mathcal{P}_{\phi} = (H/2\pi)^2. \quad (42)$$

The corresponding curvature perturbation is given by $\mathcal{R} = (-H/\dot{\phi})\delta\phi$ (valid in linear perturbation theory independently of slow-roll). Using Eqs. (35) and (31), this is equivalent to

$$\frac{4}{25}\mathcal{P}_{\mathcal{R}}(k) \equiv \delta_H^2(k) = \frac{1}{75\pi^2 M_{\text{P}}^6} \frac{V^3}{V'^2} = \frac{1}{150\pi^2 M_{\text{P}}^4} \frac{V}{\epsilon}. \quad (43)$$

In this expression, the potential and its derivative are evaluated at the epoch of horizon exit for the scale k , which is defined by $k = aH$.²⁰

This prediction, for the spectrum of $\mathcal{R}(k)$ a few Hubble times after horizon exit, is of no use as it stands. But one can show that $\mathcal{R}(k)$ is time-independent between that epoch and the approach of horizon entry long after inflation ends. As we saw in Section 2, this allows one to calculate observable quantities.

²⁰Eq. (43) becomes valid only a few Hubble times after horizon exit, but its right hand side is slowly varying and we might as well evaluate it actually *at* horizon exit. The difference this makes is of the same order as the error in Eq. (43).

Comparing Eq. (43) with the value Eq. (23) deduced from the COBE observation of the cmb anisotropy gives²¹

$$M_{\text{P}}^{-3}V^{3/2}/V' = 5.3 \times 10^{-4}. \quad (44)$$

This relation provides a useful constraint on the parameters of the potential. It can be written in the equivalent form

$$V^{1/4}/\epsilon^{1/4} = .027M_{\text{P}} = 6.7 \times 10^{16} \text{ GeV}. \quad (45)$$

Since ϵ is much less than 1, the inflationary energy scale $V^{1/4}$ is at least a couple of orders of magnitude below the Planck scale [212].

The scale leaving the horizon at a given epoch is directly related to the number $N(\phi)$ of e -folds of slow-roll inflation, that occur after the epoch of horizon exit. Indeed, since H is slowly varying we have $d \ln k = d(\ln(aH)) \simeq d \ln a = H dt$. From the definition Eq. (38) this gives

$$d \ln k = -dN(\phi), \quad (46)$$

and therefore

$$\ln(k_{\text{end}}/k) = N(\phi), \quad (47)$$

where k_{end} is the scale leaving the horizon at the end of slow-roll inflation. As we shall see, this relation is very useful when working out the prediction for a given form of the potential.

This is a good place to insert a historical footnote, about the origin of the slow-roll prediction for $\mathcal{P}_{\mathcal{R}}$. As we noted already, it comes in two parts. One is the formula Eq. (43) for $\mathcal{P}_{\mathcal{R}}$ a few Hubble times after horizon exit, and the other is the statement that \mathcal{R} (hence $\mathcal{P}_{\mathcal{R}}$) is time-independent while k is well outside the horizon.

Both parts were, in essence, given at about the same time in Refs. [132, 278, 128, 23]. (Related work [233] had been done earlier.) To be precise, these authors gave results which become more or less equivalent after the spectrum has been defined, though that last step was not explicitly made and except for the last work only a particular potential is discussed.²² Soon afterwards the results were given again, this time with an explicitly defined spectrum [212].

Strictly speaking none of these five derivations is completely satisfactory. The first three make simplifying assumptions. Regarding the constancy of \mathcal{R} , all except the third assume something equivalent to it without adequate proof. We discuss the constancy of \mathcal{R} in Section 4. Regarding Eq. (43), none of these early derivations properly considers the effect of the inflaton field perturbation on the metric, but as we noted already that turns out to be negligible.

²¹This relation ignores any gravitational wave contribution, but there is no point in including their effect in the present context. The reason is that the prediction for δ_H that is being used has an error of at least the same order. If necessary one could include [287, 197] the effect of the gravitational waves using the more accurate formula Eq. (78).

²²The last three works give results equivalent to the one we quote, and the first gives a result which is approximately the same.

3.2.2 The spectral index

We have an expression for $\mathcal{P}_{\mathcal{R}}(k)$ in terms of V and V' , and we want to calculate the spectral index defined by $n(k) - 1 \equiv d\mathcal{P}_{\mathcal{R}}/d\ln k$. From Eqs. (39) and (46),

$$\frac{d\phi}{d\ln k} = -M_{\text{P}}^2 \frac{V'}{V}, \quad (48)$$

where, as always, $k = aH$. We shall need the following expressions

$$\frac{d\epsilon}{d\ln k} = 2\epsilon\eta - 4\epsilon^2 \quad (49)$$

$$\frac{d\eta}{d\ln k} = -2\epsilon\eta + \xi^2 \quad (50)$$

$$\frac{d\xi^2}{d\ln k} = -2\epsilon\xi^2 + \eta\xi^2 + \sigma^3, \quad (51)$$

where

$$\xi^2 \equiv M_{\text{P}}^4 \frac{V'(d^3V/d\phi^3)}{V^2} \quad (52)$$

$$\sigma^3 \equiv M_{\text{P}}^6 \frac{V'^2(d^4V/d\phi^4)}{V^3}. \quad (53)$$

Following for instance [28], we have introduced respectively the square and the cube of a quantity, even though the quantity itself never appears in an equation. As we shall see, this is a convenient device. Also, in the case $V' \propto \phi^p$, with $p \neq 1$ or 2 , one has $|\eta| \sim |\xi| \sim |\sigma|$. The hierarchy can be continued [28], each new equation introducing a new quantity $M_{\text{P}}^{2n} V^{n-1} (d^{n+1}V/d\phi^{n+1})$.

Using Eqs. (49) and (43) one finds [193, 64, 269]

$$n - 1 = -6\epsilon + 2\eta, \quad (54)$$

and using Eqs. (49) and (50) [180],

$$\frac{dn}{d\ln k} = -16\epsilon\eta + 24\epsilon^2 + 2\xi^2. \quad (55)$$

Practically all models proposed so far (Section 6) have $V' \propto \phi^p$ or $V' \propto \phi^p \ln \phi$, and in most cases one also has $\phi \ll M_{\text{P}}$. Then $\epsilon \sim (\phi/M_{\text{P}})|\eta|$ is negligible, and one can write

$$n - 1 = 2\eta, \quad (56)$$

$$\frac{dn}{d\ln k} = 2\xi^2. \quad (57)$$

More generally, one can argue that ϵ is small irrespectively of the form of the potential, provided that $\phi \ll M_{\text{P}}$. To see this, take the cosmological range of scales to span four decades, corresponding to $\Delta \ln k \simeq 9$. This corresponds to 9 e -folds of inflation. In slow-roll inflation ϵ has negligible variation over one e -fold and in typical models it has only small variation over the 9 e -folds. Taking that to be the case, and assuming that $\phi \ll M_{\text{P}}$, one learns from (39) that $\epsilon \ll \frac{1}{2} \times (1/9)^2 = 6 \times 10^{-3}$.

3.2.3 Error estimates for the slow-roll predictions

In deriving the prediction for $\mathcal{P}_{\mathcal{R}}$ we used the flatness conditions $\epsilon \ll 1$ and $|\eta| \ll 1$, as well as the slow-roll approximation whose fractional error is $\epsilon - \eta$ (Eq. (36)). As a result one expects $\mathcal{P}_{\mathcal{R}}$ to pick up fractional errors of order ϵ and η ,

$$\frac{\Delta \mathcal{P}_{\mathcal{R}}}{\mathcal{P}_{\mathcal{R}}} = O(\epsilon, \eta). \quad (58)$$

Using Eqs. (49), (50) and (51) one therefore expects

$$n - 1 = 2\eta - 6\epsilon + O(\xi^2) \quad (59)$$

$$\frac{dn}{d \ln k} = -16\epsilon\eta + 24\epsilon^2 + 2\xi^2 + O(\sigma^3). \quad (60)$$

In the first expression we ignored errors that are quadratic in ϵ and η , because barring cancellations the corresponding fractional errors are small by virtue of the flatness conditions $\epsilon \ll 1$ and $|\eta| \ll 1$. In the second expression we ignored errors that are cubic in ϵ , η and ξ . Barring cancellations, the accuracy of the prediction for $n - 1$ requires

$$|\xi^2| \ll \max(\epsilon, |\eta|), \quad (61)$$

and the accuracy of the prediction for its derivative requires in addition

$$|\sigma^3| \ll \max(\epsilon^2, \epsilon|\eta|, |\xi^2|). \quad (62)$$

3.3 Beyond the slow-roll prediction

The slow-roll predictions given in the last subsection are very convenient, because they involve only V and its low derivatives evaluated at the epoch of horizon exit. The use of slow-roll is not however mandatory; on the contrary, one can obtain [232, 272] predictions using essentially no assumptions beyond linear perturbation theory.

In linear perturbation theory, the quantity $u = a\delta\phi$ satisfies the following exact equation

$$\frac{\partial^2 u}{\partial \tau^2} + \left(k^2 - \frac{1}{z} \frac{d^2 z}{d\tau^2} \right) u = 0. \quad (63)$$

Here, τ is conformal time defined by $d\tau = dt/a$, and

$$z \equiv a\dot{\phi}/H \quad (64)$$

$$\frac{d^2 z}{d\tau^2} = 2a^2 H^2 \left(1 + \epsilon_H + \frac{3}{2}\delta + \frac{1}{2}\delta^2 + \frac{1}{2}\epsilon_H\delta + \frac{1}{2H} \frac{d\epsilon_H}{dt} + \frac{1}{2H} \frac{d\delta}{dt} \right), \quad (65)$$

where

$$\epsilon_H \equiv \frac{1}{2} \frac{\dot{\phi}^2}{H^2} = -\frac{\dot{H}}{H^2} \quad (66)$$

$$\delta \equiv \frac{\ddot{\phi}}{H\dot{\phi}}, \quad (67)$$

and an overdot denotes d/dt .

It is convenient to set $\tau = 0$ at the end of slow-roll inflation. In the extreme slow-roll limit $\dot{H} = 0$, this corresponds to

$$\tau = -1/(aH). \quad (68)$$

One assumes that inflation is near enough slow-roll that $k|\tau| \gg 1$ a few Hubble times before horizon exit, and $k|\tau| \ll 1$ a few Hubble times after. Then, there is a solution $u = w$ of Eq. (63) which satisfies

$$w = (2k)^{-1/2} e^{-ik\tau}, \quad (69)$$

a few Hubble times before horizon exit. A few Hubble times after horizon exit this solution has the behaviour

$$w/z \rightarrow \text{constant}. \quad (70)$$

One can show that the spectrum of \mathcal{R} is then given by

$$\mathcal{P}_{\mathcal{R}}(k) = \frac{k^3}{2\pi^2 z^2} |w(k)|^2, \quad (71)$$

Given an inflationary trajectory defined by $a(\tau)$ and $\dot{\phi}(\tau)$, this method gives a practically unique, and accurate, result in all reasonable cases. The trajectory in turn follows from the potential practically independently of the initial conditions, if slow-roll becomes very accurate at some early epoch.

We noted earlier that in the regime where the slow-roll predictions for $\mathcal{P}_{\mathcal{R}}$, $n - 1$ and $dn/d \ln k$ are approximately valid, the four flatness conditions Eqs. (31), (32), (61) and (62) are also valid. In that case, the ‘exact’ solution yields an improved version of the slow-roll predictions for $\mathcal{P}_{\mathcal{R}}$ and $n - 1$ [287]. Let us see how this goes.

Eqs. (36), (49) and (50) and the flatness conditions give the approximation

$$\frac{d^2 z}{d\tau^2} = 2a^2 H^2 \left(1 + \epsilon_H + \frac{3}{2} \delta \right), \quad (72)$$

with ϵ_H and δ slowly varying on the Hubble timescale. This leads to the approximation [287]

$$\mathcal{P}_{\mathcal{R}}^{1/2}(k) = [1 - (2C + 1) \epsilon_H - C\delta] \frac{H^2}{2\pi |\dot{\phi}|}, \quad (73)$$

where $C = -2 + \ln 2 + b \simeq -0.73$, with b the Euler-Mascheroni constant. As always, the right hand side is evaluated at $k = aH$.

We want an expression involving V and its derivatives. Substituting Eq. (35) into Eq. (27) gives

$$\frac{3M_{\text{P}}^2 H^2}{V} = 1 + \frac{1}{3} \epsilon, \quad (74)$$

and substituting Eq. (36) into Eq. (29) gives

$$-\frac{3H\dot{\phi}}{V'} = 1 - \frac{1}{3} \epsilon + \frac{1}{3} \eta. \quad (75)$$

These are improvements in the slow-roll formulas, valid to linear order in ϵ and η . Squaring the last equation gives

$$\frac{\epsilon_H}{\epsilon} = 1 - \frac{2}{3}\epsilon + \frac{1}{3}\eta, \quad (76)$$

and Eq. (36) is

$$\delta = \epsilon - \eta. \quad (77)$$

Inserting these four expressions into Eq. (73) gives

$$\delta_H = \frac{2}{5}\mathcal{P}_{\mathcal{R}}^{1/2}(k) = \frac{1}{5\sqrt{3}\pi M_{\text{p}}^3} \left(\frac{V^{3/2}}{|V'|} \right) \left[1 - \left(2C + \frac{1}{6} \right) \epsilon + \left(C - \frac{1}{3} \right) \eta + O(\xi^2) \right]. \quad (78)$$

The fractional error in this improved expression for $\mathcal{P}_{\mathcal{R}}$ is expected to be of order $O(\xi^2)$, plus terms quadratic in ϵ and η that we did not display. The ξ^2 term will be present, because it contributes to the variation per Hubble time of η (Eq. (50)) which is being ignored.

Using $k = aH$ with Eqs. (30), (74) and (75) gives the improved formula

$$\frac{d\phi}{d \ln k} = \sqrt{2\epsilon} \left(1 + \frac{1}{3}\epsilon + \frac{1}{3}\eta \right). \quad (79)$$

This leads to

$$\frac{1}{2}(n-1) = -3\epsilon + \eta - \left(\frac{5}{3} + 12C \right) \epsilon^2 + (8C-1)\epsilon\eta + \frac{1}{3}\eta^2 - \left(C - \frac{1}{3} \right) \xi^2 + O(\sigma^3). \quad (80)$$

The fractional error of order σ^3 comes from differentiating the error of order ξ^2 in Eq. (78) ($d\xi^2/d \ln k$ is given by Eq. (51)). Contrary to what is stated in [179], the actual coefficient of σ^3 cannot be evaluated without going back to the exact equation. There will also be error terms cubic in ϵ , η and ξ , that we do not display.

The improved solution becomes exact in the case of power-law inflation ($a \propto \phi^p$) when $\epsilon_H = -\delta$ is constant, and in the case of $V = V_0 \pm \frac{1}{2}m^2\phi^2$ in the limit $\phi \rightarrow 0$ when $\epsilon \rightarrow 0$ and δ becomes constant.

In some models, the improvement is big enough to measure *with fixed values of the parameters in the potential*. But in the cases that have been examined to date, this change can be practically cancelled by varying the parameters. As a result, the improvement is probably going to be useful only if gravitational waves are detected (Section 3.5).

3.4 The number of e -folds of slow-roll inflation

A model of inflation will give us an inflationary potential $V(\phi)$, and a prescription for the value ϕ_{end} of the field at the *end* of slow-roll inflation. This is not enough to work out the prediction for $\mathcal{P}_{\mathcal{R}}(k)$, because we need to know the value of ϕ when a given cosmological scale k leaves the horizon. Using Eq. (40), we can do this if we know the number $N(\phi)$ of e -folds of slow-roll inflation taking place after that epoch. The model will give $d \ln k / d\phi$, (through Eq. (48)) so we need this information for just one cosmological scale.

For definiteness, let us consider the scale $k^{-1} = H_0^{-1} = 3000h^{-1}\text{Mpc}$, which is the biggest cosmological scale of interest.²³ As this is more or less the scale probed by COBE, we denote it by a subscript COBE. The number of e -folds of inflation after this scale leaves the horizon is

$$N_{\text{COBE}} = \ln(a_{\text{end}}/a_{\text{COBE}}). \quad (81)$$

Since this scale is the one entering the horizon now, $a_{\text{COBE}}H_{\text{COBE}} = a_0H_0$ where the subscript 0 indicates the present epoch. This leads to

$$N_{\text{COBE}} = \ln\left(\frac{a_{\text{end}}H_{\text{end}}}{a_0H_0}\right) - \ln\left(\frac{H_{\text{end}}}{H_{\text{COBE}}}\right). \quad (82)$$

The second term will be given by the model of slow-roll inflation and is usually $\lesssim 1$; for simplicity let us ignore it. The first term depends on the evolution of the scale factor between the end of slow-roll inflation and the present.

Assume first that slow-roll inflation gives way promptly to matter domination ($a \propto t^{2/3}$), which is followed by a radiation dominated era ($a \propto t^{1/2}$) lasting until the present matter dominated era begins. Then one has [194, 195]

$$N_{\text{COBE}} = 62 - \ln(10^{16}\text{ GeV}/V_{\text{end}}^{1/4}) - \frac{1}{3}\ln(V_{\text{end}}^{1/4}/\rho_{\text{reh}}^{1/4}), \quad (83)$$

(ρ_{reh} is the ‘reheat’ temperature, when radiation domination begins.) With $V^{1/4} \sim 10^{16}\text{ GeV}$ and instant reheating this gives $N_{\text{COBE}} \simeq 62$, the biggest possible value. In fact, ρ_{reh} should probably be no bigger than 10^{10} GeV to avoid too many gravitinos [271], and using that value gives $N_{\text{COBE}} = 58$, perhaps the biggest reasonable value. With $V^{1/4} = 10^{10}\text{ GeV}$, the lowest scale usually considered, one finds $N_{\text{COBE}} = 48$ with instant reheating, and $N_{\text{COBE}} = 39$ if reheating is delayed to just before nucleosynthesis.

The smallest cosmological scale that will be directly probed in the foreseeable future is perhaps six orders of magnitude lower than H_0^{-1} , which corresponds to replacing N_{COBE} by $N_{\text{COBE}} - 6\ln 10 = N_{\text{COBE}} - 14$.

The estimates for N_{COBE} are valid only if there is no additional inflation, after slow-roll inflation ends. In fact, there are at least two possibilities for additional inflation. One is that slow-roll gives way smoothly to a significant amount of fast-roll inflation. This does not happen in most models, but it does happen in the rather attractive model described in Sections 6.9 and 8.6. Its effect is to reduce N_{COBE} by some amount N_{fast} , which is highly model-dependent.²⁴ The other possibility is that there is a separate, late era of thermal inflation, as described in footnote 60 of Section 6.10. The minimal assumption of one bout of thermal inflation will reduce N_{COBE} by $N_{\text{thermal}} \sim 10$.

We want slow-roll inflation to generate structure on all cosmological scales. Taking the smallest one to correspond to $N_{\text{COBE}} - 15$, and remembering that without thermal inflation

²³The absolute limit of direct observation is $2H_0^{-1}$, the distance to the particle horizon in a flat, matter-dominated Universe. Since the prediction is made for a randomly placed observer in a much bigger patch, bigger scales in principle contribute to it, but sensitivity rapidly decreases outside our horizon. Only if the spectrum increases sharply on very large scales [126, 110] might there be a significant effect. This Grishchuk-Zeldovich effect is not present in any model of inflation that has been proposed so far.

²⁴The quantity N_{fast} defined in this way is not identical with the number of e -folds of fast-roll inflation, since H is not constant during such inflation. But the latter provides a rough approximation to N_{fast} if slow-roll is only marginally violated, as in Section 6.9.

N_{COBE} is in the range 40 to 60, we learn that the amount of additional inflation must certainly satisfy

$$N_{\text{fast}} + N_{\text{thermal}} < 25 \text{ to } 45. \quad (84)$$

In many models of inflation, $n(k)$ is strongly dependent on $N(\phi)$ at the epoch of horizon exit (see for instance the table on page 80). Then a more stringent limit upper limit may come from the requirement that $|1 - n| < 0.2$.

From now on, we shall usually denote N_{COBE} simply by N . The more generally quantity $N(\phi)$, referring to an arbitrary field value, will always have its argument ϕ displayed.

3.5 Gravitational waves

Inflation also generates gravitational waves, with two independent components $h_{+,\times}$. Perturbing the Einstein action, one finds that each of quantities $(M_{\text{P}}/\sqrt{2})h_{+,\times}$ has the same action as a massless scalar field. It follows that $h_{+,\times}$ are independent gaussian perturbations, whose spectrum on scales far outside the horizon has the time-independent value [279, 267]

$$\mathcal{P}_{\text{grav}}(k) = \frac{2}{M_{\text{P}}^2} \left(\frac{H}{2\pi} \right)^2. \quad (85)$$

As usual, the right hand side is evaluated at the epoch of horizon exit $k = aH$. According to the analytic approximation mentioned earlier [280], the relative contribution $C_{\text{grav}}(\ell)/C_{\mathcal{R}}(\ell)$, of gravitational waves to the low multipoles, is equal to

$$r \equiv 12.4\epsilon. \quad (86)$$

We are using r defined by this equation as a convenient measure of the relative importance of the gravitational waves.

Using the slow-roll conditions, the spectral index is

$$n_{\text{grav}} = -2\epsilon. \quad (87)$$

This is the fourth quantity we calculated from the three quantities V , ϵ and η , so it will provide a consistency check if gravitational waves are ever detected.

We noted earlier that the primordial gravitational waves will not be detectable by Planck unless $r \gtrsim 0.1$, and are unlikely to be detected in the foreseeable future unless $r \gtrsim 0.01$. Most models of inflation give a much smaller value [216]. To see why, note first that the waves are significant only up to $\ell \sim 100$, corresponding to the first 4 or so e -folds of inflation after our Universe leaves the horizon. From Eq. (39), this means that the field variation is at least of order the Planck scale,

$$\Delta\phi \simeq 4\sqrt{2\epsilon}M_{\text{P}} = 0.5M_{\text{P}}(r/0.1)^{1/2}. \quad (88)$$

Afterwards, we have say ~ 50 e -folds more inflation, which will increase the total $\Delta\phi$. In models where ϵ increases with time this gives

$$\Delta\phi \gtrsim \frac{25}{4}M_{\text{P}}(r/0.1)^{1/2}. \quad (89)$$

Then detectable gravitational waves require $\Delta\phi \gtrsim 2$ to $6M_{\text{P}}$, placing the inflation model out of theoretical control. In models where ϵ decreases with time, the extra change in ϕ need not be significant, making it possible to generate detectable gravitational waves in models with $\Delta\phi \gtrsim 0.2$ to $0.5M_{\text{P}}$. Of the models proposed so far in the framework of particle theory, only tree-level hybrid inflation is of the latter type ($V = V_0 + \frac{1}{2}m^2\phi^2$ with the first term dominating, or the same thing with a higher power of ϕ .) But in most versions of hybrid inflation the field is small, the only exception so far being reference [219].

Another viewpoint is to look at the COBE normalization Eq. (45). It can be written

$$V^{1/4} = (2.0 \times 10^{16} \text{ GeV})(r/0.1)^{1/4}, \quad (90)$$

so detectable waves require $V^{1/4} \gtrsim 1 \times 10^{16} \text{ GeV}$. Such a big value is the exception rather than the rule for existing models.

We conclude that a detectable gravitational wave signal is unlikely. If such a signal is present, Eqs. (43), (54), (86) and (87) and more accurate versions of them will allow one to deduce $V(\phi)$ and its low derivatives. This is the ‘reconstruction’ programme [197]. Note that it will estimate $V(\phi)$ only on the limited portion of the trajectory corresponding to the ten or so e -folds occurring while cosmological scales leave the horizon.

3.6 Before observable inflation

The only era of inflation that is directly relevant for observation is the one beginning when the observable Universe leaves the horizon. This era of ‘observable’ inflation will undoubtedly be preceded by more inflation, but all memory of earlier inflation is lost apart from the starting values of the fields at the beginning of observable inflation. Nevertheless, one ought to try to understand the earlier era if only to check that the assumed starting values are not ridiculous.

A complete history of the Universe will presumably start when the energy density is at the Planck scale.²⁵ (Recall that $V^{1/4}$ is at least two orders of magnitude lower during observable inflation.) The usual hypothesis is that the scalar fields at that epoch take on chaotically varying values as one moves around the universe, inflation occurring in patches where conditions are suitable [199, 202]. The observable Universe is located in one of these patches, and from now on we consider only it.

One would indeed like to start the descent from the Planck scale with an era of inflation, for at least two reasons. One, which applies only to the case of positive spatial curvature, is to avoid having the Universe collapse in a few Planck times (or fine-tune the initial density parameter Ω). The other, which applies in any case, is to have an event horizon so that the homogeneous patch within which we are supposed to live is not eaten up by its inhomogeneous surroundings. However, there is no reason to suppose that this initial era of inflation is of the slow-roll variety. The motivation for slow-roll comes from the observed

²⁵We discount, for the moment, the fascinating possibility that additional space dimensions open up well below the Planck scale. We also do not consider the idea that a complete (open or closed) inflating universe is created by a quantum process, with energy density already far below the Planck scale [133, 207, 295, 42]. A more modest proposal [118] is that our Universe is located within a bubble, which nucleated at a low energy scale [118]; but the universe within which that bubble originated is still supposed to have begun at the Planck scale.

fact that δ_H is almost scale-independent, which applies only during the relatively brief era when cosmological scales are leaving the horizon. In the context of supergravity, where achieving slow-roll inflation requires rather delicate conditions, it might be quite attractive to suppose that non-slow-roll inflation takes the Universe down from the Planck scale with slow-roll setting in only much later. A well known potential that can give non-slow-roll inflation is $V \propto \exp(\sqrt{2/p}\phi/M_{\text{P}})$, which gives $a \propto t^p$ and corresponds to non-slow-roll inflation in the regime where p is bigger than 1 but not much bigger.

Well before observable inflation, it is possible to have an era of ‘eternal inflation’ during which the motion of the inflaton field is dominated by the quantum fluctuation.²⁶ The condition for this to occur is that the predicted spectrum $\mathcal{P}_{\mathcal{R}}$ be formally bigger than 1 [282].

With all this in mind, let us ask what might precede observable inflation, with a view to seeing what initial conditions for the latter might be reasonable. Going back in time, one might find a smooth inflationary trajectory going all the way back to an era when V is at the Planck scale (or at any rate much bigger than its value during observable inflation). In that case the inflaton field will probably be decreasing during inflation. Another natural possibility is for the inflaton to find itself near a maximum of the potential before observable inflation starts. Then there may be eternal inflation followed by slow-roll inflation. If the maximum is a fixed point of the symmetries it is quite natural for the field to have been driven there by its interaction with other fields. Otherwise it could arrive there by accident, though this is perhaps only reasonable if the distance from the maximum to the minimum is $\gtrsim M_{\text{P}}$ (see for instance Ref. [163] for an example). In this latter case, the fact that eternal inflation occurs near the maximum may help to enhance the probability of inflation starting there [200]. If the maximum is a fixed point, the inflaton field might be placed there through a coupling with another field, with that field initially inflating [147].²⁷ Alternatively, it may be that the inflaton field is placed at the origin through thermal corrections to the potential [198, 6], but this mechanism is difficult to implement.

In summary, two kinds of initial condition seem reasonable. One is to have the inflaton moving towards the origin, the idea being that the field value is initially at least of order M_{P} . The other is to have the inflaton moving away from a maximum of the potential, preferably located at the origin. We emphasize that these are just speculations; to make a definite statement, one needs a definite model going back to the Planck scale.

4 Calculating the curvature perturbation generated by inflation

This section is somewhat specialized, and may be omitted by the general reader. It concerns the calculation of the spectrum $\mathcal{P}_{\mathcal{R}}$ of the primordial curvature perturbation \mathcal{R} . We first consider the standard case of a single-component inflaton; essentially all of the models

²⁶Eternal inflation taking place at large field values is discussed in detail in Ref. [201, 208]. The corresponding phenomenon for inflation near a maximum was noted earlier by a number of authors.

²⁷The potential will be something like Eq. (147), with ψ the field corresponding to observable inflation. One initially has hybrid inflation, but in contrast with the usual case the destabilized field takes so long to roll down that it becomes the single inflaton field of observable inflation.

considered in the text are of this kind. Then we explain the concept of a multi-component inflaton, and see how to extend the calculation to that case.

In both cases we use an approach that has only recently been developed [270, 273, 195], though its starting point can already be seen in the first calculations [132, 278, 128]. This starting point consists of the following assumption. During any era of the early Universe, the evolution of the relevant quantities along each comoving worldline is practically the same as in an unperturbed Universe, after smoothing on a comoving scale that is well outside the horizon (Hubble distance $H^{-1}(t)$).²⁸

The assumed condition seems very reasonable. There needs to be *some* smoothing scale that makes the perturbations negligible or it would not make sense to talk about an unperturbed Universe. The horizon scale will be big enough, unless there is dramatic new physics on a much bigger scale, and the absence of an observed Grishchuk–Zel’dovich effect [126] or tilted Universe effect [294] more or less assures us that there is no such scale.

We shall see how a comparison of the evolution of different comoving regions provides a simple and powerful technique for calculating the density perturbation. As we discuss later, this approach is quite different from the usual one of writing down, and then solving, a closed set of equations for the perturbations in the relevant degrees of freedom (for instance the components of the inflaton field during inflation). Roughly speaking the present approach replaces the sequence ‘perturb then solve’ by the far simpler sequence ‘solve then perturb’, though it is actually more general than the other approach. For the case of a single-component inflaton it gives a very simple, and completely general, proof of the constancy of \mathcal{R} on scales well outside the horizon. For the multi-component case it allows one to follow the evolution of \mathcal{R} , knowing only the evolution of the *unperturbed* universe corresponding to a given value of the initial inflaton field. So far it has been applied to three multi-component models [270, 109, 111].

4.1 The case of a single-component inflaton

We begin with a derivation of the usual result for the single-component case. The assumption about the evolution along each comoving worldline is invoked only at the very end, when it is used to establish the constancy of \mathcal{R} which up till now has only been demonstrated for special cases. Otherwise the proof is the standard one [194, 195], but it provides a useful starting point for the multi-component case.

A few Hubble times after horizon exit during inflation, when $\mathcal{R}(k, t)$ can first be regarded as a classical quantity, its spectrum can be calculated using the relation [165, 194, 195]²⁹

$$\mathcal{R}(\mathbf{x}) = H \Delta\tau(\mathbf{x}), \tag{91}$$

where $\Delta\tau$ is the separation of the comoving hypersurface (with curvature \mathcal{R}) from a spatially flat one coinciding with it on average. The relation is generally true, but we apply it at an epoch a few Hubble times after horizon exit during inflation.

²⁸‘Smoothing’ on a scale R means that one replaces (say) the energy density $\rho(\mathbf{x})$ by $\int d^3x' W(|\mathbf{x}' - \mathbf{x}|) \rho(\mathbf{x}')$ with $W(y) \simeq 1$ for $y \lesssim R$ and $W \simeq 0$ for $y \gtrsim R$. A simple choice is to take $W = 1$ for $y < R$ and $W = 0$ for $y > R$ (top-hat smoothing).

²⁹In [194] there is an incorrect minus sign on the right hand side.

On a comoving hypersurface the inflaton field ϕ is uniform, because the momentum density $\dot{\phi}\nabla\phi$ vanishes. It follows that

$$\Delta\tau(\mathbf{x}) = -\delta\phi(\mathbf{x})/\dot{\phi}, \quad (92)$$

where $\delta\phi$ is defined on the flat hypersurface. Note that the comoving hypersurfaces become singular (infinitely distorted) in the slow-roll limit $\dot{\phi} \rightarrow 0$, so that to first order in slow-roll any non-singular choice of hypersurface could actually be used to define $\delta\phi$.

The spectrum of $\delta\phi$ is calculated by assuming that well before horizon exit (when the particle concept makes sense) $\delta\phi$ is a practically massless free field in the vacuum state. Using the flatness and slow-roll conditions one finds, a few Hubble times after horizon exit, the famous result [194, 195] $\mathcal{P}_\phi = (H/2\pi)^2$, which leads to the usual formula (43) for the spectrum.

However, this result refers to \mathcal{R} a few Hubble times after horizon exit, and we need to check that \mathcal{R} remains constant until the radiation dominated era where we need it. To calculate the rate of change of \mathcal{R} we proceed as follows [218, 194, 195].

In addition to the energy density $\rho(\mathbf{x}, t)$ and the pressure $P(\mathbf{x}, t)$, we consider a locally defined Hubble parameter $H(\mathbf{x}, t) = \frac{1}{3}D_\mu u^\mu$ where u^μ is the four-velocity of comoving worldlines and D_μ is the covariant derivative. (The quantity $3H$ is often denoted by θ in the literature.) The Universe is sliced into comoving hypersurfaces, and each quantity is split into an average ('background') plus a perturbation,

$$\rho(\mathbf{x}, t) = \rho(t) + \delta\rho(\mathbf{x}, t) \quad (93)$$

and so on. (We use the same symbol for the local and the background quantity since there is no confusion in practice.) As usual, \mathbf{x} is the Cartesian position-vector of a comoving worldline and t is the time. To first order, perturbations 'live' in unperturbed spacetime, since the inclusion of the perturbation in the space time metric when describing the *evolution* of a perturbation would be a second order effect.³⁰

We ignore the anisotropic stress of the early Universe, since it is unlikely to affect the constancy of \mathcal{R} [195]. The locally defined quantities satisfy [131, 212, 218, 194, 195]

$$H^2(\mathbf{x}, t) = M_{\text{P}}^{-2}\rho(\mathbf{x}, t)/3 + \frac{2}{3}\nabla^2\mathcal{R}. \quad (94)$$

The laplacian acts on comoving hypersurfaces. This is the Friedmann equation except that $K(\mathbf{x}, t) \equiv -(2/3)a^2\nabla^2\mathcal{R}$ need not be constant. The evolution along each worldline is

$$\frac{d\rho(\mathbf{x}, t)}{d\tau} = -3H(\mathbf{x}, t)(\rho(\mathbf{x}, t) + P(\mathbf{x}, t)), \quad (95)$$

$$\frac{dH(\mathbf{x}, t)}{d\tau} = -H(\mathbf{x}, t)^2 - \frac{1}{2}M_{\text{P}}^{-2}(\rho(\mathbf{x}, t) + 3P(\mathbf{x}, t)) - \frac{1}{3}\frac{\nabla^2\delta P}{\rho + P}. \quad (96)$$

Except for the last term these are the same as in an unperturbed universe. If that term vanishes \mathcal{R} is constant, but otherwise one finds

$$\dot{\mathcal{R}} = -H\delta P/(\rho + P). \quad (97)$$

³⁰This includes the case that the perturbation being evolved is itself a perturbation in the metric, such as the gravitational wave amplitude or the spatial curvature perturbation \mathcal{R} .

In this equation we have in mind that ρ and P are the unperturbed quantities, depending only on t , though as we are working to first order in the perturbations it would make no difference if they were the locally defined quantities.

According to Eq. (97), \mathcal{R} will be constant if δP is negligible. We now show that this is so, by first demonstrating that $\delta\rho$ is negligible, and then using the new viewpoint to see that P will be a practically unique function of ρ making δP also negligible.

From now on, we work with Fourier modes, represented by the same symbol, and replace ∇^2 by $-(k/a)^2$. Extracting the perturbations from Eq. (94) gives

$$2\frac{\delta H}{H} = \frac{\delta\rho}{\rho} - \frac{2}{3}\left(\frac{k}{aH}\right)^2 \mathcal{R}. \quad (98)$$

This allows one to calculate the evolution of $\delta\rho$ from Eq. (95), but we have to remember that the proper-time separation of the hypersurfaces is position-dependent. Writing $\tau(\mathbf{x}, t) = t + \delta\tau(\mathbf{x}, t)$ we have [165, 218, 220]

$$\delta(\dot{\tau}) = -\delta P/(\rho + P). \quad (99)$$

Writing $\delta\rho/\rho \equiv (k/aH)^2 Z$ one finds [218]

$$(fZ)' = f(1+w)\mathcal{R}. \quad (100)$$

Here a prime denotes $d/d(\ln a)$ and $f'/f \equiv (5+3w)/2$ where $w \equiv P/\rho$. With w and \mathcal{R} constant, and dropping a decaying mode, this gives

$$Z = \frac{2+2w}{5+3w}\mathcal{R}. \quad (101)$$

More generally, integrating Eq. (100) will give $|Z| \sim |\mathcal{R}|$ for any reasonable variation of w and \mathcal{R} . Even for a bizarre variation there is no scale dependence in either w (obviously) or in \mathcal{R} (because Eq. (107) gives it in terms of δP , and we will see that if δP is significant it is scale-independent). In all cases $\delta\rho/\rho$ becomes negligible on scales sufficiently far outside the horizon.³¹

The discussion so far applies to each Fourier mode separately, on the assumption that the corresponding perturbation is small. To make the final step, of showing that δP is also negligible, we need to consider the full quantities $\rho(\mathbf{x}, t)$ and so on. But we still want to consider only scales that are well outside the horizon, so we suppose that all quantities are smoothed on a comoving scale somewhat shorter than the one of interest. The smoothing removes Fourier modes on scales shorter than the smoothing scale, but has practically no effect on the scale of interest.

Having done this, we invoke the assumption that the evolution of the Universe along each worldline is practically the same as in an unperturbed universe. In the context of

³¹As it stands, this analysis fails if a single oscillating field dominates (as might happen just after inflation) because $1+w$ then passes through zero. In that case one can consider $(1+w)\mathcal{R}$. Combining Eqs. (97) and (100), one sees that it satisfies a non-singular differential equation, which means that it will change by a negligible amount during each of the brief episodes when the right hand side of Eq. (97) becomes non-negligible and formally goes through infinity. During such an episode, the comoving hypersurfaces become infinitely distorted and \mathcal{R} briefly loses its meaning, but what matters is that \mathcal{R} is practically constant except for these episodes.

slow-roll inflation, this means that the evolution is determined by the inflaton field at the ‘initial’ epoch a few Hubble times after horizon exit. To high accuracy, ρ and P are well defined functions of the initial inflaton field and *if it has only one component* this means that they are well defined functions of each other. Therefore δP will be very small on comoving hypersurfaces because $\delta\rho$ is.³²

Finally, we note for future reference that δH is also negligible because of Eq. (98).

4.2 The multi-component case

So far we have assumed that the slow-rolling inflaton field is essentially unique. What does ‘essentially’ mean in this context? A strictly unique inflaton trajectory would be one lying in a steep-sided valley in field space. This is not very likely in a realistic model. Rather there will be a whole family of possible inflaton trajectories, lying in the space of two or more real fields ϕ_1, ϕ_2, \dots . Usually, though, the different trajectories are completely equivalent, so that we still have an ‘essentially’ unique inflaton field. For instance, in many cases the inflaton field is the modulus of a complex field, with V independent of the phase. Each choice of the phase gives a different but equivalent inflaton trajectory in the space of the complex field. Also, there may be a field(s) a , unrelated to the inflaton field, which has practically zero mass. Different choices of a lead to different inflaton trajectories in field space, but in the usual case that a has no cosmological effect these trajectories will again be equivalent.

In both of these cases, one can modify things so that the trajectories are *inequivalent*. In the case of the complex field, it might be that V is a function of both the real and imaginary parts, call them ϕ_1 and ϕ_2 , with V satisfying the flatness conditions Eqs. (31) and (32) as a function of each field separately. Then there will in general be a family of curved inflaton trajectories, corresponding to the lines of steepest descent, which are inequivalent. In this case, it is useful to think of the inflaton as a two-component object (ϕ_1, ϕ_2) . More generally, there might be a family of curved inflaton trajectories in the space of several fields, so that there is a *multi-component* inflaton.

In the case of an unrelated massless field a , that field might survive and be stable, to become dark matter after it starts to oscillate about its minimum. The inflaton trajectories are now inequivalent, but the inequivalence shows up only when the oscillation starts. The vacuum fluctuation of a during inflation then turns into an *isocurvature density perturbation*. Extensions of the standard model typically contain a field which can have just these properties, namely the axion. Postponing until later the discussion of this case, we continue discussion of the multicomponent case.

Multi-component inflaton models generally have just two components, and are called double inflation models because the trajectory can lie first in the direction of one field, then in the direction of the other. They were first proposed in the context of non-Einstein gravity [281, 168, 234, 119, 305, 235, 120, 68, 69, 284, 108, 109]. By redefining the fields and the spacetime metric one can recover Einstein gravity, with fields that are not small on the

³²If k/a is the smoothing scale, the assumption that the evolution is the same as in an unperturbed universe with the same initial inflaton field has in general errors of order $(k/aH)^2$. In the single-component case, where δP is also of this order, we cannot use the assumption to actually calculate it, but neither is it of any interest.

Planck scale and in general non-canonical kinetic terms and a non-polynomial potential. Then models with canonical kinetic terms were proposed [268, 255, 256, 254, 258, 24, 169, 136, 138, 104, 257, 270], with potentials such as $V = \lambda_1 \phi_1^p + \lambda_2 \phi_2^q$. These potentials too inflate in the large-field regime where theory provides no guidance about the form of the potential. However there seems to be no bar to having a multi-component model with $\phi \ll M_{\text{P}}$, and one may yet emerge in a well-motivated particle theory setting. In that case a hybrid model might emerge, though the models proposed so far are all of the non-hybrid type (ie., the multi-component inflaton is entirely responsible for the potential).

In this brief survey we have focussed on the era when cosmological scales leave the horizon. In the hybrid inflation model of Ref. [262, 111], the ‘other’ field is responsible for the last several e -folds of inflation, so one is really dealing with a two-component inflaton (in a non-hybrid model). The scales corresponding to the last few e -folds are many orders of magnitude shorter than the cosmological scales, but it turns out that the perturbation on them is big so that black holes can be produced. This phenomenon was investigated in Refs. [262, 111]. The second reference also investigated the possible production of topological defects, when the first field is destabilized.

4.3 The curvature perturbation

It is assumed that while cosmological scales are leaving the horizon all components of the inflaton have the slow-roll behaviour

$$3H\dot{\phi}_a = -V_{,a}. \quad (102)$$

(The subscript $,a$ denotes the derivative with respect to ϕ_a .) Differentiating this and comparing it with the exact expression $\ddot{\phi}_a + 3H\dot{\phi}_a + V_{,a} = 0$ gives consistency provided that

$$M_{\text{P}}^2 (V_{,a}/V)^2 \ll 1, \quad (103)$$

$$M_{\text{P}}^2 |V_{,ab}/V| \ll 1. \quad (104)$$

(The second condition could actually be replaced by a weaker one but let us retain it for simplicity.) One expects slow-roll to hold if these flatness conditions are satisfied. Slow-roll plus the first flatness condition imply that H (and therefore ρ) is slowly varying, giving quasi-exponential inflation. The second flatness condition ensures that $\dot{\phi}_a$ is slowly varying.

It is not necessary to assume that all of the fields continue to slow-roll after cosmological scales leave the horizon. For instance, one or more of the fields might start to oscillate, while the others continue to support quasi-exponential inflation, which ends only when slow-roll fails for all of them. Alternatively, the oscillation of some field might briefly interrupt inflation, which resumes when its amplitude becomes small enough. (Of course these things might happen while cosmological scales leave the horizon too, but that case will not be considered.)

The expression (91) for \mathcal{R} still holds in the multi-component case. Also, one still has $\Delta\tau = -\delta\phi/\dot{\phi}$ if $\delta\phi$ denotes the component of the vector $\delta\phi_a$ parallel to the trajectory. A few Hubble times after horizon exit the spectrum of every component of the vector $\delta\phi_a$, in particular the parallel one, is still $(H/2\pi)^2$. If \mathcal{R} had no subsequent variation this would lead to the usual prediction, but we are considering the case where the variation is significant.

It is given in terms of δP by Eq. (97), and when δP is significant it can be calculated from the assumption that the evolution along each worldline is the same as for an unperturbed universe with the same initial inflaton field. This will give

$$\delta P = P_{,a} \delta \phi_a, \quad (105)$$

where $\delta \phi_a$ is evaluated at the initial epoch and the function $P(\phi_1, \phi_2, \dots, t)$ represents the evolution of P in an unperturbed universe. Choosing the basis so that one of the components is the parallel one, and remembering that all components have spectrum $(H/2\pi)^2$, one can calculate the final spectrum of \mathcal{R} . The only input is the evolution of P in the unperturbed universe corresponding to a generic initial inflaton field (close to the classical initial field).

In this discussion we started with Eq. (91) for the initial \mathcal{R} , and then invoked Eq. (97) to evolve it. The equations can actually be combined to give

$$\mathcal{R} = \delta N, \quad (106)$$

where $N = \int H d\tau$ is the number of Hubble times between the initial flat hypersurface and the final comoving one on which \mathcal{R} is evaluated. This remarkable expression was given in Ref. [281] and proved in Refs. [270, 273]. The approach we are using is close to the one in the last reference.

The proof that Eqs. (91) and (97) lead to $\mathcal{R} = \delta N$ is very simple. First combine them to give

$$\mathcal{R}(\mathbf{x}, t) = H_1 \Delta \tau_1(\mathbf{x}) - \int_{t_1}^t H(t) \frac{\delta P}{\rho + P}, \quad (107)$$

where t_1 is a few Hubble times after horizon exit. Then use Eq. (99) to give

$$\mathcal{R}(\mathbf{x}, t) = H_1 \Delta \tau_1(\mathbf{x}) + \int_{t_1}^t H(t) \delta \dot{\tau}(\mathbf{x}, t) dt. \quad (108)$$

As we remarked at the end of Section 4.1, δH is negligible. As a result, this can be written

$$\mathcal{R}(\mathbf{x}, t) = H_1 \Delta \tau_1(\mathbf{x}) + \delta \int_{t_1}^t H(\mathbf{x}, t) \dot{\tau}(\mathbf{x}, t) dt. \quad (109)$$

Finally redefine $\tau(\mathbf{x}, t)$ so that it vanishes on the initial *flat* hypersurface, which gives the desired relation $\mathcal{R} = \delta N$.

In Ref. [273] this relation is derived using an arbitrary smooth interpolation of hypersurfaces between the initial and final one, rather than by making the sudden jump to a comoving one. Then H is replaced by the corresponding quantity \tilde{H} for worldlines orthogonal to the interpolation (incidentally making $\delta \tilde{H}$ non-negligible). One then finds $\mathcal{R} = \delta \tilde{N}$. One also finds that the right hand side is independent of the choice of the interpolation, as it must be for consistency. If the interpolating hypersurfaces are chosen to be comoving except very near the initial one, $\tilde{N} \simeq N$ which gives the desired formula $\mathcal{R} = \delta N$.³³

³³The last step is not spelled out in Ref. [273]. The statement that \tilde{N} is independent of the interpolation is true only on scales well outside the horizon, and its physical interpretation is unclear though it drops out very simply in the explicit calculation.

4.4 Calculating the spectrum and the spectral index

Now we derive explicit formulas for the spectrum and the spectral index, following [273]. Since the evolution of H along a comoving worldline will be the same as for a homogeneous universe with the same initial inflaton field, N is a function only of this field and we have

$$\mathcal{R} = N_{,a} \delta\phi_a. \quad (110)$$

(Repeated indices are summed over and the subscript $,a$ denotes differentiation with respect to ϕ_a .) The perturbations $\delta\phi_a$ are Gaussian random fields generated by the vacuum fluctuation, and have a common spectrum $(H/2\pi)^2$. The spectrum $\delta_H^2 \equiv (4/25)\mathcal{P}_{\mathcal{R}}$ is therefore

$$\delta_H^2 = \frac{V}{75\pi^2 M_{\text{P}}^2} N_{,a} N_{,a}. \quad (111)$$

In the single-component case, $N' = M_{\text{P}}^{-2} V/V'$ and we recover the usual expression. In the multi-component case we can always choose the basis fields so that while cosmological scales are leaving the horizon one of them points along the inflaton trajectory, and then its contribution gives the standard result with the orthogonal directions giving an additional contribution. Since the spectrum of gravitational waves is independent of the number components (being equal to a numerical constant times V) the relative contribution r of gravitational waves to the cmb is always *smaller* in the multi-component case.

The contribution from the orthogonal directions depends on the whole inflationary potential after the relevant scale leaves the horizon, and maybe even on the evolution of the energy density after inflation as well. This is in contrast to the contribution from the parallel direction which depends only on V and V' evaluated when the relevant scale leaves the horizon. The contribution from the orthogonal directions will be at most of order the one from the parallel direction provided that all $N_{,a}$ are at most of order $M_{\text{P}}^{-2} V/V'$. We shall see later that this is a reasonable expectation at least if \mathcal{R} stops varying after the end of slow-roll inflation.

To calculate the spectral index we need the analogue of Eqs. (48) and (39). Using the chain rule and $dN = -H dt$ one finds

$$\frac{d}{d \ln k} = -\frac{M_{\text{P}}^2}{V} V_{,a} \frac{\partial}{\partial \phi_a}, \quad (112)$$

$$N_{,a} V_{,a} = M_{\text{P}}^{-2} V. \quad (113)$$

Differentiating the second expression gives

$$V_{,a} N_{,ab} + N_{,a} V_{,ab} = M_{\text{P}}^{-2} V_{,b}. \quad (114)$$

Using these results one finds

$$n - 1 = -\frac{M_{\text{P}}^2 V_{,a} V_{,a}}{V^2} - \frac{2}{M_{\text{P}}^2 N_{,a} N_{,a}} + 2 \frac{M_{\text{P}}^2 N_{,a} N_{,b} V_{,ab}}{V N_{,d} N_{,d}}. \quad (115)$$

Again, we recover the single field case using $N' = M_{\text{P}}^{-2} V/V'$.

Differentiating this expression and setting $M_{\text{P}} = 1$ for clarity gives

$$\begin{aligned} \frac{dn}{d \ln k} &= -\frac{2}{V^3} V_a V_b V_{,ab} + \frac{2}{V^4} (V_a V_{,a})^2 + \frac{4}{V} \frac{(V - N_{,a} N_{,b} V_{,ab})^2}{(N_{,d} N_{,d})^2} \\ &+ \frac{2}{V} \frac{N_{,a} N_{,b} V_{,c} V_{,abc}}{N_{,d} N_{,d}} + \frac{4}{V} \frac{(V_{,c} - N_{,a} V_{,ac}) N_{,b} V_{,bc}}{N_{,d} N_{,d}}. \end{aligned} \quad (116)$$

A correction to the formula for $n - 1$ has also been worked out [239]. Analogously with the single-component case, both this correction and the variation of $n - 1$ involve the first, second and third derivatives of V . Provided that the derivatives of N in the orthogonal directions are not particularly big, and barring cancellations, a third flatness condition $V_{,abc} V_{,c} / V^2 \ll \max\{\sum_a (V_{,a})^2, \sum_{ab} |V_{,ab}|\}$ ensures that both the correction and the variation of $n - 1$ in a Hubble time are small. (One could find a weaker condition that would do the same job.)

These formulas give the spectrum and spectral index of the density perturbation, if one knows the evolution of the homogeneous universe corresponding both to the classical inflaton trajectory and to nearby trajectories. An important difference in principle from the single-component case, is that the classical trajectory is not uniquely specified by the potential, but rather has to be given as a separate piece of information. However, if there are only two components the classical trajectory can be determined from the COBE normalization of the spectrum, and then there is still a prediction for the spectral index.

This treatment can be generalized straightforwardly [273] to the case of non-canonical kinetic terms described by Eq. (135). However, in the regime where all fields are $\ll M_{\text{P}}$ one expects the ‘curvature’, associated with the ‘metric’ H_{ab} in Eq. (135) to be negligible, and then one can recover the canonical normalization $H_{ab} = \delta_{ab}$ by redefining the fields.

4.5 When will \mathcal{R} become constant?

We need to evaluate N up to the epoch where $\mathcal{R} = \delta N$ has no further time dependence. When will that be?

As long as all fields are slow-rolling, \mathcal{R} is constant if and only if the inflaton trajectory is straight. If it turns through a small angle θ , and the trajectories have not converged appreciably since horizon exit, the fractional change in \mathcal{R} is in fact 2θ .³⁴ Since slow-roll requires that the change in the vector $\dot{\phi}_a$ during one Hubble time is negligible, the total angle turned is $\ll N$. Hence the relative contribution of the orthogonal directions cannot be orders of magnitude bigger than the one from the parallel direction, if it is generated during slow-roll inflation. (In two dimensions the angle turned cannot exceed 2π of course, but there could be say a corkscrew motion in more dimensions.) Later slow-roll may fail for one or more of the fields, with or without interrupting inflation, and things become more complicated, but in general there is no reason why \mathcal{R} should stop varying before the end of inflation.

³⁴Thinking in two dimensions and taking the trajectory to be an arc of a circle, a displacement $\delta\phi$ towards the center decreases the length of the trajectory by an amount $\theta\delta\phi$, to be compared with the decrease $\delta\phi$ for the same displacement along the trajectory. (The rms displacements will indeed be the same if the trajectories have not converged.) The speed along the new trajectory is faster in inverse proportion to the length since it is proportional to V' and V is fixed at the initial and final points on the trajectory. Thus the perpendicular displacement increases N by 2θ times the effect of a parallel displacement, for $\theta \ll 1$.

Now let us ask what happens after the end of inflation (or to be more precise, after significant particle production has spoiled the above analysis, which may happen a little before the end). The simplest case is if the relevant trajectories have practically converged to a single trajectory $\phi_a(\tau)$, as in Ref. [111]. Then \mathcal{R} will not vary any more (even after inflation is over) as soon as the trajectory has been reached. Indeed, setting $\tau = 0$ at the end of inflation, this unique trajectory corresponds to a post-inflationary universe depending only on τ . The fluctuation in the initial field values causes a fluctuation $\Delta\tau$ in the arrival time at the end of inflation, leading to a time-independent $\mathcal{R} = \delta N = H_{\text{end}}\Delta\tau$.

What if the trajectory is not unique at the end of inflation? Immediately following inflation there might be a quite complicated situation, with ‘preheating’ [171, 172, 153, 11, 177, 154, 155, 156, 173, 174] or else the quantum fluctuation of the ‘other’ field in hybrid models [60] converting most of the inflationary potential energy into marginally relativistic particles in much less than a Hubble time. But after at most a few Hubble times one expects to arrive at a matter-dominated era so that \mathcal{R} is constant. Subsequent events will not cause \mathcal{R} to vary provided that they occur at definite values of the energy density, since again P will have a definite relation with ρ . This is indeed the case for the usually-considered events, such as the decay of matter into radiation and thermal phase transitions (including thermal inflation). The conclusion is that it is reasonable to suppose that \mathcal{R} achieves a constant value at most a few Hubble times after inflation, which is maintained until horizon entry except possibly for the large-scale isocurvature effect mentioned in Section 4.7. On the other hand one cannot exclude the possibility that one of the orthogonal components of the inflaton provides a significant additional degree of freedom, allowing \mathcal{R} to have additional variation before we finally arrive at the radiation-dominated era preceding the present matter-dominated era.

4.6 Working out the perturbation generated by slow-roll inflation

If \mathcal{R} stops varying by the end of inflation, the final hypersurface can be located just before the end (not necessarily at the very end because that might not correspond to a hypersurface of constant energy density). Then, knowing the potential and the hypersurface in field space that corresponds to the end of inflation, one can work out $N(\phi_1, \phi_2, \dots)$ using the equations of motion for the fields, and the expression

$$3M_{\text{P}}^2 H = \rho = V + \frac{1}{2} \frac{d\phi_a}{d\tau} \frac{d\phi_a}{d\tau}. \quad (117)$$

To perform such a calculation it is not necessary that all of the fields continue to slow-roll after cosmological scales leave the horizon. In particular, the oscillation of some field might briefly interrupt inflation, which resumes when its amplitude becomes small enough. If that happens it may be necessary to take into account ‘preheating’ during the interruption.

In general all this is quite complicated, but there is one case that may be extremely simple, at least in a limited regime of parameter space. This is the case

$$V = V_1(\phi_1) + V_2(\phi_2) + \dots \quad (118)$$

with each V_a proportional to a power of ϕ_a . For a single-component inflaton this gives inflation ending at $\phi_{\text{end}} \simeq M_{\text{P}}$, with cosmological scales leaving the horizon at $\phi \gg \phi_{\text{end}}$. If

the potentials V_a are identical we recover that case. If they are different, slow-roll may fail in sequence for the different components, but in some regime of parameter space the result for N (at least) might be the same as if it failed simultaneously for all components. If that is the case one can derive simple formulas [255, 270], provided that cosmological scales leave the horizon at $\phi_a \gg \phi_a^{\text{end}}$ for all components.

One has

$$H dt = -M_{\text{P}}^{-2} \frac{V}{V'_1} d\phi_1 = -M_{\text{P}}^{-2} \sum_a \frac{V_a}{V'_a} d\phi_a. \quad (119)$$

It follows that

$$N = M_{\text{P}}^{-2} \sum_a \int_{\phi_a^{\text{end}}}^{\phi_a} \frac{V_a}{V'_a} d\phi_a. \quad (120)$$

Since each integral is dominated by the endpoint ϕ_a , we have $N_{,a} = M_{\text{P}}^{-2} V_a / V'_a$ and

$$\delta_H^2 = \frac{V}{75\pi^2 M_{\text{P}}^6} \sum_a \left(\frac{V_a}{V'_a} \right)^2. \quad (121)$$

The spectral index is given by Eq. (115), which simplifies slightly because $V_{,ab} = \delta_{ab} V_a''$.

The simplest case is $V = \frac{1}{2} m_1^2 \phi_1^2 + \frac{1}{2} m_2^2 \phi_2^2$. Then n is given by the following formula

$$1 - n = \frac{1}{N} \left[\frac{(1+r)(1+\mu^2 r)}{(1+\mu r)^2} + 1 \right], \quad (122)$$

where $r = \phi_2^2 / \phi_1^2$ and $\mu = m_2^2 / m_1^2$. If $\mu = 1$ this reduces to the single-component formula $1 - n = 2/N$. Otherwise it can be much bigger, but note that our assumptions will be valid if at all in a restricted region of the r - μ plane.

4.7 An isocurvature density perturbation?

Following the astrophysics usage, we classify a density perturbation as adiabatic or isocurvature with reference to its properties at some epoch *during the radiation-dominated era preceding the present matter-dominated era*, while it is still far outside the horizon. For an adiabatic density perturbation, the density of each particle species is a unique function of the total energy density. For an isocurvature density perturbation the total density perturbation vanishes, but those of the individual particle species do not. The most general density perturbation is the sum of an adiabatic and an isocurvature perturbation, with \mathcal{R} specifying the adiabatic density perturbation only.

For an isocurvature perturbation to exist the universe has to possess more than the single degree of freedom provided by the total energy density. If the inflaton trajectory is unique, or has become so by the end of inflation, there is only the single degree of freedom corresponding to the fluctuation back and forth along the trajectory and there can be no isocurvature perturbation. Otherwise one of the orthogonal fields can provide the necessary degree of freedom. The simplest way for this to happen is for the orthogonal field to survive, and acquire a potential so that it starts to oscillate and becomes matter.³⁵ The start of the

³⁵If the potential of the ‘orthogonal’ field already exists during inflation the inflaton trajectory will have a tiny component in its direction, so that it is not strictly orthogonal to the inflaton trajectory. This makes no practical difference. In the axion case the potential is usually supposed to be generated by QCD effects long after inflation.

oscillation will be determined by the total energy density, but its amplitude will depend on the initial field value so there will be an isocurvature perturbation. It will be compensated, for given energy density, by the perturbations in the other species of matter and radiation which will continue to satisfy the adiabatic condition $\delta\rho_m/\rho_m = \frac{3}{4}\delta\rho_r/\rho_r$.

The classic example of this is the axion field [202, 176, 215], which is simple because the fluctuation in the direction of the axion field causes no adiabatic density perturbation, at least in the models proposed so far. The more general case, where one of the components of the inflaton may cause both an adiabatic and an isocurvature perturbation has been looked at in for instance Ref. [257], though not in the context of specific particle physics. If an isocurvature perturbation in the non-baryonic dark matter density exists, it must not conflict with observation and this imposes strong constraints on, for instance, models of the axion [215, 204].

An isocurvature perturbation in the density of a species of matter may be defined by the ‘entropy perturbation’ [165, 220, 194, 195]

$$S = \frac{\delta\rho_m}{\rho_m} - \frac{3}{4} \frac{\delta\rho_r}{\rho_r}, \quad (123)$$

where ρ_m is the non-baryonic dark matter density. Equivalently, $S = \delta y/y$, where $y = \rho_m/\rho_r^{3/4}$. Since we are dealing with scales far outside the horizon, ρ_m and ρ_r evolve as they would in an unperturbed universe which means that y is constant and so is S . Provided that the field fluctuation is small S will be proportional to it, and so will be a Gaussian random field with a nearly flat spectrum [215, 194, 195].

For an isocurvature perturbation, \mathcal{R} vanishes during the radiation dominated era preceding the present matter dominated era. But on the very large scales entering the horizon well after matter domination, S generates a nonzero \mathcal{R} during matter domination, namely $\mathcal{R} = \frac{1}{3}S$. A simple way of seeing this, which has not been noted before, is through the relation (97). Since $\delta\rho = 0$, one has $S = -(\rho_m^{-1} + \frac{3}{4}\rho_r^{-1})\delta\rho_r$. Then, using $\delta P = \delta\rho_r/3$, $\rho_r/\rho_m \propto a$ and $Hdt = da/a$ one finds the quoted result by integrating Eq. (97).

As discussed for instance in Ref. [194, 195], the large-scale cmb anisotropy coming from an isocurvature perturbation is $\Delta T/T = -(\frac{1}{3} + \frac{1}{15})S$, where S is evaluated on the last-scattering surface. The second term is the Sachs-Wolfe effect coming from the curvature perturbation we just calculated, and the first term is the anisotropy $\frac{1}{4}\delta\rho_r/\rho_r$ just after last scattering (on a comoving hypersurface). By contrast the anisotropy from an adiabatic perturbation comes only from the Sachs-Wolfe effect, so for a given large-scale density perturbation the isocurvature perturbation gives an anisotropy six times bigger. As a result an isocurvature perturbation with a flat spectrum cannot be the dominant contribution to the cmb, though one could contemplate a small contribution [291].

5 Field theory and the potential

All models of inflation assume the validity of field theory, and in particular the existence of a potential V which is a function of the scalar fields. In this section we discuss, in an elementary way, the form of the scalar field potential that one might expect on the basis of particle theory.

5.1 Renormalizable versus non-renormalizable theories

A given field theory, like the Standard Model or a supersymmetric extension of it, is nowadays regarded as an effective theory. Such a theory is valid when the (biggest) relevant energy scale is less than some ‘ultraviolet cutoff’, which we shall denote by Λ_{UV} . In the context of collider physics, the relevant energy scale is usually the collision energy. In the context of inflation, it is usually the value of the inflaton field. It will be helpful to keep these two cases in mind.

In the most optimistic case, Λ_{UV} will be the Planck scale M_{P} . For field theory in three space dimensions, Λ_{UV} presumably cannot be higher than M_{P} , since at that scale the theory will be invalidated by effects like the quantum fluctuation of the spacetime metric. But it might be lower. In weakly coupled heterotic string theory it is suggested (Section 7.9.3) that Λ_{UV} is the string scale $M_{\text{str}} \simeq g_{\text{str}}^2 M_{\text{P}}$, where $g_{\text{str}}^2 \sim 1$ to 0.1 is the gauge coupling at the string scale.

At high scales, n compactified space dimensions may become relevant. In that case, the biggest possible value of Λ_{UV} is presumably the Planck scale M_{4+n} for gravity with these extra dimensions. It is typically lower than M_{P} , as the following argument shows. If R is the size of the compactified dimensions (assumed to be all equal) the Newtonian gravitational force $1/(M_{\text{P}}r)^2$ is valid only for $r \gg R$, and for $r \ll R$ it turns into $1/(M_{4+n}r)^{2+n}$ where M_{4+n} is the Planck scale for gravity with the n extra dimensions. Matching these expressions at the scale $r \sim R$ one learns that $(M_{4+n}/M_{\text{P}})^{2+n} \sim (M_{\text{P}}R)^{-n}$. The right hand side is less than 1, or it would not make sense to talk about the extra dimensions.

At least if one is dealing with a field theory in which the fields are confined to the three space dimensions, M_{4+n} may be a useful estimate of the appropriate renormalization scale. This is what happens in Horava-Witten M-theory [139, 303] (there are two sets of fields, each confined to a different three dimensional space). There is one extra dimension (plus much smaller ones that we do not consider), with $(M_5/M_{\text{P}}) \sim 0.1$ and therefore $(M_{\text{P}}R)^{-1} \sim 10^{-3}$. Another proposal (Reference [14, 12] and earlier ones cited there) invokes $n = 2$, $M_6 \sim 1$ TeV and therefore $R \sim 1$ mm.

If the fields of the inflation model are confined to three space dimensions, extra dimensions *per se* should make no difference provided that their size is much less than the Hubble distance during inflation. As one easily verifies, this is automatic for $n \geq 2$, given the condition $V^{1/4} < M_{4+n}$ that certainly needs to be imposed. It will also be the case in Horava-Witten M theory, since the COBE bound Eq. (45) requires $H \lesssim 10^{-3} M_{\text{P}}$.

There is also the proposal that the cutoff is inversely related to the size L of the region that is to be described. For instance, [59] suggest that one needs $\Lambda_{\text{UV}} \lesssim \sqrt{M_{\text{P}}/L}$; with a box size a few times bigger than the Hubble distance (used when calculating the vacuum fluctuation of the inflaton) this would give $\Lambda_{\text{UV}} \sim V^{1/4}$ [59].

All of this refers to the cutoff for a field theory including all of the fields in Nature. In the context of terrestrial and astrophysics, one often considers an effective theory, obtained by integrating out fields with mass $\gtrsim \Lambda_{\text{UV}}$.³⁶ In the simpler context of inflation model-

³⁶ For the present purpose, integrating fields out (of the action) can be taken to mean that the scalar field potential is minimized with respect to them, at fixed values of the fields which are not integrated out. This gives a well-defined field theory, if the motion of the integrated-out fields about this minimum is negligible. That will always be the case if their masses are much bigger than those of the fields that are not integrated out. It is also the case when the coupling between the two sets of fields is of only gravitational strength,

building though, it is usually supposed that Λ_{UV} is associated with the breakdown of field theory itself. In general, we shall assume that $\Lambda_{\text{UV}} \sim M_{\text{P}}$, while recognizing occasionally the important possibility of a lower value.

A field theory is specified by the lagrangian (density) \mathcal{L} , such that the action is $S = \int d^4x \mathcal{L}$. It has dimension [energy]⁴, and is a function of the fields and their derivatives with respect to space and time. In a given theory the lagrangian will contain parameters, that define the masses of the particles and their interactions. In a renormalizable theory, the number of parameters is finite, even after quantum effects are included; the Standard Model is such a theory. Nowadays, a renormalizable theory is regarded as an approximation to a non-renormalizable one. The non-renormalizable theory is supposed to be a complete description of nature, on energy scales $\lesssim M_{\text{P}}$.

The non-renormalizable theory contains an infinite number of parameters, which may be thought of as summarizing the unknown Planck-scale physics, and it can be replaced by the renormalizable theory in any situation where M_{P} can be regarded as infinite.

We are focussing on supersymmetric theories, which can be either renormalizable or non-renormalizable. Supergravity, which is presumed to be the version of supersymmetry chosen by nature, is non-renormalizable. A simpler version, called global supersymmetry, can be renormalizable. Following the usual practice, we shall take the term ‘global supersymmetry’ to denote a version that is renormalizable, with the possible exception of terms appearing in the the superpotential; see Section 7.8.

In the usual situations, including most models of inflation, global supersymmetry is supposed to be valid. Global supersymmetry may be broken either explicitly or softly (see below) and both possibilities are considered for inflation models. An important consideration for inflation model-building is the fact that soft susy breaking coming from the underlying supergravity theory (gravity-mediated susy breaking) has to be weaker than would be expected for a generic theory. Several proposals have been made for achieving this, and at present there is no consensus about which one is correct.

5.2 The lagrangian

The fields can be classified according to the spin of the corresponding particles; in the Standard Model one has spin 0 (Higgs), spin 1/2 (quarks and leptons) and spin 1 (gauge bosons). Fields with these spins are ubiquitous in extensions of the Standard Model. There is also the graviton with spin 2, and according to supergravity the gravitino with spin 3/2. At the particle physics level, a model of inflation consists of the relevant part of the lagrangian.

The spin-0 fields are called scalar fields, and they are what we need for inflation. Happily, there are lots of scalar fields in supersymmetric extensions of the Standard Model. This is because every spin 1/2 field is accompanied by either a spin 0 or a spin 1 field, with the first case ubiquitous.

During inflation, only scalar fields exist in the Universe. At the classical level, their evolution is determined by that part of the Lagrangian \mathcal{L} containing only the scalar fields.

even if the integrated-out fields are not particularly heavy; an example is provided by the dilaton and bulk moduli of string theory, which are usually integrated out when considering the other (‘matter’) fields.

If only a single, real, scalar field is relevant, the Lagrangian in flat spacetime is of the form

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi). \quad (124)$$

In this expression, $V(\phi)$ is the potential. The other term is called the kinetic term, and in it ∂_μ denotes the spacetime derivative $\partial/\partial x^\mu$. Up to a field redefinition, this is the only Lorentz-invariant expression containing first derivatives but no higher. The resulting equation of motion is

$$\ddot{\phi} - \nabla^2\phi + V'(\phi) = 0, \quad (125)$$

where the prime denotes $d/d\phi$.

For a spatially homogeneous field this becomes

$$\ddot{\phi} + V'(\phi) = 0. \quad (126)$$

This is the same as for a particle moving in one dimension, with position $\phi(t)$ and potential $V(\phi)$.

The assumption of flat spacetime corresponds to Special Relativity and negligible gravity. In the expanding Universe we need General Relativity, describing curved spacetime. Its effect on the field equation is to introduce an extra term $-3H\dot{\phi}$ on the left hand side, so that we get Eq. (29). This is analogous to a friction term for particle motion. The extra term is significant only in the context of cosmology.

With a suitable choice of the origin, a non-interacting (free) field has the potential $V = m^2\phi^2/2$ where m is the mass of the corresponding particle. The field equations has a time-independent, spatially homogeneous, solution $\phi = 0$, which represents the vacuum. Plane waves, corresponding to oscillations around the vacuum state, correspond after quantization to non-interacting particles of the species ϕ , which have mass m . Self interactions correspond to higher-order terms in $V(\phi)$. In a renormalizable theory, only cubic and quartic terms are allowed. The cubic term is usually forbidden by a symmetry, and dropping it the potential is

$$V = \frac{1}{2}m^2\phi^2 + \frac{1}{4}\lambda\phi^4. \quad (127)$$

It is assumed that $\lambda \lesssim 1$, because otherwise the interaction would become so strong that ϕ would not correspond to a physical particle (the non-perturbative regime). On the other hand, values of λ very many orders of magnitude less than 1 are not usually envisaged since they would represent fine-tuning.

The full potential will have an infinite number of terms, and including the cubic one for generality one can write

$$V(\phi) = V_0 + \frac{1}{2}m^2\phi^2 + \lambda_3 M_P\phi^3 + \frac{1}{4}\lambda\phi^4 + \sum_{d=5}^{\infty} \lambda_d M_P^{4-d}\phi^d + \dots. \quad (128)$$

The non-renormalizable ($d > 4$) couplings λ_d are generically of order 1, though they may be suppressed in a supersymmetric theory as we shall discuss.

All this extends to the case of several scalar fields ϕ, ψ , etc. With two fields, the simplest lagrangian density is

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi + \frac{1}{2}\partial_\mu\psi\partial^\mu\psi - V(\phi, \psi). \quad (129)$$

The field equations are

$$\ddot{\phi} - \nabla^2 \phi + \frac{\partial V(\phi, \psi)}{\partial \phi} = 0, \quad (130)$$

$$\ddot{\psi} - \nabla^2 \psi + \frac{\partial V(\phi, \psi)}{\partial \psi} = 0. \quad (131)$$

The extension to further fields is similar.

The potential as a function of all the fields will be a power series. With the origin in field space chosen to be the vacuum, as we are assuming at the moment, the power series for each field will have the form Eq. (128) (no linear term) provided that the other fields are fixed at the origin.

It is often appropriate to combine two real fields ϕ_1 and ϕ_2 into a single complex field, defined by convention as

$$\phi = \frac{1}{\sqrt{2}} (\phi_1 + i\phi_2). \quad (132)$$

The kinetic term corresponding to Eq. (129) is

$$\mathcal{L}_{\text{kin}} = \partial_\mu \phi^* \partial^\mu \phi. \quad (133)$$

The use of a complex field is particularly appropriate if the potential depends only on $|\phi|$. Then Eq. (127) is replaced by

$$V(\phi) = V_0 + m^2 |\phi|^2 + \frac{1}{4} \lambda |\phi|^4. \quad (134)$$

Complex fields are, in any case, part of the language of supersymmetry.

With two or more real fields, it is no longer true that the most general Lorentz-invariant lagrangian density \mathcal{L} can be reduced to the above form, Eq. (129), by a field redefinition. For several real fields ϕ_n , the most general kinetic term involving derivatives is

$$\mathcal{L}_{\text{kin}} = \sum_{m,n} H_{mn} \partial_\mu \phi_m \partial^\mu \phi_n, \quad (135)$$

where H_{mn} is an arbitrary function of the fields. In a supersymmetric theory, all fields are complex and the most general kinetic term has the more restricted form

$$\mathcal{L}_{\text{kin}} = \sum_{m,n} K_{mn^*} \partial_\mu \phi_m \partial^\mu \phi_n^*. \quad (136)$$

where $K_{mn^*} \equiv \partial^2 K / \partial \phi_m \partial \phi_n^*$ and K is called the Kähler potential.

We recover the canonical expression Eq. (129) only with the canonical choice if $K_{mn^*} = \delta_{mn}$. With more than one field it is not in general possible to recover this form by a field redefinition. If it is impossible, the space of the fields is said to be curved. One expects that the curvature scale will be of order M_{P} , allowing one to choose $K_{mn^*} = \delta_{mn}$ to high accuracy in the regime $|\phi_n| \ll M_{\text{P}}$.³⁷

A non-canonical kinetic term modifies the field equations, so that the slope of the potential no longer has its usual significance. Canonical normalization is assumed when describing slow-roll inflation.

³⁷This is the case if the origin $\phi_n = 0$ is chosen to so that the vacuum values of the fields are $\ll M_{\text{P}}$. As we shall see soon, a different choice is more natural for certain fields predicted by string theory.

5.3 Internal symmetry

5.3.1 Continuous and discrete symmetries

In addition to Lorentz invariance, the action will usually be invariant under a group of transformations acting exclusively on the fields, with no effect on the spacetime indices. This is called an internal symmetry.

Consider first the case of a single real field ϕ , with V a function of ϕ^2 as for example in Eq. (127). Then there is invariance under the Z_2 group $\phi \rightarrow -\phi$. Invariance under a group like this, which has only discrete elements, is called a discrete symmetry.

Now consider the case of a single complex field, with V depending on $|\phi|$ as for example in Eq. (134). Then there is invariance under the $U(1)$ group

$$\phi \rightarrow e^{i\chi}\phi \quad \chi \text{ arbitrary}, \quad (137)$$

with χ an arbitrary real number. This is the case for Eq. (134). Alternatively, there might be invariance under the Z_N group

$$\phi \rightarrow e^{i\chi}\phi \quad \chi = 2\pi n/N, \quad (138)$$

with n an arbitrary integer. In the limit where the integer N goes to infinity, the $U(1)$ group is recovered. Invariance under a continuous group like $U(1)$ is said to be a continuous symmetry.

A given symmetry group acts on some of the fields, but not on others. The action of a given Z_N or $U(1)$ on the full set of fields may be given by

$$\phi_n \rightarrow e^{iq_n\chi}\phi_n, \quad (139)$$

which defines the charge q_n of each field under the given symmetry.

In these expressions, the origin in field space has been taken to be the fixed point of the symmetry group. The gradient of the potential vanishes at the fixed point, which therefore represents a maximum, minimum or saddle point of the potential.

In a supersymmetric theory, it is usual to take all scalar fields to be complex. (Each of them is the partner of a spin-half field that has two components, corresponding to the two possible spin values.) If such a theory emerges from string theory, there are two kinds of field. The most numerous, usually called matter fields, transform under groups built out of $U(1)$'s (continuous symmetries) and Z_N 's (discrete symmetries). As in Eqs. (137) and (138) there is a unique fixed point in field space, which is generally chosen as the origin. For a given $U(1)$ (say) the transformation can be brought into the form Eq. (137) with a suitable choice of the directions in field space that define the ϕ_n , but in general this cannot be done for all of them simultaneously. We then have a non-abelian group (one whose elements do not commute) such as $SU(N)$.

In addition to the matter fields, there are special fields namely the dilaton s , and certain fields called bulk moduli. In the example we shall discuss in Sections 7.9 and 8.3 there are three of the latter, t_I with $I = 1$ to 3. The dilaton and bulk moduli are charged under discrete symmetry groups that are not built out of Eq. (137), and the most convenient choice of origin for these fields is not the fixed point of these symmetry groups.

A $U(1)$ symmetry is said to be global, if χ in Eq. (137) is independent of spacetime position. This is mandatory if no gauge field transforms under the $U(1)$, because then the spacetime derivatives in the kinetic term inevitably spoil the symmetry. If gauge fields have a suitable transformation under the $U(1)$, we can allow χ to depend on position, because the change in the kinetic term is cancelled by a change in the part of the action involving the gauge fields. The symmetry is then said to be a local symmetry, or a gauge symmetry. An example is the electromagnetic gauge field (electromagnetic potential) A_μ . This generalizes to non-abelian groups. There is an electromagnetic-like interaction associated with each gauge symmetry. The Standard Model is invariant under the gauge symmetry group $SU(3)_C \otimes SU(2)_L \otimes SU(1)_Y$, the factors corresponding respectively to the colour, left-handed electroweak and hypercharge interactions.

Generalizing from Eq. (139), the fields not affected by a given symmetry group are said to be uncharged under the group, or to be singlets.³⁸ It is usually supposed that every field is charged under some symmetry, though the opposite possibility of a ‘universal singlet’ is sometimes considered [246].

5.3.2 Spontaneously broken symmetry and vevs

Any minimum of the potential represents a possible vacuum state, with the scalar fields having the time-independent value corresponding to the minimum. (Such values are indeed solutions of the field equation Eq. (125)). In the examples encountered so far there is a unique minimum, but matters can be more complicated.

As a simple example, consider Eq. (127) with the sign of the mass term reversed,

$$V(\phi) = V_0 - \frac{1}{2}m^2\phi^2 + \frac{1}{4}\lambda\phi^4. \quad (140)$$

It has the same Z_2 symmetry as the original potential, corresponding to invariance under $\phi \rightarrow -\phi$. But as shown in Figure 3, the minimum at the origin is replaced by minima at $\phi = \pm(m/\sqrt{\lambda})$. Taking, say, the positive sign, one can define a new field $\tilde{\phi} = \phi - (m/\sqrt{\lambda})$. Then, if the constant V_0 is chosen appropriately, one has near the minimum

$$V = \frac{1}{2}\tilde{m}^2\tilde{\phi}^2 + A\tilde{\phi}^3 + B\tilde{\phi}^4, \quad (141)$$

where $\tilde{m} = \sqrt{2}m$ and we are not interested in the precise values of A and B . The minima represent possible vacuum expectation values (vevs) of the field. Each of them represents a possible vacuum of the theory, around which are small oscillations corresponding (after quantization) to particles. The oscillations correspond to an almost-free field if the cubic and quadratic terms in Eq. (141) are small. (It turns out that the criterion for this is $\lambda \lesssim 1$, which as in the previous case one assumes to be valid.) On the other hand, the original Z_2 symmetry will not be evident in this almost-free field theory, and one says that it has been spontaneously broken.

The vev of a field is denoted by angle brackets, so that in the above case one has $\langle\phi\rangle = \pm m/\sqrt{\lambda}$.

³⁸The latter terminology originated with the case of non-abelian groups, where each field charged under the group is necessarily part of a multiplet of charged fields.

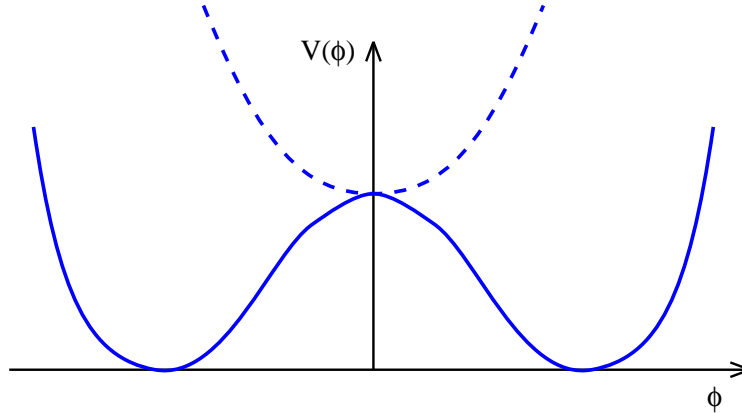


Figure 3: The full line illustrates schematically the potential Eq. (140). The dashed line shows the same potential with the sign of m^2 reversed (symmetry restoration). This Figure is taken from reference [195].

Now consider Eq. (134) with the sign of the mass term reversed,

$$V(\phi) = V_0 - m^2|\phi|^2 + \frac{1}{4}\lambda|\phi|^4. \quad (142)$$

The vacuum now consists of the circle $|\phi| = \langle|\phi|\rangle \equiv 2m/\sqrt{\lambda}$. About any point in the vacuum, there is a ‘radial’ mode of oscillation corresponding to the one we already considered, plus an ‘angular’ mode with zero frequency.

For a global symmetry, the particle corresponding to the angular mode is called the Goldstone boson of the symmetry, while the particle corresponding to the radial mode has no particular name. As we discuss in a moment, continuous global symmetries are usually broken, so that their Goldstone bosons acquire mass and become pseudo-Goldstone bosons. Examples are the pion (corresponding to the chiral symmetry of QCD) and the axion (corresponding to the hypothetical Peccei-Quinn symmetry that is proposed to ensure the CP invariance of QCD).

For a gauge symmetry, the particle corresponding to the radial mode is called a Higgs particle, while the would-be Goldstone boson loses its identity to become one of the degrees of freedom of the gauge boson. This case, generalized to the $SU(2)$ group, occurs in the electroweak sector of the Standard Model, and supersymmetric generalizations of it. More Higgs fields occur in GUT models.

The field which spontaneously breaks the symmetry, that we have denoted by ϕ , need not be one of the elementary fields appearing in the lagrangian. Instead it can be a product of these fields, called a condensate. The fields can be spin half, so if all symmetry-breaking scalars were condensates one would have no need of elementary scalars. The pion field is a condensate, and in some models so is the axion. In this case there need be no particle corresponding to the radial mode.

Higgs fields are usually taken to be elementary, because this is the simplest possibility. The desire to have elementary scalar fields is one of the most important motivations for supersymmetry.

The above discussion applies to matter fields, but a similar one applies to any internal symmetry and in particular to the dilaton and bulk moduli. The general criterion for spontaneous symmetry breaking is that the vacuum (the minimum of the potential) does not correspond to a fixed point of the symmetry; as a result of this there is more than one copy of the vacuum, different copies being related by the symmetry.

5.3.3 Explicitly broken global symmetries

Global (but not gauge) symmetries can be explicitly broken. This means that the action is not precisely invariant under the symmetry group.

Consider first the Z_2 symmetry acting on a real field, $\phi \rightarrow -\phi$. It is broken if one adds to the potential Eq. (127) or Eq. (140) an odd term.

Now consider a global $U(1)$ acting on a complex field, according to Eq. (137). It is broken if one adds to the the potential Eq. (134) or Eq. (142) a term that depends on the phase of ϕ . For instance, there might be a contribution of the form

$$\Delta V = \lambda_d M_{\text{P}}^{4-d} \left(\frac{\phi^d + \phi^{*d}}{2} \right). \quad (143)$$

Instead of being generated from the tree-level potential in this way, $\Delta V(\theta)$ can come from a non-perturbative effect (to be precise, an instanton).

With explicit breaking, a Goldstone boson acquires mass, to become a pseudo Goldstone boson. This case occurs in QCD, where the pion is a pseudo-Goldstone boson. The axion (if it exists) is also a pseudo-Goldstone boson. If we write $\phi = \langle |\phi| \rangle e^{i\theta}$, the canonically normalized pseudo-Goldstone boson field is $\psi \equiv \sqrt{2} |\phi| \theta$. Its potential $V(\theta)$ has period $2\pi/N$ where N is some integer. For $N \geq 2$, the original $U(1)$ symmetry Eq. (137) has been broken down to the residual symmetry Z_N Eq. (138). For $N = 1$, there is no residual symmetry.

In the above example

$$\Delta V(\theta) = \lambda_d M_{\text{P}}^4 \left(\frac{\langle |\phi| \rangle}{M_{\text{P}}} \right)^d \cos(d\theta). \quad (144)$$

Defining the zero of ψ to be at a minimum of V , one finds

$$m_\psi^2 = d^2 \lambda_d M_{\text{P}}^2 \left(\frac{\langle |\phi| \rangle}{M_{\text{P}}} \right)^{d-2}. \quad (145)$$

Provided that m_ψ is much less than m_ϕ , the radial part of ϕ which has the latter mass can remain practically at the vev while ψ oscillates. For much bigger values this becomes impossible, and we have completely lost the original symmetry.

It is usually supposed that all continuous global symmetries are approximate. One reason is that this seems to be the case for field theories derived from string theory [56, 20, 75]. In contrast, they typically contain many discrete symmetries [302].

5.3.4 The restoration of a spontaneously broken internal symmetry

In the early Universe, the scalar fields will be displaced from their vacuum expectation values (vevs). In particular, a field with a nonzero vev, corresponding to a spontaneously broken symmetry, may have zero value in the early Universe. Then the symmetry is restored at early times. This may happen during inflation, and also during the subsequent hot big bang.

A simple example which can illustrate both cases is the following potential, involving real fields ϕ and ψ .

$$V = V_0 - \frac{1}{2}m_\psi^2\psi^2 + \frac{1}{4}\lambda\psi^4 + \frac{1}{2}m^2\phi^2 + \frac{1}{2}\lambda'\psi^2\phi^2 \quad (146)$$

$$= \frac{1}{4}\lambda(M^2 - \psi^2)^2 + \frac{1}{2}m^2\phi^2 + \frac{1}{2}\lambda'\psi^2\phi^2. \quad (147)$$

Comparing the two ways of writing the potential, one sees that the parameters are related by

$$m_\psi^2 = \lambda M^2, \quad (148)$$

$$V_0 = \frac{1}{4}\lambda M^4 = \frac{1}{4}M^2 m_\psi^2. \quad (149)$$

The minimum of V , corresponding to the vacuum, is at $\phi = 0$ and $\psi = M$. The latter field has a nonzero vev, spontaneously breaking the discrete symmetry $\psi \rightarrow -\psi$. But now suppose that, in the early Universe, ϕ^2 has a nonzero value, bigger than a critical value $\phi_c^2 = m_\psi^2/\lambda'$. Then the minimum with respect to ψ lies at $\psi = 0$, and the symmetry is restored. With the relabelling $\psi \rightarrow \phi$, this is illustrated by the dashed line in Figure 3.

In an appropriate region of parameter space, the fields can be in thermal equilibrium at temperature $T \gtrsim \phi_c$, making ϕ^2 typically of order T^2 . Then the symmetry $\psi \rightarrow -\psi$ is restored for $T \gtrsim \phi_c$, and spontaneously breaks as T falls below that value.

Alternatively, ϕ might be the inflaton. Then the symmetry $\psi \rightarrow -\psi$ is restored until ϕ falls below ϕ_c , after which it spontaneously breaks. If V_0 dominates, this signals the end of inflation and we have hybrid inflation [204]. Even if it does not, the change might correspond to a feature in the spectrum $\mathcal{P}_{\mathcal{R}}$, or topological defects [166, 296, 167, 304, 170, 268, 214, 137, 238]. (This is an alternative to the familiar Kibble mechanism [157] of defect formation, which applies if the symmetry is restored by thermal effects.)

5.4 The true vacuum and the inflationary vacuum

The different vacua, that occur when a symmetry is spontaneously broken, are physically equivalent, and are simply referred to as *the vacuum*.³⁹

³⁹For simplicity we are supposing that the vacuum so-defined is unique. In general the potential might have another minimum (or set of minima related by a spontaneously broken symmetry) in which V has a different value; or there might be three or more minima with different values of V . In these cases, it is not clear whether the vacuum corresponding to our Universe (the one with $V = 0$) must be the global minimum (the one with the lowest value of V). If it is not the global minimum, the lifetime for tunneling to the latter should presumably be much bigger than the age of the Universe. Examples of multiple vacua are shown in Figures 5 and 6.

During inflation, the spatially-averaged inflaton field ϕ is not at a minimum of the potential, and it varies slowly with time. The spatially-averaged non-inflaton fields mostly adjust themselves to be at the instantaneous minimum of the potential with ϕ at the current value, which may or may not be the same as the vacuum value. Others may have extremely flat potentials, giving negligible motion for the spatial average.

These spatially-averaged fields provide a classical background, around which are the quantum fluctuations described by quantum field theory.⁴⁰ The classical background may be taken to be constant, because the variation of V is slow on the Hubble timescale. It defines an effective vacuum for a quantum field theory, which we shall call the inflationary vacuum. To emphasize the distinction, we shall often call the actual vacuum, corresponding to the minimum of the potential, the true vacuum.

In some applications, such as when calculating the vacuum fluctuation of the inflaton field, it is necessary to formulate quantum field theory in the setting of curved spacetime (the expanding Universe). The main difference though, between the inflationary vacuum and the true one, is the value of the vacuum energy density V . In the true vacuum it is practically zero ($|V|^{1/4} \lesssim 10^{-3} \text{ eV}$, corresponding to the bound on the cosmological constant). During inflation it is big.

From this perspective *two separate searches are in progress, for quantum field theories beyond the Standard Model*. There is the search for the field theory that applies in the true vacuum, and the search for the field theory that applies during inflation. In some proposals these theories are very different, whereas in others they are almost the same. Roughly speaking, the former proposals predict that the inflationary energy scale is $V^{1/4} \gg 10^{10} \text{ GeV}$, and the latter predict that it is in the range $10^5 \text{ GeV} \lesssim V^{1/4} \lesssim 10^{10} \text{ GeV}$.

5.5 Supersymmetry

Practically all viable extensions of the Standard Model invoke supersymmetry. The main reason is that they invoke fundamental scalar fields, which look natural only in the context of supersymmetry. Indeed, supersymmetry eliminates the quadratic divergences in the mass m^2 of fundamental light scalar fields, $\delta m^2 \sim \Lambda_{\text{UV}}^2$, Λ_{UV} being the scale beyond which the low energy theory no longer applies. In about ten years, the Large Hadron Collider (LHC) at CERN will either discover supersymmetry, if it has not been discovered before then, or practically kill it. In the latter eventuality the task of understanding whatever *is* observed at the LHC will take precedence over such relatively trivial matters as inflation model-building, so let us suppose optimistically that supersymmetry is valid.

We shall consider supersymmetry in detail in Section 7, but let us note a few important points. Supersymmetry is an extension of Lorentz invariance, and therefore not an internal symmetry. It relates bosons and fermions. In the ‘ $N = 1$ ’ version generally adopted, there are ‘chiral’ supermultiplets each containing a complex scalar field (spin 0) plus a chiral fermion (spin 1/2) field, and ‘gauge’ supermultiplets each containing a gauge field (spin 1) and a gaugino (spin 1/2) field. Each Standard Model particle has an undiscovered superpartner; there are squarks and sleptons with spin 0, Higgsinos with spin 1/2 and

⁴⁰The averaging is to be done over the comoving box within which quantum field theory is formulated. It should be large compared with the comoving scale presently equal to the size of the observable Universe, but it is neither necessary nor desirable to make it exponentially bigger.

gauginos with spin 1/2. (It turns out that at least two Higgs fields are required.)

Supersymmetry is expected to be local as opposed to global, and local supersymmetry is called supergravity because it automatically incorporates gravity. In $N = 1$ supergravity, the graviton (spin 2) is accompanied by the gravitino (spin 3/2). Global supersymmetry provides a good approximation to supergravity for most purposes.

To decide between different possible forms of field theory, and in particular supergravity, one may look to a hopefully more fundamental theory like weakly coupled string theory or Horava-Witten M-theory.

Unbroken supersymmetry would require that each particle has the same mass as its partner. This is not observed, so supergravity is spontaneously broken in the true vacuum. (A local symmetry cannot be explicitly broken.) The scale of this breaking is conveniently characterized by a scale M_S , related to the gravitino mass $m_{3/2}$ by

$$M_S^2 = \sqrt{3}M_P m_{3/2}. \tag{150}$$

To have a viable phenomenology, the spontaneous breaking is supposed to occur in a ‘hidden sector’ of the theory, communicating only weakly with the ‘visible’ sector containing particles with Standard Model interactions. In the visible sector, one has for most purposes global supersymmetry with explicit breaking of a special kind, called ‘soft supersymmetry breaking’. Soft susy breaking must give the squarks and sleptons masses

$$\tilde{m} \sim 100 \text{ GeV to } 1 \text{ TeV}. \tag{151}$$

(Gauginos may also have such masses, or they may be lighter.) This typical ‘soft’ mass \tilde{m} is an important parameter for model building. It cannot be much above 1 TeV or susy would not do its job of allowing us to understand the existence of the Standard Model Higgs field. Nor can it be much less than 100 GeV, or the squarks and sleptons would have been observed.

The relation between M_S and \tilde{m} is model-dependent. In a class of theories known as gravity-mediated one has

$$M_S \simeq \sqrt{\tilde{m}M_P} \sim 10^{10} \text{ to } 10^{11} \text{ GeV}. \tag{152}$$

(For definiteness we usually take 10^{10} GeV in what follows.) Then $m_{3/2} \sim \tilde{m}$. In another class, called gauge-mediated, M_S can be anywhere between 10^6 GeV and 10^{11} GeV, corresponding to $1 \text{ keV} \lesssim m_{3/2} \lesssim 1 \text{ TeV}$.

All this refers to the true vacuum. During inflation, susy is also necessarily broken. In most models the mechanism of susy breaking during inflation has nothing to do with the mechanism of susy breaking in the true vacuum (and is much simpler). In an interesting class of models, the mechanism is supposed to be the same. As a rough guide, inflation models with $V^{1/4} \gg M_S$ fall into the first class, while models with a lower V fall into the second.

5.6 Quantum corrections to the potential

So far we specified the part of the Lagrangian involving only the scalar fields. When quantum effects are included, this is not enough to describe these fields; we need the rest

of the lagrangian, that describing higher-spin fields that can couple to scalar fields. During inflation, when the scalar fields are almost independent of position, these effects can be summarized by giving an effective potential V and (if necessary) an effective kinetic function K_{mn*} , which are to be used in the field equation Eq. (125) or its non-canonical equivalent. Note that we are using the same symbols for the effective objects and the ones that appear in the lagrangian.

Quantum effects are determined by the couplings of the fields (as well as their masses). Gauge couplings (couplings to gauge fields) are characterized by a dimensionless constant g , or equivalently by $\alpha = g^2/4\pi$. (For electromagnetism, g is the electron charge and α evaluated at low energy is the fine structure constant $\alpha_{\text{em}} = 1/137$.) Couplings not involving gauge fields, called Yukawa couplings, can again be characterized by dimensionless constants. Complex scalar fields with no gauge couplings are called gauge singlets, and both their radial and angular components (pseudo-Goldstone bosons) are favourite candidates for the inflaton field.

Quantum effects are of two kinds; the perturbative effects represented by Feynman graphs, and the non-perturbative effects represented by things like instanton contributions to the path integral. This separation is meaningful only if the relevant couplings are small, in particular if gauge couplings satisfy $\alpha \lesssim 1$. At large couplings the theory is completely non-perturbative.

Gauge couplings are not supposed to be extremely small, and one should take $g \sim 1$ for crude order of magnitude estimates (making α one or two orders of magnitude below 1). For renormalizable Yukawa couplings, values a few orders of magnitude below unity are generally regarded as reasonable, at least for the renormalizable couplings in an effective field theory.

5.6.1 Gauge coupling unification and the Planck scale

With quantum effects included, the masses and couplings to be used in the lagrangian depend on the relevant energy scale Q . The dependence on Q (called ‘running’) can be calculated through the renormalization group equations (RGE’s), and is logarithmic. In the context of collider physics, Q can be taken to be the collision energy, if there are no bigger relevant scales (particle masses). In the context of inflation, Q can be taken to be the value of the inflaton field if, again, there are no bigger relevant scales (particle masses, or values of other relevant fields).

For the Standard Model there are three gauge couplings, α_i where $i = 3, 2, 1$, corresponding respectively to the strong interaction, the left-handed electroweak interaction and electroweak hypercharge. (The electromagnetic gauge coupling is given by $\alpha^{-1} = \alpha_1^{-1} + \alpha_2^{-1}$.) In the one-loop approximation, ignoring the Higgs field, their running is given by

$$\frac{d\alpha_i}{d\ln(Q^2)} = \frac{b_i}{4\pi}\alpha_i^2. \quad (153)$$

The coefficients b_i depend on the number of particles with mass $\ll Q$. Including all particles in the minimal supersymmetric Standard Model particle gives $b_1 = 11$, $b_2 = 1$ and $b_3 = -3$.

Using the values of α_i measured by collider experiments at a scale $Q \simeq 100$ MeV, one

finds that all three couplings become equal at a scale [10, 117, 92, 184, 230]⁴¹ $Q = M_{\text{GUT}}$, where

$$M_{\text{GUT}} \simeq 2 \times 10^{16} \text{ GeV}. \quad (154)$$

The unified value is

$$\alpha_{\text{GUT}} \simeq 1/25. \quad (155)$$

One explanation of this remarkable experimental result may be that there is a Grand Unified Theory (GUT), involving a higher symmetry with a single gauge coupling, which is unbroken above the scale M_{GUT} . Another might be that field theory becomes invalid above the unification scale, to be replaced by something like weakly coupled string theory or Horava-Witten M-theory, which is the source of unification. At the time of writing there is no consensus about which explanation is correct [70].

5.6.2 The one-loop correction

The perturbative part of the effective potential is given by a sum of terms, corresponding to the number of loops in the Feynman graphs. The no-loop term is called the tree-level term (because the Feynman graphs look like trees) and it has the power-series form Eq. (128).

In any given situation, one can usually choose the renormalization scale Q so that the loop corrections are small. Then, the 1-loop correction typically dominates, and only it has so far been considered in connection with inflation model-building.

We now discuss the form of the 1-loop correction, initially making the choice $Q = M_{\text{P}}$. In a supersymmetric theory, in the usual case that ϕ is much bigger than the masses of the particles in the loop, two cases arise.

If the relevant part of the Lagrangian is supersymmetric, corresponding to spontaneous susy breaking, the loop correction is typically of the approximate form

$$\delta V \simeq V c \ln(\phi/M_{\text{P}}), \quad (156)$$

where V is the tree-level potential. In this expression, c is related to the dimensionless coupling g (between ϕ and the field in the loop) by something like $c \sim g^2/(8\pi^2)$. It is often called a loop suppression factor, because in a typical situation each additional loop gives another factor c . If the field in the loop is a gauge supermultiplet, c is expected to be of order 10^{-1} to 10^{-2} . If it contains a chiral supermultiplet, g is a Yukawa coupling and c might be of the same order, or it might be a few orders of magnitude smaller.

The other case typically arises if there is soft susy breaking in the relevant sector. The loop correction in this case is of the approximate form

$$\delta V \simeq \frac{1}{2} c \mu^2 \phi^2 \ln(\phi/M_{\text{P}}). \quad (157)$$

Assuming that we are dealing with a flat direction the complete potential is now

$$V = V_0 + \frac{1}{2} m^2(\phi) \phi^2 + \dots, \quad (158)$$

⁴¹To be precise, $\frac{5}{3}\alpha_1 = \alpha_2 = \alpha_3 = \alpha_{\text{GUT}}$, the factor $5/3$ arising because the historical definition of α_1 is not very sensible. In passing we note that the unification fails by many standard deviations in the absence of supersymmetry, which may be construed as evidence for supersymmetry and anyhow highlights the remarkable accuracy of the experiments leading to this result.

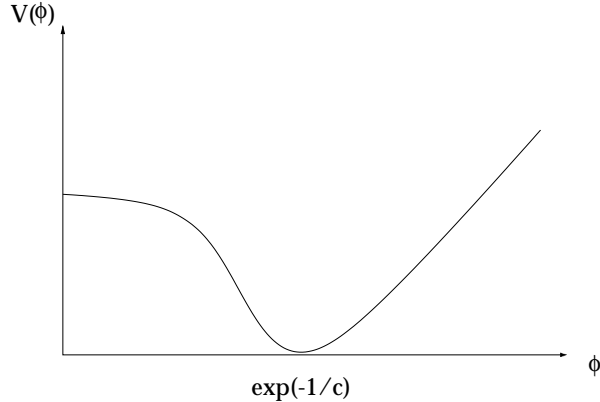


Figure 4: A non-perturbative loop correction generates a minimum in the potential. The minimum corresponds to $\phi \sim \exp(-1/c)M_{\text{P}}$, where $c \ll 1$ is a loop suppression factor.

where

$$m^2(\phi) = m^2 + c\mu^2 \ln(\phi/M_{\text{P}}), \quad (159)$$

and the dots represent non-renormalizable terms.

Let us consider a typical case, where the parameter μ is of order m . Because the loop suppression factor c is $\ll 1$, V will have a maximum or minimum at $\phi \sim e^{-1/c}M_{\text{P}}$. The minimum occurs if the mass-squared is positive at the Planck scale. This case is illustrated in Figure 4. The maximum occurs if the mass-squared is negative at the Planck scale. In that case there is a minimum at $\phi = 0$, and another at some value $\phi_{\text{min}} \gtrsim \exp(-1/c)M_{\text{P}}$ which is determined by the non-renormalizable terms of the tree-level potential. Typically, ϕ_{min} is the global minimum. If $\phi = 0$ is our vacuum, V vanishes there as shown in Figure 5. In that case, the lifetime for tunneling to the global minimum had better be much longer than the age of the Universe. If $\phi = 0$ is not our vacuum, one can have either of the situations shown in Figures 6 and 7. We shall see in Section 8.6 how they permit one to construct models of inflation.

A notable feature of these expressions is that they generate a scale many orders or magnitude less than M_{P} without fine-tuning. This occurs because the loop suppression factor, say a couple of orders of magnitude below 1, is exponentiated. This phenomenon is known as dimensional transmutation, and optimistically one may suppose that with its help all mass scales can be generated more or less directly from the Planck scale.

For an accurate calculation of the potential $V(\phi)$ we should abandon the choice $Q = M_{\text{P}}$ for the renormalization scale. With a general scale Q , the potential becomes

$$V(Q, \phi) = V_0 + \frac{1}{2}m^2(Q)\phi^2 + \frac{1}{2}c(Q)\mu^2(Q) \ln(\phi/Q). \quad (160)$$

At a given value of ϕ , the 1-loop correction vanishes if we set $Q \simeq \phi$. The 2-loop and higher corrections are then hopefully small, and we obtain Eq. (158) with $m^2(\phi) \equiv m^2(Q \simeq \phi)$ now given by the RGE's instead of by Eq. (159). The RGE for m^2 is

$$\frac{dm^2(Q)}{d \ln Q} = c(Q)\mu^2(Q). \quad (161)$$

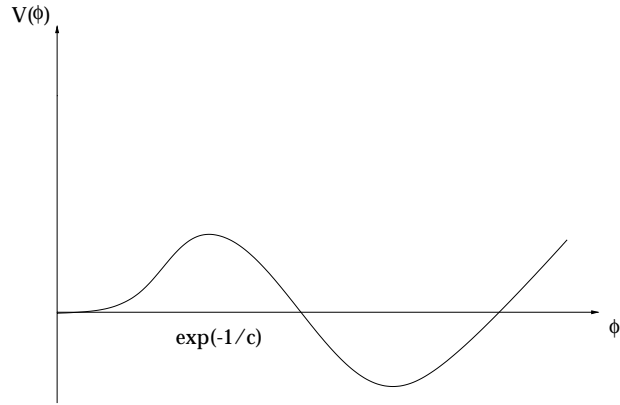


Figure 5: A non-perturbative loop correction generates a maximum in the potential, at a value $\phi \sim \exp(-1/c)M_{\text{P}}$ hierarchically smaller than the Planck scale. Non-renormalizable terms generated a minimum, at a bigger ϕ which may or may not be of order M_{P} . There is another minimum at $\phi = 0$, which typically corresponds to a bigger value of ϕ . In the true vacuum, V vanishes. As shown in the graph, the true vacuum may be at $\phi = 0$.

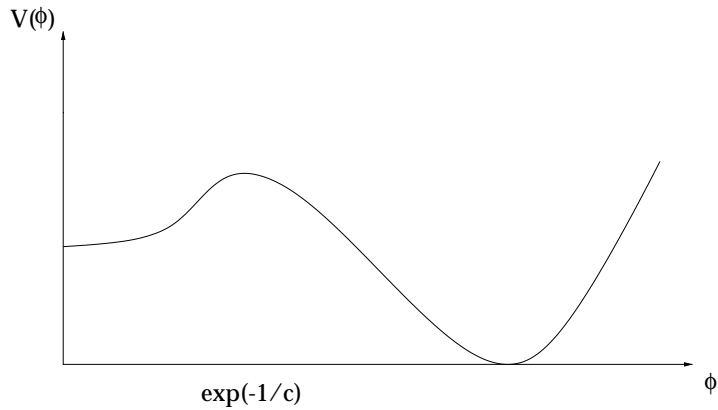


Figure 6: Alternatively, the true vacuum may be at the minimum with nonzero ϕ .

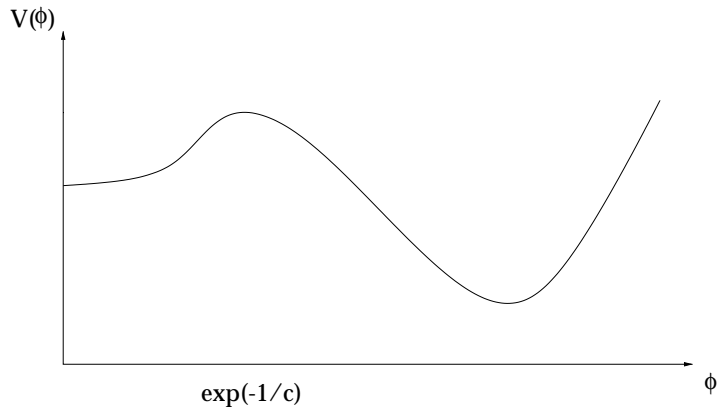


Figure 7: A third possibility is that neither of the minima correspond to the true vacuum. Rather, it lies in another field direction, ‘out of the paper’.

Those of c and μ will also be first order differential equations, and $m(Q)$ is determined by solving the equations simultaneously as in Section 8.6. If c and μ have negligible running we recover Eq. (159).

Being a physical quantity, $V(Q, \phi)$ should actually be independent of Q , so that $\partial V/\partial Q$ vanishes. Including only 1-loop corrections, this is guaranteed at the point $Q = \phi$ by the RGE of $m(Q)$. Well away from the point $Q = \phi$, $\partial V/\partial Q$ becomes significantly different from zero if we include only the 1-loop correction. This is what one would expect, since the 1-loop correction then becomes big which indicates that the 2-loop and higher corrections need to be included.

5.7 Non-perturbative effects

5.7.1 Condensation and dynamical supersymmetry breaking

Since b_3 is negative, the QCD coupling $\alpha_3(Q)$ increases as Q is decreases, and it becomes of order 1 at the scale $Q = \Lambda_{\text{QCD}} \sim 100 \text{ MeV}$.⁴² On smaller scales we are in the non-perturbative regime. In this regime, quarks and gluons bind into hadrons, and should no longer be included in a perturbative calculation. Also, products of two quark fields acquire nonzero vevs,

$$\langle q\bar{q} \rangle \sim \Lambda_{\text{QCD}}^3, \quad (162)$$

(spin 1/2 fields have the energy dimension 3/2, whereas scalar fields have dimension 1).

Note that the large hierarchy between the GUT scale, and the scale $\Lambda_{\text{QCD}} \sim 100 \text{ MeV}$ corresponding to $\alpha_3 = 1$, is generated naturally by the RGE's. Indeed, from Eq. (153)

$$\frac{\Lambda_{\text{QCD}}}{M_{\text{GUT}}} = \exp\left(-\frac{1}{b_3 c}\right) \quad (163)$$

with

$$c = \frac{\alpha_{\text{GUT}}}{2\pi}. \quad (164)$$

With a view to generating supersymmetry breaking, it is usually supposed that the behaviour exhibited by QCD occurs also for some other gauge interaction. The particles with this interaction should not possess the Standard Model interactions, and correspond to the hidden sector mentioned earlier. One can again have spin-1/2 condensates $\langle \lambda\bar{\lambda} \rangle \sim \Lambda_c^3$, where λ can be either a chiral fermion field as in QCD, or a gaugino field. The condensation scale Λ_c of the hidden sector may be far bigger than Λ_{QCD} .

5.7.2 A non-perturbative contribution to the potential

Above the condensation scale, the effect on the potential is to introduce a term like Λ_c^{4+p}/ϕ^p . In a flat direction, it can be stabilized by a non-renormalizable tree-level term $\phi^{4+m}/M_{\text{P}}^m$, to generate a large vev given by

$$\langle \phi \rangle \sim \left(\frac{\Lambda_c}{M_{\text{P}}}\right)^{\frac{4+p}{4+p+m}} M_{\text{P}} \sim e^{-1/c} M_{\text{P}}. \quad (165)$$

(To obtain the final equality, we used the generalization of Eq. (163).)

⁴²For $Q \lesssim 100 \text{ GeV}$, the value of b_3 changes as massive particles cease to be effective, but it remains negative.

5.8 Flatness requirements on the tree-level inflation potential

So far our discussion of the potential has been quite general. Now we want to specialize to the case where ϕ is the inflaton field. We shall formulate conditions on the tree-level potential that ought to be satisfied in any model of inflation, and ask how they can be satisfied in a supersymmetric theory.

During inflation, the tree-level potential with all other fields fixed will be of the form Eq. (128). The mass-squared (and for that matter the coefficients of higher-order terms) can have either sign; equivalently we can make the convention that all coefficients are positive and there is a plus or minus sign in front of them. We adopt the latter convention so that $V = V_0 \pm \frac{1}{2}m^2\phi^2 \dots$.

In Eq. (128) the origin has been chosen as a point where V' vanishes, and before proceeding we want to comment on this choice. As mentioned earlier (page 41) string-derived field theories contain matter fields on the one hand, and the dilaton and bulk moduli on the other.

In the space of the matter fields, the origin is usually chosen to be the (unique) fixed point of the internal symmetries. The derivatives of V vanish there. In most models of inflation, the inflaton is supposed to be the radial part of a matter field, with this choice of origin. Then V' vanishes at the origin, provided that any other matter fields coupling to the inflaton vanish during inflation.

If there are nonzero matter fields coupling to the inflaton, or if the inflaton is a pseudo-Goldstone boson (corresponding essentially to the real part of a matter field with a displaced origin), we simply define the origin as a point where V' vanishes.

Finally we come to the case that the inflaton is the real or imaginary part of a bulk modulus or the dilaton, with some choice of the origin. For these fields the usual choice of origin is not at all useful, so we again choose the origin as a point where V' vanishes. In this case we expect ϕ to be of order M_{P} during inflation, whereas if ϕ is a matter field it is usually much smaller.

Assuming canonical normalization of the fields, inflation requires that the potential satisfies the flatness conditions $\epsilon \ll 1$ and $|\eta| \ll 1$, where $\epsilon \equiv \frac{1}{2}M_{\text{P}}^2(V'/V)^2$ and $\eta \equiv M_{\text{P}}^2V''/V$ (Eqs. (31) and (32)). As mentioned in Section 5.2, canonical normalization is not expected to hold if $\phi \gtrsim M_{\text{P}}$, but should be a good approximation if ϕ is significantly below M_{P} . In what follows, we assume at least approximate canonical normalization, and $\phi \lesssim M_{\text{P}}$.⁴³

We want to see how the two flatness conditions constrain the tree-level potential Eq. (128). As we have seen, quantum corrections have to be added to the tree-level expression. They may give a significant or even dominant contribution to the slope of the potential. But it is reasonable to assume that this contribution does not accurately cancel the tree-level contribution, over the whole relevant range of ϕ values (the values corresponding to horizon exit for cosmological scales).⁴⁴ By the same token, one can assume that there is no accurate

⁴³A different case, where inflation happens at $\phi \sim M_{\text{P}}$ and the kinetic function becoming singular at slightly higher values, is discussed briefly in Section 6.6.

⁴⁴In this context, we are regarding the use of a running inflaton mass (Section 6.16) as still a tree-level effect. Note also that in mutated hybrid inflation (Section 6.13) there is an additional contribution to the inflaton potential, coming from the implicit dependence in $V(\phi) = V(\phi, \psi(\phi))$. In that case our discussion can be taken to apply to $V(\phi, 0)$.

cancellation between different terms of the tree-level potential.

The assumptions that $\phi \lesssim M_{\text{P}}$ and that there are no cancellations lead to considerable simplification. The flatness conditions require $V \simeq V_0$, since if a single term were to dominate V the flatness conditions would certainly be violated. Also, the second flatness condition implies the first for each tree-level term.

We will consider a slightly more precise version of the second flatness condition, $|\eta| \lesssim 0.1$. (Barring a cancellation between the two terms in Eq. (54) this follows from the observational bound Eq. (24).) With the stated assumptions this gives the following bound on each coefficient

$$\frac{M_{\text{P}}^2 m^2}{V_0} \lesssim 0.1 \quad (166)$$

$$3\lambda \left(\frac{\phi}{M_{\text{P}}}\right)^2 \lesssim 0.1 \frac{V_0}{M_{\text{P}}^4}, \quad (167)$$

$$d(d-1)\lambda_d \left(\frac{\phi}{M_{\text{P}}}\right)^{d-2} \lesssim 0.1 \frac{V_0}{M_{\text{P}}^4}. \quad (168)$$

These bounds simply say that the contribution of each term to V is at most of order $0.1(\phi/M_{\text{P}})^2 V_0/d^2$; this is essentially the same as our assumption that these contributions are $\ll V_0$, given our other assumption $\phi \lesssim M_{\text{P}}$.

The first constraint Eq. (166) gives a constraint on the inflaton mass, which is independent of the field.

It looks at first sight as if the other inequalities can always be ensured by making ϕ very small, but matters are not so simple because Eqs. (44) and (39) require ϕ to vary appreciably. To be precise, the biggest and smallest scales probed by COBE differ by a factor of 50 or so, corresponding to $\Delta N \simeq 4$. Using Eqs. (44) and (39), we learn that while these scales are leaving the horizon,

$$\frac{V_0}{M_{\text{P}}^2 \phi^2} \lesssim \frac{V_0}{M_{\text{P}}^2 (\Delta\phi)^2} \simeq \frac{(.027)^2}{2\Delta N} \simeq 2 \times 10^{-8}. \quad (169)$$

Putting this into Eq. (167) we find

$$3\lambda \lesssim 2 \times 10^{-9}, \quad (170)$$

Putting it into Eq. (168) gives

$$d(d-1)\lambda_d \lesssim 2 \times 10^{-9} \left(\frac{2 \times 10^{-8} M_{\text{P}}^4}{V_0}\right)^{\frac{d-4}{2}}. \quad (171)$$

5.9 Satisfying the flatness requirements in a supersymmetric theory

These constraints are quite strong in the context of received ideas about particle theory. Consider first the constraint Eq. (166), on the inflaton mass. In a globally supersymmetric theory (or a non-supersymmetric theory) the constraint poses no particular problem since the mass can be set to an arbitrarily small value. Unfortunately, the corrections to global

susy coming from a generic supergravity theory are not small during inflation; rather, they give $m^2 \gtrsim V_0/M_{\text{P}}^2$ for every scalar field and in particular for the inflaton [60, 285].⁴⁵ Therefore, to construct a model of inflation in the context of supergravity, one must either invoke an accidental cancellation, or a non-generic supergravity theory. We shall have much more to say about the problem of keeping the inflation mass small in the context of supergravity.

For the constraints on λ and λ_d , we need to consider separately the case that the inflaton is the radial part of a matter field (the usual case), and the case that it is a bulk modulus or the dilaton (more precisely, the real or imaginary part of one of these with respect to some origin).

5.9.1 The inflaton a matter field

Consider first the constraint Eq. (170) on λ . If ϕ is a generic field, one does not expect λ to be so small. But in a globally supersymmetric theory, the potential is typically independent of some of the fields, when the others are held at the origin. Such fields are called ‘flat directions’ (in field space).⁴⁶ This makes $\lambda = 0$ in the globally supersymmetric theory. When we go the full supergravity theory, we generically find in a flat direction that λ is of order V_0/M_{P}^4 . Then, the flatness condition Eq. (167) is satisfied provided that $\phi \ll M_{\text{P}}$.

Now we consider Eq. (168), omitting the cubic term $d = 3$ since it is usually forbidden by a symmetry. Even in a flat direction, the non-renormalizable couplings λ_d are generically of order 1, at least for d not too large.⁴⁷ In that case, Eq. (171) becomes an upper bound on V_0 . For $d = 5$ it gives $V_0^{1/4} < 3 \times 10^{11}$ GeV, and for $d = 6$ it gives $V_0^{1/4} \lesssim 1 \times 10^{14}$ GeV. For $d \rightarrow \infty$ it becomes $V^{1/4} \lesssim 1.5 \times 10^{16}$ GeV which is anyhow more or less demanded by the COBE normalization. (Not a coincidence, as one sees by examining the argument that led to Eq. (171).)

For low d these bounds are violated in many models of inflation. In these cases, at least some of the λ_d must be below the generic value $\lambda_d \sim 1$. But provided that ϕ is well below M_{P} , this will be needed only for the first few coefficients, and it is enough to make these of order V_0/M_{P}^4 . As we shall see in Section 8.2, that can be achieved in a supersymmetric theory by imposing a discrete symmetry.

In some models, notably D -term inflation, ϕ is of order M_{P} rather than much less. In that case [178] *all* of the λ_d (as well as λ) need to be significantly *less* than V_0/M_{P}^4 . This

⁴⁵This fact was first recognized in References [248, 61, 81], but the last two did not consider the case of the inflaton. The first, working actually in the context of minimal supergravity, took the view that a sufficiently small mass will occur through an accidental cancellation.

⁴⁶The direction is flat only when other fields coupling to it vanish; for instance if ϕ is a flat direction, and there is a term $\phi^2\psi^2$ in the potential, a nonzero value of ψ will lift the flatness. At this point, we should make it clear that the flat direction can be a linear combination of the fields that one would naturally choose; for instance in the above example the natural fields (with say definite charges under a $U(1)$ symmetry) might be $(\phi \pm \psi)$.

⁴⁷For large d , λ_d might be suppressed by a large d -dependent factors. For instance, if supergravity were obtained by integrating out heavy fields, from some renormalizable field theory valid on scales bigger than M_{P} , then one might expect $|\lambda_d| \sim 1/d!$ [178]. Such is not the case, but we are reminded that d -dependent factors might be present when supergravity is matched to say a string theory. As our estimates of the λ_d apply only if d is not too large, and are anyhow very rough, the factor $d(d-1)$ in Eq. (168) cannot be taken seriously, and we set it equal to 1 in what follows.

can be achieved by imposing an exact $U(1)$ (or higher) symmetry, but in the usual case that the inflaton is a gauge singlet the symmetry would have to be global and as we noted on page 44 global continuous symmetries do not seem to be present in field theories derived from string theory [56, 20, 75]. Accordingly, models of inflation with $\phi \sim M_{\text{P}}$ are at present quite speculative. One possibility [178] is that the coefficients λ_d actually fall off rapidly at large d , so that only the first few need be suppressed.

According to these estimates, the power series expansion Eq. (128) ceases to be reliable for $\phi \gg M_{\text{P}}$. In this regime one has in general no idea what form the potential will take.

5.9.2 The inflaton a bulk modulus or the dilaton

If the inflaton ϕ is the real or imaginary part of a bulk modulus (with some choice of origin) its potential during inflation will be of the form

$$V = A + Bf(x). \quad (172)$$

Here, $x = \phi/M_{\text{P}}$ and $f(x)$ and its derivatives are generically of order 1 in magnitude. Also, ϕ is typically of order M_{P} during inflation.

The constant term A can be negligible, or can dominate V . If it is negligible, it is clear that the flatness conditions $|\eta| \ll 1$ and $\epsilon \ll 1$ are marginally violated. (In terms of the coefficients we have $m^2 \sim V_0/M_{\text{P}}^2$ and $\lambda \sim \lambda_d \sim V_0/M_{\text{P}}^4$.) If the constant term dominates, the flatness conditions are satisfied.

The potentials of the real and imaginary parts of the dilaton are very model dependent, but they are often supposed also to be of the above form, with A negligible.

6 Forms for the potential; COBE normalizations and predictions for n

At the lowest level, a ‘model of inflation’ is simply a specification of the form of the potential relevant during inflation; this will be $V(\phi)$ for a single-field model, or $V(\phi, \psi_1, \psi_2 \dots)$ for a hybrid inflation model.⁴⁸ In this section we give a survey of ‘models’ in this sense, that have been proposed in the literature. The particle theory background will be mentioned only briefly, pending the full discussion of Sections 8 and 9.

The potential of a given model will contain one, two or more parameters. Discounting particle theory, these are constrained only by observation. The most fundamental constraint is the COBE normalization Eq. (44). The corresponding upper bound was known (to order of magnitude) long before the cmb anisotropy was actually observed, and was therefore available when inflation was first proposed. It ruled out the first viable models of inflation [198, 6] (or to be precise, required that the dimensionless coupling is tiny, Eq. (185)) and

⁴⁸The other, irrelevant, fields are supposed to give a negligible contribution to V . Most of them will have masses during inflation that are big enough to anchor them at the vacuum values. (The criterion for this is $M_{\text{P}}^2 V''/V \gg 1$ where the prime is the derivative with respect to the relevant field, or equivalently $m \gg H$.) At the other extreme, some of the irrelevant fields may correspond to field directions which are even flatter than that of the inflaton. Such fields may be displaced far from their vacuum values during inflation, with possibly observable effects. The classic examples are the dilaton and the bulk moduli of string theory, and the QCD axion.

has been imposed as a constraint on all models of inflation since then.⁴⁹ The COBE normalization typically determines the magnitude of the potential, as opposed to its shape.

The other important constraint is provided by the spectral index n , given by Eq. (54) or more usually Eq. (56). The spectral index can often be calculated just from the shape of the potential, and is a powerful discriminator between models.

One can also calculate the scale-dependence of n and the relative contribution r of gravitational waves. The latter is too small ever to observe in most models, but the former may well provide additional discrimination in the future.

Without going into detail, we shall try to give some indication of the extent to which each form for the potential is attractive in the context of current ideas about particle theory. In particular, we shall indicate whether there is a mechanism for keeping the inflaton mass small in the context of supergravity (page 55), or whether an accidental cancellation is invoked.

6.1 Single-field and hybrid inflation models

As we already pointed out, there are two broad classes of ‘model’. In **single-field** models, the slow-rolling inflaton field ϕ gives the dominant contribution to the potential, and inflation ends when ϕ starts to oscillate about its vacuum value. In **hybrid** models, the dominant contribution to the potential V comes from some field ψ which is *not* slow-rolling, but is fixed by its interaction with ϕ .⁵⁰

There are two, very different, kinds of single-field model. In what are usually called *chaotic inflation models* ϕ is moving towards the origin, and its magnitude during observable inflation is several times M_{P} . In what are usually called *new inflation* models, ϕ is moving away from the origin, and during observable inflation its magnitude is at most of order M_{P} .⁵¹ In hybrid models, ϕ may be moving in either direction, but its magnitude is again at most of order M_{P} .

Both in single-field and hybrid inflation, one will have a potential $V(\phi)$ during inflation, which depends on one or more parameters. One will also know the value ϕ_{end} at the end of slow-roll inflation.⁵² Given this information, the recipe for obtaining the predictions is simple.

⁴⁹Before the COBE observation one did not have a precise normalization, but the approximate one was known from galaxy surveys. In the early days one entertained the possibility that the cmb anisotropy and large-scale structure had a non-inflationary origin, in which case the normalization became an upper bound.

⁵⁰In mutated hybrid inflation, ψ is a function of the inflaton field. Then the potential during inflation is $V(\phi, \psi(\phi))$. At this point, we should note that the definition of a ‘field’ is in principle not unique. However, we are supposing that the fields can be taken to be canonically normalized, so that the field-space ‘metric’ K_{mn} is Euclidean. Then, apart from the choice of origin, the choice of fields corresponds to a choice of orthogonal directions in field space. In the context of particle physics there is usually a naturally preferred choice (up to gauge transformations) making the definition of the fields essentially unique in that context. On the other hand, the ‘inflaton field’ ϕ may be a linear combination of the particle physics fields.

⁵¹We shall generally avoid the terms ‘chaotic’ and ‘new’, since they are also used to indicate initial conditions long before observable inflation starts (respectively chaotically varying fields, and fields in thermal equilibrium).

⁵²In single-field models it always corresponds to the failure of one of the flatness conditions (Eqs. (31) and (32)). In hybrid inflation, this may be the case, or alternatively it may correspond to ϕ arriving at the critical value ϕ_c at which the non-inflaton field is destabilized.

- Calculate the number of e -folds $N(\phi)$ to the end of slow-roll inflation using Eq. (40). In many cases, this integral is insensitive to ϕ_{end} in which case the predictions are independent of that quantity.
- The value of $N(\phi)$ when the observable Universe leaves the horizon, denoted simply by N with no argument, depends on the history of the Universe after slow-roll inflation ends. We saw in Section 3.4, that a reasonable estimate is $N \sim 50$, unless there is significant inflation after slow-roll inflation ends. Using this, or a lower, estimate of N , calculate the corresponding slow-roll parameters η and ϵ .
- Use ϵ to impose the COBE normalization Eq. (44) on the model.
- See if there are significant gravitational waves. As discussed in Section 3.5, this requires $\epsilon \gtrsim 0.01$ which is hardly ever satisfied. We shall mention gravitational waves only in the rare models where they are significant.
- Check that $\epsilon \ll |\eta|$. If it is, the full expression $n - 1 = 2\eta - 6\epsilon$ may be replaced by $n - 1 = 2\eta$. As discussed after Eq. (56), this is usually the case, and we shall mention the full expression for η only for those rare models where it is needed. Using one expression or the other, calculate n . As shown in the table on page 80, it often depends only on the shape of the potential.
- Check to see if n has significant variation on cosmological scales, corresponding to $N - 10 \lesssim N(\phi) \lesssim N$.

6.2 Monomial and exponential potentials

Now we begin our survey of models, starting with single-field models and going on to hybrid models. We start with the simplest potential of all. It is

$$V = \frac{1}{2}m^2\phi^2. \quad (173)$$

Almost as simple are $V = \frac{1}{4}\lambda\phi^4$, and $V = \lambda M_{\text{Pl}}^{4-p}\phi^p$ with $p/2 \geq 3$. These monomial potentials were proposed as the simplest realizations of chaotic initial conditions (Section 3.6) at the Planck scale [199].

Inflation ends at $\phi_{\text{end}} \simeq pM_{\text{P}}$, after which ϕ starts to oscillate about its vev $\phi = 0$. When cosmological scales leave the horizon $\phi = \sqrt{2Np}M_{\text{P}}$. Since the inflaton field is then of order 1 to $10M_{\text{P}}$, there is no particle physics motivation for a monomial potential.

The model gives $n - 1 = -(2 + p)/(2N)$ (using the full expression $n = 1 + 2\eta - 6\epsilon$), and gravitational waves are big enough to be eventually observable with $r = 2.5p/N = 5(1 - n) - 2.5/N$. The COBE normalization Eq. (44) corresponds to $m = 1.8 \times 10^{13}$ GeV for the quadratic case. For $p = 4, 6, 8$ it gives respectively $\lambda = 2 \times 10^{-14}$, $\lambda = 8 \times 10^{-17}$, $\lambda = 6 \times 10^{-20}$ and so on. The COBE normalization gives $V^{1/4} \sim 10^{16}$ GeV. The same prediction is obtained for a more complicated potential, provided that it is proportional to ϕ^p during cosmological inflation, and in particular ϕ could have a nonzero vev $\ll M_{\text{P}}$ [186, 189, 190, 171].

Inflation at $\phi \gg M_{\text{P}}$ which ends at $\phi \sim M_{\text{P}}$ is the prediction of a wide variety of monotonically increasing potentials [134, 135], but they are seldom considered because there is too much freedom and no guidance from particle theory.

The limit of a high power is an exponential potential, of the form $V = \exp(\sqrt{2/q}\phi)$. This gives $\epsilon = \eta/2 = 1/q$ which lead to $n - 1 = -2/q$ and $r = 10/q$. This is the case of ‘extended inflation’, where the basic interaction involves non-Einstein gravity but the exponential potential occurs after transforming to Einstein gravity [182, 175]. However, simple versions of this proposal are ruled out by observation, because the end of inflation corresponds to a first order phase transition, and in order for the bubbles not to spoil the cmb isotropy one requires $n \lesssim 0.75$. With the effect of gravitational waves included, this strongly contradicts observation [193, 121].

6.3 The paradigm $V = V_0 + \dots$

The models we have just considered are the only ones that have ϕ well in excess of M_{P} . *In all of the other models that we shall described it is assumed that $\phi \lesssim M_{\text{P}}$ during observable inflation.* As a result of this condition, the potential is always of the form $V = V_0 + \dots$, with the constant V_0 dominating. To avoid repetition we shall take all this for granted in what follows.

6.4 The inverted quadratic potential

Another simple potential leading to inflation is [34, 200, 103, 250, 2, 181, 163, 159, 146, 21, 160]

$$V = V_0 - \frac{1}{2}m^2\phi^2 + \dots, \quad (174)$$

with the constant V_0 dominating. We shall call this the ‘inverted’ quadratic potential, to distinguish it from the same potential with the plus sign which comes from the simplest version of hybrid inflation. The dots indicate the effect of higher powers, that are supposed to come in after cosmological scales leave the horizon.

This potential gives $1 - n = 2\eta = 2M_{\text{P}}^2 m^2 / V_0$. If m and V_0 are regarded as free parameters, the region of parameter space permitting slow-roll inflation corresponds to $1 - n \ll 1$. Thus n is indistinguishable from 1 except on the edge of parameter space. However, there are two reasons why the edge might be regarded as favoured. One is the fact that a generic supergravity theory gives $m^2 \sim V_0 / M_{\text{P}}^2$. Since slow-roll inflation requires $|\eta| \ll 1$, either η is somewhat reduced from its natural value by accident, or it is suppressed because the theory has a non-generic form. One might argue that η should be as big as possible in models that rely on an accident, corresponding to n significantly different from 1.

The other reason for expecting n to be significantly below 1, which is specific to this potential, has to do with the position of the minimum, ϕ_{m} . If the inverted quadratic form for the potential holds until V_0 ceases to dominate, one expects

$$\phi_{\text{m}} \sim \frac{V_0^{1/2}}{m} = \left(\frac{2}{1-n} \right)^{1/2} M_{\text{P}}. \quad (175)$$

(This is also an estimate of ϕ_{end} in that case.) To have any hope of understanding the potential within the context of particle theory, ϕ_{m} should not be more than a few times M_{P} , which requires n to be well below 1.

The second reason for expecting n to be significantly below 1 does not hold if the potential steepens drastically soon after cosmological scales leave the horizon, as in the model at the end of Section 6.5, or if inflation ends through a hybrid mechanism as in Section 6.11. In some of these models the first reason does not hold either, and n is in fact indistinguishable from 1.

The COBE normalization Eq. (44) for the inverted quadratic potential is

$$\frac{2}{1-n} \frac{V_0^{1/2}}{M_{\text{P}}\phi} = 5.3 \times 10^{-4}. \quad (176)$$

The field ϕ is evaluated when COBE scales leave the horizon, N e -folds before the end of slow-roll inflation at some epoch ϕ_{end} . It is given by $\phi = \phi_{\text{end}}e^{-x}$ where

$$x \equiv N|1-n|/2 < 5. \quad (177)$$

(The bound comes from $N < 50$ and $|1-n| < 0.2$. At the moment we are dealing with the case $n < 1$ but we shall use the variable x also for the case $n > 1$.) This gives

$$\frac{V_0^{1/2}}{M_{\text{P}}^2} = 5.3 \times 10^{-4} \frac{1-n}{2} e^{-x} \frac{\phi_{\text{end}}}{M_{\text{P}}}. \quad (178)$$

If the inverted quadratic form holds until V_0 ceases to dominate, $\phi_{\text{end}} \gtrsim M_{\text{P}}$, and $V_0^{1/4} \gtrsim 1 \times 10^{15}$ GeV. If it fails earlier, as in the two cases mentioned, V_0 can be much lower.

Since the field variation is bigger than M_{P} , this type of model is unattractive in the context of particle theory. Let us consider the proposals that have been made.

Modular inflation If ϕ is the (real or imaginary part of the) dilaton or a bulk modulus of string theory, and other fields are not significantly displaced from their vacuum values, its potential will be given by Eq. (172) with A negligible, $V = Bf(\phi/M_{\text{P}})$, with $f(x)$ and its derivatives roughly of order 1 in the regime $|x| \lesssim 1$.⁵³ In that regime one expects the flatness parameters $\eta \equiv M_{\text{P}}^2 V''/V$ and $\epsilon \equiv M_{\text{P}}^2 (V'/V)^2/2$ to be both roughly of order 1, and they might both be significantly below 1 near some value of ϕ so that slow-roll inflation can occur there. One favours the case that this value would be a maximum of the potential so that ‘eternal’ inflation would set the initial condition. Then the potential will be of the inverted quadratic form. So far, investigations using specific models [34, 2, 225, 106] have actually concluded that viable inflation does not occur.⁵⁴

Radial part of a matter field Alternatively one could take ϕ to be the radial part of a matter field, but this is problematic in the context of string theory for the reasons discussed on page 56.

⁵³The case of A dominating would correspond to hybrid inflation, which we are not considering at the moment.

⁵⁴Ref. [32] claims to have been successful, but an analytic calculation of that model reviewed in Ref. [224] finds that it is not viable.

Angular part of a matter field Instead of taking ϕ to be radial part of a matter field, one might take it to be a pseudo-Goldstone boson, corresponding to the angular part of a matter field whose radial part is fixed. This was first proposed in Ref. [103], and dubbed ‘natural’ inflation. It has subsequently been considered by several authors [163, 250, 2, 159, 111]. One might think that this proposal avoids the problem mentioned on page 56 but this turns out not to be the case. The potential of the pseudo-Goldstone boson, coming say from instanton effects, is typically of the form

$$V(\phi) = V_0 \cos^2(\phi/M). \quad (179)$$

where $\frac{1}{2}M/\sqrt{2}$ is the magnitude of the corresponding complex field. Near the top of the potential, inflation takes place and to sufficient accuracy we have an inverted quadratic potential with $m^2 = 2V_0/M^2$, and $1 - n = 4(M_P/M)^2$, and to have viable inflation we need M to be significantly bigger than M_P . From Eq. (144), non-renormalizable terms will then give a ‘correction’ $\Delta V \gg V(\phi)$ unless they are suppressed to all orders. The difficulty of understanding such a suppression is precisely the problem stated in Section 5.9.1.

6.5 Inverted higher-order potentials

If the quadratic term is heavily suppressed or absent, one will have

$$V \simeq V_0(1 - \mu\phi^p + \dots) \quad (180)$$

with $p \geq 3$. For this potential one expects that the integral (40) for N is dominated by the limit ϕ leading to [160]

$$\phi^{p-2} = [p(p-2)\mu N M_P^2]^{-1} \quad (181)$$

and

$$n \simeq 1 - 2 \left(\frac{p-1}{p-2} \right) \frac{1}{N} \quad (182)$$

It is easy to see that the integral is indeed dominated by the ϕ limit, if higher terms in the potential (180) become significant only when V_0 ceases to dominate at $\phi^p \sim \mu^{-1}$. Then, in the regime where V_0 dominates, $\eta = [(p(p-1)M_P^2/\phi^2)\mu\phi^p]$, and if this expression becomes of order 1 in that regime inflation presumably ends soon after. Otherwise inflation ends when V_0 ceases to dominate. At the end of inflation one therefore has $M_P^2\mu\phi_{\text{end}}^{p-2} \sim 1$ if $\phi_{\text{end}} \ll M_P$, otherwise one has $\mu\phi_{\text{end}}^p \sim 1$. (We are supposing for simplicity that p is not enormous, and dropping it in these rough estimates.) The integral (40) is dominated by the limit ϕ provided that

$$N M_P^2 \mu \phi_{\text{end}}^{p-2} \gg 1. \quad (183)$$

This is always satisfied in the first case, and is satisfied in the second case provided that $\phi_{\text{end}} \ll \sqrt{N}M_P$ which we shall assume. If higher order terms come in more quickly than we have supposed, or if inflation ends through a hybrid inflation mechanism then ϕ_{end} will be smaller than these estimates, and one will have to see whether the criterion (183) is satisfied. If it is satisfied, the COBE normalization Eq. (44) is [160]

$$5.3 \times 10^{-4} = (p\mu M_P^p)^{\frac{1}{p-2}} [N(p-2)]^{\frac{p-1}{p-2}} V_0^{\frac{1}{2}} M_P^{-2}. \quad (184)$$

For $p = 4$, this becomes a bound on the dimensionless coupling λ defined by $V = V_0 - \frac{1}{4}\lambda\phi^4 + \dots$, which is independent of V_0 ;

$$\lambda = 3 \times 10^{-15} (50/N)^3. \quad (185)$$

Such a tiny number can hardly be a fundamental parameter, but it can be generated if λ is a function of some heavy fields which are integrated out as in the example of Section 8.5.

A practically equivalent form for the potential is

$$V = V_0 + \frac{1}{4}\lambda\phi^4 \log(\phi/Q). \quad (186)$$

The logarithm comes from the loop correction ignoring ϕ 's supersymmetric partner. This was the first viable model of inflation [198, 6] (see also [274]). The constraint $\lambda \sim 10^{-15}$ presumably rules out the model if λ is a fundamental parameter though there is a dissenting view [185]. In any case, the model does not survive with supersymmetry, since the fermionic partner of ϕ then gives an equal and opposite loop contribution (Section 7.7.1).

A dynamical mechanism for suppressing the mass-squared term has been proposed [3]. The potential is

$$V = V_0(1 + \beta\phi^2\psi - \gamma\phi^3 + \dots), \quad (187)$$

where ψ is another field. Then, with β and γ of order 1 in Planck units, and initial values $\psi \sim M_{\text{P}}$ and $\phi \simeq 0$ one can check that the quadratic term is driven to a negligible value before cosmological inflation begins. For this proposal to work, the mass m_ψ has to have a negligible effect, which requires $m_\psi^2 \ll V_0/M_{\text{P}}^2$. As with the inflaton mass, this is violated in a generic supergravity theory. In Reference [3] ψ is supposed to be a pseudo-Goldstone boson, but as we noted earlier this is not an attractive mechanism for keeping the mass small in the context of string theory.

The above proposal gives ϕ a vev of order M_{P} . Some particle-physics motivation for a vev $\ll M_{\text{P}}$ is given in Refs. [159, 160], though not in the context of supergravity.

One could contemplate models in which more than one power of ϕ is significant while cosmological scales leave the horizon, but this requires a delicate balance of coefficients. Models of this kind were also discussed a long time ago [91, 247], again with a vev of order M_{P} , but their motivation was in the context of setting the initial value of ϕ through thermal equilibrium and has disappeared with the realization that this 'new inflation' mechanism is not needed.

A more recent proposal is described in Section 8.5. It gives

$$V(\phi) \simeq V_0 - \frac{m^2}{2}\phi^2 - \frac{\lambda}{4}\phi^4, \quad (188)$$

This gives

$$-V' = m^2\phi + \lambda\phi^3 + \dots \quad (189)$$

and the two terms are equal at $\phi = \phi_* \equiv m/\sqrt{\lambda}$. It is supposed that the first term dominates while cosmological scales are leaving the horizon, but that the second term dominates before the end of inflation. For an estimate of Eq. (40), one can keep only the first term of Eq. (189) when the integration variable is less than ϕ_* , and only the second term when it is bigger.

In the latter case, one can also take the integral to be dominated by its lower limit ϕ_* . This gives

$$\frac{\phi}{\phi_*} \simeq \exp\left(\frac{1}{2} - x\right), \quad (190)$$

where x is defined by Eq. (177). The COBE normalization Eq. (176) then gives

$$\lambda = 3 \times 10^{-15} (50/N)^3 (2x)^3 \exp(1 - 2x). \quad (191)$$

Using the constraint

$$\frac{1}{2} < x < 5, \quad (192)$$

this becomes

$$4 \times 10^{-16} < \lambda < 3 \times 10^{-15}. \quad (193)$$

In this model, the tiny value of λ occurs because it is of the form F/M_{P}^2 , where F is a function of fields that have been integrated out.

6.6 Another form for the potential

Another potential that has been proposed is

$$V \simeq V_0(1 - e^{-q\phi/M_{\text{P}}}) \quad (194)$$

with q of order 1. This form is supposed to apply in the regime where V_0 dominates, which is $\phi \gtrsim M_{\text{P}}$. Inflation ends at $\phi_{\text{end}} \sim M_{\text{P}}$, and when cosmological scales leave the horizon one has

$$\phi = \frac{1}{q} \ln(q^2 N) M_{\text{P}}, \quad (195)$$

$$n - 1 = -2\eta = -2/N. \quad (196)$$

This potential is mimicked by $V = V_0(1 - \mu\phi^{-p})$ with $p \rightarrow \infty$ (Table 1). Gravitational waves are negligible. The COBE normalization Eq. (44) is now $V_0^{1/4} \simeq 7 \times 10^{15}$ GeV.

This potential occurs in what one might call non-minimal inflation [285]. Here, the original potential is not particularly flat, but the kinetic term given by Eq. (136) becomes singular at a field value of order M_{P} leading to a flat potential after converting to a canonically-normalized inflaton field. Suppose, for example, that K is given by Eqs. (349) and (350), and purely for convenience suppose that $t + t^* = M_{\text{P}}$ (it is expected to be of this order). Suppose also that all other fields vanish except for some field ϕ_1 , and set $M_{\text{P}} = 1$. Then $K = -3 \ln(1 - |\phi_1|^2)$, and assuming that V is independent of the phase of ϕ_1 it is easy to show [285] that the potential is given by Eq. (194) with $q = \sqrt{2}$ and the canonically normalized field

$$\phi = \tanh^{-1} \sqrt{2} |\phi_1| - \frac{1}{\sqrt{2}} \ln \frac{2dV/d|\phi_1|}{V} \Big|_{|\phi_1|=1} \quad (197)$$

Another derivation [283, 48] modifies Einstein gravity by adding a large R^2 term to the usual R term, but with a huge coefficient, and a third [25] uses a variable Planck mass. In both cases, after transforming back to Einstein gravity one obtains the above form with $q = \sqrt{2/3}$. These proposals too invoke large field values, making it difficult to see how V can be sufficiently small (and how the kinetic terms can be almost canonical, as is assumed).

6.7 Hybrid inflation

We now turn to hybrid inflation models. In these models, the slowly rolling inflaton field ϕ is not the one responsible for most of the energy density. That role is played by another field ψ , which is held in place by its interaction with the inflaton field until the latter falls below a critical value ϕ_c . When that happens ψ is destabilized and inflation ends.

This paradigm has proved very fruitful, since its introduction by Linde [204] in 1991. Early treatments of it are references [194] (1993), [205, 60, 231, 87, 297] (1994), [285, 286, 188, 265] (1995), and [262, 88, 111, 224, 35, 130] (1996); by now it is the standard paradigm of inflation.

In a related class of models the inflaton field is rolling away from the origin, and inflation ends when it *rises* above some critical value ϕ_c . This paradigm, now known as inverted hybrid inflation, is less useful as we shall discuss in Section 6.11. It was introduced by Ovrut and Steinhardt [249] in 1984, but has received little attention.

Note that the essential feature of hybrid inflation is the *dominance* of the potential, by the field that is held fixed. Potentials of the form proposed by Linde had been considered earlier by several authors, starting with Kofman and Linde [166]. But they presumed the parameters to be such that the other field gives only a small contribution to the potential. As we noted at the end of Section 5.3.4, such models are interesting because they might produce topological defects, or a feature in the spectrum, but they are not hybrid inflation models.⁵⁵

6.8 Hybrid inflation with a quadratic potential

We begin with the case that the potential during inflation has the simplest possible tree-level form,

$$V = V_0 + \frac{1}{2}m^2\phi^2. \quad (198)$$

The first term is supposed to dominate, and inflation occurs provided that the condition

$$m^2 \ll V_0/M_{\text{P}}^2 \quad (199)$$

is at least marginally satisfied (this is the condition $\eta \ll 1$). We shall assume unless otherwise stated that $\phi \ll M_{\text{P}}$, so that $\epsilon \ll \eta$ and

$$n - 1 = 2\eta = 2M_{\text{P}}^2 m^2 / V_0. \quad (200)$$

By itself, the above potential has no mechanism for ending inflation, since the flatness parameters ϵ and η become smaller as ϕ decreases. Inflation is supposed to end through a hybrid inflation mechanism as described in a moment, when ϕ falls below some critical value ϕ_c . When the observable Universe leaves the horizon

$$\frac{\phi}{\phi_c} = e^x, \quad (201)$$

where x is given by Eq. (177). At least with the two prescriptions for ϕ_c discussed below, Eq. (201) is consistent with the assumption $\phi \ll M_{\text{P}}$.

⁵⁵The earlier model of [93, 94] seems also to be of this kind.

We emphasize at this point that the loop correction, ignored when one considers this potential, often dominates in reality. Several examples will be given later.

Proceeding with the assumption of a tree-level potential, the COBE normalization Eq. (44) is

$$\frac{2}{n-1} \frac{V_0^{1/2}}{M_{\text{P}}\phi} = 5.3 \times 10^{-4}, \quad (202)$$

or

$$\frac{V_0^{1/2}}{M_{\text{P}}^2} = 5.3 \times 10^{-4} \frac{n-1}{2} e^x \frac{\phi_c}{M_{\text{P}}}. \quad (203)$$

To work out ϕ_c , we need to include the non-inflaton field ψ that is responsible for V_0 . The full potential for the original model [204] is Eq. (147) that we already considered.

$$V = V_0 - \frac{1}{2} m_\psi^2 \psi^2 + \frac{1}{4} \lambda \psi^4 + \frac{1}{2} m^2 \phi^2 + \frac{1}{2} \lambda' \psi^2 \phi^2 \quad (204)$$

$$= \frac{1}{4} \lambda (M^2 - \psi^2)^2 + \frac{1}{2} m^2 \phi^2 + \frac{1}{2} \lambda' \psi^2 \phi^2. \quad (205)$$

Comparing the two ways of writing the potential, one sees that the parameters are related by

$$m_\psi^2 = \lambda M^2, \quad (206)$$

$$V_0 = \frac{1}{4} \lambda M^4 = \frac{1}{4} M^2 m_\psi^2. \quad (207)$$

This gives

$$\phi_c^2 = m_\psi^2 / \lambda' = \lambda M^2 / \lambda'. \quad (208)$$

It is useful to define

$$\eta_\psi \equiv \frac{m_\psi^2 M_{\text{P}}^2}{V_0}. \quad (209)$$

To have inflation end promptly when ϕ falls below ϕ_c , as is assumed in this model, one needs η_ψ significantly bigger than 1.⁵⁶ In terms of η_ψ , the COBE normalization becomes

$$\lambda' = 2.8 \times 10^{-7} e^{2\eta N} \eta^2 \eta_\psi. \quad (210)$$

A different prescription [262] is to replace the renormalizable coupling $\frac{1}{4} \lambda \psi^2 \phi^2$ by a non-renormalizable coupling

$$\frac{1}{2} \psi^2 \phi^4 / \Lambda_{\text{UV}}^2. \quad (211)$$

The COBE normalization is now

$$\frac{V_0^{1/2}}{M_{\text{P}} \Lambda_{\text{UV}}} = 2.8 \times 10^{-7} e^{2\eta N} \eta^2 \sqrt{\eta_\psi}. \quad (212)$$

⁵⁶This is obvious if ψ remains homogeneous, but the same result can actually be established [60] even without that assumption.

If one takes Λ_{UV} significantly below M_{P} , the flatness conditions on the potential discussed in Section 5.8 may become more stringent.⁵⁷

With ϕ_c given by either of these prescriptions, Eq. (203) implies [60] a limit $n \lesssim 1.3$ (assuming that V_0 dominates the potential and that $M \lesssim M_{\text{P}}$). In that case, the present observational limit $|n - 1| < 0.2$ is more or less predicted.

Different prescriptions will be considered in Sections 6.13, 8.3.4 and 8.7, and on page 125. In the last case, ϕ is of order M_{P} when the observable Universe leaves the horizon.

6.9 Masses from soft susy breaking

When hybrid inflation is implemented in a supersymmetric theory, the slope of the potential is often dominated by a loop correction. But there are cases where a tree-level slope $\frac{1}{2}m^2\phi^2$ can dominate and we mention one of them now.

The crucial feature of the model [262] is that the parameters η and η_ψ are both very roughly of order 1. (This is what one might expect if the masses m and m_ψ both vanish in the limit of global supersymmetry, and come only from supergravity corrections.) The vev of ψ is therefore roughly $M \sim M_{\text{P}}$. It is presumed that this is achieved by replacing the first term of Eq. (205) by a more complicated function of ψ , rather than by making λ tiny as would be required by Eq. (207). If ψ is the radial part of a matter field, λ is presumably negligible while the non-renormalizable terms are suppressed.

The simplest thing is to assume that ψ is the dilaton or a bulk modulus, whose potential is of the form Eq. (172). Alternatively it might be a matter field with non-renormalizable coupling suppressed to high order as a result of a discrete symmetry. In any case, $\psi = 0$ is presumably a fixed point of the relevant symmetries.

A less crucial feature is the assumption that $V_0^{1/4}$ is very roughly of order 10^{10} GeV. This is motivated by an assumption that there is a gravity-mediated mechanism of susy breaking in the true vacuum, which operates also during inflation with essentially the same strength.

As we have seen, the observational constraint $|n - 1| < 0.2$ actually requires $|\eta| < 0.1$. The reduction of η below its natural value of order 1 is supposed to come from an accidental cancellation in this model. To minimize the cancellation required, one prefers n to be significantly above 1.⁵⁸

With the choice $\eta_\psi \sim 1$ some number N_ψ of e -folds of inflation occur after ϕ reaches ϕ_c . As discussed in Section 3.4, one has to require that N_ψ is less than the total number of e -folds after cosmological scales leave the horizon, since the fluctuation while ψ is rolling does not generate the flat spectrum required in this regime. In fact, it gives a spike in the spectrum [262, 111], and one must require that it does not lead to excessive black hole formation. Typically this reduces the already significant upper limit on n , that follows from the same requirement in the absence of a spike [49, 262, 122].

Assuming that ψ remains almost homogeneous after ϕ falls below ϕ_c , one can calculate

⁵⁷The scale Λ_{UV} is presumably supposed to come from integrating out some sector of the full theory. The non-renormalizable terms relevant for ϕ may or may not have the same effective scale Λ_{UV} . See Section footnote 5.1.

⁵⁸Of the six examples displayed in the Figures of Ref. [262] only one actually has n significantly bigger than 1, and therefore it should be regarded as a favoured parameter choice.

the number of e -folds of inflation that occur while ψ rolls down to its vacuum value $\psi = M$.⁵⁹ The result is [290]

$$N_\psi = \frac{1}{2\eta_\psi} \left(1 + \sqrt{1 + \frac{4\eta_\psi}{3}} \right) \ln \frac{M}{\psi_{\text{initial}}}. \quad (213)$$

Here $\psi_{\text{initial}} \sim H$ is the initial value of ψ , given by its quantum fluctuation. Since $V_0^{1/4}$ is supposed to be of order 10^{10} GeV, in this model, $\psi_{\text{initial}} \sim 10^{-16} M_{\text{P}}$, leading to [290]

$$N_\psi \sim \frac{37}{2\eta_\psi} \left(1 + \sqrt{1 + \frac{4\eta_\psi}{3}} \right). \quad (214)$$

Requiring $N_\psi < 10$ leads to $\eta_\psi > 8$, and requiring $N_\psi < 30$ leads to $\eta_\psi > 1.7$.

In this model, the COBE normalization requires λ' in Eq. (210), or $\Lambda_{\text{UV}}/M_{\text{P}}$ in Eq. (212), to be a few orders of magnitude below unity. These small couplings are consistent with the assumption that loop corrections are negligible. On the other hand, the inflaton could still have large couplings to other fields, which could give a large loop correction. If that happens, one arrives at the running inflaton mass model of Section 6.16.

6.10 Hybrid thermal inflation

Related to the scheme we just described, is a radical proposal [4], which would have a distinctive observational signature. Its basic ingredients are fairly natural, though the particular combination required may be difficult to arrange.

The idea is to have a hot big bang during the era immediately preceding observable inflation, with all relevant fields in thermal equilibrium as was proposed in the early models of inflation. (This primordial hot big bang is presumably preceded by more inflation as described in Section 3.6.) Let us begin with the simplest version of the proposal. Including the finite temperature T , the potential during inflation is something like

$$V(\phi, \psi) = V_0 + T^4 + T^2\psi^2 - \frac{1}{2}m_\psi^2\psi^2 + T^2\phi^2 - \Delta V(\phi). \quad (215)$$

As in the previous case, it is supposed that very roughly $m_\psi^2 \sim V_0/M_{\text{P}}^2$, corresponding to a true vacuum value ψ very roughly of order M_{P} (but maybe some orders of magnitude less). The last term, which will determine the motion of the the inflaton field ϕ , is not specified in detail.

The temperature falls roughly like $1/a$, and an epoch of what one might call ‘hybrid thermal inflation’ begins when the potential is dominated by V_0 at $T \sim V_0^{1/4}$, and ends when ψ is destabilized at $T \sim m_\psi$.⁶⁰ This lasts for $N_{\text{thermal}} \sim 10$ e -folds. After a further

⁵⁹In the case $\eta_\psi \ll 1$ one has slow-roll inflation, and the homogeneity can be checked by calculating the vacuum fluctuation. It seems reasonable that it will hold to sufficient accuracy also if $\eta_\psi \sim 1$.

⁶⁰The phenomenon of ordinary thermal inflation was noted in References [34, 187], and discussed in detail in References [222, 223, 288, 26]. Ordinary thermal inflation is identical with the phenomenon we are describing now, except that the field ϕ is not present. Ordinary thermal inflation is supposed to happen long after ordinary inflation is over, with the susy breaking scale the same as in the vacuum. This makes m_ψ a typical soft mass of order 100 GeV, and assuming $\psi \ll M_{\text{P}}$ it makes $V_0^{1/4} \ll 10^{10}$ GeV.

N_ψ e -folds, given by Eq. (214), ψ arrives at its true vacuum value and inflation ends. Meanwhile, ϕ rolls slowly, and is supposed to be the dominant source of the primordial curvature perturbation. (This last feature would need checking case by case, as the other field ψ may be significant—see Section 4.)

To avoid unacceptable relics of the thermal era, at least a few e -folds of inflation have to occur before the observable Universe leaves the horizon [194, 294], which will probably use up all of the e -folds of thermal inflation. In that case, we just have a hybrid inflation model with the unspecified potential $V = V_0 - \Delta V(\phi)$. (Different from the usual case though, in that the other field ψ is already destabilized when the observable Universe leaves the horizon.) However, there could well be several of the other fields ψ_n , taking different numbers of e -folds to reach their vacuum values. As each one does so, a feature in the spectrum could be generated, because the inflaton mass coming from supergravity may change. As a more complicated variant of the scheme, one may suppose that the destabilization of one field affects the stability of another.

6.11 Inverted hybrid inflation

One can also construct hybrid inflation models where ϕ is rolling away from the origin, under the influence of the inverted quadratic potential Eq. (174). A simple potential $V(\phi, \psi)$ which achieves this is [224]

$$V = V_0 - \frac{1}{2}m_\phi^2\phi^2 + \frac{1}{2}m_\psi^2\psi^2 - \frac{1}{2}\lambda\phi^2\psi^2 + \dots \quad (216)$$

The dots represent terms which give V a minimum where it vanishes, but which play no role during inflation. At fixed ϕ there is a minimum at $\psi = 0$ provided that

$$\phi < \phi_c = \frac{m_\psi}{\sqrt{\lambda}}. \quad (217)$$

A better-motivated potential leading to inverted hybrid inflation will be described in Section 8.7. A more complicated one appears in Reference [249], but the inflaton trajectory turns out to be unstable [247].

Inverted hybrid inflation is characterised by the appearance of a negative coupling $-\phi^n\psi^m$, in contrast with the usual positive coupling $\phi^n\psi^m$. Such a negative coupling, for fields in thermal equilibrium, corresponds to high temperature symmetry restoration [298]. In the context of supersymmetry it is more difficult to arrange than the positive coupling. In any case, one has to ensure that the potential remains bounded from below in its presence.

6.12 Hybrid inflation with a cubic or higher potential

Instead of the quadratic potential Eq. (198), one might consider a potential

$$V = V_0 (1 + c\phi^p), \quad (218)$$

with $p \geq 3$ (and $c > 0$).

This case is similar to the one that we discuss in some detail in Section 6.14. One has

$$\eta = cM_{\text{P}}^2 p(p-1)\phi^{p-2}, \quad (219)$$

and inflation is possible [205] only in the regime $\eta \ll 1$.⁶¹ It is not clear how the inflaton is supposed to get into this regime.

The number of e -folds to the end of inflation is

$$N(\phi) \simeq \left(\frac{p-1}{p-2}\right) \left(\frac{1}{\eta(\phi_c)} - \frac{1}{\eta(\phi)}\right). \quad (220)$$

For $\phi \gg \phi_c$, $N(\phi)$ approaches a constant

$$N_{\text{max}} \equiv \left(\frac{p-1}{p-2}\right) \frac{1}{\eta(\phi_c)} \quad (221)$$

The spectral index is given by

$$\frac{n-1}{2} = \left(\frac{p-1}{p-2}\right) \frac{1}{N_{\text{max}} - N}. \quad (222)$$

The quartic case has been considered in some detail [265], including the regime $\phi \gg M_{\text{P}}$ that we are ignoring.

One may also consider the case where (say) quadratic, cubic and quartic terms are all important during observable inflation [297], but that will clearly involve considerable fine-tuning.

6.13 Mutated hybrid inflation

In both ordinary and inverted hybrid inflation, the other field ψ is precisely fixed during inflation. If it varies, an effective potential $V(\phi)$ can be generated even if the original potential contains no piece that depends only on ϕ . This mechanism was first proposed in Ref. [286], where it was called mutated hybrid inflation. The potential considered was

$$V = V_0 (1 - \psi/M) + \frac{1}{4}\lambda\phi^2\psi^2 + \dots \quad (223)$$

The dots represent one or more additional terms, which give V a minimum at which it vanishes but play no role during inflation. All of the other terms are significant, with V_0 dominating. For suitable choices of the parameters inflation takes place with ψ held at the instantaneous minimum, leading to a potential

$$V = V_0 \left(1 - \frac{V_0}{\lambda^2 M^2 \phi^2}\right). \quad (224)$$

This gives

$$n - 1 = -\frac{3}{2N}, \quad (225)$$

⁶¹We are assuming that $V \simeq V_0$ as long as the right hand side of the above expression is $\ll 1$. As usual, we consider only the case $\phi \lesssim M_{\text{P}}$.

and the COBE normalization Eq. (44) is

$$5.2 \times 10^{-4} = (2N)^{3/4} \sqrt{\lambda} \frac{V_0^{1/4} \sqrt{M}}{M_{\text{P}}^{3/2}}. \quad (226)$$

A different version of hybrid inflation [188] was called ‘smooth’ hybrid inflation emphasizing that any topological defects associated with ψ will never be produced. In this version, the potential is $V = V_0 - A\psi^4 + B\psi^6\phi^2 + \dots$. It leads to $V = V_0(1 - \mu\phi^{-4})$.

Retaining the original name, the most general mutated hybrid inflation model with only two significant terms is [224]

$$V = V_0 - \frac{\sigma}{p} M_{\text{P}}^{4-p} \psi^p + \frac{\lambda}{q} M_{\text{P}}^{4-q-r} \psi^q \phi^r + \dots. \quad (227)$$

In a suitable regime of parameter space, ψ adjusts itself to minimize V at fixed ϕ , and $\psi \ll \phi$ so that the slight curvature of the inflaton trajectory does not affect the field dynamics. Then, provided that V_0 dominates the energy density, the effective potential during inflation is

$$V = V_0(1 - \mu\phi^{-\alpha}), \quad (228)$$

where

$$\mu = M_{\text{P}}^{4+\alpha} \left(\frac{q-p}{pq} \right) \frac{\sigma^{\frac{q}{q-p}} \lambda^{-\frac{p}{q-p}}}{V_0} > 0, \quad (229)$$

$$\alpha = \frac{pr}{q-p}. \quad (230)$$

For $q > p$, the exponent α is positive as in the examples already mentioned, but for $p > q$ it is negative with $\alpha < -1$. In both cases it can be non-integral, though integer values are the most common for low choices of the integers p and q . This potential is supposed to hold until V_0 ceases to dominate at

$$\phi_{\text{end}} \sim \mu^{1/\alpha}, \quad (231)$$

after which slow-roll inflation ends.

The situation in the regime $-2 < \alpha < -1$ is similar to the one that we discussed already for the case $\alpha = -2$; the prediction for n covers a continuous range below 1 because it depends on the parameters, but to have a model with $\phi \ll M_{\text{P}}$ the potential has to be steepened after cosmological scales leave the horizon. The COBE normalization in this case is [224]

$$5.3 \times 10^{-4} = \frac{M_{\text{P}}^{\alpha-2} V_0^{1/2}}{|\alpha| \mu} \left[M_{\text{P}}^{|\alpha|-2} \phi_{\text{c}}^{2-|\alpha|} - |\alpha| (2 - |\alpha|) M_{\text{P}}^{\alpha} \mu N \right]^{-\frac{|\alpha|-1}{2-|\alpha|}}. \quad (232)$$

In the cases $\alpha < -2$ and $\alpha > -1$, the situation is similar to the the one that we encountered in Section 6.5 (except for the special cases $\alpha \simeq -2$ and $\alpha \simeq -1$, which we do not consider). In the case $\alpha < -2$, the integral (40) is dominated by the limit ϕ provided that $\phi_{\text{end}} \ll \sqrt{N} M_{\text{P}}$, which we assume. In the case $\alpha > -1$ one has $\phi_{\text{end}} < \phi$, and assuming $\phi \ll M_{\text{P}}$ while cosmological scales leave the horizon again means that Eq. (40) is dominated

by the limit ϕ . In all of these cases, the COBE normalization Eq. (184) and the prediction Eq. (182) are valid, with p replaced by $-\alpha$.

Of the various possibilities regarding α , some are preferred over others in the context of supersymmetry. One would prefer [224] q and r to be even if $\alpha > 0$ (corresponding to $q > p$) and p to be even if $\alpha < 0$. Applying this criterion with $p = 1$ or 2 and q and r as low as possible leads [224] to the original mutated hybrid model, along with the cases $\alpha = -2$ and $\alpha = -4$ that we discussed earlier in the context of inverted hybrid and single-field models.

A different example of a mutated hybrid inflation potential is given in Ref. [111], where ψ is a pseudo-Golstone boson with the potential (179).

Mutated hybrid inflation with explicit ϕ dependence So far we have assumed that the original potential has no piece that depends only on ϕ . If there is such a piece it has to be added to the inflationary potential (228). If it dominates while cosmological scales leave the horizon, the only effect that the ψ variation has on the inflationary prediction is to determine $\phi_c = \phi_{\text{end}}$ through Eq. (231).

6.14 Hybrid inflation from dynamical supersymmetry breaking

In Section 5.7.2, we noted that non-perturbative effects, such as those associated with dynamical supersymmetry breaking, could give a potential proportional to $1/\phi^p$ where p is some integer,

$$V(\phi) = V_0 + \frac{\Lambda^{p+4}}{\phi^p} + \dots, \quad (233)$$

where the dots represent terms that are negligible during inflation. This potential has been proposed [161, 162] as a model of inflation. It is convenient to define a dimensionless quantity $\alpha \equiv \Lambda^{p+4} M_{\text{P}}^{-p} V_0^{-1}$, so that

$$V = V_0 \left(1 + \alpha \left(\frac{M_{\text{P}}}{\phi} \right)^p + \dots \right). \quad (234)$$

This gives

$$\eta = \alpha p(p+1) (M_{\text{P}}/\phi)^{p+2}. \quad (235)$$

The potential satisfies the flatness conditions in the regime $\eta \ll 1$.⁶² Inflation is supposed to end when ϕ reaches a critical value ϕ_c , through some unspecified hybrid inflation mechanism.

The number of e-folds to the end of inflation is

$$N(\phi) \simeq \left(\frac{p+1}{p+2} \right) \left(\frac{1}{\eta(\phi_c)} - \frac{1}{\eta(\phi)} \right), \quad \epsilon \ll \eta, \quad (236)$$

For $\phi \ll \phi_c$ $N(\phi)$ approaches a constant

$$N_{\text{tot}} \equiv \left(\frac{p+1}{p+2} \right) \frac{1}{\eta(\phi_c)} = \frac{1}{p(p+2)} \alpha^{-1} \left(\frac{\phi_c}{M_{\text{P}}} \right)^{p+2}. \quad (237)$$

This is quite an unusual feature. Most models of inflation have no intrinsic upper limit on the total amount of expansion that takes place during the inflationary phase, although only

⁶²We are assuming that $V \simeq V_0$ as long as the right hand side of the above expression is $\ll 1$.

the last 50 or 60 e-folds are of direct observational significance. Here the total amount of inflation is bounded from above, although that upper bound can in principle be very large.

The COBE normalization Eq. (44) is

$$\delta_H \simeq \frac{(p+2)}{2\pi} \sqrt{\frac{1}{3}} \left(\frac{V_0^{1/2}}{M_{\text{P}} \phi_c} \right) N_{\text{tot}} \left(1 - \frac{N}{N_{\text{tot}}} \right)^{(p+1)/(p+2)}, \quad (238)$$

where $N \lesssim 50$ corresponds to the epoch when COBE scales leave the horizon. The spectral index is given by

$$n - 1 \simeq \left(\frac{p+1}{p+2} \right) \frac{2}{N_{\text{tot}} - N}. \quad (239)$$

The spectrum turns out to be blue ($n > 1$), but for $N_{\text{tot}} \gg 50$ the spectrum approaches scale-invariance ($n = 1$). If one takes the case of $p = 2$ and $\phi_c \sim V_0^{1/4}$, the COBE constraint Eq. (44) is met for $V_0^{1/4} \simeq 10^{11}$ GeV and $\Lambda \simeq 10^6$ GeV.

In this class of models, n is indistinguishable from 1 in most of parameter space. A value of n significantly above 1 is however possible for properly tuned values of the parameters. Taking $N = 50$ and $p = 2$, a spectral index of $n > 1.1$ requires N_{tot} given by Eq. (237) to be less than 65. In the context of supergravity, it is more comfortable to be in this regime since an accidental cancellation is being invoked to avoid the generic contributions of order 1 to the quantity $2\eta = n - 1$.

Such a small amount of inflation could have observationally important consequences. Also, unlike standard hybrid inflation models, dynamical supersymmetric inflation allows a measurable deviation from a power-law spectrum of fluctuations, with a variation in the scalar spectral index $|dn/d(\ln k)|$ that may be as large as 0.05 [162].

It is important to note that this upper limit on the total amount of inflation can potentially lead to difficulties with initial conditions: how does the field end up in the correct region of the potential with a small enough rate of change to initiate slow-roll? While this sort of problem with initial conditions is in fact common to many models of inflation, it is mitigated to a certain degree by the existence of classical solutions which admit a formally *infinite* amount of inflation. No such solution exists in this case. It is reasonable to expect that the field will initially be at small values, $\phi \ll \langle \phi \rangle$, since the term ϕ^{-p} in the potential will generically appear only at scales smaller than Λ , with a phase transition connecting the high energy and low energy behaviours. However, in the absence of a detailed model for this phase transition, the question of initial conditions remain quite obscure.

6.15 Hybrid inflation with a loop correction from spontaneous susy breaking

The models considered so far work at tree level. This is valid only if the couplings of the inflaton to other fields are strongly suppressed. In particular, the inflaton presumably has to be a gauge singlet (no coupling to gauge fields) since gauge couplings are not supposed to be suppressed.

In the absence of supersymmetry, the couplings should indeed be suppressed. The reason is that the loop correction is then $\Delta V \propto \phi^4 \ln(\phi/Q)$ which would spoil inflation as in Eq. (186). But with supersymmetry, there is no reason to suppose that the inflaton couplings are suppressed.

As we saw in Sections 5.6.2 and 7.7, the 1-loop correction in a supersymmetric theory typically has one of two forms, $\Delta V \propto \ln(\phi/Q)$ or $\Delta V \propto \phi^2 \ln(\phi/Q)$. We discuss the first form in this subsection, and the second form in the next one.

This form typically arises if susy is broken spontaneously. Assuming that tree-level terms are negligible during inflation, the potential is of the form

$$V = V_0 \left(1 + \frac{Cg^2}{8\pi^2} \ln(\phi/Q) \right). \quad (240)$$

In this expression, C may be taken to be the number of possible 1-loop diagrams, in other words the number of fields which have significant coupling to the inflaton. The other factor g is a typical coupling of these fields (times a numerical factor of order 1). It may be a gauge coupling (D -term inflation, Section 9) or a Yukawa coupling (Section 8.4). In the former case C might be of order 100, which as we shall see would be bad news.

In both cases, this potential occurs as part of a hybrid inflation model. Depending on the parameters, inflation ends when either slow-roll fails ($\eta \sim 1$) or the critical value is reached, whichever is earlier.⁶³ However, the precise value of ϕ_{end} is irrelevant because the integral Eq. (40) is dominated by the limit ϕ . It gives

$$\phi \simeq \sqrt{\frac{NCg^2}{4\pi^2}} M_{\text{P}} \quad (241)$$

$$= 11 \sqrt{\frac{N}{50} \frac{C}{100} \frac{g^2}{1.0}} M_{\text{P}} \quad (242)$$

$$= 0.2 \sqrt{\frac{N}{20} C \frac{g^2}{0.1}} M_{\text{P}}. \quad (243)$$

This makes ϕ comparable with the Planck scale, and maybe bigger. As we discussed in Section 5.9 one needs $\phi \lesssim M_{\text{P}}$ and preferably $\phi \ll M_{\text{P}}$, in order to keep the theory under control and in particular to justify the assumption of canonical normalization for the fields. Let us proceed on the assumption that ϕ is not too big.

Assuming that the loop dominates the slope, and using Eq. (40), the flatness parameters are

$$\eta = -\frac{1}{2N}, \quad (244)$$

$$\epsilon = C \frac{g^2}{8\pi^2} |\eta|. \quad (245)$$

The COBE normalization Eq. (44) is

$$V^{1/4} = 6.0 \left(\frac{50}{N} \right)^{\frac{1}{4}} C^{1/4} g \times 10^{15} \text{ GeV}. \quad (246)$$

The spectral index is given by

$$1 - n = \frac{1}{N} \left(1 + \frac{3Cg^2}{16\pi^2} \right). \quad (247)$$

⁶³If slow-roll fails at a value $\phi_{\text{end}} > \phi_c$ inflation will continue until the amplitude of the oscillation becomes of order ϕ_c . The number of e -folds of this type of inflation is $\Delta N \sim \ln(\phi_{\text{end}}/\phi_c)$, which is typically negligible.

Taking the bracket to be close to 1, and N to be in the range 25 to 50, one obtains the distinctive prediction $n = .96$ to $.98$. With $g = 1$ and $C = 100$, $1 - n$ is increased by a factor $\simeq 2$, but it is clear that anyhow n is close to 1. This prediction will eventually be tested.

6.16 Hybrid inflation with a running mass

Now we turn to the case, that the loop correction is of the form $\phi^2 \ln(\phi/Q)$, which typically arises when susy is softly broken. Models of inflation invoking such a correction have been proposed by Stewart [289, 290].

As we noted in Section 5.6.2, this type of loop correction is equivalent to replacing the inflaton mass by a slowly varying (running) mass $m^2(\phi)$. At $\phi = M_{\text{P}}$, the running mass is supposed to have the magnitude $|m^2| \sim V_0/M_{\text{P}}^2$, which is the minimum one in a generic supergravity theory. The inflaton is supposed to have couplings (gauge, or maybe Yukawa) that are not too small, and for the most part we assume that $m^2(\phi)$ passes through zero before it stops running.⁶⁴ Because the couplings are small compared with unity, V' then vanishes at some relatively nearby point, which we denote by ϕ_* .

6.16.1 General formulas

It is useful to write

$$V(\phi) = V_0 \left(1 - \frac{1}{2} M_{\text{P}}^{-2} \mu^2(\phi) \phi^2 \right), \quad (248)$$

where

$$\mu^2(\phi) \equiv -M_{\text{P}}^2 m^2(\phi)/V_0. \quad (249)$$

We are supposing that V_0 dominates, since this is necessary for inflation in the regime $\phi \lesssim M_{\text{P}}$ where the field theory is under control. Then

$$M_{\text{P}} \frac{V'}{V_0} = -\phi \left[\mu^2 + \frac{1}{2} \frac{d\mu^2}{dt} \right] \quad (250)$$

$$\eta \equiv M_{\text{P}}^2 \frac{V''}{V_0} = - \left[\mu^2 + \frac{3}{2} \frac{d\mu^2}{dt} + \frac{1}{2} \frac{d^2\mu^2}{dt^2} \right], \quad (251)$$

where $t \equiv \ln(\phi/M_{\text{P}})$.

We assume that while observable scales are leaving the horizon one can make a linear expansion in $\ln \phi$,⁶⁵

$$\mu^2 \simeq \mu_*^2 + c \ln(\phi/\phi_*), \quad (252)$$

where $|c| \ll 1$ is related to the couplings involved. This gives

$$M_{\text{P}} \frac{V'}{V_0} = c\phi \ln(\phi_*/\phi) \quad (253)$$

$$\eta \equiv M_{\text{P}}^2 \frac{V''}{V_0} = c [\ln(\phi_*/\phi) - 1]. \quad (254)$$

⁶⁴The running associated with a given loop will stop when ϕ falls below the mass of the particle in the loop.

⁶⁵This is equivalent to writing $\mu^2 = c \ln(\phi/Q)$ as in the table on page 80, the free parameter Q then replacing the free parameter ϕ_* . In turn, this is equivalent to using a loop correction, with the renormalization scale Q fixed at the point where m^2 vanishes.

Note that $\mu_*^2 = -\frac{1}{2}c$, and that $\mu^2 = 0$ at $\ln(\phi_*/\phi) = -\frac{1}{2}$ while $V'' = 0$ at $\ln(\phi_*/\phi) = 1$.

The number $N(\phi)$ of e -folds to the end of slow-roll inflation is given by

$$N(\phi) = M_{\text{P}}^{-2} \int_{\phi_{\text{end}}}^{\phi} \frac{V}{V'} d\phi. \quad (255)$$

Using the linear approximation near ϕ_* , this gives

$$N(\phi) = -\frac{1}{c} \ln \left(\frac{c}{\sigma} \ln \frac{\phi_*}{\phi} \right), \quad (256)$$

or

$$(\sigma/c)e^{-cN} = \ln(\phi_*/\phi). \quad (257)$$

Knowing the functional form of $m^2(\phi)$, and the value of ϕ_{end} , the constant σ can be evaluated by taking the limit $\phi \rightarrow \phi_*$ in the full expression Eq. (255). We shall see that in most cases one expects

$$|c| \lesssim |\sigma| \lesssim 1. \quad (258)$$

The spectral index $n = 1 + 2\eta$ is given in terms of c and σ by

$$\frac{n-1}{2} = \sigma e^{-cN} - c. \quad (259)$$

The COBE normalization is

$$\frac{V_0^{1/2}}{M_{\text{P}}^2} = 5.3 \times 10^{-4} M_{\text{P}} \frac{|V'|}{V_0}, \quad (260)$$

In our case it is convenient to define a constant τ by

$$\ln(M_{\text{P}}/\phi_*) \equiv \tau/|c|. \quad (261)$$

Assuming that $|m^2|$ has the typical value V_0/M_{P}^2 at the Planck scale, the linear approximation Eq. (252) applied at that scale would give $\tau \simeq 1$. Will the linear approximation apply at that scale? If *all* relevant masses at the Planck scale are of order V_0/M_{P}^2 , one expects on dimensional grounds that the linear approximation will be valid in the regime $|c \ln(\phi/\phi_*)| \ll 1$. Then the approximation will be just beginning to fail at the Planck scale. At least in this case, one expects τ to be very roughly of order 1.

Using the definition of τ , Eqs. (253) and (257) give

$$\frac{V_0^{1/2}}{M_{\text{P}}^2} = e^{-\tau/|c|} \exp \left(-\frac{\sigma}{c} e^{-cN_{\text{COBE}}} \right) |\sigma| e^{-cN_{\text{COBE}}} \times 5.3 \times 10^{-4}. \quad (262)$$

In these models, the spectral index may be strongly scale-dependent. In fact, using $d \ln k = -dN$ one finds

$$\frac{dn}{d \ln k} = 2c\sigma e^{-cN} = 2c \left(\frac{n-1}{2} + c \right). \quad (263)$$

For it to be eventually observable we need $|dn/d \ln k| \gtrsim 10^{-3}$, and this condition is satisfied in a large part of the parameter space.

Let us discuss the regime of validity of Eqs. (259) and (263), using Eqs. (59) and (60). The quantities appearing in these expressions are

$$\xi^2 = c\sigma e^{-cN} \quad (264)$$

$$\sigma_V^3 = -\xi^4/c = -\xi^2 c \ln(\phi/\phi_*) . \quad (265)$$

(We relabelled the quantity σ in Eq. (53) as σ_V .)

Eq. (259) will be a good approximation if

$$|\xi|^2 \ll |\eta| . \quad (266)$$

In contrast to the other models we have discussed (where $V' \propto \phi^p$), this condition is not guaranteed. But in this model, ξ^2 is slowly varying. As a result Eq. (51) (with ϵ negligible) implies that the condition will hold except within a few e -folds of a point where η changes sign.

The error of order ξ^2 just represents a small change in the effective value of σ , which can be cancelled by a small change in the underlying parameters (couplings and masses). The improved slow-roll approximation Eq. (80) shows that the error actually corresponds to changing σ by an amount $1.06c$. In the present state of theory the precise amount is not of interest. It would become so only if the underlying parameters were predicted by something like string theory.

When cosmological leave the horizon, $|\sigma_V^3| \ll |\xi^2|$, so the slow-roll formula for $dn/d \ln k$ will also be valid.

6.16.2 The four models

Four types of inflation model are possible, corresponding to whether ϕ_* is a maximum or a minimum, and whether ϕ during inflation is smaller or bigger than ϕ_* .

In the case that ϕ_* is a maximum, one expects the potential to have the form shown in Figures 6 and 7. There is a minimum at $\phi = 0$, and the non-renormalizable terms will ensure that there is a minimum also at some value $\phi_{\min} > \phi_*$. The latter will generally be lower than the one at the origin, and we assume that this is the case. This lowest minimum represents the true vacuum if V vanishes there as in Figure 6. If instead V is positive as in Figure 7, the vacuum lies in some other field direction, ‘out of the paper’. In this case, it is supposed that ϕ arrives near the maximum by tunneling from the minimum that lies on the opposite side.

In the case that ϕ_* is a minimum, the potential will be like the one in Figure 4. The unique minimum represented by $\phi_* = 0$ is the vacuum if V vanishes there (the case shown in Fig. 4). If instead V is positive at the minimum, the vacuum lies in some other field direction.

Model (i); ϕ_* a maximum with $\phi < \phi_*$ This model [290, 62] corresponds to $m^2(M_P) < 0$, $c > 0$ and $\sigma > 0$, with ϕ decreasing during inflation. The spectral index increases as the scale k^{-1} decreases, and can be either bigger or less than 1.

For inflation to end, the form Eq. (248) of $V(\phi)$ must be modified when ϕ falls below some critical value ϕ_c , presumably through a hybrid inflation mechanism. On the other

hand, if the inflaton mass continues to run until $m^2 \simeq V_0/M_{\text{P}}^2$, *slow-roll* inflation will end then. Let us suppose first that this is the case, and define ϕ_{fast} by

$$m^2(\phi_{\text{fast}}) = V_0/M_{\text{P}}^2. \quad (267)$$

This is equivalent to defining $\eta(\phi_{\text{fast}}) = 1$, up to corrections of order c which presumably should not be included in a one-loop calculation. The end of slow-roll inflation corresponds to $\phi_{\text{end}} = \phi_{\text{fast}}$, and the linear approximation Eq. (252) gives the rough estimate $|\ln(\phi_{\text{end}}/\phi_*)| \sim 1/c$, making $\sigma \sim 1$.

Now consider the case where inflation ends at some value ϕ_c , with $|m^2(\phi_c)| < V_0/M_{\text{P}}^2$. If the mass is still running at that point, the linear estimate Eq. (256) gives $\sigma \sim c \ln(\phi_*/\phi_c) < 1$. Values $\sigma \ll c$ can be achieved only with ϕ_c very close to ϕ_* which would represent fine-tuning. Therefore we expect in this case $c \lesssim \sigma \lesssim 1$.

If the mass stops running before ϕ_c is reached, at some point ϕ_{low} , then m^2 has a constant value $m_{\text{low}}^2 = m^2(\phi_{\text{low}})$ in the regime $\phi_c < \phi < \phi_{\text{low}}$. In this regime, some number ΔN of e -folds of slow-roll inflation occur. We are assuming that cosmological scales leave the horizon while the mass is still running, which requires

$$\Delta N < N_{\text{COBE}} - 10 \quad (268)$$

$$< 38 + \ln(V_0^{1/4}/10^{10} \text{ GeV}). \quad (269)$$

Retaining the estimate of the previous paragraph for the e -folds of inflation before the mass stops running, the constant σ to be used in Eq. (257) will be in the range

$$c \lesssim \sigma \lesssim e^{c\Delta N}. \quad (270)$$

After imposing observational constraints [62, 63], one finds that $e^{c\Delta N}$ is no more than one or two orders of magnitude above unity.

Model (ii); ϕ_* a maximum with $\phi > \phi_*$ Like the previous model, this one corresponds to $m^2(M_{\text{P}}) < 0$ and $c > 0$, but now $\sigma < 0$ and ϕ increases during inflation. The spectral index is less than 1, and decreases as the scale decreases.

In contrast with the previous case, inflation can end without any need for a hybrid inflation mechanism, or a change in the form of the potential Eq. (248), if the minimum at $\phi > \phi_*$ is the true vacuum. If the form Eq. (248) holds until ϕ reaches the value ϕ_{fast} defined by $\eta(\phi_{\text{fast}}) = -1$, slow roll inflation will end there. To leading order in c this corresponds to⁶⁶

$$m^2(\phi_{\text{fast}}) = -V_0/M_{\text{P}}^2. \quad (271)$$

Setting $\phi_{\text{end}} = \phi_{\text{fast}}$, and using the crude linear approximation one finds $\phi_{\text{end}} \sim e^{1/|c|}\phi_* \sim M_{\text{P}}$, and $\sigma \sim -1$.

On the other hand, slow-roll inflation might end at some earlier point ϕ_c . In the true-vacuum case illustrated in Figure 6, this may happen through a steepening in the form of

⁶⁶This estimate of ϕ_{fast} assumes that quartic and higher terms in the tree-level potential are negligible at ϕ_{fast} . Assuming that only one such term is significant, one easily checks that the estimate is roughly correct, unless the dimension of the term is not extremely large. We do not consider that case, or the case where more than one term is significant.

$V(\phi)$. Otherwise it may happen through an inverted hybrid inflation mechanism. In both cases, we expect $c \lesssim |\sigma| \lesssim 1$.

In contrast with the previous model, this one also makes sense if m^2 stops running (as ϕ decreases) before it changes sign; in other words, if it stops running at ϕ_{low} with $m^2(\phi_{\text{low}}) < 0$, but very small. In this case the maximum of the potential is at the origin and η is small and constant up to $\phi = 0$. The above treatment remains valid if m^2 has started to run before cosmological scales leave the horizon (remember that in this model, ϕ increases during inflation). Otherwise, one has a different model that we shall not consider.

Model (iii); ϕ_* a minimum with $\phi < \phi_*$ This corresponds to $m^2(M_{\text{P}}) > 0$, $c < 0$ and $\sigma < 0$, and ϕ increases during inflation. The spectral index can be either above or below 1, and it increases as the scale decreases.

Now $|m^2|$ decreases during inflation, and slow-roll inflation ends only when the potential Eq. (248) ceases to hold at some value $\phi_{\text{end}} = \phi_c$. In a single-field model, corresponding to V vanishing at the minimum, this can occur through a steepening of the form of the tree-level potential, as higher powers of ϕ become important. Alternatively, if V is positive at the minimum it can occur through a hybrid inflation mechanism (inverted hybrid inflation).

To estimate σ in this case, suppose first that (as ϕ decreases) the mass continues to run until $m^2 = -V_0/M_{\text{P}}^2$, and denote the point where this happens by ϕ_{fast} . Slow roll inflation can then only occur in the regime $\phi \gtrsim \phi_{\text{fast}}$. It follows that

$$\phi_{\text{end}} \gtrsim \phi_{\text{fast}}, \quad (272)$$

and the linear approximation $\phi_{\text{fast}} \sim e^{-1/|c|}\phi_*$ then gives $|\sigma| \lesssim 1$. As before $|\sigma| \gtrsim |c|$ is required to avoid the fine-tuning $\ln(\phi_*/\phi_c) \ll 1$.⁶⁷

If the mass stops running at some point ϕ_{low} , with $|m^2(\phi_{\text{low}})| \ll 1$, inflation can begin at arbitrarily small field values. If cosmological scales start to leave the horizon only after the mass has started to run, Eq. (272) still applies and the estimate for σ is unchanged. We do not consider the opposite case.

Model (iv); ϕ_* a minimum with $\phi > \phi_*$ Like the previous case this one corresponds to $m^2(M_{\text{P}}) > 0$ and $c < 0$, but now $\sigma > 0$ and ϕ decreases during inflation. The spectral index is bigger than 1, and it decreases as the scale decreases.

Everything is the same as in the previous case, except that a hybrid inflation mechanism will definitely be needed to end inflation, since higher-order terms in ϕ can hardly become more important as ϕ decreases. We again expect $|c| \lesssim \sigma \lesssim 1$, with the lower limit needed to avoid the fine-tuning $\ln(\phi_c/\phi_*) \ll 1$. As a result we expect $|c| \lesssim \sigma \lesssim 1$.

Like Model (iii), this one can still make sense if the mass stops running before ϕ_* is reached. The above treatment applies if cosmological scales leave the horizon while the mass is still running. We do not consider the opposite case.

⁶⁷Stewart [289] took the view that models (iii) and (iv) require a fine-tuning of ϕ_c over the whole range of parameter space. As with all views on fine-tuning, this is a matter of taste.

6.16.3 Observational constraints

In this model, the spectral index can change very significantly on cosmological scales. The usual constraint $|n - 1| < 0.2$ may therefore not apply, but as a crude procedure [63] one can impose this constraint at both N_{COBE} and $N_{\text{COBE}} - 10$. In all four models one finds a viable range of parameter space.

6.17 The spectral index as a discriminator

The point of contact with observation is the spectral index $n(k)$. The Planck satellite will measure it with an accuracy $\Delta n \sim 0.01$ over a range $\Delta \ln k \simeq 6$, and will measure $dn/d \ln k$ if it exceeds a few times 10^{-3} . Let us summarise the predictions of the various models, and see how well the Planck measurement will discriminate between them.

In most models of inflation, the potential is of the form $V(\phi) = V_0 + \dots$, with the constant first term dominating and $\phi \lesssim M_{\text{P}}$. With certain qualifications stated in the text (notably a requirement $\phi \ll M_{\text{P}}$ that needs to be imposed in certain cases) the spectrum of the gravitational waves is too small ever to observe. With similar qualifications, the spectral index for various models is shown in Tables 1 and 2, along with its scale-dependence $dn/d \ln k$.

The simplest cases are $V = V_0 \pm \frac{1}{2}m^2\phi^2$, which give a scale-independent spectral index that may or may not be close to 1.

Next in simplicity come the cases $V = V_0(1 - c\phi^p)$. Here p can be an integer ≥ 3 , corresponding to self-coupling of the inflaton at tree-level, or it can be in the ranges $2 < p < \infty$ or $-\infty < p < 1$ (not necessarily an integer) corresponding to mutated hybrid inflation. Related to these, as far as the prediction is concerned, are the cases $V = V_0(1 - e^{-q\phi})$ (Section 6.6) which corresponds to $p \rightarrow -\infty$ and $V = V_0(1 + c \ln(\phi/Q))$ (Section 6.15) which corresponds to $p \rightarrow 0$. In all these cases the predictions are

$$\frac{1}{2}(n - 1) = - \left(\frac{p - 1}{p - 2} \right) \frac{1}{N} \quad (273)$$

$$\frac{1}{2} \frac{dn}{d \ln k} = - \left(\frac{p - 1}{p - 2} \right) \frac{1}{N^2}. \quad (274)$$

The second expression can be written

$$\frac{1}{2} \frac{dn}{d \ln k} = - \left(\frac{p - 2}{p - 1} \right) \left(\frac{n - 1}{2} \right)^2. \quad (275)$$

Excluding the cases $p \simeq 1$ and $p \simeq 2$, the factor $(p - 1)/(p - 2)$ is of order 1. As a result, $(n - 1)$ is far enough below zero to be eventually observable. The scale-dependence will probably be too small to measure if N is around 50, but should be observable if N is significantly smaller.

Next consider the case $V = V_0(1 + c\phi^p)$ with p an integer ≥ 3 (tree-level self-coupling) or ≤ -1 (dynamical symmetry breaking). In these cases there is a maximum possible number of e -folds of inflation, whose value is unknown. If it is not too big, $n - 1$ may be far enough above zero to eventually detect. The scale-dependence is given by Eq. (275), and will be observable if $|n - 1|$ is more than a few times 0.01. Note that in these models, it is (more

| p | $1 - n$ | | $-10^3 dn/d \ln k$ | |
|---------------------------|----------|----------|--------------------|----------|
| | $N = 50$ | $N = 20$ | $N = 50$ | $N = 20$ |
| $p \rightarrow 0$ | 0.02 | 0.05 | (0.4) | 2.6 |
| $p = -2$ | 0.03 | 0.075 | (0.6) | 3.8 |
| $p \rightarrow \pm\infty$ | 0.04 | 0.10 | (0.8) | 5.0 |
| $p = 4$ | 0.06 | 0.15 | (1.2) | 5.4 |
| $p = 3$ | 0.08 | 0.20 | (1.6) | 10.0 |

Table 1: Predictions for the spectral index n and its variation $dn/d \ln k$, are displayed for some potentials of the form $V_0(1 - c\phi^p)$ that are discussed in the text. The variation will be detectable by Planck if $|dn/d \ln k| \gtrsim 2.0 \times 10^{-3}$. The case $p \rightarrow 0$ corresponds to the potential $V_0(1 - c \ln \phi)$, and the case $p \rightarrow -\infty$ corresponds to $V_0(1 - e^{-q\phi})$.

| Comments | $V(\phi)/V_0$ | $\frac{1}{2}(n - 1)$ | $\frac{1}{2} \frac{dn}{d \ln k}$ |
|---|---|--|--|
| Mass term | $1 \pm \frac{1}{2}c \frac{\phi^2}{M_{\text{P}}^2}$ | $\pm c$ | 0 |
| Softly broken susy | $1 \pm \frac{1}{2}c \frac{\phi^2}{M_{\text{P}}^2} \ln \frac{\phi}{Q}$ | $\pm c + \sigma e^{\pm cN}$ | $\mp c\sigma e^{\pm cN}$ |
| Spont. broken susy | $1 + c \ln \frac{\phi}{Q}$ | $-\frac{1}{2N}$ | $-\frac{1}{2} \frac{1}{N^2}$ |
| $p > 2$ or $-\infty < p < 1$ (self-coupling or hybrid) | $1 - c\phi^p$ | $-\left(\frac{p-1}{p-2}\right) \frac{1}{N}$ | $-\left(\frac{p-1}{p-2}\right) \frac{1}{N^2}$ |
| Various models | $1 - e^{-q\phi}$ | $-\frac{1}{N}$ | $-\frac{1}{N^2}$ |
| p integer ≤ -1 (dyn. s. b.) or ≥ 3 (self-coupling) | $1 + c\phi^p$ | $\frac{p-1}{p-2} \frac{1}{N_{\text{max}} - N}$ | $-\left(\frac{p-2}{p-1}\right) \left(\frac{n-1}{2}\right)^2$ |

Table 2: Predictions for the spectral index $n(k)$. Wavenumber k related to number of e -folds N by $d \ln k = -dN$. Constants c, q and Q are positive while σ and p can have either sign. In the first three cases, there is a theoretical constraint $|c| \ll 1$. In the second case, one expects $|\sigma| \gtrsim |c|$.

than usually) unclear how the inflaton is supposed to arrive at the inflaton part of the potential.

Finally we come to the case of a running inflaton mass (Section 6.16). This gives a distinctive prediction for the shape of n , and in contrast with the other models the predicted magnitude of $dn/d \ln k$ can be of order $(n - 1)$.

7 Supersymmetry

7.1 Introduction

In the last section we looked at some ‘models’ of inflation, taken to mean forms for the inflationary potential that look reasonable from the viewpoint of particle theory. Now we go deeper, taking on board present ideas about what might lie beyond the Standard Model. The eventual goal is to see whether deeper considerations favour one form of the potential over another. We begin by reviewing supersymmetry, which is the almost universally accepted

framework for constructing extensions of the Standard Model.

Supersymmetry can be formulated either as a global or a local symmetry. In the latter case it includes gravity, and is therefore called supergravity. Supergravity is presumably the version chosen by Nature.

7.2 The motivation for supersymmetry

It is widely accepted that the standard model of gauge interactions describing the laws of physics at the weak scale is extraordinarily successful. The agreement between theory and experimental data is very good. Yet, we believe that the present structure is incomplete. Only to mention a few drawbacks, the theory has too many parameters, it does not describe the fermion masses and why the number of generations is three. It contains fundamental scalars, something difficult to reconcile with our current understanding of non-supersymmetric field theory. Finally, it does not incorporate gravity.

It is tempting to speculate that a new (but yet undiscovered) symmetry, supersymmetry [243, 129, 299, 19], may provide answers to these fundamental questions. Supersymmetry is the only framework in which we seem to be able to understand light fundamental scalars. It addresses the question of parameters: first, unification of gauge couplings works much better with than without supersymmetry; second, it is easier to attack questions such as fermion masses in supersymmetric theories, in part simply due to the presence of fundamental scalars. Supersymmetry seems to be intimately connected with gravity. So there are a number of arguments that suggest that nature might be supersymmetric, and that supersymmetry might manifest itself at energies of order the weak interaction scale.

Is supersymmetry expected to play a fundamental role at the early stages of the evolution of the Universe and, more specifically, during inflation? The answer is almost certainly yes. For one thing, the mere fact that we are invoking scalar fields (the inflaton, and at least one other in the case of hybrid inflation) means that supersymmetry is involved. More concretely, the potential needs to be very flat in the direction of the inflaton, and supersymmetry can help here too. We noted earlier that supersymmetric theories typically possess many flat directions, in which the dangerous quartic term of the potential vanishes. It helps in a more general sense too. While the necessity of introducing very small parameters to ensure the extreme flatness of the inflaton potential seems very unnatural and fine-tuned in most non-supersymmetric theories, this technical naturalness may be achieved in supersymmetric models. Indeed, the nonrenormalization theorem guarantees that a fundamental object in supersymmetric theories, the superpotential, is not renormalized to all orders of perturbation theory [125]. In other words, the nonrenormalization theorems in unbroken, renormalizable global supersymmetry guarantee that we can fine-tune any parameter at the tree level and this fine-tuning will not be destabilized by radiative corrections at any order in perturbation theory. Therefore, inflation in the context of supersymmetric theories seems, at least technically speaking, more natural than in the context of non-supersymmetric theories.

7.3 The susy algebra and supermultiplets

We begin with some basics, that apply to both global susy and supergravity.

In the low-energy regime, phenomenology requires the type of supersymmetry known as $N = 1$ (one generator). This is usually assumed to be the case also in the higher energy regime relevant during inflation (though see [107]). In this section, we present some features of $N = 1$ supersymmetric theories, that are likely to be relevant for inflation. The reader interested in more details is referred to the excellent introductions by Nilles [243], Bailin and Love [19] and Wess and Bagger [299]. Except where stated, we use the conventions of Wess and Bagger except that some of their symbols are replaced by more modern ones (for instance, the superpotential is denoted by W instead of P .)

The basic supersymmetry algebra is given by

$$\{Q_\alpha, \bar{Q}_\beta\} = 2\sigma_{\alpha\dot{\beta}}^\mu P_\mu, \quad (276)$$

where Q_α and \bar{Q}_β are the supersymmetric generators (bars stand for conjugate), α and β run from 1 to 2 and denote the two-component Weyl spinors (quantities with dotted indices transform under the $(0, \frac{1}{2})$ representation of the Lorentz group, while those with undotted indices transform under the $(\frac{1}{2}, 0)$ conjugate representation). σ^μ is a matrix four vector, $\sigma^\mu = (-\mathbf{1}, \vec{\sigma})$ and P_μ is the generator of spacetime displacements (four-momentum).

The chiral and vector superfields are two irreducible representations of the supersymmetry algebra containing fields of spin less than or equal to one. Chiral fields contain a Weyl spinor and a complex scalar; vector fields contain a Weyl spinor and a (massless) vector. In superspace a chiral superfield may be expanded in terms of the Grassmann variable θ [299]

$$\phi(x, \theta) = \phi(x) + \sqrt{2}\theta\psi(x) + \theta^2 F(x). \quad (277)$$

Here x denotes a point in spacetime, $\phi(x)$ is the complex scalar, ψ the fermion, and F is an auxiliary field. As in this expression, we shall generally use the same symbol to represent a superfield and its scalar component. Under a supersymmetry transformation with anticommuting parameter ζ , the component fields transform as

$$\delta\phi = \sqrt{2}\zeta\psi, \quad (278)$$

$$\delta\psi = \sqrt{2}\zeta F + \sqrt{2}i\sigma^\mu\bar{\zeta}\partial_\mu\phi, \quad (279)$$

$$\delta F = -\sqrt{2}i\partial_\mu\psi\sigma^\mu\bar{\zeta}. \quad (280)$$

Here and in the following, for any generic two-component Weyl spinor λ , $\bar{\lambda}$ indicates the complex conjugate of λ . For a gauge theory one has to introduce vector superfields and the physical content is most transparent in the Wess-Zumino gauge. In this gauge and for the simplest case of an abelian group $U(1)$, the vector superfield may be written as

$$V = -\theta\sigma^\mu\bar{\theta}A_\mu + i\theta^2\bar{\theta}\bar{\lambda} - i\bar{\theta}^2\theta\lambda + \frac{1}{2}\theta^2\bar{\theta}^2 D. \quad (281)$$

Here A_μ is the gauge field, λ_α is the gaugino, and D is an auxiliary field. The analog of the gauge invariant field strength is a chiral field:

$$W_\alpha = -i\lambda_\alpha + \theta_\alpha D - \frac{i}{2}(\sigma^\mu\bar{\sigma}^\nu\theta)_\alpha F_{\mu\nu} + \theta^2\sigma_{\alpha\dot{\beta}}^\mu\partial_\mu\bar{\lambda}^{\dot{\beta}}, \quad (282)$$

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$, and where $\bar{\sigma}^\mu = (-\mathbf{1}, -\vec{\sigma})$. Regarding the supersymmetry transformations, let us just note that

$$\delta\lambda = i\zeta D + \zeta\sigma^\mu\bar{\sigma}^\nu F_{\mu\nu}. \quad (283)$$

Global supersymmetry is defined as invariance under these transformations with ξ independent of spacetime position, and local supersymmetry (supergravity) as invariance with ξ depending on spacetime position. In the latter case one has to introduce another supermultiplet containing the graviton and gravitino.

Global supersymmetry need not be renormalizable (Section 7.8). But the usual convention is that ‘global supersymmetry’ refers to a theory which is renormalizable, except possibly for the superpotential W defined below. For the most part we follow that convention.⁶⁸

As we discuss in Section 7.8, global supersymmetry may be regarded as a limit of supergravity, in which roughly speaking gravity is made negligible by taking M_{P} to infinity. *For most purposes* it is a good approximation if the vevs of all relevant scalar fields and auxiliary fields are much less than M_{P} . (Relevant here means that they have not been integrated out (page 37).) There are however two notable exceptions.

In the true vacuum, global susy (whether renormalizable or not) would predict a large positive value for V , instead of the practically zero value observed in our Universe. According to supergravity, a negative contribution of unknown magnitude should be subtracted from the global susy value. It is assumed that this value makes V practically zero in the true vacuum, though one does not understand the origin of this exact cancellation. (This is called the cosmological constant problem.)

During inflation, the naive limit $M_{\text{P}} \rightarrow \infty$ makes no sense [290], because as we saw in Section 3 M_{P} plays an essential role. The approximation of global supersymmetry can be justified only in special circumstances, by methods more subtle than simply taking M_{P} to infinity. As we shall see, this is a problem for inflation model-building, because a generic supergravity theory does not give a potential that is sufficiently flat for inflation. By contrast a generic globally supersymmetric theory works perfectly well.

7.4 The lagrangian of global supersymmetry

We focus first on global susy, with the usual restriction that it be renormalizable except for possible non-renormalizable terms in the superpotential.

To write down the action for a set of chiral superfields, ϕ_i , transforming in some representation of a gauge group G , one introduces, for each gauge generator, a vector superfield, V^a . Defining the matrix $V = T^a V_a$, where T^a are the hermitian generators of the gauge group G in the representation defined by the scalar fields and excluding the possible Fayet-Iliopoulos term to be discussed later, the most general renormalizable lagrangian, written in superspace, is then

$$\mathcal{L} = \sum_n \int d^4\theta \phi_n^\dagger e^V \phi_n + \frac{1}{4k} \int d^2\theta W_\alpha^2 + \int d^2\theta W(\phi_n) + \text{h.c.}, \quad (284)$$

where in the adjoint representation $\text{Tr}(T^a T^b) = k\delta^{ab}$ and $W(\phi_n(x, \theta))$ is a fundamental object known as superpotential. The corresponding function of the scalar components

⁶⁸One can also consider the fully non-renormalizable version of global susy, which includes a non-trivial Kähler potential and/or a non-trivial gauge kinetic function. At this point, let us make it clear that we are talking about the Kähler potential, and the gauge kinetic function, of the fundamental lagrangian, giving the tree-level potential.

$\phi_n(x)$, denoted by the same name and symbol, is a holomorphic function of the ϕ_n . For simplicity, we shall pretend that there is a single gauge $U(1)$ interaction, with coupling constant g . This is adequate since such an interaction is the only one that we consider in detail. (To be precise, we consider a $U(1)$ with a Fayet-Iliopoulos term.) In the case of several $U(1)$'s, there are no cross-terms in the potential from the D -terms, i.e. V_D is simply expressed as $\sum_n (V_D)_n$.

To write this down in terms of component fields, we need the covariant derivative

$$D_\mu = \partial_\mu - \frac{i}{2}gA_\mu. \quad (285)$$

In terms of the component fields, the lagrangian takes the form:

$$\begin{aligned} \mathcal{L} &= \sum_n \left(D_\mu \phi_n^* D^\mu \phi_n + i D_\mu \bar{\psi}_n \bar{\sigma}^\mu \psi_n + |F_n|^2 \right) \\ &- \frac{1}{4} F_{\mu\nu}^2 - i \lambda \sigma^\mu \partial_\mu \bar{\lambda} + \frac{1}{2} D^2 + \frac{g}{2} D \sum_n q_n \phi_n^* \phi_n \\ &- \left[i \sum_n \frac{g}{\sqrt{2}} \bar{\psi}_n \bar{\lambda} \phi_n - \sum_{nm} \frac{1}{2} \frac{\partial^2 W}{\partial \phi_n \partial \phi_m} \psi_n \psi_m \right. \\ &\left. + \sum_n F_n \left(\frac{\partial W}{\partial \phi_n} \right) \right] + \text{c.c.} \end{aligned} \quad (286)$$

At the end of the second line, q_n are the $U(1)$ -charges of the fields ϕ_n . The equations of motion for the auxiliary fields F_n and D are the constraints:

$$F_n = - \left(\frac{\partial W}{\partial \phi_n} \right)^* \quad (287)$$

$$D = -\frac{g}{2} \sum_n q_n |\phi_n|^2. \quad (288)$$

Eq. (286) contains the gauge invariant kinetic terms for the various fields, which specify their gauge interactions. It also contains, after having made use of Eqs. (287) and (288), the scalar field potential,

$$V = V_F + V_D, \quad (289)$$

$$V_F \equiv \sum_n |F_n|^2, \quad (290)$$

$$V_D \equiv \frac{1}{2} D^2. \quad (291)$$

This separation of the potential into an F term and a D term is crucial for inflation model-building, especially when it is generalized to the case of supergravity.

The potential specifies the masses of the scalar fields, and their interactions with each other. The first term in the third line specifies the interactions of gaugino and scalar fields, while the second specifies the masses of the chiral fermions and their interactions with the scalars. All of these non-gauge interactions are called Yukawa couplings.

To have a renormalizable theory, W is at most cubic in the fields, corresponding to a potential which is at most quartic. However, one commonly allows W to be of higher order, producing the kind of potentials that were mentioned in Section 5.9.

From the above expressions, in particular Eq. (290), one sees that the overall phase of W is not physically significant. An internal symmetry can either leave W invariant, or alter its phase. The latter case corresponds to what is called an R-symmetry. Because W is holomorphic, the internal symmetries restrict its form much more than is the case for the actual potential V . In particular, terms in W of the form $\frac{1}{2}m\phi_1^2$ or $m\phi_1\phi_2$, which would generate a mass term $m^2|\phi_1|^2$ in the potential, are usually forbidden.⁶⁹ As a result, scalar particles usually acquire masses only from the vevs of scalar fields (ie., from the spontaneous breaking of an internal symmetry) and from supersymmetry breaking. The same applies to the spin-half partners of scalar fields, with the former contribution the same in both cases.

In the case of a $U(1)$ gauge symmetry, one can add to the above lagrangian what is called a Fayet-Iliopoulos term [99],

$$- 2\xi \int d^4\theta V. \quad (292)$$

This corresponds to adding a contribution $-\xi$ to the D field, so that Eq. (288) becomes

$$D = -\frac{g}{2} \sum_n q_n |\phi_n|^2 - \xi. \quad (293)$$

The D term of the potential therefore becomes

$$V_D = \frac{1}{2} \left(\frac{g}{2} \sum_n q_n |\phi_n|^2 + \xi \right)^2. \quad (294)$$

From now on, we shall use a more common notation, where ξ and the charges are redefined so that

$$V_D = \frac{1}{2} g^2 \left(\sum_n q_n |\phi_n|^2 + \xi \right)^2. \quad (295)$$

This is equivalent to

$$D = -g \left(\sum_n q_n |\phi_n|^2 + \xi \right). \quad (296)$$

A Fayet-Iliopoulos term may be present in the underlying theory from the very beginning,⁷⁰ or appears in the effective theory after some heavy degrees of freedom have been integrated out.

It looks particularly intriguing that an anomalous $U(1)$ symmetry is usually present in weakly coupled string theories [124]. (Anomalous in this context means that $\sum q_n \neq 0$.) In this case [83, 17, 84]

$$\xi = \frac{g_{\text{str}}^2}{192\pi^2} \text{Tr} \mathbf{Q} M_{\text{P}}^2. \quad (297)$$

⁶⁹An exception is the μ term of the MSSM, $\mu H_U H_D$, which gives mass to the Higgs fields.

⁷⁰It is allowed by a gauge symmetry, unless the $U(1)$ is embedded in some non-Abelian group. $\xi = 0$ can be enforced by charge conjugation symmetry which flips all $U(1)$ charges. Such symmetry is possible in nonchiral theories.

Here $\text{Tr} \mathbf{Q} = \sum q_n$, which is typically [102, 164] of order 100. One expects the string-scale gauge coupling g_{str} (Section 7.9.3) to be of order 1 to 10^{-1} , making $\xi \simeq 10^{-1}$ to $10^{-2} M_{\text{P}}$.

In the context of the strongly coupled $E_8 \otimes E_8$ heterotic string [139], anomalous $U(1)$ symmetries may appear and have a nonperturbative origin, related to the presence, after compactification, of five-branes in the five-dimensional bulk of the theory. There is, at the moment, no general agreement on the relative size of the induced Fayet-Iliopoulos terms on each boundary compared to the value of the universal one induced in the weakly coupled case [227, 41].

7.5 Spontaneously broken global susy

7.5.1 The F and D terms

Global supersymmetry breaking may be either spontaneous or explicit. Let us begin with the first case. For spontaneous breaking, the lagrangian is supersymmetric as given in the last subsection. But the generators Q_α fail to annihilate the vacuum. Instead, they produce a spin-half field, which may be either a chiral field ψ_α or a gauge field λ_α . The condition for spontaneous susy breaking is therefore to have a nonzero vacuum expectation value for $\{Q_\alpha, \psi_\beta\}$ or $\{Q_\alpha, \lambda_\beta\}$.

The former quantity is defined by Eq. (279), and the latter by Eq. (283). The quantities $\partial_\mu \phi$ and $F_{\mu\nu}$ contain derivatives of fields, and are supposed to vanish in the vacuum. It follows that susy is spontaneously broken if, and only if, at least one of the auxiliary fields F_n or D has a non-vanishing vev.

In the true vacuum, one defines the scale M_{S} of global supersymmetry breaking by

$$M_{\text{S}}^4 = \sum_n |F_n|^2 + \frac{1}{2} D^2, \quad (298)$$

or equivalently

$$M_{\text{S}}^4 = V. \quad (299)$$

(In the simplest case D vanishes and there is just one F_n .)

When we go to supergravity, part of V is still generated by the supersymmetry breaking terms, but there is also a contribution $-3|W|^2/M_{\text{P}}^2$. This allows V to vanish in the true vacuum as is (practically) demanded by observation.

During inflation, V is positive so the negative term is smaller than the susy-breaking terms. In most models of inflation it is negligible. In any case, V is at least as big as the susy breaking term, so the search for a model of inflation is also a search for a susy-breaking mechanism in the early Universe.

Spontaneous symmetry breaking can be either tree-level (already present in the lagrangian) or dynamical (generated only by quantum effects like condensation). The spontaneous breaking in general breaks the equality between the scalar and spin- $\frac{1}{2}$ masses, in each chiral supermultiplet. But at tree level the breaking satisfies a simple relation, which can easily be derived from the lagrangian (286). Ignoring mass mixing for simplicity, one finds in the case of symmetry breaking by an F -term,

$$\sum_n (m_{n1}^2 + m_{n2}^2 - 2m_{nf}^2) = 0. \quad (300)$$

Here n labels the chiral supermultiplets, m_{nf} is the fermion mass while m_{n1} and m_{n2} are the scalar masses.⁷¹ In the case of symmetry breaking by a D term, coming from a $U(1)$, the right hand side of Eq. (300) becomes $D\text{Tr}\mathbf{Q}$. But in order to cancel gauge anomalies, it is often desirable that $\text{Tr}\mathbf{Q} = 0$ which recovers Eq. (300).

7.5.2 Tree-level spontaneous susy breaking with an F term

Models of tree-level spontaneous susy-breaking where only F terms have vevs are called O’Raifeartaigh models. We consider them now, postponing until Section 7.6.1 the case of D -term susy breaking.

The simplest O’Raifeartaigh model involves a single field X ,

$$W = m^2 X + \dots, \quad (301)$$

where the dots represent terms independent of X . The potential is given by $V = m^4 + \dots$, and $F_X = m^2$; thus supersymmetry is broken for nonvanishing m . Some models of inflation invoke such a linear superpotential.

We shall encounter more complicated O’Raifeartaigh models for inflation later. At this point let us give the following example, which is probably of only pedagogical interest. It involves three singlet fields, X, ϕ and Y , with superpotential:

$$W = \lambda_1 X(\phi^2 - \mu^2) + \lambda_2 Y \phi^2. \quad (302)$$

With this superpotential, the equations

$$F_X = \frac{\partial W}{\partial X} = \lambda_1(\phi^2 - \mu^2) = 0, \quad F_Y = \frac{\partial W}{\partial Y} = \lambda_2 \phi^2 = 0 \quad (303)$$

are incompatible. Note that at this level not all of the fields are fully determined, since the equation

$$\frac{\partial W}{\partial \phi} = 0 \quad (304)$$

can be satisfied provided

$$\lambda_1 X + \lambda_2 Y = 0. \quad (305)$$

This vacuum degeneracy is accidental and is lifted by quantum corrections. Since either $\langle F_X \rangle$ or $\langle F_Y \rangle$ are nonvanishing, supersymmetry is broken at the tree-level.

7.5.3 Dynamically generated superpotentials

It has been known for a long time that global, renormalizable supersymmetry may be dynamically broken in four dimensions [5, 241]. There already exist excellent reviews of this subject and the reader is referred to [76, 275, 142, 293, 241, 116] for more details. Several mechanisms have been proposed, but only two have so far been invoked for inflation model-building. These are a dynamically generated superpotential, and a quantum moduli space, which we look at now starting with the former.

⁷¹More generally, if the mass-squared matrix is non-diagonal the left hand side of Eq. (300) is the supertrace defined in Eq. (300).

In some cases, the dynamically generated superpotential occurs in a theory characterized by many classically flat directions. Typically, the potentials generated along these flat directions fall down to zero at large values of the fields. These potentials, however, must be stabilized by some mechanism and so far no compelling model has been proposed.

Alternatively, models are known in which supersymmetry is broken without flat directions [5] and no need of complicated stabilization mechanisms. In some directions, non-perturbative effects might raise the potential at small field values, while tree-level terms raise it at large values. If some F -term is nonzero in the ground state, supersymmetry is spontaneously broken.

To provide an explicit example, let us consider the model discussed in [85] in which the tree-level terms are non-renormalizable. The gauge group is $SU(6) \otimes U(1) \otimes U(1)_m$ and the chiral superfields are $A(15, 1, 0)$, $\bar{F}^\pm(\bar{6}, -2, \pm 1)$, $S^0(1, 3, 0)$ and $S^\pm(1, 3, \pm 2)$. $U(1)_m$ is irrelevant for supersymmetry breaking but may play the role of messenger hypercharge. The gauge symmetries forbid a cubic superpotential in the model. At the level of dimension five operators, the unique term allowed $W = \frac{1}{M} A \bar{F}^+ \bar{F}^- S^0$, where M may be identified with M_P . Along the $SU(6)$ and $U(1)$ D -flat directions the gauge symmetry is broken down to $Sp(4)$. Gluino condensation at the scale Λ leads to a nonperturbative superpotential whose form follows uniquely from symmetry considerations: $W_{np} = \frac{\Lambda^5}{\mathcal{O}^{1/3}}$, where $\mathcal{O} = \bar{F}_i^+ \bar{F}_j^- A^{ij} \epsilon_{klmnop} A^{kl} A^{mn} A^{op}$. Turning on the nonrenormalizable superpotential lifts the flat directions and the value of the potential at the minimum turns out to be $V_0 \sim \Lambda^5/M$ and F -terms are of order of $\Lambda^{15/6} M^{-1/2}$ signalling the breaking of supersymmetry.

A generic prediction of dynamical supersymmetry breaking models is the appearance of a superpotential $W \simeq \Lambda^{3+q}/\phi^q$, leading to a potential $V(\phi) = (\Lambda^{p+4})/(|\phi^p|)$, where the index p and the scale Λ depend upon the underlying gauge group.

7.5.4 Quantum moduli spaces

Recent developments have also shown that many supersymmetric theories may have other types of non-perturbative dynamics which lead to degenerate quantum moduli spaces of vacuum instead of dynamically generated superpotentials [76, 275, 142, 293, 143, 145]. The quantum deformation of a classical moduli space constraint may lead to supersymmetry breaking. This happens because the patterns of breakings of global and gauge symmetries on a quantum moduli space may differ from those on the classical moduli space and the quantum deformed constraint associated with the moduli space is inconsistent with a stationary superpotential. Indeed, moduli generally transform under global symmetries and there is a point on the classical moduli space at which *all* the fields have zero vev and global symmetries are unbroken. However, at the quantum level points which are part of the classical moduli space may be removed. If tree-level interactions have vanishing potential, and auxiliary fields, only at points on the classical moduli space which are not part of the quantum deformed moduli space, supersymmetry gets broken.

We consider the following simple example. The gauge theory considered is an $SU(2)$ gauge theory with matter consisting of four doublet chiral superfields Q_I, \bar{Q}^J , where $I, J = 1, 2$ are flavour indices. The theory also contains a singlet superfield S and the superpotential reads

$$W = gS(Q_1 \bar{Q}_1 + Q_2 \bar{Q}_2), \quad (306)$$

where g is a Yukawa coupling constant. At the classical level, in the absence of this superpotential ($g = 0$), the space of vacua (D -flat directions) is parameterized by a set of complex fields consisting of S plus the following 6 $SU(2)$ invariants (mesons and baryons)

$$M_I^J = Q_I \bar{Q}^J, \quad B = \epsilon^{IJ} Q_I Q_J, \quad \bar{B} = \epsilon_{IJ} \bar{Q}^I \bar{Q}^J. \quad (307)$$

The invariants are however subject to the constraint

$$\det M - \bar{B} B = 0 \quad (308)$$

so that in the end the space of vacua at $g = 0$ has complex dimension 6. In the presence of the superpotential, the classical moduli space has two branches: *a)* $S \neq 0$, with $M_I^J = B = \bar{B} = 0$. On this branch the quarks get a mass $\sim gS$ from the superpotential and the gauge symmetry is unbroken; *b)* $S = 0$, with non-zero mesons and baryons satisfying two constraints. One is Eq. (308) while the other is $F_S = \text{Tr} M = 0$. Here the gauge group is broken.

This moduli space is however reduced by quantum effects. In particular a non-zero vacuum energy is generated along the $S \neq 0$ branch. This is established by considering the effective theory far away along $S \neq 0$. Here the quark fields get masses of order S and decouple. The effective theory consists of the (free) singlet S plus a pure $SU(2)$ gauge sector. The effective scale Λ_L of the low-energy $SU(2)$ along this trajectory is given to all orders by the 1-loop matching of the gauge couplings at the quarks' mass gS and reads

$$\Lambda_L^6 = g^2 S^2 \Lambda^4, \quad (309)$$

where Λ is the scale of the original theory with massless quarks. In the pure $SU(2)$ gauge theory gauginos condense and an effective superpotential $\sim \Lambda_L^3$ is generated

$$W_{eff} = gS\Lambda^2. \quad (310)$$

Thus

$$F_S = g\Lambda^2 \quad (311)$$

and supersymmetry is broken, with a vacuum energy density F_S^2 which is independent of S . As we mention later, it has been suggested [73] that $|S|$ is the inflaton.

7.6 Soft susy breaking

In the effective theory, which describes the interactions of the Standard Model particles and their superpartners at energies $\lesssim 1$ TeV, supersymmetry is taken to be broken explicitly. In order to preserve the theoretical motivation for supersymmetry (the absence of quadratic divergences and the naturalness of the theory) only certain 'soft' susy-breaking terms are allowed. These are

- Masses (and mass-mixing terms) for scalars, whose typical value will be denoted by \tilde{m} .
- Masses for gauginos, whose typical value will be denoted by $m_{\tilde{g}}$.

- Cubic terms in the scalar field potential, of the form $(A_{ijk}\phi_i\phi_j\phi_k + \text{c.c.})$. The typical value of the couplings A_{ijk} will be denoted by A .

There are no soft chiral fermion masses, nor any soft quartic terms. Both of these have their unbroken susy values; in particular, the quartic term vanishes in a flat direction of unbroken susy. For susy to do its job one requires that the mass scales $m_{\tilde{g}}$, \tilde{m} and A are all $\lesssim 1$ TeV.

The squark and slepton masses come almost entirely from the soft susy breaking (except for the stop), and to have escaped detection they have to be $\gtrsim 100$ GeV. So at least \tilde{m} should be in the range roughly 100 GeV to 1 TeV.

The effective theory, with explicit soft susy breaking, describes only the ‘visible’ sector of the theory that consists of the fields possessing the Standard Model gauge interactions. In the full theory, spontaneous susy breaking is supposed to take place, but in a ‘hidden’ sector, consisting of fields which do not possess the Standard Model gauge interactions. When the hidden sector is integrated out (footnote 36) one obtains the effective theory in the visible sector.

The spontaneous breaking is usually of the F -term type. Models are classified as ‘gravity-mediated’ if the interaction between the two sectors is only of gravitational strength, or as ‘gauge-mediated’ if it is stronger (usually involving a gauge interaction). In the gauge-mediated case, the entire theory including the mechanism of spontaneous susy breaking is supposed to be describable in terms of global susy. In the gravity-mediated case, the mechanism of spontaneous susy breaking is usually supposed to involve supergravity in an essential way, since that theory is anyhow needed to describe the interaction between the two sectors. (One is however free to suppose that in this case too, the mechanism of spontaneous susy breaking is describable in terms of global supersymmetry [261].)

7.6.1 Soft susy breaking from a D term

Before dealing with the gauge-mediated case, we look at a proposal [36, 260, 228, 229, 242, 97, 33, 38, 15, 150] that invokes a D term. The D term comes from a $U(1)$ with a Fayet-Iliopoulos term, which is usually considered to have a stringy origin as described in Section 7.4. As we shall see, such a term has also been widely used for building models of inflation, but for now we are concerned with the true vacuum.

The hidden sector consists of two fields ϕ_{\pm} . The part of the superpotential depending only on them is $W = m\phi_+\phi_-$. Ignoring the rest of the superpotential for the moment, the potential is

$$\begin{aligned}
 V &= m^2(|\phi_+|^2 + |\phi_-|^2) \\
 &+ \frac{g^2}{2} \left(\sum_i q_i |\tilde{Q}_i|^2 + |\phi_+|^2 - |\phi_-|^2 + \xi \right)^2.
 \end{aligned}
 \tag{312}$$

The scalar fields of the visible sector are denoted by \tilde{Q}_i , and we shall see in a moment that they have masses of order m . Accordingly, we take $m \simeq (1 - 10)$ TeV, without enquiring into the origin of m .

Let us consider the part of V setting $\tilde{Q} = 0$. It is easy to see that its minimum breaks supersymmetry as well as the anomalous $U(1)$ gauge symmetry with [260]

$$\langle \phi_- \rangle = \left(\xi - \frac{m^2}{g^2} \right)^{1/2}, \quad \langle \phi_+ \rangle = 0 \quad (313)$$

$$\langle F_{\phi_+} \rangle = m \left(\xi - \frac{m^2}{g^2} \right)^{1/2}, \quad \langle D \rangle = m^2. \quad (314)$$

If we parameterize $\xi = \epsilon M_{\text{Pl}}^2$, we have $\langle \phi_- \rangle \simeq \epsilon^{1/2} M_{\text{Pl}}$ and $\langle F_{\phi_+} \rangle \simeq \epsilon^{1/2} m M_{\text{Pl}}$. In weakly coupled string theory, ϵ is given by Eq. (297) and is of order 10^{-1} to 10^{-2} . Integrating out⁷² ϕ_{\pm} generates soft susy breaking mass terms of order m for the scalar fields charged under $U(1)$ ⁷³

$$\tilde{m}_{\tilde{Q}_i}^2 = q_i \langle D \rangle = q_i m^2. \quad (315)$$

The charges q_i are required to be positive to avoid color/charge breaking. Invariance under the anomalous $U(1)$ will require, therefore, that terms in the superpotential involving visible-sector fields with nonzero charges are multiplied by appropriate powers of ϕ_-/M_{Pl} [228, 229].

If m is large enough and if the first two generations of squarks are (equally) charged under the $U(1)$, the harmful flavour-changing neutral currents (FCNC's) are suppressed and trilinear soft breaking mass terms are also suppressed by powers of ϵ so that large supersymmetric CP-violating phases pose no problem [260, 228, 229].

If the Fayet-Iliopoulos has a stringy origin, it is directly proportional to g_{str}^2 , see Eq. (297). As such, it depends on the vacuum expectation value of (the real part of) the dilaton field s , $g_{\text{str}}^2 = M_{\text{P}}/(\text{Re } s)$ (see subsection 7.9.3 for more details).

It has been recently argued [16] that, within some particular mechanisms for stabilizing the dilaton in string theories, the supersymmetry breaking contribution to the soft masses of sfermions coming from the the dilaton F -term always dominates over the D -term supersymmetry breaking contribution from the anomalous $U(1)$.

However, other mechanisms for stabilizing the dilaton may not have this effect. For instance, if the dilaton is stabilized by the contributions to the superpotential, the dilaton F -term vanishes and the soft supersymmetry breaking mass terms only comes from the D -term.

Finally, we would like to point out that the class of model with D -term supersymmetry breaking may have some problems on the cosmological side, as far as the dark matter abundance is concerned [113].

7.6.2 Gauge-mediated susy breaking

Global susy models involving only the F term are called gauge-mediated models [77, 71, 78, 79, 80, 9, 240, 72, 74, 18], because communication between the hidden and visible sectors is usually through a gauge interaction. A review of these models is given in Reference [116].

⁷²See footnote 36

⁷³The term $\langle F_{\phi_+} \rangle$ will give a gravity-mediated contribution which is smaller by a factor ϵ .

The minimal gauge mediated supersymmetry breaking models are defined by three sectors: (i) a hidden sector (often called a secluded sector in this context) that breaks supersymmetry; (ii) a messenger sector that serves to communicate the SUSY breaking to the standard model and (iii) the standard model sector. The minimal messenger sector consists of a single $\mathbf{5} + \bar{\mathbf{5}}$ of $SU(5)$ (to preserve gauge coupling constant unification), *i.e.* color triplets, q and \bar{q} , and weak doublets ℓ and $\bar{\ell}$ with their interactions determined by the following superpotential:

$$W = \lambda_1 X \bar{q} q + \lambda_2 X \bar{\ell} \ell. \quad (316)$$

When the field X acquires a vacuum expectation value for both its scalar and auxiliary components, $\langle X \rangle$ and $\langle F_X \rangle$ respectively, the fields $q \pm q^*$ acquire masses $\lambda_1^2 \langle X \rangle^2 \pm \lambda_1 \langle F_X \rangle$, and similarly for the fields $\ell \pm \ell^*$. This supersymmetry breaking in the messenger sector gives gaugino masses at one loop and scalar masses at two loops (with messengers and gauge bosons in the loops). At the scale $\langle X \rangle$, the gaugino masses are given approximately by

$$M_j(\langle X \rangle) = k_j \frac{\alpha_j(\langle X \rangle)}{4\pi} \Lambda, \quad j = 1, 2, 3, \quad (317)$$

where $\Lambda \equiv \langle F_X \rangle / \langle X \rangle$, $k_1 = 5/3$, $k_2 = k_3 = 1$ and α_i are the three standard model gauge couplings in Eq. (153). The scalar masses are given approximately by

$$\tilde{m}^2(\langle X \rangle) = 2 \sum_{j=1}^3 C_j k_j \left[\frac{\alpha_j(\langle X \rangle)}{4\pi} \right]^2 \Lambda^2, \quad (318)$$

where $C_3 = 4/3$ for color triplets, $C_2 = 3/4$ for weak doublets (and equal to zero otherwise) and $C_1 = Y^2$ with $Y = Q - T_3$. To have squarks and gaugino masses of order 100 GeV, we need

$$\Lambda \equiv \langle F_X \rangle / \langle X \rangle \sim 10^5 \text{ GeV}. \quad (319)$$

Because the scalar masses are functions of only the gauge quantum numbers, the flavour-changing-neutral-current processes are naturally suppressed in agreement with experimental bounds. The reason for this suppression is that the gauge interactions induce flavour-symmetric supersymmetry-breaking terms in the visible sector at the scale $\langle X \rangle$ and, because this scale is usually much smaller than the Planck scale, only a slight asymmetry is introduced by renormalization group extrapolation to low energies. This is in contrast to the supergravity scenarios where one generically needs to invoke additional flavor symmetries to achieve the same goal.

Notice that there is no need to have $\sqrt{\langle F_X \rangle} \sim \langle X \rangle$. The only requirement is Eq. (319), and the hierarchy

$$\Lambda \ll \sqrt{\langle F_X \rangle} \ll \langle X \rangle \quad (320)$$

is certainly allowed [261, 263]. In fact, $\sqrt{\langle F_X \rangle}$ can take any value between 10^4 , and 10^{10} to 10^{11} GeV . This corresponds to $10^4 \lesssim \langle X \rangle \lesssim 10^{15}$ to 10^{17} GeV . If the upper bound is saturated, the gravity-mediated susy breaking that is always present (Section 7.10) becomes of the same order as the gauge-mediated susy breaking; if it were exceeded, gravity-mediated susy breaking would make the soft susy breaking parameters too big ($\gg 1 \text{ TeV}$). The upper bound is also required from considerations about nucleosynthesis [114].

To obtain the hierarchy $\sqrt{\langle F_X \rangle} \ll \langle X \rangle$, one can suppose that nonrenormalizable operators are involved, as in Section 5.9, or [261] that X has a soft susy breaking mass which runs, as in Section 5.6.2.

In the latter case, the mass may come from supergravity corrections. Alternatively, \tilde{m}_X^2 may receive contribution from one-loop Yukawa interactions. To illustrate this idea, we can consider the following toy model

$$W = \lambda_1 A \bar{\Psi} \Psi + B \left(\bar{\Psi} \Psi + \lambda_2 \Phi^+ \Phi^- + \lambda_3 B^2 \right) \quad (321)$$

where A and B are singlets, Φ^\pm have charge ± 1 under a messenger $U(1)$ and $\bar{\Psi}$ and Ψ are charged under some gauge group G . We assume that some susy breaking occurs in a hidden sector dynamically and is transmitted directly to Φ^\pm via the messenger $U(1)$ resulting in a negative mass squared m^2 for these two states. Minimizing the potential, one can show that there is a flat direction represented by $X \equiv \lambda_1 A + B$ whose VEV is undetermined at the tree-level and that supersymmetry is broken with $F_X = \frac{m^2}{\lambda_2} \frac{1}{(2 - \lambda_2/3\lambda_3)}$. \tilde{m}_X^2 gets a one-loop contribution proportional to $\lambda_2^2 m^2$ through the Yukawa interaction $W = \lambda_2 B \Phi^+ \Phi^-$.

Arguably, the cosmological constant problem is worse in the case of gauge-mediated susy breaking, than in the gravity-mediated case. To achieve the (practically) vanishing potential that is required by observation, the global supersymmetry result $V = \sum |W_n|^2$ must be cancelled by a term $-3|W|^2/M_{\text{P}}^2$ in the full supergravity theory. But if W is dominated by the sector of the theory responsible for gauge-mediated susy breaking, one will typically have $|W_n| \sim |W|/|\phi_n|$ with $|\phi_n| \ll M_{\text{P}}$. The conclusion is that $|W|$ must come from some other sector of the theory, or else be identified with the constant W_0 in the expansion of W (Eq. (331)) which might perhaps come from a string theory. In contrast, with gravity-mediated susy breaking the sector of the theory responsible for susy breaking usually gives $|W|$ of the right order, because the relevant ϕ_n are usually of order M_{P} .

7.7 Loop corrections and running

This is a good place to discuss the loop corrections in more detail.

Perhaps the most convincing reason for believing supersymmetry is its solution to the hierarchy problem [292]. In a theory where the largest interesting energy scale is the Planck mass or unification scale, light fundamental scalars (like a single Higgs doublet) get quadratically divergent contributions to their masses via one-loop diagrams where other heavy scalar or gauge fields are running in the loop. The scalar mass is given by $m_\phi^2 = (m_\phi^2)_0 + c\Lambda_{\text{UV}}^2$, where $(m_\phi^2)_0$ is the tree-level mass term, Λ_{UV} is the ultraviolet cutoff scale of the theory to be identified with some extremely large scale and c is a loop suppression factor. The Higgs mass can only be small if there is a delicate fine-tuning between classical and quantum effects. The only known symmetry which can suppress the quadratically divergent corrections is supersymmetry. Indeed, the way supersymmetry works is to cancel the leading Λ_{UV}^2 contribution by adding extra degrees of freedom into the game. The cancellation works because the number of degrees of freedom is basically doubled in a supersymmetric theory: each spin 0 or 1 field is accompanied by its fermionic partner. This amounts to adding an extra contribution to m_ϕ^2 which is equal in magnitude, but opposite in sign to the original one. The cancellation is exact in the limit of exact supersymmetry.

7.7.1 One-loop corrections

Let us address this issue more formally and imagine one is interested in the computation of the one-loop effective potential $V_{1\text{-loop}}(\phi)$ of a given scalar field of the supersymmetric theory. In the dimensional reduction with modified minimal subtraction (\overline{DR}) scheme of renormalization, it reads

$$V_{1\text{-loop}}(\phi) = \frac{Q^2}{32\pi^2} \text{Str} \mathcal{M}^2 + \frac{1}{64\pi^2} \text{Str} \left[\mathcal{M}^4(\phi) \left(\ln \frac{\mathcal{M}^2(\phi)}{Q^2} - \frac{3}{2} \right) \right], \quad (322)$$

where $\mathcal{M}^2(\phi)$ is the field dependent mass-squared matrix for the particles contributing to the loop correction. These particles will in general have spins $j = 0, 1/2$ or 1 , and the supertrace is defined as

$$\text{Str} A = \sum_j (-1)^{2j} (1 + 2j) \text{Tr} A_j, \quad (323)$$

Here, A denotes either \mathcal{M}^2 or the square bracket, and A_j is the ordinary trace for particles of spin j .

The scale Q is the renormalization scale, at which all the parameters (masses, gauge and Yukawa couplings, etc.) entering the tree-level and the one-loop potential (322) must be evaluated.

In Eq. (322) we have explicitly written the quadratic divergent piece proportional to $\text{Str} \mathcal{M}^2$. In non-supersymmetric theories this term is field dependent and is the source of the divergent corrections to the squared mass m_ϕ^2 . On the contrary, in supersymmetric (and anomaly free) theories, this term is *independent* of the fields and proportional to the soft breaking masses of the fields contributing to the effective potential. It therefore contributes only to the cosmological constant, and we drop it giving

$$V_{1\text{-loop}}(\phi) = \frac{1}{64\pi^2} \text{Str} \left[\mathcal{M}^4(\phi) \left(\ln \frac{\mathcal{M}^2(\phi)}{Q^2} - \frac{3}{2} \right) \right], \quad (324)$$

With unbroken supersymmetry, the loop correction vanishes, and the tree-level scalar potential of the field ϕ is not renormalized at all (in particular, there is no one-loop contribution to the squared mass m_ϕ^2). Notice that, in the case of global supersymmetric theory, this property is true at any order of perturbation theory as a result of the nonrenormalization theorem. If supersymmetry is broken, the supertrace as well as the one-loop potential usually no longer vanish.

As an example, we consider a simple situation that can give Eqs. (156) and (157). The loop correction comes from a single complex field ψ (with masses m_1 and m_2 for the real and imaginary parts) and its fermionic partner (with mass m_f). The interaction is supposed to be $\frac{1}{2}\lambda\phi^2|\psi^2|$. When ϕ (taken to be real) is much bigger than the masses the total loop correction is

$$\Delta V \simeq \frac{1}{32\pi^2} \left[\sum_{i=1,2} \left(m_i^2 + \frac{1}{2}\lambda\phi^2 \right)^2 - 2 \left(m_f^2 + \frac{1}{2}\lambda\phi^2 \right)^2 \right] \ln \frac{\phi}{Q} \quad (325)$$

The coefficient of ϕ^4 vanishes by virtue of the supersymmetry. Two cases commonly arise for the other terms.

The first case occurs when there is soft susy breaking in the relevant sector, with zero (or negligible) fermion masses. Then the quadratic term dominates and one has

$$\Delta V \simeq \frac{1}{32\pi^2} \lambda (m_1^2 + m_2^2) \phi^2 \ln(\phi/Q). \quad (326)$$

The second case occurs when there is spontaneous susy breaking in the relevant sector, giving $2m_f^2 = m_1^2 + m_2^2$. Then the coefficient of ϕ^2 vanishes leaving

$$\Delta V \simeq \frac{(m_1^2 - m_2^2)^2}{64\pi^2} \ln \frac{\phi}{Q}. \quad (327)$$

Including more chiral supermultiplets and/or gauge supermultiplets gives similar results; softly broken susy makes $\Delta V \propto \phi^2 \ln(\phi/Q)$, but spontaneously broken susy makes $\Delta V \propto \ln(\phi/Q)$ because $\text{Str } \mathcal{M}^2$ vanishes.

7.7.2 The Renormalization Group Equations (RGE's)

In the perturbative regime, the potential V is given by the tree-level expression, plus 1-loop, 2-loop etc quantum corrections,

$$V = V_{\text{tree}}(Q) + V_{1\text{-loop}}(Q) + V_{2\text{-loop}}(Q) + \dots \quad (328)$$

It depends on the parameters appearing in the Lagrangian (masses and couplings), but in addition each individual term depends on the the renormalization scale Q . This amounts to a choice of energy unit, which has to be made within any renormalization scheme. Physical quantities like V do not depend on Q , and this is ensured by a set of linear differential equations for the parameters, known as Renormalization Group Equation's (RGE's).

The 1-loop correction, for a given particle in the loop, was displayed in Eq. (324). If ϕ is much larger than any relevant mass scales, the typical contribution to \mathcal{M} will be of order ϕ (the only relevant scale). As a result, the loop correction will vanish for some choice $Q \sim \phi$. The potential is then given just by the tree-level contribution,

$$V(\phi) \simeq V_{\text{tree}}(\phi, Q = c\phi), \quad (329)$$

where the coefficient $c \sim 1$ depends upon the details of the theory.

7.8 Supergravity

So far we have considered global supersymmetry, taken to be renormalizable except possibly for terms in the superpotential. In the usual context of collider physics, particle detectors and astrophysics, this is adequate for most purposes. But during inflation one needs to consider supergravity, which contains within it the most general non-renormalizable version of global susy.

A non-renormalizable field theory is an effective one, valid below some ultra-violet cutoff Λ_{UV} . With all of the fields and interactions in Nature included, Λ_{UV} is generally identified with M_{P} (Section 5.1), and we shall do this in the end. But for clarity of exposition we initially leave Λ_{UV} unspecified.

7.8.1 Specifying a supergravity theory

In Section 7.3 we defined the chiral and gauge supermultiplets, and their supersymmetry transformations. These formulas remain valid in supergravity, but the lagrangian is different. In addition to the superpotential W one now needs two more functions. These are the Kähler potential K , and the gauge kinetic function f .⁷⁴ Both W and f are holomorphic function of the complex scalar fields, but the real function K is not holomorphic; it is regarded as a function of the fields and their complex conjugates.

Only the combination

$$G \equiv M_{\text{P}}^{-2}K + \ln \frac{|W|^2}{M_{\text{P}}^6} \quad (330)$$

is physically significant. So we have invariance under the Kähler transformation $M_{\text{P}}^{-2}K \rightarrow M_{\text{P}}^{-2}K - X - \bar{X}$, $W \rightarrow e^X W$ where X is any holomorphic function of the fields.

We shall adopt the following conventions [299]. The scalar components ϕ^n and auxiliary components F^n of chiral supermultiplets are labelled by a superscript. A subscript n denotes $\partial/\partial\phi^n$, and a subscript n_* denotes $\partial/\partial\phi^{n*}$. (Note that $K_{nm*} = G_{nm*}$.) Occasionally one lowers components, $\phi_n \equiv K_{nm*}\phi^{m*}$ and $F_n \equiv K_{nm*}F^{m*}$; the inverse matrix of K_{nm*} , which raises components, is denoted by K^{m*n} . A summation over repeated indices is implied.

We first consider the expansion of W , K and f about a suitable origin in field space. It may be chosen to be the position of the vacuum or, in the case of matter fields, to be the fixed point of the symmetries.

The superpotential We already considered the superpotential, in the context of global susy. Since it is holomorphic in the fields, it is of the form

$$W = W_0 + \Lambda^2 W_1(\phi^n) + m W_2(\phi^n) + W_3(\phi^n) + \Lambda_{\text{UV}}^{3-d} \sum_{d=4}^{\infty} W_d(\phi^n). \quad (331)$$

Each quantity W_d is the sum of dimension d terms; in other words, it is a sum of terms, each of which is a product of d fields times a coefficient. For the non-renormalizable terms ($d \geq 4$), the coefficient is expected to be of order 1, *unless* it is forbidden by internal symmetries.

As we noted on page 85, W is strongly constrained by internal symmetries, because it is holomorphic. For a given field, if one starts with an expression in which the field only occurs at low order, one can forbid additional terms up to a finite order by imposing a discrete Z_N symmetry, and one can forbid additional terms up to all orders by imposing a continuous symmetry. However, in the case of a gauge singlet the continuous symmetry would have to be global, and as we noted on page 44 global continuous symmetries do not seem to exist in string theory. Therefore, in the case of a gauge singlet, it may be unreasonable to forbid additional terms to all orders. As we shall see, this is a problem for models of inflation where the inflaton field has a value of order M_{P} .

⁷⁴Presumably there are also functions specifying terms involving second and higher spacetime derivatives. It is reasonable to suppose that such terms are negligible compared with the kinetic term unless the spacetime derivatives are of order 1 in Planck units. But then field theory will break down anyway.

The Kähler potential The Kähler potential determines the kinetic terms of the scalar fields, according to the formula

$$\mathcal{L}_{\text{kin}} = (\partial_\mu \phi^{n*}) K_{n^*m} (\partial^\mu \phi^m). \quad (332)$$

It is a function of the fields and their complex conjugates, and can be chosen to have the expansion

$$K = K_{nm^*} \phi^n \phi^{m*} + \Lambda_{\text{UV}}^{2-d} \sum_{d=3}^{\infty} K_d(\phi^n, \phi^{n*}), \quad (333)$$

where K_{nm^*} is evaluated at the origin. For simplicity we have assumed that any constant or linear term has been absorbed into the superpotential by a Kähler transformation, which is always possible. One can choose the scalar fields to be canonically normalized at the origin, corresponding to $K_{nm^*} = \delta_{nm^*}$.

As in the previous expression, each K_d is a sum with each term in the sum a product of d fields, times a coefficient which is expected to be of order 1 unless it is forbidden by a symmetry. As K is not holomorphic, symmetries do not constrain it very strongly. It can, for instance, be an arbitrary function of the $|\phi^n|^2$, and the coefficient of a monomial built out of such terms will generically be of order 1. As we shall see, this is a problem for inflation model-building.

The gauge kinetic function The gauge kinetic function determines the kinetic terms of the gauge and gaugino fields. One can choose them to be canonically normalized when the scalar fields are at the origin, which corresponds to

$$f = 1 + \Lambda_{\text{UV}}^{-d} \sum_{d=1}^{\infty} f_d(\phi^n). \quad (334)$$

As is the case with W , symmetries powerfully constrain the form of f because it is holomorphic. We need to consider f because it appears in the scalar field potential.

7.8.2 The scalar potential and spontaneously broken supergravity

Supergravity can be broken only spontaneously, not explicitly like global susy. The transformation equations Eqs. (279) and (283) hold in supergravity, so it remains true that the condition for spontaneous breaking is a non-vanishing vev for one or more of the auxiliary fields F^n and D .

In contrast with the case of global susy, the vevs of F^n and D can receive contributions from fermion condensates as well as from scalar fields. A favoured possibility for susy breaking (in the vacuum) is gaugino condensation, but as discussed later one can in that case add an effective non-perturbative contribution to W instead of including the condensate explicitly. Assuming that this has been done, the auxiliary fields are given by⁷⁵

$$D = -g(q_n K_n \phi^n + \xi), \quad (335)$$

$$F^n = -e^{K/2} K^{nm^*} \left(W_m + M_{\text{P}}^{-2} W K_m \right)^*. \quad (336)$$

⁷⁵The Kähler invariance of the first expression is guaranteed by the gauge invariance. Indeed, one can replace K_n by G_n , because the gauge invariance requires $\sum_n q_n W_n \phi^n = 0$.

The tree-level potential is given by

$$V = V_D + F^2 - 3M_{\text{P}}^{-2} e^{K/M_{\text{P}}^2} |W|^2, \quad (337)$$

where

$$V_D \equiv \frac{1}{2} (\text{Re } f)^{-1} g^2 (q_n K_n \phi^n + \xi)^2, \quad (338)$$

and

$$F^2 \equiv F^n K_{nm^*} F^{m^*} = F_n K^{m^*n} F_m \quad (339)$$

$$= e^{K/M_{\text{P}}^2} \left(W_m + M_{\text{P}}^{-2} W K_m \right)^* K^{m^*n} \left(W_n + M_{\text{P}}^{-2} W K_n \right). \quad (340)$$

In the second line, we defined $F_n \equiv K_{nm^*} F^{m^*}$, and K^{m^*n} is the inverse of the matrix K_{nm^*} .

As in global supersymmetry, V_D is proportional to D^2 , while F^2 is equal to $\sum |F_n|^2$ if we choose $K_{nm^*} = \delta_{nm}$. The last term in Eq. (337) allows the true vacuum energy to vanish, as is (practically) demanded by observation.

It is usual to define

$$V_F = F^2 - 3e^{K/M_{\text{P}}^2} M_{\text{P}}^{-2} |W|^2 \quad (341)$$

$$= e^{K/M_{\text{P}}^2} \left[\left(W_n + M_{\text{P}}^{-2} W K_n \right) K^{m^*n} \left(W_m + M_{\text{P}}^{-2} W K_m \right)^* - 3M_{\text{P}}^{-2} |W|^2 \right]. \quad (342)$$

Then

$$V = V_D + V_F, \quad (343)$$

and one calls V_F the F term even though it does not come only from the auxiliary fields F_n .

Taking M_{P} to infinity with Λ_{UV} fixed gives

$$V_F = W_n K^{m^*n} (W_m)^*. \quad (344)$$

We then have non-renormalizable global supersymmetry. Renormalizable global supersymmetry is obtained by taking Λ_{UV} to infinity as well.

The other possible limit is $\Lambda_{\text{UV}} \rightarrow \infty$ with M_{P} fixed. This is minimal supergravity, characterised by canonical kinetic terms. It has no motivation from string theory.

In the usually-considered case that Λ_{UV} is identified with M_{P} , one simply says that (renormalizable) global supersymmetry is obtained from supergravity in the limit $M_{\text{P}} \rightarrow \infty$. From now on, we make this identification except where stated.

The scale of susy breaking in the true vacuum is denoted by M_{S} and defined by

$$M_{\text{S}}^4 = F^2 + V_D. \quad (345)$$

An equivalent definition is

$$V = M_{\text{S}}^4 - M_{\text{P}}^{-2} e^{K/M_{\text{P}}^2} |W|^2. \quad (346)$$

Since V (practically) vanishes in the true vacuum, this is equivalent to

$$M_{\text{S}}^4 = M_{\text{P}}^{-2} e^{K/M_{\text{P}}^2} |W|^2. \quad (347)$$

One can show that the gravitino mass is given by

$$M_{\text{S}}^4 = 3m_{3/2}^2 M_{\text{P}}^2. \quad (348)$$

7.9 Supergravity from string theory

One hopes that the lagrangian describing field theory, will eventually be derivable from some more fundamental theory. Candidates under consideration at present include weakly coupled (heterotic) string theory [123] and Horava-Witten M-theory [303, 139]. In this section we look at the form of supergravity predicted by weakly coupled string theory. Then we briefly mention the case of M-theory, which has not so far been invoked for inflation model-building.

A crucial role is played by special fields, namely the dilaton and the bulk moduli. The dilaton, usually denoted by s , specifies the gauge coupling at the string scale, and the bulk moduli specifying the radii of the compactified dimensions. (Weakly coupled strings live in nine space dimensions, so six of them have to be compactified.) We consider the cases where there is just one bulk modulus t , and where there are three bulk moduli t^I .

For simplicity, we ignore the Green-Schwarz term needed to cancel the modular anomaly induced by field theory loop corrections, and initially we ignore the dilaton as well. In this section, we set $M_P = 1$ unless otherwise stated.

7.9.1 A single modulus t

The simplest case corresponds to compactification on a six-torus [301]. It should be regarded as a toy model, since it permits only one generation in the Standard Model. In units of the string scale (slightly below M_P , see footnote 101) the radius of the six-torus is $(2x)^{-1/2}$ where

$$x \equiv t + t^* - \sum_n |\phi^n|^2. \quad (349)$$

Here t is a bulk modulus, and ϕ^n are a subset of the matter fields, called the untwisted sector. (The other matter fields are said to belong to the twisted sector.) The Kähler potential derived from string theory is

$$K = -3 \ln x. \quad (350)$$

If we ignore the twisted sector, and assume that W is independent of t , Eq. (342) takes the remarkably simple form

$$V_F = \frac{3}{x^2} \sum_n |W_n|^2. \quad (351)$$

It is assumed that the vacuum of the globally supersymmetric theory (minimum of its potential) is at $W_n = V = 0$, corresponding to unbroken global supersymmetry. Then, the vacuum of the supergravity theory is also at $V = 0$, as is required by observation, but supersymmetry is now in general spontaneously broken. At the tree level under consideration here, the scale of supersymmetry breaking given by Eq. (347) is undetermined. (V in the vacuum is independent of x and therefore e^K is undetermined.) This corresponds to what is called a no-scale supergravity theory [183].

Although supersymmetry is broken, the scalar masses given by this tree-level expression do not feel the effect of the breaking as is clear from the fact that the potential V has the same form as in global susy.

The no-scale model is a consequence of the assumptions about W . In general one expects that W will depend on t , and the twisted sector may be important. We shall look at these issues in the next subsection, in the context of the more realistic model that has three bulk moduli.

For future reference, we note that if the D term Eq. (338) involves only the untwisted sector it is of the form

$$V_D = \frac{1}{2}(\text{Re } f)^{-1} g^2 \left(x^{-1} \sum_n q_n |\phi^n|^2 + \xi \right)^2, \quad (352)$$

7.9.2 Three moduli t^I

Compactification on the six-torus is not phenomenologically viable, because it allows only one generation in the Standard Model. To obtain the three generations that are observed, one can use [140, 102, 66, 100, 67, 13, 141, 164] orbifold compactifications with three tori.

There are now three moduli t^I ($I = 1$ to 3). This theory possesses invariance under the modular transformations. Acting on the moduli, these transformations are generated by $t^I \rightarrow 1/t^I$ and $t^I \rightarrow t^I \pm i$. A matter field ϕ^α transforms like $\prod_I \eta^{-2q_I^\alpha}(t^I)$, where η is the Dedekind function and q_I^α are the weights of the field. Modular symmetry has a fixed point (up to modular transformations) at which the matter fields vanish and $t^I = e^{i\pi/6}$. At this point, the derivative of V with respect to every field vanishes.

The matter fields are divided into fields ϕ^{AJ} , that belong to the untwisted sector, ϕ^{AJ} having modular weight $q_I^{AJ} = \delta_I^J$, and fields ϕ^A belonging to the twisted sector, that have weights $q_I^A > 0$ (typically less than 1). In units of the string scale, the radius of the I th torus is $(2x_I)^{-1/2}$, where

$$x_I = t^I + t^{I*} - \sum_A |\phi^{AI}|^2. \quad (353)$$

We expect $|t^I| \sim 1$ with all matter fields $\ll 1$, both in the true vacuum and during inflation.

The superpotential has a power series expansion in the matter fields, of the form

$$W = \sum_m \lambda_m \prod_\alpha (\phi^\alpha)^{n_m^\alpha} \prod_I \eta(t^I)^{2(\sum_\alpha n_m^\alpha q_I^\alpha - 1)}, \quad (354)$$

where n_m^α are positive integers or zero. The t^I dependence of each coefficient is dictated by modular invariance, which requires that W transforms like $\prod_I \eta^{-2}(t^I)$ (up to a modular-invariant holomorphic function, which we do not consider because it would have singularities). Using this expression one sees that

$$\frac{\partial W}{\partial t^I} \equiv W_I = 2\xi(t^I) \left(\sum_\alpha q_I^\alpha \phi^\alpha W_\alpha - W \right). \quad (355)$$

The Kähler potential is

$$K = - \sum_{I=1}^3 \ln x_I + \sum_A \left(\prod_I x_I^{-q_I^A} \right) |\phi^A|^2 + \dots. \quad (356)$$

The first term comes directly from string theory, and it gives the part of K that is independent of the twisted fields. The second term comes from an expansion of the S -matrix as a power series in matter fields. The additional terms are restricted by modular invariance, but they could in general include terms like

$$\left(\prod_I x_I^{-q_I^A} \right) |\phi^A|^2 \frac{|\phi^{BI}|^2}{t^I + t^{I*}}. \quad (357)$$

Such terms would generically have coefficients of order 1, and as we shall see they could spoil the flatness of the inflationary potential. They can be eliminated if we assume that K depends on the moduli and untwisted fields only through the combinations x_I , as advocated in [106].

If the twisted fields and the W_A are negligible, the potential Eq. (342) becomes

$$V_F = e^K \left[\sum_I \left(x_I \sum_A |W_{AI} + \phi^{AI*} W_I|^2 + |x_I W_I - W|^2 \right) - 3|W|^2 \right]. \quad (358)$$

In this expression, $W_I \equiv \partial W / \partial t^I$.

If W is a sum of cubic terms, each containing just one field from each untwisted sector, then W does not depend on the moduli and we have simply

$$V_F = \frac{\sum_I x_I \sum_A |W_{AI}|^2}{x_1 x_2 x_3}. \quad (359)$$

This expression is similar to Eq. (351), that we wrote down earlier. It has all the properties that we described then, and is also called a no-scale model.

For future reference, we note that with Eq. (356) the D term Eq. (338) becomes

$$V_D = \frac{1}{2} (\text{Re } f)^{-1} g^2 \left(\sum_\alpha q_\alpha \left(\prod_I x_I^{-q_I^\alpha} \right) |\phi^\alpha|^2 + \xi \right)^2. \quad (360)$$

Here, α runs over both twisted and untwisted fields.

7.9.3 The dilaton

At the the string scale,⁷⁶ the gauge coupling is related to the dilaton field s by

$$g_{\text{str}}^2 = M_{\text{P}} / (\text{Re } s). \quad (361)$$

This expression takes the real part of the gauge kinetic function to be 1. Equivalently, g_{str} can be absorbed into f . Then at the string scale

$$f(s) = s / M_{\text{P}}. \quad (362)$$

⁷⁶ The string scale is the one below which, in weakly coupled string theory, field theory will become a valid approximation. At this scale, the gauge couplings in the true vacuum are supposed to have a common value g_{str} , presumably of order 1. The scale and coupling are related by $M_{\text{str}} \simeq g_{\text{str}} M_{\text{P}}$. The value $g_{\text{str}}^2 \simeq 0.5$ would correspond to the value $\alpha_{\text{str}} \equiv g_{\text{str}}^2 / 4\pi \simeq 1/25$, which with naive running of the couplings is suggested by observation at a scale of order 10^{16} GeV.

Ignoring Green-Schwarz terms, the contribution of the dilaton to the Kähler potential is

$$\Delta K = -\ln(s + s^*) \dots \quad (363)$$

This gives an extra contribution to the potential

$$\Delta V = \frac{|F^s|^2}{(s + s^*)^2} \quad (364)$$

$$= e^K |(s + s^*)W_s - W|^2. \quad (365)$$

(Of course it also contributes an overall factor $(s + s^*)^{-1}$ from the e^K in front of everything in Eq. (342).)

In the true vacuum Eq. (361) requires $s \sim 1$ to $10M_{\text{P}}$, and during inflation the order of magnitude of s is presumably not very different, so as to be within the domain of attraction of the true vacuum.

The contribution of s to the superpotential is non-perturbative, and very model-dependent. It is often supposed to be something like

$$W(s) = M_{\text{P}}^3 e^{-s/(bM_{\text{P}})}. \quad (366)$$

Since $e^K \propto 1/\text{Re } s$, these expressions make $\text{Re } s$ run away to infinity at least with a single term in Eq. (366). There is no consensus about what stabilizes the dilaton either in the true vacuum or [45, 21, 158] during inflation. The simplest possibility is to invoke an additional (non-perturbative) contribution to the Kähler potential.

All this assumes that the dilaton is part of a chiral supermultiplet, like the other scalar fields. An alternative description [37] puts the real part of the dilaton in a linear supermultiplet. The situation then is qualitatively similar to the one that we have described, but different in important details.

7.9.4 Horava-Witten M-theory

In Horava-Witten M-theory [303, 139], K receives an extra contribution [211, 192]. For the untwisted fields, this is

$$\Delta K = \frac{1}{3} \frac{1}{s + s^*} \left(\sum_I \alpha_I (t^I + t^{I*}) \right) \left(\sum_I \frac{\sum_A |\phi^{IA}|^2}{t^I + t^{I*}} \right). \quad (367)$$

The parameters α_I are expected to be roughly of order 1. The gauge coupling in the visible sector (at the string scale) becomes

$$f = s + \sum_I \alpha_I t^I. \quad (368)$$

The ‘string’ scale at which this expression is valid will be lower than in weakly coupled string theory.

7.10 Gravity-mediated soft susy breaking

This is a good place to give a brief account of gravity-mediated soft susy breaking.

7.10.1 General features

The basic features are the same as for gauge-mediated susy breaking (Section 7.6.2). The softly broken global susy, that describes the visible sector, is supposed to be only an effective theory. In the full theory, supersymmetry is spontaneously broken. The spontaneous breaking takes place in a hidden sector, whose fields do not possess the Standard Model gauge interactions. The spontaneous breaking mechanism is supposed to involve an F -term.

In contrast with the gauge-mediated case, the mechanism of spontaneous susy breaking in the hidden sector is usually supposed to involve supergravity in an essential way. The defining difference, though, is that the mechanism of *transmission* of susy breaking to the visible sector comes only from interactions of gravitational strength. In other words, each interaction term is multiplied by a power of M_{P}^{-1} . Some interaction terms of this type will be present as non-renormalizable terms in the expansions Eqs. (331) and (333); for instance, no symmetry can prevent the appearance of a term in K like

$$K = \dots \lambda M_{\text{P}}^{-4} |\phi|^2 |y|^2 \dots, \quad (369)$$

where ϕ belongs to the hidden sector and y belongs to the visible sector, and the coupling λ of such a term will generically be of order 1. Additional interaction terms will arise in the potential because of the form of the supergravity expression Eq. (342).

Given the values of the auxiliary fields that spontaneously break susy, and those of the fields themselves, one can calculate the soft susy masses-squared m_n^2 (or more generally the soft mass-matrix) and the $A_{nm\ell}$ parameters that define the soft trilinear terms. One finds generically $m_n^2 \sim A_{nm\ell}^2 \sim M_S^4/M_{\text{P}}^2 (= 3m_{3/2}^2)$, where M_S is the susy breaking scale defined by Eq. (345), (346) or (347), and $m_{3/2}$ is the gravitino mass defined by Eq. (348). One can see this by making rough estimates, as in the similar analysis of Section 8.2.1. A classic explicit calculation, with some specific assumptions, is given in Section 7.10.3.

The gaugino masses are given by

$$m_{1/2} = \sum_n \frac{\partial f}{\partial \phi^n} \frac{F^n}{2\text{Re}f} \quad (370)$$

where f is the gauge kinetic function for the visible sector.

The simplest example of gravity-mediated susy breaking was given in an unpublished paper by Polonyi [259]. The superpotential is split into the sum of two functions

$$W = W(\phi) + W(y^a), \quad (371)$$

where y^a denote the visible sector fields and ϕ denotes a hidden sector field which is a gauge singlet. Its superpotential is taken to be

$$W(\phi) = M_S^2(\phi + \beta). \quad (372)$$

If gravitational effects are ignored, $W(\phi)$ leads to a flat potential independent of ϕ

$$V = M_S^4, \quad (373)$$

susy is broken, but the vev of ϕ is undermined. Once gravity is turned on, the presence of the negative terms produces a minimum of the potential at

$$\langle\phi\rangle \sim M_{\text{P}}, \quad (374)$$

$$F_{\phi} = \frac{\partial W}{\partial\phi} \sim M_{\text{S}}. \quad (375)$$

The constant $\beta = (2 - \sqrt{3})M_{\text{P}}$ is chosen to make the cosmological constant vanishing in the true vacuum, $V(\langle\phi\rangle) = 0$.

7.10.2 Gravity-mediated susy breaking from string theory

The nonvanishing auxiliary fields of the hidden sector are usually taken to be those of the dilaton and/or the bulk moduli. Also, the bulk moduli t^I and their auxiliary fields F^{t^I} are usually set equal to common values, t and F^t . Finally, the weakly coupled string theory expression Eq. (356) is assumed. Then the scalar masses are [151, 44]

$$m_n^2 = m_{3/2}^2 \left[(3 + q_n \cos^2 \theta) C^2 - 2 \right], \quad (376)$$

where $q_n = \sum_I q_I^n$ and $\tan^2 \theta = (K_{ss^*}/K_{tt^*})|F^s/F^t|^2$. The constant C is given by $C^2 - 1 = V_0/(3M_{\text{P}}^2 m_{3/2}^2)$, and it is equal to 1 in the true vacuum case that we are dealing with at the moment. As usual $m_{3/2}^2 = e^{K/M_{\text{P}}^2}|W|^2/M_{\text{P}}^4$.

At a deeper level, the vevs of the auxiliary fields are usually supposed to mimic some dynamical effect, often originating in string theory with extra space dimensions. A favoured mechanism is gaugino condensation, which is supposed to generate a superpotential $W(s)$ looking something like Eq. (366). (With several hidden sectors there is a sum of such terms.) The value of b has to be such that

$$W \sim \Lambda_{\text{c}}^3 \sim (10^{13} \text{ GeV})^3. \quad (377)$$

This gives the right soft susy breaking scale, $M_{\text{S}}^2 \sim \Lambda_{\text{c}}^3/M_{\text{P}} \sim (10^{10} \text{ GeV})^2$. With this mechanism F^s vanishes, since once s is stabilized the perfect square Eq. (365) is driven to zero. In weakly coupled string theory, Eqs. (362) and (370) then make the gaugino masses vanish at the string scale. This is probably forbidden by observation, but it is avoided in Horava-Witten M-theory where Eq. (362) is replaced by Eq. (368).

A particular version of gravity-mediated susy breaking is the no-scale theory, corresponding to Eq. (359). In this case, the masses of untwisted fields vanish at tree level, though running them from the string scale can still give masses of order 100 GeV at the electroweak scale.⁷⁷

In the context of weakly-coupled string theory, no-scale gravity corresponds to the assumption that the superpotential in the relevant sector of the theory is independent of the bulk moduli. Because of the modular invariance encapsulated in Eq. (354), this may be

⁷⁷To be more correct, the relevant coefficients in the expansion Eq. (333) are supposed to be of order 1 at the Planck scale. They run, which is equivalent to running the susy breaking parameters even though the latter may really be defined only below a lower scale where supersymmetry breaks.

difficult to arrange in the true vacuum under consideration at present, since W is necessarily nonzero. (During inflation the no-scale form is easier to achieve as we discuss later, provided that W is negligible.)

In Horava-Witten M-theory, no-scale gravity will presumably be a valid approximation only if some of the α_I in Eq. (367) are significantly below 1.

7.10.3 Formalism for gravity-mediated supersymmetry breaking

This subsection is more technical, and can be skipped by the general reader. It gives a formalism for calculating the soft scalar terms explicitly, with some assumptions, and an example of how gaugino condensation can generate an effective contribution to W .

For the formalism, we follow the original notation [277, 115], in which the complex conjugate of a field is labelled by a subscript. The visible sector fields are y^a (collectively y) and the hidden sector fields are ϕ^i (collectively ϕ). It is supposed that $\phi \gg y$, and $\xi^i \equiv \phi^i/M_{\text{P}}$ is defined (collectively ξ). The soft susy breaking parameters are calculated in the limit $M_{\text{P}} \rightarrow \infty$, with ξ fixed.

Requiring that the low-energy lagrangian for the visible sector is not multiplied by powers of M_{P} defines the dependence on M_{P} of W and K [277]

$$\begin{aligned} W(\xi, y) &= M_{\text{P}}^2 W^{(2)}(\xi) + M_{\text{P}} W^{(1)}(\xi) + W^{(0)}(\xi, y), \\ K(\xi, \xi^\dagger, y, y^\dagger) &= M_{\text{P}}^2 K^{(2)}(\xi, \xi^\dagger) + M_{\text{P}} K^{(1)}(\xi, \xi^\dagger) + K^{(0)}(\xi, \xi^\dagger, y, y^\dagger). \end{aligned} \quad (378)$$

In addition, the y^a are supposed to be canonically normalized [277],

$$K^{(0)}(\xi, \xi^\dagger, y, y^\dagger) = y^a \Lambda_a^b(\xi, \xi^\dagger) y_b^\dagger + \left(\Gamma(\xi, \xi^\dagger, y) + \text{h.c.} \right), \quad (379)$$

with the vacuum expectation value $\langle \Lambda_a^b \rangle = \delta_a^b$. Finally, the ϕ^i fields are gauge singlets, so that gauge invariance requires Λ_a^b to be diagonal.

If there are no mass scales in the theory other than M_{P} and those induced by some spontaneous symmetry breaking (this is what happens in string-inspired theories), the renormalizable self couplings of the light fields y^a is of the form [115]

$$\begin{aligned} W^{(0)}(\xi, y) &= \sum_n c_n(\xi) g_n^{(3)}(y), \\ \Gamma(\xi, \xi^\dagger, y) &= \sum_m c'_m(\xi, \xi^\dagger) g_m^{(2)}(y), \end{aligned} \quad (380)$$

where $g_n^{(3)}(y)$ and $g_m^{(2)}(y)$ are, respectively, the trilinear and bilinear terms in y^a allowed by the symmetries of the theory.

After taking the limit $M_{\text{P}} \rightarrow \infty$ [115], we obtain for the visible sector a renormalizable global susy theory, with explicit soft breaking terms. The scalar potential is of the form

$$\begin{aligned} V &= \left| \frac{\partial \hat{g}}{\partial y^a} \right|^2 + m_{3/2}^2 y^a S_a^b y_b^\dagger + [m_{3/2}^\dagger (y^a R_a^b \frac{\partial \hat{g}}{\partial y^b} + \sum_n (A_n - 3) \hat{g}_n^{(3)}) \\ &+ \sum_n (B_n - 2) \mu_n g_m^{(2)}] + \text{h.c.}] + \text{D - terms} . \end{aligned} \quad (381)$$

The first term is the unbroken susy result, the second term is the soft mass matrix for the complex fields, and the term in square brackets contains soft trilinear terms, as well as bilinear terms that complete the specification of the mass matrix of the real fields.

We have imposed the constraint $V = 0$ appropriate for the vacuum, and the gravitino mass is the modulus of

$$m_{3/2} \equiv \langle e^{K^{(2)}/2} W^{(2)} \rangle. \quad (382)$$

The soft parameters are determined by the following formulas.

$$S_a^b = \delta_a^b + \left\langle \rho^{\dagger i} \left(\frac{\partial \Lambda_c^b}{\partial \xi_j^\dagger} \frac{\partial \Lambda_a^c}{\partial \xi^i} - \frac{\partial^2 \Lambda_a^b}{\partial \xi_j^\dagger \partial \xi^i} \right) \rho_j \right\rangle, \quad R_a^b = \delta_a^b - M_{\text{P}} \left\langle \rho^{\dagger i} \frac{\partial \Lambda_a^b}{\partial \xi^i} \right\rangle, \quad (383)$$

where

$$\rho_j \equiv \left(\frac{\partial^2 K^{(2)}}{\partial \xi^i \partial \xi_j^\dagger} \right)^{-1} \frac{\partial}{\partial \xi_j} (\ln W^{(2)} + K^{(2)}). \quad (384)$$

Here \hat{g} is the superpotential for the light fields defined by

$$\hat{g}(y) = \sum_n \hat{g}_n^{(3)}(y) + \sum_m \mu_m g_m^{(2)}(y), \quad (385)$$

with

$$\hat{g}_n^{(3)}(y) = \langle e^{K^{(2)}/2} \rangle c_n(\langle \xi \rangle) g_n^{(3)}(y), \quad \mu_m = m_{3/2} \left\langle \left(1 - \rho_i \frac{\partial}{\partial \xi_i^\dagger} \right) c'_m(\xi, \xi^\dagger) \right\rangle. \quad (386)$$

Also,

$$\begin{aligned} A_n &= \left\langle \rho^{\dagger i} \frac{\partial}{\partial \xi^i} [K^{(2)} + \ln c_n(\xi)] \right\rangle, \\ B_m &= \left\langle \left[2 + \left(\rho^{\dagger i} \frac{\partial}{\partial \xi^i} - \rho_i \frac{\partial}{\partial \xi_j^\dagger} \right) - \rho^{\dagger i} \rho_j \frac{\partial^2}{\partial \xi^i \partial \xi_j^\dagger} \right] \frac{c'_m(\xi, \xi^\dagger)}{(1 - M \rho_i \frac{\partial}{\partial \xi_i^\dagger} c'_m(\xi, \xi^\dagger))} \right\rangle. \end{aligned} \quad (387)$$

Identifying the ultra-violet cutoff Λ_{UV} in Eqs. (331) and (333) with M_{P} , one will have generically $|S_a^b| \sim |R_a^b| \sim |A_n| \sim |B_m| \sim 1$, making the soft susy breaking mass matrix-squared of order $m_{3/2}^2$ and the trilinear terms of order $m_{3/2}$.

Next we see how gaugino condensation can give an effective superpotential. We consider an extension of susy-QCD based on the gauge group $SU(N_c)$ in the hidden sector with $N_f \leq N_c$ flavors of ‘‘quarks’’ Q^i in the fundamental representation and ‘‘antiquarks’’ \tilde{Q}_i in the antifundamental representation of $SU(N_c)$ [5]. The gauge kinetic function may be chosen to be $f = ks$, where s is the dilaton superfield and k is the Kac-Moody level of the hidden gauge group.

Because of the gauge structure, the gauge group $SU(N_c)$ enters the strong-coupling regime at the scale

$$\Lambda_c = M_{\text{P}} e^{-\frac{ks}{2b_0}}, \quad (388)$$

where $b_0 = (3N_c - N_f)/(16\pi^2)$ is the one-loop beta function for the hidden sector gauge group.

Below the scale Λ_c the appropriate degrees of freedom for $N_f < N_c$ are the mesons $M_i^j = Q^i \tilde{Q}_j$. The effective superpotential is fixed uniquely by the global symmetries as follows [5]

$$W = (N_c - N_f) \langle \lambda \bar{\lambda} \rangle, \quad (389)$$

where the gaugino condensation scale is

$$\langle \lambda \bar{\lambda} \rangle = \left(\frac{\Lambda_c^{3N_c - N_f}}{\text{Det}M} \right)^{\frac{1}{N_c - N_f}}. \quad (390)$$

8 *F*-term inflation

8.1 Preserving the flat directions of global susy

Let us recall the discussion of Section 5.9. We saw there that in any model of inflation, the quartic term of the potential $V(\phi)$ should be small. One can ensure this by choosing the inflaton to be a flat direction of global supersymmetry, but one still has to ensure that the mass term and non-renormalizable terms are sufficiently small. At least for the mass term, this does not happen in a generic supergravity theory. The following strategies have been proposed to get around this problem.

1. The potential is dominated by the F term, but the inflaton mass is suppressed because K and W have special forms.
2. The potential is dominated by the F term, whose form is generic. However, the inflaton mass is suppressed because of an accidental cancellation between different terms.
3. The potential is dominated by the F term, whose form at the Planck scale is generic. However, the inflaton mass is suppressed in the regime where inflation takes place, because it runs strongly with scale.
4. The potential is dominated by a Fayet-Iliopoulos D term.
5. The potential is dominated by the F term, whose form is generic. However, the kinetic term of the inflaton field becomes singular near the region where inflation takes place, so that after going to a canonically-normalized the potential becomes flat even though it was not originally.

We mentioned the last possibility in Section 6.6 and it will not be considered further. We consider in this section the three F -term possibilities, and then go on to the D -term.

8.2 The generic F -term contribution to the inflaton potential

In this section we show that in a generic model of F -term inflation, the flatness parameter $\eta \equiv M_{\text{P}}^2 V''/V$ of the would-be inflaton potential is at least of order 1, in contrast with the requirement $|\eta| \lesssim 0.1$. We are continuing the discussion of Section 5.9, and supposing that the inflaton is the radial part of a matter field.

The full potential is given by Eq. (342), and as it contains more than one complex field we cannot adopt the assumption of Section 5.9 of exact canonical normalization; this would correspond to the condition $K_{nm^*} = \delta_{nm^*}$ which will be impossible to arrange for all field values. However, we assume this condition at the origin for the inflaton field ($n = m = i$), in order to calculate the inflaton mass-squared. We also assume that it provides at least a rough approximation for all of the fields, and that in addition $|K_n| \lesssim M_{\text{P}}$ and $e^{K/M_{\text{P}}^2} \sim 1$. These assumptions are valid in the string theory examples that are usually considered.

By analogy with Eq. (346), we define the scale M_{inf} of susy breaking during inflation by

$$V \simeq V_0 = M_{\text{inf}}^4 - M_{\text{P}}^{-2} e^{K/M_{\text{P}}^2} |W|^2. \quad (391)$$

(V_0 is the first term of Eq. (128), which we taking to dominate during inflation.) In contrast with the case for the true vacuum, there is no need for a strong cancellation between the first and second terms. If there is no strong cancellation,

$$V_0 \simeq M_{\text{inf}}^4. \quad (392)$$

8.2.1 The inflaton mass

We are mainly concerned with the contribution to η of the quadratic term in Eq. (128), which is $\eta = m^2 M_{\text{P}}^2 / V$. Purely for simplicity, we suppose that V depends only on $|\phi_i|$ so that $m^2 = V_{ii^*}$ evaluated at the origin.

To get off the ground, we first assume that all field values are $\ll M_{\text{P}}$, with $K_{nm^*} = \delta_{nm}$ at the origin. Then we find from Eq. (342), assuming that the inflaton is a flat direction corresponding to $W_{ni} = 0$,

$$m^2 = M_{\text{P}}^{-2} V_0 - M_{\text{P}}^{-2} |W_i|^2 + \sum_{nm} K_{ii^*}^{m^*n} W_n^* W_m. \quad (393)$$

The right hand side is evaluated with all fields at the origin. The contribution of the first term to η is precisely 1. For the other terms, take first the case $V_0 \sim M_{\text{inf}}^4$. Then the (negative) contribution of the second term to η is at most of order 1. For the third term, we use Eq. (333), and set $\Lambda_{\text{UV}} = M_{\text{P}}$. Then $K_{ii^*}^{\bar{n}m}$ will be of order M_{P}^{-2} , and the contribution of the third term to η is also of order 1 (with either sign). Generically, there is no reason to expect an accurate cancellation of the contribution +1 coming from the first term.

The case that one or more fields have values of order M_{P} is more model dependent, but the generic contributions to η are still at least of order V_0/M_{P}^2 . In particular, one gets a contribution to m^2 analogous to the third one of Eq. (393), $\sum_{nm} K_{ii^*}^{m^*n} F_n^* F_m$, that is generically of this order.

If we abandon the assumption $V_0 \simeq M_{\text{inf}}^4$, the estimate becomes bigger;

$$m^2 \sim \frac{M_{\text{inf}}^4}{M_{\text{P}}^2} \sim \left(\frac{V_0}{M_{\text{P}}^2} \right) \left(\frac{M_{\text{inf}}^4}{V_0} \right). \quad (394)$$

8.2.2 The quartic coupling and non-renormalizable terms

The expansion Eq. (331) of W will generically give coefficients $\lambda \sim \lambda_d \sim 1$ in Eq. (128). According to Section 5.8, $\lambda \lesssim 10^{-9}$. To achieve this, the inflaton is chosen to be a flat

direction, so that the relevant renormalizable terms of Eq. (331) vanish. Repeating the above discussion one then finds $\lambda \sim V_0/M_{\text{P}}^4$.

At least the first few λ_d should also be suppressed, by eliminating the relevant non-renormalizable terms in Eq. (331). These coefficients are then also of order $\sim V_0/M_{\text{P}}^4$.

As before, these estimates assume $V_0 \simeq M_{\text{inf}}^4$ and more generally we have

$$\lambda \sim \lambda_d \sim \frac{M_{\text{inf}}^4}{M_{\text{P}}^4} \sim \left(\frac{V_0}{M_{\text{P}}^4} \right) \left(\frac{M_{\text{inf}}^4}{V_0} \right). \quad (395)$$

8.3 Preserving flat directions in string theory

8.3.1 A recipe for preserving flat directions

A strategy for keeping the F -term flat was given by Stewart [285] (see also [60]).

The basic idea is to ensure that the potential has almost the same form as in global susy. This is done by imposing some simple conditions on W and the fields, and choosing a rather special form for K . The required form occurs in weakly coupled string theory, though apparently not in Horava-Witten M-theory.

The fields are divided into three classes, which we shall label ϕ , ψ and χ . During inflation, it is required that the following relations are satisfied to sufficient accuracy

$$\begin{aligned} W &= W_\phi = W_\psi = \chi = 0 \\ W_\chi &\neq 0. \end{aligned} \quad (396)$$

The inflaton is going to be one of the ϕ fields, which means that the others are constant during inflation; as a result the requirement $\chi = 0$ can always be imposed by a choice of origin, though it may not be a natural one. With these assumptions, the potential Eq. (342) becomes during inflation

$$V_F = e^K \sum_{nm} W_n K^{m^*n} (W_m)^*, \quad (397)$$

where the sum goes only over the χ fields. The required form for K is

$$K = -\ln \left[f(\phi, \phi^*) - \sum_{nm} \chi_n^* C_{nm}(\psi, \psi^*) \chi_m \right] + \tilde{K}(\psi, \psi^*) + O(\chi^2, \chi^{*2}), \quad (398)$$

where f and \tilde{K} are arbitrary functions, and C is a matrix which might be the unit matrix. Then the potential during inflation is

$$V(\phi) = e^{\tilde{K}} \sum_n W_n (C^{-1})_{nm} (W_m)^*. \quad (399)$$

We see that the dependence of V on the fields ϕ comes only from the W_n . For such fields, flat directions of global susy are preserved, provided that they are not spoiled by fields that are displaced from the origin. We can have viable inflation by choosing the inflaton to be one of these flat directions. Note, though, that the ψ fields have to be stabilized in the presence of the \tilde{K} and C_{nm} factors.

One can quantify [285] the required accuracy of the assumptions by looking at Eq. (342). A slight violation of the conditions on W typically gives $|\eta| \sim M_{\text{P}}^{-2} |W/W_\chi|^2$, $|W_\phi/W_\chi|^2$ or $|W_\psi/W_\chi|^2$. A small contribution $\delta K(\phi, \chi)$ to K gives, assuming $\chi = 0$,

$$\epsilon \sim M_{\text{P}}^{-2} \delta K'^2 \quad (400)$$

$$|\eta - 2\epsilon| \sim \delta K'', \quad (401)$$

where the prime denotes a typical partial derivative of δK .

8.3.2 Preserving the flatness in weakly coupled string theory

In weakly coupled string theory, ignoring Green-Schwarz terms, K given by Eqs. (356) and (363) is of the required form if the ϕ and χ fields constitute a single untwisted sector (with the modulus a ϕ field), and the twisted fields vanish to sufficient accuracy. Accordingly we can require the following conditions, to sufficient accuracy during inflation [60, 106].

1. All derivatives of W with respect to matter fields vanish, except for the one corresponding to a single untwisted field, say W_{C3} . (One could allow more untwisted fields from the $I = 3$ sector without changing anything, and of course the choice $I = 3$ is arbitrary.)
2. $W = W_s = W_I = \phi_{C3} = 0$. (The easiest way of ensuring $W_I = 0$ is to suppose that every term in the expansion (354) of W vanishes.)
3. The twisted fields vanish.

From Eqs. (400) and (401) it is actually enough to have the twisted fields fixed at values $\ll M_{\text{P}}$. Also, condition 2 is accurate enough if $|W|/M_{\text{P}}$, $|W_s|$ and $|W_I|$ are all $\ll |W_{C3}|$. These conditions are straightforward to achieve if one ignores the dilaton, which is reasonable for models with $V^{1/4} \gg 10^{10}$ GeV, provided that the dilaton contribution $W(s)$ is the same during inflation as it is in the true vacuum. The present scheme may not work for models with $V^{1/4} \lesssim 10^{10}$ GeV.

With these conditions in place, Eq. (342) gives

$$V = \frac{|W_{C3}|^2}{x_1 x_2}. \quad (402)$$

Flat directions in the untwisted $I = 3$ sector are preserved, if their flatness is not spoiled by coupling to fields with nonzero values, and one of them can be the inflaton. It could also be t_3 , or a combination. Note that the analogous procedure in the case of a single modulus would not work, because of the factor 3 in front of Eq. (350).

The above strategy preserves the flatness of the globally supersymmetric potential at all values of the inflaton field. This is possible because the inflaton is supposed to belong to an untwisted sector, and string theory gives the part of the Kähler potential depending only on the sector for all field values. If the inflaton field is small it may be enough to keep the inflaton mass small, and this can be achieved provided that one knows the relevant part of the Kähler potential up to quartic terms. For the twisted fields, Eq. (356) gives the required information if we assume that K depends on the untwisted fields and bulk

moduli only through the x_I . In general Eq. (356) gives the usual result $m^2 \gtrsim V_0/M_{\text{P}}^2$, but an exception has been noted [53, 54]; if Eq. (376) applies, with $F^s = 0$ and $m_{3/2}^2 \equiv e^K |W|^2$ negligible, then m^2 vanishes provided that the inflaton field has weight $q_n = 3$.

All this is in weakly coupled string theory. In Horava-Witten M-theory, K receives an extra contribution Eq. (367). If the α_I are of order 1 this contribution will presumably give us back the generic result $m^2 \gtrsim V_0/M_{\text{P}}^2$.

8.3.3 Case of a linear superpotential

Returning to Eq. (402), we have to ensure the stability of t_1 and t_2 . This is achieved [60] if W_{C3} comes from a term $\Lambda^2 \phi_{C3}$, with Λ independent of the matter fields. Then, modular invariance requires $\Lambda^2 \propto \eta^{-2}(t_1) \eta^{-2}(t_2)$, and

$$V \propto \left[|\eta(t_1) \eta(t_2)|^4 x_1 x_2 \right]^{-1}. \quad (403)$$

To discuss the stability of the moduli, we can set the matter fields equal to zero so that $x_I = t_I + \bar{t}_I$. As shown in [60], V is stabilized at $t_1 = t_2 = e^{i\pi/6}$ up to modular transformations. The masses-squared of the canonically normalized t_1 and t_2 turn out to be precisely V/M_{P}^2 , which presumably hold them in place during inflation.

The value $t_I = e^{i\pi/6}$ corresponds to a fixed point⁷⁸ of the modular transformations. Since it must be an extremum of the potential, it is not particularly surprising to find that it represents the minimum during inflation. In the model of [37], it also represents a possible true vacuum value. In that case, the moduli stabilized at this point during inflation will remain there, and will not be produced in the early Universe.

If our assumptions are exactly satisfied, the linear superpotential will make the tree-level potential absolutely flat during inflation. The slope might come from loop corrections or from the assumptions not being exactly satisfied. (A contribution to K from Green-Schwarz terms has been shown [60] to give a slope corresponding to n significantly less than 1.) The slope might also come from the nonzero D -term that we are about to invoke, through the K_n factors in Eq. (338).

8.3.4 Generating the F term from a Fayet-Iliopoulos D -term

Instead of putting in the mass scale Λ by hand, one might generate it using a Fayet-Iliopoulos D -term [106].⁷⁹

Suppose that $W = \lambda \phi_1 \phi_2 \phi_3$, with each field from a different untwisted sector. We suppose that ϕ_1 and ϕ_2 acquire vevs when the D term is driven to (practically) zero.⁸⁰ From Eq. (352), one sees that the vevs $|\phi_1|^2$ and $|\phi_2|^2$ will be proportional to respectively x_1 and x_2 , making V given by Eq. (402) independent of these quantities.

⁷⁸The other fixed point in the fundamental domain, namely $t_I = 1$, is a saddle point of potential (403); see eg. [101]. (To be more precise, $t_I = 1$ is a fixed point if in addition to modular invariance there is symmetry under $\text{Im } t_I \rightarrow -\text{Im } t_I$, which is the case in the present model.)

⁷⁹This was considered in [60, 285], but the factors K_n in the D term Eq. (338) were not considered whereas they are in fact crucial.

⁸⁰The ratio $|\phi_1|/|\phi_2|$ can be fixed, for instance, by gauging a non-anomalous $U(1)$ symmetry under which ϕ_1 and ϕ_2 have opposite and equal charges.

Flat directions are now preserved in all of the untwisted sectors, provided that they are not spoiled by the displacement of fields from the origin. Any of them is a candidate for the inflaton, and so are each of the moduli t_I .

The same thing actually works [106] in the toy model with a single modulus t ; taking all three fields to belong to the (single) untwisted sector one eliminates the x dependence appearing in Eq. (351). Other authors using Eq. (351) for inflation suppose that x is fixed, either by an ad hoc functional form for $K(x)$ [202, 247, 237, 31], or by a loop correction [105]. The first option seems unsatisfactory, and in the second option the status of the loop correction during inflation is not clear.

We have not yet considered the stability of the dilaton, either in the D -term model or in the one with a linear superpotential. This has been investigated [106] with the (real part of the) dilaton in a linear multiplet, using a model [37] which stabilizes the dilaton in the true vacuum. The dilaton is stabilized in the model with a linear superpotential, but not in the D -term model in the simple form given above. However, the vevs induced by the D term can then induce additional vevs through the F term. It was shown [106] that this can stabilize the dilaton, while preserving the flatness in one or more of the untwisted directions. (By ‘stability’ we mean existence of a minimum in the potential, with all fields except the dilaton fixed. Starting from a wide variety of initial conditions, the dilaton will typically settle down to the minimum [27].)

To have a complete model, one also needs to end inflation, and because of the form we are imposing on W this will probably require hybrid as opposed to single-field inflation. No complete example has yet been given for the particular superstring-derived theory that we are considering, but one can presumably be constructed along the lines of the following model [285].

The model works with a superpotential that has the general properties listed at the beginning of the last subsection. The Kähler potential is assumed to be of the form Eq. (398), with for simplicity $C = e^{\tilde{K}} = 1$ so that the potential is the same as in global susy, but its detailed form is not considered. Also, $K_n = \phi_n^*$ is used when calculating the D term. The model contains one χ field and three ψ fields.

Working with units $M_P = 1$, the superpotential is

$$W = \lambda_1 \phi \psi_1 \psi_3 + \lambda_2 \psi_2^n \chi \tag{404}$$

with $n \geq 2$. The D -term is taken to be⁸¹

$$V_D = \frac{1}{2} g^2 \left(\xi - \psi_1^2 - \psi_2^2 + \psi_3^2 + n \chi^2 \right)^2, \tag{405}$$

and it is assumed that

$$\lambda_2 \xi^{n-2} \ll g. \tag{406}$$

It is assumed that during inflation $\psi_1^2 + \psi_2^2 < \xi$. Then χ and ψ_3 will be driven to zero, and so will the derivatives of W with respect to ϕ , ψ_1 and ψ_2 . The potential then becomes

$$V = \lambda_1^2 \phi^2 \psi_1^2 + \lambda_2^2 \psi_2^{2n} + \frac{1}{2} g^2 \left(\xi - \psi_1^2 - \psi_2^2 \right)^2. \tag{407}$$

⁸¹We use the same symbol for the square and the modulus-squared of a field since it is obvious from the context which is intended.

During inflation it is assumed that

$$\phi^2 > \frac{g^2}{\lambda_1^2} (\xi - \psi_2^2). \quad (408)$$

Then ψ_1 is driven to zero, along with the derivative of W with respect to ψ_3 . From Eq. (406) ψ_2 has a constant value given approximately by

$$\xi - \psi_2^2 \simeq \frac{n\lambda_2^2 \xi^{n-1}}{g^2} \ll \xi. \quad (409)$$

Restoring M_{P} , we conclude that during inflation, there is an exactly flat potential with magnitude given by

$$V_0^{1/4} \simeq \sqrt{\lambda_2} \left(\frac{\xi}{M_{\text{P}}^2} \right)^{\frac{n-2}{4}} \sqrt{\xi}, \quad (410)$$

in the regime

$$\phi > \phi_c \equiv (\lambda_2/\lambda_1) \sqrt{n} (\xi/M_{\text{P}}^2)^{\frac{n-2}{2}} \sqrt{\xi}. \quad (411)$$

This scheme is similar to the scheme of D -term inflation that we consider later, but differs from it in two crucial respects. One is that the loop correction is much smaller, because the D -term is much smaller. As a result, there is no need for the inflaton field to have the dangerous value $\phi \sim M_{\text{P}}$. The other is that the the COBE normalization $V_0^{1/4} \lesssim 10^{-2} M_{\text{P}}$ can be achieved without supposing that $\sqrt{\xi}$ is so small.

8.3.5 Simple global susy models of inflation

In these examples we took seriously the requirement of modular invariance. We end by considering a couple of models that ignore this requirement, while using a superpotential of the required form 396. It would not be difficult to generalize them so that modular invariance is satisfied, though the stability of the moduli and dilaton may require care.

The mutated hybrid inflation model Eq. (223) is generated by [224]

$$W = \Lambda^2 \chi_1 \left(1 - \frac{1}{2} \psi/M \right) + \sqrt{\frac{\lambda}{4}} \phi \psi \chi_2. \quad (412)$$

The χ fields are driven to zero, giving Eq. (223) with $V_0 = \Lambda^4$. The COBE normalization Eq. (226), with $M \sim M_{\text{P}}$, corresponds to $\Lambda \sim 10^{13}$ to 10^{14} GeV. It was suggested [286] that Λ could be identified with gaugino condensation scale, though it is not clear how that might be achieved.

To obtain inverted hybrid inflation one can use [224]

$$W = \Lambda^2 \left(1 - \frac{\lambda \phi^2 \psi^2}{\Lambda^4} \right) \chi. \quad (413)$$

This drives χ to zero, and after adding suitable mass terms one obtains Eq. (216). The mass terms can come from the supergravity corrections. (A more complicated superpotential was given much earlier [249], but the inflaton trajectory turned out to be unstable [247].)

8.4 Models with the superpotential linear in the inflaton

We next turn to models where the superpotential during inflation is linear in the inflaton field [60, 87, 88, 191, 210, 73], or linear except for small corrections [181, 146, 147]. The first case gives hybrid inflation with a potential whose slope is dominated by a loop correction. The second case gives single-field inflation with an inverted quadratic, or higher-order, potential.

This paradigm has been widely regarded as a way of keeping supergravity corrections under control. Unfortunately, the analysis leading to that viewpoint is likely to be incorrect, since it assumes that all of the fields in Nature have values $\ll M_{\text{P}}$ during inflation. To see what is going on, first suppose that this assumption is correct. Then the inflaton mass-squared is given by Eq. (393), and one can see [60] that indeed the first two terms cancel if the superpotential is linear in the inflaton field. So to achieve a sufficiently small mass one need only tune down the coefficient of the relevant quartic term in K , below its natural value $\sim M_{\text{P}}^{-2}$. Arguably, this is preferable to arranging an accurate cancellation. But now suppose that there are fields ϕ_n , with values M_{P} . One sees from Eq. (342) that with the minimal form $K = \sum |\phi_n|^2$, each such field contributes $\eta = M_{\text{P}}^{-2} |\phi_n|^2 \simeq 1$. There is no reason to suppose that the non-minimal form presumably holding in reality will give a much smaller contribution. So one is back with a cancellation and the paradigm has no special virtue. Specific examples of fields with values of order M_{P} are the dilaton and bulk moduli that emerge from string theory.

We focus on the hybrid inflation model, where the superpotential during inflation is exactly linear in the inflaton field. The field whose radial part will be the inflaton is a gauge singlet, denoted by S . The original version of the model [60] used the superpotential

$$W = S(\kappa\psi - \mu^2), \quad (414)$$

where κ is a dimensionless coupling. This form does not allow ψ to be charged under any symmetry, but one can change it to [87]

$$W = S(\kappa\bar{\psi}\psi - \mu^2). \quad (415)$$

Here, ψ and $\bar{\psi}$ are oppositely charged under all symmetries so that their product is invariant. The absence of additional terms involving S is enforced to all orders if S is charged under a global $U(1)$ R-symmetry, and up to a finite order if it is charged under a discrete (Z_N) symmetry. As we noted before, only the latter seems to be allowed in the context of string theory.

Instead of putting in the scale μ by hand, one may derive it [73] from dynamical supersymmetry breaking by a quantum moduli space (Section 7.5.4).

The canonically-normalized inflaton field is $\phi \equiv \sqrt{2}|S|$, and the global susy potential is

$$V = \frac{\kappa^2\phi^2}{2}(|\psi|^2 + |\bar{\psi}|^2) + |\kappa\bar{\psi}\psi - \mu^2|^2. \quad (416)$$

This has the same general form as original tree-level hybrid inflation potential Eq. (205), with zero inflaton mass. The interaction with ϕ then gives ψ and $\bar{\psi}$ identical 2×2 mass matrices for their real and imaginary parts, and after diagonalizing one finds masses

$$m_{\pm}^2 = \frac{1}{2}\kappa^2\phi^2 \pm \mu^2\kappa. \quad (417)$$

The critical value is therefore given by $\phi_c^2 = 2\mu^2/\kappa$. For $\phi > \phi_c$, $|\psi| = |\bar{\psi}| = 0$ and we have slow-roll inflation.

The potential is exactly flat at tree-level, but the loop correction gives a significant slope [87]. Indeed, using Eq. (327) and remembering that there are two chiral multiplets, one finds the potential we wrote down in Eq. (240), with $\Lambda = \mu$ and $Cg^2 = \kappa^2$. We already worked out the prediction of this potential for n , and its COBE normalization.

Some authors [251, 210] have considered the possibility that quadratic and quartic tree-level terms are significant, with the former assumed to come only from the quartic term of the Kähler potential and the latter assumed to come only from the factor e^{K/M_{P}^2} . According to the analysis of Section 8.2, neither of these assumptions is very reasonable.

8.5 A model with gauge-mediated susy breaking

Now we consider a global susy model [86, 263] in which W does not have the form (396). Our discussion somewhat extends the original one.

In this model, the supergravity corrections are presumed to be small because of an accident. As we shall see, a very severe cancellation is required. The model assumes that there is gauge-mediated susy breaking in the true vacuum, which also operates during inflation. It uses a particle physics model [82] which replaces the μ parameter of the MSSM by a term $\lambda_\mu M_{\text{P}}^{-n} S^{n+1}$. The field $\phi \equiv \sqrt{2}\text{Re}S$ becomes the inflaton. In this model, gravitinos do not pose a cosmological problem, while the moduli problem is ameliorated.

The superpotential is supposed to be

$$W = -\frac{\beta X S^{2+p}}{M_{\text{P}}^p} + \frac{S^{m+3}}{M_{\text{P}}^m} + \lambda_\mu M_{\text{P}}^{-n} S^{n+1} H_U H_D + \dots \quad (418)$$

This structure can be enforced by discrete symmetries. The dots represent the contributions to W that do not involve S . They generate among other things the vev F_X , which we take to be real and positive, and close to the vacuum value discussed in Section 7.6.2. The third term generates the μ term, but plays no role during inflation. The case $p = m = 2$ is considered.

Adding a negative mass-squared term that is supposed to come from supergravity, the potential along the real component of S (denoted by the same symbol) is

$$V \simeq V_0 - m^2 S^2 - \frac{1}{4} \lambda S^4 + \left(S^4 - 4\beta X S^3 \right)^2, \quad (419)$$

with

$$\lambda = 8\beta M_{\text{P}}^{-2} F_X. \quad (420)$$

The constant term V_0 is given by

$$V_0 = F_X^2 - 3M_{\text{P}}^{-2} e^{K/M_{\text{P}}^2} |W|^2, \quad (421)$$

and as is usual in gauge-mediated models the origin of the last term is not unspecified.

One can determine the vacuum value of S by minimizing this potential, and using the vacuum value $X \simeq F_X/\Lambda$ with $\Lambda \sim 10^5$ GeV. Assuming $X \ll S$, one finds $S^4 \sim \beta M_{\text{P}}^2 F$ and $F_X^3 \lesssim M_{\text{P}}^2 \Lambda^4$ or $\sqrt{F_X} \lesssim 10^9$ GeV. By setting $V = 0$ in the vacuum, one finds that

in this case $V_0 \sim \beta^2 F^2$. In the opposite case $X \gg S$, one finds $S^2 \sim M_{\text{P}}^2 \Lambda^2 / F_X$ and $\sqrt{F_X} \gtrsim 10^9 \text{ GeV}$. Then,

$$V_0 \sim \frac{\Lambda M_{\text{P}}^2}{\beta X^3} F_X^2. \quad (422)$$

One can check that X is negligible during inflation (assuming that like F_X it has almost the same value as in the true vacuum). The potential then becomes the one that we analyzed in Section 6.5. We found that the COBE normalization requires $\sqrt{\beta F_X} \sim 10^{11} \text{ GeV}$, marginally consistent with the upper limit for gauge-mediated susy breaking if β is close to 1. This corresponds to $X \sim 10^{17} \text{ GeV}$.

Using Eq. (422) one finds

$$V_0 \sim 10^{-10} F_X^2. \quad (423)$$

The generic supergravity contributions to m^2 are of order $F_X^2 / M_{\text{P}}^2 \sim 10^{10} V_0 / M_{\text{P}}^2$. In contrast with the usual situation, the generic contributions have to be suppressed to at one part in 10^{11} , even if n is significantly different from 1 ($n - 1 = m^2 M_{\text{P}}^2 / V_0 \simeq 0.1$).

8.6 The running inflaton mass model revisited

Now we look in more detail at the theory behind the running mass model of Section 6.16.

8.6.1 The basic scenario

The fundamental assumption of the model is that the sector of the theory occupied by the inflaton is hidden from the sector where supersymmetry is spontaneously broken, and communicates with it only through interactions of gravitational strength. We shall call the former the *inflaton sector*, and the latter the *inflationary SSB sector*. The inflaton sector is supposed to be described by a renormalizable global susy theory, with soft susy breaking terms. In other words, there is supposed to be gravity-mediated supersymmetry breaking, in the inflaton sector during inflation.

It is not necessary, for the viability of the model, to assume anything about the inflationary SSB sector. But the simplest thing is to identify the inflationary SSB sector with the one that generates susy breaking in the true vacuum, which we call the *vacuum SSB sector*. Also, one might suppose that the susy-breaking scales are the same, $M_{\text{inf}} \sim M_S$. In that case, we shall have $M_{\text{inf}} \sim 10^{10} \text{ GeV}$ if there is gravity-mediated susy breaking in the visible sector, and $10^5 \lesssim M_{\text{inf}} \lesssim 10^{10} \text{ GeV}$ if there is gauge-mediated susy breaking in the visible sector. (Presumably the inflaton sector is different from the visible sector, though they might conceivably be identical if there is gravity-mediated susy breaking in the visible sector.)

Even if the inflationary and vacuum SSB sectors are identical, it is not inevitable that the susy-breaking scales are the same. Take for instance the case of gaugino condensation, where that scale is determined by $W(s)$. Even if $W(s)$ has the same functional form in the two cases, it will not have the same value because s will be different. But $W(s)$ might be a different function during inflation. For instance, gaugino condensation might occur only after inflation. If it does occur, the value of b might be different, because during inflation some of the fields which contribute to the running of the gauge coupling and are light in the

true vacuum, become heavy and no longer contribute to the renormalization group equation of the gauge coupling [158].

At the Planck scale, the inflaton mass-squared $m^2(\phi)$ (along with other soft susy breaking parameters in the inflaton sector) is supposed to have its generic magnitude given by Eq. (394) with $M_{\text{inf}}^4 \simeq V_0$,⁸²

$$|m^2(M_{\text{P}})| \sim V_0/M_{\text{P}}^2. \quad (424)$$

The mass-squared is supposed to run strongly with scale, so that it becomes small which allows inflation to occur.

8.6.2 Directions for model-building

Although a complete model is far from being written down, one can see some basic features that will be needed.

The complete potential might look roughly like Eq. (205). If the mass m_ψ is also generated by soft susy breaking, then as we noted in Section 6.9 ψ would have a vev of order M_{P} ; it might be something like the dilaton or a bulk modulus, or a matter field with non-renormalizable terms suppressed to high order by a discrete symmetry. On the other hand, m_ψ might be bigger and come from some other mechanism, in which case ψ could be a more ordinary field.

The quartic coupling of Eq. (205) could come from a term $\sqrt{\lambda'} S \phi \psi$ in the superpotential, with S some field that vanishes during inflation. The alternative coupling in Eq. (211), plus an identical term with $\phi \rightarrow \psi$ that we did not consider for simplicity, could come [262] from a term $\phi^2 \psi^2 / \Lambda_{\text{UV}}$ in the superpotential.

One will have to avoid the strong cancellation between the terms of Eq. (391), that is present in the true vacuum. In the case of gauge-mediated susy breaking in the true vacuum, this might require an understanding of the origin of the sector that generates the magnitude of W in the true vacuum, which is so far something of a puzzle. In the opposite case, the situation maybe under better control, since one could use an explicit model (such as the one of Reference [37]) which already specifies all of the relevant quantities in the true vacuum.

8.6.3 Running with a gauge coupling

Following [290, 62], we calculate the running inflaton mass, on the assumption that the inflaton is charged under a gauge group and that its Yukawa couplings have a negligible effect.

The RGE's have the same form as the well-known ones that describe the running of the squark masses with only QCD included,

$$\frac{d\alpha}{dt} = \frac{b}{2\pi} \alpha^2 \quad (425)$$

$$\frac{d}{dt} \left(\frac{\tilde{m}}{\alpha} \right) = 0 \quad (426)$$

⁸²To be more correct, the relevant coefficients in the expansion Eq. (333) are supposed to be of order 1 at the Planck scale, in Planck units. They run, which is equivalent to running the susy breaking parameters even though the latter may really be defined only below a lower scale where supersymmetry breaks.

$$\frac{dm_\phi^2}{dt} = -\frac{2c}{\pi}\alpha\tilde{m}^2 \quad (427)$$

Here α is related to the gauge coupling by $\alpha = g^2/4\pi$, \tilde{m} is the gaugino mass, and $t \equiv \ln(\phi/M_{\text{P}}) < 0$. The numbers b and c depend on the group; c is the Casimir quadratic invariant of the inflaton representation under the gauge group, for instance $c = (N^2 - 1)/2N$ for any fundamental representation of $SU(N)$, and $b = -3N + N_{\text{f}}$ for a supersymmetric $SU(N)$ with N_{f} pairs of fermions in the fundamental/antifundamental representation.

The Renormalization Group Equations can easily be solved. The result is

$$m^2(\phi) = m_0^2 + \frac{2c}{b}\tilde{m}_0^2 \left[1 - \frac{1}{\left[1 - \frac{b\alpha_0}{2\pi} \ln(\phi/M_{\text{P}}) \right]^2} \right]. \quad (428)$$

Here m_0 is the inflaton mass, \tilde{m}_0 is the gaugino mass, α_0 is the gauge coupling, all evaluated at the Planck scale.

We want the magnitude of m^2 to decrease as one goes down from the Planck scale. This requires $m_0^2 < 0$, corresponding to model (i) or model (ii) of Section 6.16.

We evaluate c , σ and τ to leading order in α , which is presumably all that is justified in a one-loop calculation. It is convenient to use the following definitions.

$$\mu^2 \equiv -m^2 M_{\text{P}}^2 / V_0, \quad (429)$$

$$A \equiv -\frac{2c}{b} \frac{\tilde{m}^2 M_{\text{P}}^2}{V_0}, \quad (430)$$

$$\tilde{\alpha} \equiv \frac{-b\alpha}{2\pi}, \quad (431)$$

$$y \equiv [1 + \tilde{\alpha}_0 \ln(\phi/M_{\text{P}})]^{-1}, \quad (432)$$

$$y_{**} \equiv \sqrt{1 + \frac{\mu_0^2}{A_0}}. \quad (433)$$

Applying the linear approximation one finds [63]

$$c = 2y_{**}^3 A_0 \tilde{\alpha}_0 \quad (434)$$

$$\tau = 2A_0 y_{**}^2 (y_{**} - 1). \quad (435)$$

If m^2 continues to run until the end of slow-roll inflation, σ is given by

$$\ln \sigma = 2y_{**}^2 (y_{**}^{-1} - y_{\text{end}}^{-1}) + \ln \left[\frac{4A_0 y_{**}^2 |y_{\text{end}} - y_{**}|}{y_{\text{end}} + y_{**}} \right], \quad (436)$$

where $y_{\text{end}} = (y_{**}^2 \pm A_0^{-1})$, with the plus sign for model (i) and the minus sign for model (ii).

Using this result, one can calculate the COBE normalization, and the spectral index $n(N)$. There are four cases to consider, corresponding to asymptotic freedom or not, and models (i) or (ii). Except for the case of model (ii) and no asymptotic freedom, there is [63] a region of parameter space that is allowed by the observational constraints described in Section 6.16, and includes the theoretically favoured values $\alpha_0 \sim 10^{-1}$ to 10^{-2} , $|\mu_0| \sim |A_0| \sim 1$ and $10^5 \text{ GeV} \lesssim V^{1/4} \lesssim 10^{10} \text{ GeV}$.

8.7 A variant of the NMSSM

The model we just considered supposes that there is soft susy breaking in the inflaton sector, and that the relevant soft susy breaking parameters all have their natural values at the Planck scale. In particular, the inflaton mass is supposed to satisfy $|m^2| \sim V_0/M_{\text{P}}^2$ there. We now consider a model [29, 30, 31] which also assumes soft susy breaking in the inflaton sector, with all relevant parameters *except the inflaton mass* at natural values (and actually negligible running). But the inflaton mass is supposed to vanish at the Planck scale, presumably because it occupies a special subsector in which no-scale supergravity holds. This last feature may be difficult to arrange, since the model requires an accurate cancellation in Eq. (391) and therefore a nonzero value for W (see the remarks in Section 7.10.2). (The specific proposal [31] invokes the weakly coupled string theory expressions of Section 7.9, but it requires a field with vanishing modular weight whereas one expects nonzero weights.)

In this model, the inflaton sector is actually (part of) the visible sector, and it is assumed that gravity-mediated susy-breaking holds with $M_{\text{inf}} \simeq M_{\text{S}}$.

The model [29, 30, 31] works with a variant of the next-to-minimal Standard Model [98, 245, 65]. The relevant part of the the superpotential is

$$W = \lambda N H_{\text{U}} H_{\text{D}} - k \phi N^2, \quad (437)$$

where H_{U} and H_{D} are the usual Higgs fields and N and ϕ are two standard model gauge singlet fields.

The actual next-to-minimal Standard Model is recovered if the last term of Eq. (437) becomes $-kN^3$, which leads to a Z_3 symmetry and possible cosmological problems with domain walls. In the variant, the Z_3 becomes a global $U(1)$, which is in fact the Peccei-Quinn symmetry commonly invoked to ensure the CP invariance of the strong interaction. This symmetry is spontaneously broken in the true vacuum because ϕ and N acquire vevs. The axion is the Pseudo-goldstone boson of this symmetry, and axion physics requires $\langle \phi \rangle \sim \langle N \rangle \sim 10^{10}$ GeV to 10^{13} GeV or so. (Higher values are allowed in some models, but not this one.) The latter value is adopted to make the inflation model work.

The axion is practically massless, and by a choice of the axion field one can make ϕ real.⁸³ It is going to be the inflaton, and during inflation $H_{\text{U}} H_{\text{D}}$ is negligible. Writing $\sqrt{2}N = N_1 + iN_2$, and including a soft susy breaking trilinear term $2A_k k \phi N^2 + \text{c.c}$ (with A_k taken to be real) as well as soft susy breaking mass terms, the potential is

$$V = V_0 + k^2 |N|^4 + \frac{1}{2} \sum_i m_i^2(\phi) N_i^2 + \frac{1}{2} m_\phi^2 \phi^2, \quad (438)$$

where

$$m_1^2(\phi) = m_1^2 - 2k A_k \phi + 4k^2 \phi^2, \quad (439)$$

$$m_2^2(\phi) = m_2^2 + 2k A_k \phi + 4k^2 \phi^2. \quad (440)$$

$$(441)$$

⁸³We take this real ϕ to be canonically normalized, which means that the original complex ϕ is $\sqrt{2}$ times the canonically normalized object.

The parameters m_i , A_k and m_ϕ are supposed to be generated by a gravity-mediated mechanism, which is the same as in the true vacuum, and it is supposed that the susy breaking scale is also the same. This is supposed to give generic values $m_i \sim A_k \sim 1 \text{ TeV}$ for all of these parameters except m_ϕ . The latter is supposed to vanish at the Planck scale, being generated by radiative corrections as described in a moment.

The constant term V_0 comes from some other sector of the theory, and it is supposed to dominate the potential.

The true vacuum corresponds to

$$\langle \phi \rangle = \frac{A_k}{4k}, \quad (442)$$

$$\langle N_1 \rangle = \frac{A_k}{2\sqrt{2}k} \sqrt{1 - 4\frac{m_1^2}{A_k^2}} \quad (443)$$

$$\langle N_2 \rangle = 0. \quad (444)$$

We ignored the tiny effect of m_ϕ in working out the nonzero vevs. It is assumed that $4m_1^2$ is somewhat below A_k^2 , so that

$$A_k \sim k\langle N_1 \rangle \sim k\langle \phi \rangle \sim 1 \text{ TeV}. \quad (445)$$

To have the vevs at the axion scale, say 10^{13} GeV , we require $k \sim 10^{-10}$. Also, λ should have a similar value, since $\lambda\langle N_1 \rangle$ will be the μ parameter of the MSSM.

The tiny couplings k and λ are supposed to be products of several terms like (ψ/M_{P}) where ψ is the vev of a field that is integrated out. The structure of such terms may be enforced through discrete symmetries derived from string theory. The same terms can ensure Peccei-Quinn symmetry to sufficient accuracy, without actually invoking a global symmetry. In the example given [29], ϕ is charged under a Z_5 as well as the Z_3 already encountered, which forbids terms ϕ^d up to $d = 15$ in the superpotential. One must in any case forbid them up to $d \simeq 8$, to satisfy the constraint Eq. (168).

During inflation, the fields N_i are trapped at the origin, and

$$V = V_0 + \frac{1}{2}m_\phi^2\phi^2. \quad (446)$$

The field N_1 is destabilized if ϕ lies between the values

$$\phi_c^\pm = \frac{A_k}{4k} \left(1 \pm \sqrt{1 - 4\frac{m_N^2}{A_k^2}} \right) \sim A_k/k. \quad (447)$$

If m_ϕ^2 is positive the model gives ordinary hybrid inflation ending at ϕ_c^+ , but if it is negative it gives inverted hybrid inflation ending at ϕ_c^- . We shall see that the radiative corrections actually give the latter case. The height of the potential is $V_0^{1/4} \sim A_k/\sqrt{k} \sim 10^8 \text{ GeV}$. The COBE normalization, Eq. (178) or Eq. (203), is therefore

$$A_k = 2.5 \times 10^{-4} |n-1| e^{\pm x} M_{\text{P}}, \quad (448)$$

where $x \equiv \frac{1}{2}|n-1|N$. This requires n to be completely indistinguishable from 1, $|n-1| \sim 10^{-12}$. The corresponding inflaton mass, given by $|n-1| = 2m_\phi^2 V_0/M_{\text{P}}^4$ is

$$m_\phi \sim 100 \left(\frac{A_k}{M_{\text{P}}} \right)^{2.5} M_{\text{P}} \sim \text{eV}. \quad (449)$$

The loop correction generating m_ϕ comes from the N_i , and their fermionic partner which has mass-squared $4k^2\phi^2$. In the regime $\phi \gg A_k/k (\sim m_i/k)$, one finds [29] using Eqs. (439), (440) and (327)

$$\Delta V \simeq \frac{k^2 A_k^2}{4\pi^2} \phi^2 \ln(\phi/Q), \quad (450)$$

where Q is the renormalization scale. The requirement $\partial V/\partial Q = 0$ at $\phi \sim Q$ gives, in the regime $Q \gg A_k/k$, the RGE Eq. (161),

$$\frac{dm_\phi^2}{d \ln Q} = \frac{k^2 A_k^2}{2\pi^2}. \quad (451)$$

The running of kA_k is negligible because k is small, and setting $m_\phi = 0$ at $Q = M_P$ we obtain

$$m_\phi^2(Q) = -\frac{k^2 A_k^2 \ln(M_P/Q)}{2\pi^2} \sim -k^2 A_k^2. \quad (452)$$

Strictly speaking the derivation of this result holds only for $Q \gg A_k/k$. Inflation occurs at $\phi \sim \phi_c^\pm \sim A_k/k$, and in this regime we should take $Q \sim A_k/k$ to minimize the loop correction.⁸⁴ Thus we are somewhat below the regime where the result is valid, but it hopefully gives a rough approximation. If so we are dealing with an inverted hybrid inflation model. Somewhat remarkably, the magnitude $m_\phi \sim kA_k$ agrees with the COBE normalization $m_\phi \sim \text{eV}$, within the uncertainties of A_k and k .

Finally, we note that because the running of the inflaton mass is weak, its use is optional; instead of using it, we could set $m_\phi = 0$, and generate the slope of the inflaton potential from the loop correction Eq. (450) with $Q = M_P$.

9 *D*-term inflation

D-term inflation can preserve the flat directions of global susy (and in particular keep the inflaton potential flat) provided that one of the contributions to V_D contains a Fayet-Iliopoulos term as in Eq. (294), and that all fields charged under the Fayet-Iliopoulos $U(1)$ are driven to negligible values so that $V = (g^2/2)\xi^2$. This was first pointed out by Stewart [285], who exhibited a hybrid inflation model which uses the F term to drive all of the charged fields to zero.⁸⁵ He considered only the tree-level potential without any definite proposal for its slope. Significant progress came when Binétruy and Dvali [35] and Halyo [130] pointed out, in the context of a somewhat simpler tree-level potential, that the loop correction gives a well-defined slope. This led to an explosion of interest in *D*-term inflation [148, 90, 52, 219, 217, 264, 178, 95, 40].

⁸⁴The argument of the log in the loop correction is actually $2k\phi/Q$, so one might argue that the appropriate scale is $Q \sim k\phi_c \sim A_k$ which is much lower. But the effect of including k here is the same order of magnitude as the effect of including the two-loop correction, and is presumably negligible. This is because making k small also makes the running slower.

⁸⁵A slightly different version of the model, which actually was the main focus of his paper, gives the *F*-term inflation model mentioned in Section 8.3.4. A single-field model of inflation with a Fayet-Iliopoulos *D*-term, and the inflaton charged under the relevant $U(1)$, had been considered earlier [50, 51]. It gives the inverted quadratic potential considered in Section 6.4, and is viable only under the unlikely assumption that the inflaton charge is $\ll 1$. In any case it does not preserve the flat directions of global susy.

9.1 Keeping the potential flat

We initially make the usual assumption that the fields charged under the $U(1)$ vanish exactly. Then

$$V_D = \frac{1}{2}(\text{Re } f)^{-1} g^2 \xi^2. \quad (453)$$

In this tree-level potential, the only dependence of V_D on the fields comes from the gauge kinetic function f .⁸⁶ It has non-renormalizable terms, and so does W that appears in the supposedly negligible F -term. If $|\phi| \ll M_{\text{P}}$, only low-dimensional terms are dangerous, and they can be eliminated using a suitable discrete symmetry [217, 178]. Unfortunately, we shall find that $|\phi|$ is of order M_{P} , and maybe bigger. This makes the non-renormalizable terms difficult to control [178], as well as those of K . (As well as directly affecting the potential, the latter can give a non-trivial kinetic term, which alters the potential after going to a canonically-normalized field). We proceed on the assumption that non-renormalizable terms of W and f turn out to be negligible; in particular we assume $f = 1$.

With W is under control, and the fields charged under the $U(1)$ exactly zero, the Kähler potential K can have no effect, and the coefficients λ and λ_d can be much smaller than V_0/M_{P}^4 . This will be crucial, in view of the fact that the inflaton field is of order M_{P} .

In some versions of D term inflation the charged fields are not driven to zero. If their contribution to V_D is a significant fraction of the total, the terms K_n in Eq. (338) (and similar ones from D terms involving other gauge groups under which they are charged) will generically spoil inflation [40]. The conclusion seems to be that the charged fields should be driven to sufficiently small values, even if they do not vanish.

Finally, let us mention that, if the Fayet-Iliopoulos term is to come from string theory, see Eq. (297), the corresponding D -term scales like $g_{\text{str}}^6 \propto (\text{Re } s)^{-3}$. The problem here is that, assuming that the D -term dominates over any other F -term, the potential during inflation appears to prefer $\text{Re } s \rightarrow \infty$ and therefore $V_D \rightarrow 0$. This is the D -term inflation equivalent of the dilaton runaway problem that appears in string theories in the true vacuum. However, it has been argued [158] that the physics of gaugino condensation in ten-dimensional $E_8 \otimes E_8$ superstring theories is likely to be modified during the inflationary phase in such a way as to enhance the gaugino condensation scale. This may allow the dilaton to be stabilized by the F term [158], though one has to check that the latter does not generate dangerous supergravity corrections to the inflaton potential.

9.2 The basic model

At least one of the charged fields should have negative charge q_n , so that the D term is driven to zero in the vacuum (or at least to a value much smaller than $(g^2/2)\xi^2$, as in Section 7.6.1). One has to give such negatively-charged fields couplings which drive them to small values during D -term inflation. The proposal of refs. [285, 35, 130] is that every negatively-charged field has a partner with all charges opposite. It can then couple to the inflaton in the F term, and acquire a positive mass-squared during inflation.

⁸⁶We are here taking g to be a constant, and $f = 1$ at the origin as in Eq. (334). Later we adopt the convention that $(\text{Re } f)^{-1}$ is absorbed into g^2 , making the latter a function of the fields.

Suppose for simplicity that there is just one pair, ϕ_{\pm} . The inflaton is supposed to be the radial part of an uncharged field S ($\phi = \sqrt{2}|S|$). There is a term in the superpotential

$$W = \lambda S \phi_+ \phi_- . \quad (454)$$

Since ϕ_{\pm} are going to be driven to zero, it will be enough to use the global susy expression for the D -term, giving

$$V = \frac{1}{2} \lambda^2 \phi^2 (|\phi_-|^2 + |\phi_+|^2) + \lambda^2 |\phi_+ \phi_-|^2 + \frac{g^2}{2} (|\phi_+|^2 - |\phi_-|^2 + \xi)^2 . \quad (455)$$

The global minimum is supersymmetry conserving, but the gauge group $U(1)$ is spontaneously broken

$$\langle \phi \rangle = \langle \phi_+ \rangle = 0, \quad (456)$$

$$\langle \phi_- \rangle = \xi \quad (457)$$

However, if we minimize the potential, for fixed values of ϕ , with respect to other fields, we find that for ϕ bigger than

$$\phi_c \equiv \frac{g}{\lambda} \sqrt{2\xi} \quad (458)$$

the minimum is at $\phi_+ = \phi_- = 0$. Thus, for $\phi > \phi_c$ and $\phi_+ = \phi_- = 0$ the tree level potential has a vanishing curvature in the ϕ direction and large positive curvature in the remaining two directions

$$m_{\pm}^2 = \frac{1}{2} \lambda^2 \phi^2 \pm g^2 \xi . \quad (459)$$

For $\phi > \phi_c$, the tree level value of the potential has the constant value $V = \frac{g^2}{2} \xi^2$.

This is a hybrid inflation model. At tree level, the potential $V(\phi)$ is perfectly flat, and its ϕ dependence comes from the loop correction. Supersymmetry is spontaneously broken, and inserting Eq. (459) into Eq. (327) gives

$$V = V_{1\text{-loop}} \equiv \frac{g^2}{2} \xi^2 \left(1 + \frac{g^2}{16\pi^2} \ln \frac{\lambda^2 \phi^2}{2\mu^2} \right), \quad (460)$$

where μ is the renormalization scale.

We can generalize the model by including more than one pair of fields $\phi_{n\pm}$, with charges q_n and superpotential couplings λ_n . Then the one-loop potential becomes

$$V = V_{1\text{-loop}} \equiv \frac{g^2}{2} \xi^2 \left(1 + C \frac{g^2}{16\pi^2} \ln \frac{\lambda^2 \phi^2}{2\mu^2} \right). \quad (461)$$

where

$$C = \frac{1}{2} \sum_n q_n^2 . \quad (462)$$

(In the log we took all λ_n to have a common value λ but this is not essential since λ does not affect the slope of the potential.)

Since the $U(1)$ generated by string theory is anomalous, corresponding to $\sum q_n \neq 0$, there have to be some unpaired charges. If they are positive, they will be driven to zero

and be irrelevant, and that is assumed in the paradigm under consideration. However, in weakly coupled string theory one actually expects unpaired negative charges which might ruin this paradigm [95]. That case is discussed in Section 9.3.

Inflation with this potential was discussed in Section 6.15. As noted there, slow-roll inflation will end when ϕ_c is reached, or when it gives way to fast roll, whichever is sooner. In the latter case, fast roll begins when $\eta \sim 1$, at

$$\phi_{\text{fr}} = \sqrt{\frac{Cg^2}{8\pi^2}} M_{\text{P}}. \quad (463)$$

This is about the same as ϕ_c given by Eq. (458), so it depends on the parameters which happens first. If fast roll begins first inflation will continue for an e -fold or so, ending when the oscillation amplitude falls below ϕ_c .

According to Eq. (243), ϕ is comparable with the Planck scale, and maybe bigger. If we increase the slope of V , by assuming that a tree-level contribution dominates the loop correction [219], this will increase ϕ (see Eq. (40)). The only hope of reducing it would be a cancellation between the loop and a tree-level contribution, which seems unlikely over a range of ϕ . As mentioned earlier, the large value of ϕ means that non-renormalizable terms in the potential and the kinetic function are not under good control, but we proceed on the assumption that they turn out to be harmless.

The COBE normalization is

$$\sqrt{\xi} = 8.5 \times 10^{15} \text{ GeV} \left(\frac{50C}{N} \right)^{1/4} \quad (464)$$

The scale imposed by COBE is clearly lower than the prediction Eq. (297) of weakly coupled string theory, which is a second worry for the model. Indeed, Eq. (297) requires

$$g_{\text{str}}^2 = \frac{192\pi^2}{\text{Tr } \mathbf{Q}} \left(\frac{50C}{N} \right)^{1/2} \left(\frac{5.9 \times 10^{15}}{2.4 \times 10^{18}} \right)^2 \lesssim 10^{-6}. \quad (465)$$

Such a value is unreasonable, since the dilaton during inflation would presumably have to be far away from the true vacuum value, placing it outside the domain of attraction of that value.

How can we get around this problem? The most obvious possibility is that weakly coupled string theory is replaced by something else, such as Horava-Witten M-theory, which might give a lower value for ξ . At the time of writing, it is not clear whether this is an open possibility or not [227, 41].

Another possibility is to make ξ lower by decoupling its origin from string theories. But to avoid putting it in by hand, one should generate it in some low-energy effective theory after some degrees of freedom have been integrated out. But to do this, one has presumably to break supersymmetry by some F -terms present in the sector which the heavy fields belong to and to generate the D -term by loop corrections. As a result, it turns out that $\langle D \rangle \ll \langle F^2 \rangle$, unless some fine-tuning is called for, and large supergravity corrections to η appear again. Let us give an example. Consider the following superpotential where a $U(1)$ symmetry has been imposed [263]

$$W = \lambda X \left(\bar{\Phi}_1 \Phi_1 - m^2 \right) + M_1 \bar{\Phi}_1 \Phi_2 + M_2 \bar{\Phi}_2 \Phi_1. \quad (466)$$

For $\lambda^2 m^2 \ll M_1^2, M_2^2$, the vacuum of this model is such that $\langle \phi_i \rangle = \langle \bar{\phi}_i \rangle = 0$ ($i = 1, 2$), where $\bar{\phi}_i$ and ϕ_i are the scalar components of the superfields $\bar{\Phi}_i$ and Φ_i , respectively. Supersymmetry is broken and $F_X = -\lambda^2 m^2$. This means that in the potential a term like $V = (F_X \bar{\phi}_1 \phi_1 + \text{h.c.})$ will appear. It is easy to show that, integrating out the ϕ_i and $\bar{\phi}_i$ scalar fields, induces a nonvanishing Fayet-Iliopoulos D -term

$$\xi \simeq \frac{F_X^2}{16\pi^2(M_1^2 - M_2^2)} \ln \left(\frac{M_2^2}{M_1^2} \right), \quad (467)$$

which is, however, smaller than F_X and inflation, if any, is presumably dominated by the F -term.

Staying with the high value of ξ , one might consider increasing the COBE normalization by supposing that the slope of the potential is bigger than the loop contribution. For instance, it might come from a term $\frac{1}{2}m^2\phi^2$, generated by the F term [219]. In general one has

$$g_{\text{str}}^2 = 1.9 \left(\frac{100}{\text{Tr } \mathbf{Q}} \right)^{1/3} \left(\frac{V^{1/4}}{M_{\text{P}}} \right)^{4/3}. \quad (468)$$

But even with the maximum allowed value $V^{1/4} \sim 10^{-2}M_{\text{P}}$, g_{str}^2 is still unreasonably low. This is a second problem for D -term inflation, though unlike the large-field problem it depends on details of the underlying string theory.

9.3 Constructing a workable model from string theory

The presence of the Fayet-Iliopoulos D -term (297) in weakly coupled string theory leads to the breaking of supersymmetry at the one-loop order at very high scale, the string scale. This option is generically not welcome from the phenomenological point of view because it induces too large soft susy breaking masses via gravity effects, $\tilde{m} \sim \xi/M_{\text{P}}$. The standard solution to this puzzle is to give a nonvanishing vev to some of the scalar fields which are present in the string model and are negatively charged under the anomalous $U(1)$. In such a way, the Fayet-Iliopoulos D -term is cancelled and supersymmetry is preserved. In the context of string theory, this procedure is called “vacuum shifting” since it amounts to moving to a point where the string ground state is stable⁸⁷ While maintaining the D - and F -flatness of the effective field theory, such vacuum shifting may have important consequences for the phenomenology of the string theory. Indeed, the vacuum shifting not only breaks the $U(1)$, but may also break some other gauge symmetries under which the fields which acquire a vev are charged. This is because the anomalous $U(1)$ is usually accompanied by a plethora of nonanomalous $U(1)$ ’s.

In the true vacuum, the vacuum shifting can generate effective superpotential mass terms for vector-like⁸⁸ states that would otherwise remain massless or may even be responsible for the soft mass terms of squarks and sleptons at the TeV scale.

In string theories the protection of supersymmetry against the effects of the anomalous $U(1)$ is extremely efficient. If we now apply a sort of “minimal principle” [86, 90] requiring

⁸⁷Notice, however, that if the vacuum shifting in the true vacuum is not complete, because of the presence of some non-vanishing F terms, it may give rise to interesting phenomenological implications. This is what happens for the model described in Section 7.6.1.

⁸⁸A vector-like set of fields is one having zero total charge.

that a successful scenario of D -term inflation should arise from “realistic” string models leading to the $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$ gauge structure at low energies, the cancellation of the Fayet-Iliopoulos D -term by the vacuum shifting mechanism may represent (and usually does) a serious problem. Indeed, one has to make sure that during inflation the Fayet-Iliopoulos D -term is not cancelled by one of the many scalar fields which are negatively charged under the anomalous $U(1)$ and are not coupled to the inflaton. This usually leads to the conclusion that a successful D -term inflationary scenario in string theory require many inflatons to render the vacuum shifting mechanism inoperative and it is clear that only a systematic analysis of flat directions in any specific model may answer these and similar questions. This requires the identification of possible inflatons and D - and F -flat directions for a large class of perturbative string vacua. This classification is a prerequisite to address systematically the issue of inflation in string theories as well as the phenomenological issues at low energy [57, 144].

As an illustrative example of the possible complications one has to face in building up a successful model of D -term inflation in the framework of 4D string models [95], one may consider the massless spectrum of a compactification on a Calabi-Yau manifold with Hodge numbers $h_{1,1}$, $h_{2,1}$, etc. The four-dimensional gauge group is $SO(26) \times U(1)$. There are then $h_{1,1}$ left-handed chiral supermultiplets transforming as $(\mathbf{26}, \sqrt{\frac{1}{3}}) \oplus (\mathbf{1}, -2\sqrt{\frac{1}{3}})$ and $h_{2,1}$ supermultiplets transforming as $(\mathbf{26}, -\sqrt{\frac{1}{3}}) \oplus (\mathbf{1}, 2\sqrt{\frac{1}{3}})$. In this case the $U(1)$ is anomalous because $h_{1,1}$ and $h_{2,1}$ are not equal. Indeed, suppose that $h_{1,1} - h_{2,1} > 0$. In such a case, out of the total $h_{1,1} + h_{2,1}$ chiral supermultiplets, there are only $2 h_{2,1}$ left-handed chiral supermultiplets which may form $h_{2,1}$ vector-like pairs under the $U(1)$ and give a vanishing contribution to $\text{Tr } \mathbf{Q}$. The remaining $h_{1,1} + h_{2,1} - 2 h_{2,1} = h_{1,1} - h_{2,1}$ fields will give a nonvanishing contribution to the Fayet-Iliopoulos D -term.

Taking into account the multiplicity of the fields, the one-loop D -term (297) is therefore given by

$$\xi = \frac{g_{\text{str}}^2 M_{\text{P}}^2}{192\pi^2} \cdot 2 \cdot \frac{24}{\sqrt{3}} (h_{1,1} - h_{2,1}). \quad (469)$$

We suppose the model has a gauge singlet field S which will play the role of the inflaton. Further we assume that there is a discrete R -symmetry that ensures S -flatness. These assumptions are quite ad hoc and in a realistic model we would have to demonstrate the existence of such a field, but we use this simple example to illustrate another problem that must be overcome if one is to obtain a realistic string model of D -term inflation.

With this field one may try to construct an inflationary potential. Gauge symmetries and the fact that $h_{1,1} - h_{2,1} > 0$ impose that one can generate masses only for the $h_{2,1}$ vectorlike combinations of the $SO(26)$ singlet and non-singlet fields via the couplings in the superpotential of the form

$$W = \lambda S \left[(\mathbf{26}, \sqrt{\frac{1}{3}}) \cdot (\mathbf{26}, -\sqrt{\frac{1}{3}}) + (\mathbf{1}, -2\sqrt{\frac{1}{3}}) \cdot (\mathbf{1}, 2\sqrt{\frac{1}{3}}) \right]. \quad (470)$$

Therefore only $2 h_{2,1}$ fields get a mass $\lambda \langle S \rangle$ and become very massive during inflation. This means that they decouple from the theory and do not contribute anymore to the Fayet-Iliopoulos term (469). On the other hand, the remaining $(h_{1,1} - h_{2,1})$ fields transforming as $(\mathbf{26}, \sqrt{\frac{1}{3}}) \oplus (\mathbf{1}, -2\sqrt{\frac{1}{3}})$ remain light because they cannot couple to the inflaton and give a contribution to (469), which remains, therefore, unchanged. The $(h_{1,1} - h_{2,1})$ $SO(26)$ singlet

fields with $U(1)$ charge $-2\sqrt{1/3}$, let us denote them by ϕ_i , are now available to cancel the anomalous D -term because $:\sum_i Q_i |\phi_i|^2 < 0$, as is expected if supersymmetry is not to be broken by the Fayet-Iliopoulos D -term.⁸⁹ However this prevents one from implementing D -term inflation because the scalar potential dependence on the ϕ_i fields arises only through the anomalous D -term. The vacuum expectation values of the fields ϕ_i will rapidly flow to cancel the D -term preventing inflation from occurring.

This example illustrates the problem in implementing D -term inflation in a string theory. It arises because the minimum of the potential does not generically break supersymmetry through the anomalous D -term and so there must be light fields (here the ϕ_i) with the appropriate $U(1)$ charge to cancel it. To implement D -term inflation these fields must acquire a mass for large values of S but this was not possible in this example because the ϕ_i were protected by chirality from acquiring mass by coupling to the S field.

Thus we conclude that it is crucial to consider *all* fields with non-trivial $U(1)$ quantum numbers when discussing the possible inflationary potential in the framework of string theories.

We will consider now further examples to capture other possible aspects of D -term inflation in string theories [95]. For illustrative purposes, we will use the specific string models, discussed in [55, 96] whose space of flat directions was recently analyzed in [57]. The emphasis will be on exploring the different possibilities that may be realized rather than proposing a working model of inflation. In so doing we will often restrict the analysis to some subset of the fields present in the model and ignore the rest. In view of what we concluded above, this is not consistent, but the examples that follow should only be considered as toy models attempting to capture some of the stringy characteristics one should expect when trying to construct a fully realistic model of D -term inflation in string inspired scenarios.

The presence of several (non-anomalous) additional $U(1)$ factors is a generic property of string models. For the discussion of D -term inflation, the relevant objects are thus no longer single elementary fields but rather multiple-field directions in field space along which the D -term potential of the non-anomalous $U(1)$'s vanishes [58]. These directions would be truly flat if an anomalous $U(1)_A$ (or some F -terms) were not present. To study whether a given direction remains flat in the presence of the anomalous $U(1)_A$, the important quantity is the anomalous charge Q_A along the direction. If the sign of this charge is opposite to that of the Fayet-Iliopoulos term, VEVs along the flat direction will adjust themselves to cancel the Fayet-Iliopoulos D -term and give a zero potential. If the charge has the same sign of the Fayet-Iliopoulos D -term, the potential along that direction rises steeply with increasing values of the field. The interesting case corresponds to zero anomalous charge, in which case the potential along the given direction is flat and equals, at tree level, $g_A^2 \xi^2 / 2$. In that case, the direction can be the inflaton.

The condition $Q_A = 0$, ensuring tree-level flatness of the inflaton potential, is not by itself sufficient. We must also require that the direction is stable for large values of the inflaton, that is, all non-inflaton masses deep in the inflaton direction must be positive (or zero). However the Fayet-Iliopoulos D -term in the scalar potential will give a negative

⁸⁹In this section the charges are represented by upper case letters.

contribution to the masses-squared of those fields which have a negative anomalous charge:

$$\delta m_\phi^2 = g_A^2 Q_i^A \xi. \quad (471)$$

To ensure that masses are positive in the end one can use F -term contributions (to balance the negative FI-induced masses) coming from superpotential terms of the generic form

$$\delta W = \lambda I' \Phi_+ \Phi_-, \quad (472)$$

where I' stands for some product of fields that enter the inflaton direction while Φ_\pm do not. Fields of type Φ_+ and Φ_- which couple to the inflaton direction in the superpotential terms get a large F -term mass, $\lambda \langle I' \rangle$.

Consider the simplest example, a toy model with two chiral fields S_1 and S_2 of opposite $U(1)$ charges, so that the direction $|S| = |S_1| = |S_2|$ can play the role of the inflaton. Assume that deep in this direction ($S \gg \sqrt{\xi}$) the masses of all other fields are positive (or zero) and thus no other VEVs are triggered. Then we can minimize the D -term scalar potential⁹⁰

$$V_D = \frac{1}{2} g_A^2 \left[Q_1^A (|S_1|^2 - |S_2|^2) + \sum_i Q_i^A |\phi_i|^2 + \xi \right]^2 + \frac{1}{2} \sum_\alpha g_\alpha^2 \left[Q_1^\alpha (|S_1|^2 - |S_2|^2) + \sum_i Q_i^\alpha |\phi_i|^2 \right]^2, \quad (473)$$

[where $\alpha = 1, \dots, n$ counts the additional D-term contributions of the non-anomalous $U(1)$'s] for S_1 and S_2 only.

If $\xi = 0$, $|S_1| = |S_2|$ is flat and necessarily stable, as $V = 0$. For $\xi > 0$ however, the flat direction is slightly displaced and lies at

$$\delta S^2 \equiv |S_1|^2 - |S_2|^2 = -\frac{g_A^2}{G_{11}^2} Q_1^A \xi, \quad (474)$$

where $G_{ij}^2 = g_A^2 Q_i^A Q_j^A + \sum_\alpha g_\alpha^2 Q_i^\alpha Q_j^\alpha$. This displacement is the result of the destabilization effect of ξ referred to above and occurs when the fields in the inflaton direction carry anomalous charge: as the inflaton direction must have zero anomalous charge, the fields forming it have anomalous charges of opposite signs and one of them will get a negative mass of the form (471).

Taking into account this displacement, the value of the potential along the inflaton direction is, at tree level

$$V_0 = \frac{1}{2} \frac{g_A^2}{G_{11}^2} \xi^2 \sum_\alpha g_\alpha^2 (Q_1^\alpha)^2 \equiv \frac{1}{2} g_A^2 \xi_{\text{eff}}^2 \leq \frac{1}{2} g_A^2 \xi^2. \quad (475)$$

As noted in Section 9.1, ξ_{eff} should be very close to ξ in order not to spoil inflation.

For a viable inflationary model we should ensure that the one-loop potential is appropriate to give a slow roll along the inflaton direction. Thus, we must consider the one-loop corrections proportional to the Yukawa couplings introduced in the terms of eq. (472). The

⁹⁰In writing this potential we are assuming for simplicity that kinetic mixing of different $U(1)$'s is absent. For this to be a consistent assumption the vanishing of $\text{Tr}(Q_A Q_\alpha)$ and $\text{Tr}(Q_\alpha Q_\beta)$ is a necessary condition.

field-dependent masses for the scalar components of the chiral fields Φ_{\pm} along the inflaton direction are

$$\begin{aligned} m_{\pm}^2 &= \lambda^2 \langle I' \rangle^2 + g_A^2 Q_{\pm}^A (Q_1^A \delta S^2 + \xi) + \sum_{\alpha} g_{\alpha}^2 Q_{\pm}^{\alpha} Q_1^{\alpha} \delta S^2 \\ &= \lambda^2 \langle I' \rangle^2 + G_{1\pm}^2 \delta S^2 + g_A^2 Q_{\pm}^A \xi \equiv \lambda^2 \langle I' \rangle^2 + g_A^2 a_{\pm} \xi, \end{aligned} \quad (476)$$

while the fermionic partners have masses-squared equal to $\lambda^2 \langle I' \rangle^2$. For large values of the field $\langle I' \rangle$, the one-loop potential takes the form

$$32\pi^2 \delta V_1 = 2g_A^2 (a_+ + a_-) \lambda^2 \langle I' \rangle^2 \xi \left(\log \frac{\lambda^2 \langle I' \rangle^2}{Q^2} - 1 \right) + g_A^4 (a_+^2 + a_-^2) \xi^2 \log \frac{\lambda^2 \langle I' \rangle^2}{Q^2}. \quad (477)$$

In this more complicated model the scalar direction transverse to the inflaton gains a very large mass deep in the inflaton direction. In addition, the gauge boson corresponding to the broken $U(1)$ symmetry and one neutralino also become massive. These fields arrange themselves in a massive vector supermultiplet, degenerate even if $\xi \neq 0$, and their contribution to the one-loop potential along the inflaton direction cancel exactly. The potential of Eq. (477) can be also rewritten as a RG-improved⁹¹ tree-level potential with gauge couplings evaluated at the scale $\lambda \langle I' \rangle$.

The term quadratic in $\lambda \langle I' \rangle$ would spoil the slow-roll condition necessary for a successful inflation, but it drops out because

$$\begin{aligned} g_A^2 (a_+ + a_-) &= (G_{1+}^2 + G_{1-}^2) \delta S^2 + g_A^2 (Q_+^A + Q_-^A) \xi \\ &= -G_{1I'}^2 \delta S^2 - g_A^2 Q_{I'}^A \xi \propto G_{11}^2 \delta S^2 + g_A^2 Q_1^A \xi = 0, \end{aligned} \quad (478)$$

where we have made use of the $U(1)$ invariance of $I' \Phi_+ \Phi_-$ to write the third expression which vanishes by Eq. (474).

The results just described for the simplest inflaton direction containing more than one field are generalizable to more complicated inflatons. One could have inflatons containing more than two elementary fields while still having only a one-dimensional flat direction. Another possibility is that the flat direction has more than one free VEV (multidimensional inflatons). It is straightforward to verify that the results obtained above for two mirror fields are generic provided the inflaton does not contain some subdirection capable of compensating the Fayet-Iliopoulos D -term.

As the next step in complexity one can examine the case in which, besides the inflaton VEVs $|S_1|$ and $|S_2|$, some other field φ_i is forced to take a VEV (this can be triggered by ξ in the anomalous D -term of the potential or by δS^2 in any D -term). In general, the new VEV can induce further VEVs too. For simplicity, we assume that this chain of destabilizations ends with $\langle \varphi_i \rangle$. By minimizing the D -term potential, all VEVs are determined to be

$$\delta S^2 = |S_1|^2 - |S_2|^2 = -\frac{g_A^2}{\det G^2} (G_{ii}^2 Q_1^A - G_{1i}^2 Q_i^A) \xi \quad (479)$$

$$\langle \varphi_i^2 \rangle = -\frac{g_A^2}{\det G^2} (-G_{1i}^2 Q_1^A + G_{11}^2 Q_i^A) \xi, \quad (480)$$

⁹¹In doing so, a careful treatment of the possibility of kinetic mixing of different $U(1)$'s is required. The details of our analysis are modified in the presence of such mixing but the generic results are not changed.

with $\det G^2 = G_{11}^2 G_{ii}^2 - G_{1i}^4$. The tree level potential along this direction is

$$V_0 = \frac{1}{2} g_A^2 \frac{\xi^2}{\det G^2} \sum_{\alpha, \beta} g_\alpha^2 g_\beta^2 Q_1^\alpha Q_i^\beta (Q_1^\alpha Q_i^\beta - Q_1^\beta Q_i^\alpha) \leq \frac{1}{2} g_A^2 \xi^2. \quad (481)$$

In this background, the masses of the scalar components of Φ_\pm appearing in the superpotential (472) are

$$m_\pm^2 = \lambda^2 \langle I' \rangle^2 + g_A^2 Q_\pm^A \langle D_A \rangle + \sum_\alpha g_\alpha^2 Q_\pm^\alpha \langle D_\alpha \rangle = \lambda^2 \langle I' \rangle^2 + g_A^2 a_\pm \xi, \quad (482)$$

and again, one finds $a_+ + a_- = 0$.

To illustrate the above discussion, consider the following example of a string model [96] that satisfies the conditions required for D -term inflation, at least when we restrict the analysis to a subset of the fields. The $U(1)$ charges of these fields are listed in the table (we follow the notation of ref. [57] with charges rescaled). For every listed field S_i , a "mirror" field \bar{S}_i exists with opposite charges. At trilinear order the superpotential is

$$W = \bar{S}_{11}(S_5 \bar{S}_8 + S_6 \bar{S}_9 + S_7 \bar{S}_{10} + S_{12} S_{13}) + S_{11}(\bar{S}_5 S_8 + \bar{S}_6 S_9 + \bar{S}_7 S_{10} + \bar{S}_{12} \bar{S}_{13}). \quad (483)$$

| Field | Q_A | Q_3 | Q_4 | Q_5 | Q_6 | Q_7 |
|----------|-------|-------|-------|-------|-------|-------|
| S_5 | -1 | 1 | 0 | 0 | -2 | 2 |
| S_6 | -1 | 1 | 0 | 1 | 1 | 2 |
| S_7 | -1 | 1 | 0 | -1 | 1 | 2 |
| S_8 | -1 | -1 | 0 | 0 | -2 | 2 |
| S_9 | -1 | -1 | 0 | 1 | 1 | 2 |
| S_{10} | -1 | -1 | 0 | -1 | 1 | 2 |
| S_{11} | 0 | 2 | 0 | 0 | 0 | 0 |
| S_{12} | 0 | 1 | -3 | 0 | 0 | 0 |
| S_{13} | 0 | 1 | 3 | 0 | 0 | 0 |

List of non-Abelian singlet fields with their charges under the $U(1)$ gauge groups. The charges of these fields under $U(1)_{1,2,8,9}$ are zero and not listed.

The role of the inflaton direction can be played by $\langle S_{11} \bar{S}_{11} \rangle$, formed by fields with zero anomalous charge. However for this to be viable there should be no higher order terms in the superpotential involving just the inflaton directions fields (or terms involving just a single non-inflaton direction field) for these will spoil the F -flatness of the inflaton direction. Given that slow-roll is expected to end at values of the inflaton field not much smaller than M_P , see Eq. (243), only very high dimension terms will be acceptable in the superpotential. $\langle S_{11} \bar{S}_{11} \rangle$ must be invariant under continuous gauge symmetries and so the only symmetry capable of ensuring such F -flatness is a discrete R -symmetry. Unfortunately we do not know whether the models considered have such a discrete R -symmetry and thus they may allow the dangerous terms. Henceforth we will ignore this problem and assume the dangerous terms are absent.

The rest of the fields acquire large positive masses deep in the inflaton direction due to the Yukawa couplings in (483), guaranteeing the stability of the inflaton direction $S =$

$S_{11} = \overline{S}_{11}$. One-loop corrections to the inflaton potential proportional to S^2 are absent and only the $\sim \xi^2 \log S^2$ dependence remains, providing the slow-roll condition. However, the end of inflation poses a problem for the present example: no set of VEVs for the selected fields can give zero potential. As is well known, a flat direction ($V = 0$) is always associated with an holomorphic, gauge invariant monomial built of the chiral fields. To compensate the FI-term and give $V = 0$, this monomial should have negative anomalous charge. However, in the considered subset $Q_A = Q_7/2$ and all holomorphic, gauge invariant monomials must have then $Q_A = 0$. To circumvent this problem we enlarge the field subset by adding an extra field, S_1 with $\overline{Q}(S_1) = (Q_A; Q_\alpha) = (-4; 0, 1, 0, 0, -2)$. It is easy to see that, for example, the flat direction $\langle 1^3, 5, 6, 10, \overline{13} \rangle$ can cancel the FI-term and give $V = 0$. Other flat directions exist, but clearly all of them involve S_1 . However, the superpotential (483) does not provide a large mass for S_1 when we are deep in the flat direction. Unless higher order terms in (483) provide a positive mass for S_1 , the FI-term induces a destabilization of the inflaton direction and S_1 is forced to take a VEV:

$$\langle S_1 \rangle = \frac{-g_A^2}{G_{11}^2} Q_1^A \xi, \quad (484)$$

where we use the definition $G_{ij}^2 = g_A^2 Q_i^A Q_j^A + \sum_\alpha g_\alpha^2 Q_i^\alpha Q_j^\alpha$. This is not a problem in itself because the rest of the fields are forced to have zero VEVs and so the potential cannot relax to zero. The presence of additional $U(1)$ factors prevents the vacuum shift that was problematic for the example of section 4. The value of the potential in the presence of a VEV for S_1 is

$$V = \frac{1}{2} g_A^2 \xi_{eff}^2, \quad (485)$$

with

$$\xi_{eff}^2 = \frac{\sum_\alpha g_\alpha^2 (Q_1^\alpha)^2}{G_{11}^2} \xi^2. \quad (486)$$

The masses of the rest of the fields are also affected and read:

$$m_\phi^2 = \lambda_i^2 \langle I_i^\lambda \rangle^2 + \frac{g_A^2}{G_{11}^2} (Q_i^A G_{11}^2 - Q_1^A G_{i1}) \xi, \quad (487)$$

where λ_i are some of the Yukawa couplings in (483).

In general, when all the fields in the model are included, the presence of the Fayet-Iliopoulos D -term will induce VEVs for the fields with negative anomalous charge which are not forced to have zero VEV by F -term contributions. These non-zero VEVs will in turn induce, through other D -terms, non-zero VEVs for other fields, even if they have positive anomalous charge. Finding all the VEVs requires the minimization of a complicated multifield potential that includes both F and D contributions.

In many cases the field VEVs adjust themselves to give $V = 0$ and no D -term inflation is possible. In other cases however, especially in the presence of additional $U(1)$ factors, there is a limited number of fields that must necessarily take a VEV to cancel the Fayet-Iliopoulos D -term. If the inflaton direction provides a large F -term mass for them, cancellation of the FI-term is prevented. Even if many other fields are forced to take VEVs, no configuration exists giving $V = 0$ and D -term inflation can take place in principle. To determine if that

is the case, one should minimize the effective potential for large values of the inflaton field and determine all the additional vevs triggered by the FI-term. These VEVs, of order ξ will affect the details of the potential along the inflaton direction, both at tree level (offering the possibility of reducing the effective value of ξ) and at one-loop, via their influence on the field-dependent masses of other fields.

9.4 D -term inflation and cosmic strings

Let us go back now to the basic model discussed in Section (9.2). The point we would like to comment on is the following [219]: when the field ϕ_- rolls down to its present day value $\langle\phi_-\rangle = \sqrt{\xi}$ to terminate inflation, cosmic strings may be formed since the anomalous gauge group $U(1)$ is broken to unity [148]. As it is known, stable cosmic strings arise when the manifold \mathcal{M} of degenerate vacua has a non-trivial first homotopy group, $\Pi_1(\mathcal{M}) \neq \mathbf{1}$. The fact that at the end of hybrid inflationary models the formation of cosmic strings may occur was already noticed in Ref. [149] in the context of global supersymmetric theories and in Ref. [210] in the context of supergravity theories.

It has been recently shown [39] that (at least some of) the strings formed at the breaking of the anomalous $U(1)$ are local, in the sense that their energy per unit length can be localized in a finite region surrounding the string's core, even though this energy is formally logarithmically infinite. This happens because the axion field configuration may be made to wind around the strings so that any divergence must come from the region near the core instead of asymptotically. Moreover, as we have seen in the previous Section (9.3) in realistic four-dimensional string models, there are extra local $U(1)$ symmetries that can be also spontaneously broken by the D -term. This happens necessarily if there are no singlet fields charged under the anomalous $U(1)$ only. In such a case, there may arise local cosmic strings associated with extra $U(1)$ factors.

In D -term inflation the string per-unit-length is given by $\mu = 2\pi\xi$. Cosmic strings forming at the end of D -term inflation are very heavy and temperature anisotropies may arise both from the inflationary dynamics and from the presence of cosmic strings. From recent numerical simulations on the cosmic microwave background anisotropies induced by cosmic strings [7, 8, 252] it is possible to infer that this mixed-perturbation scenario [210] leads to the COBE normalized value $\sqrt{\xi} = 4.7 \times 10^{15}$ GeV [148], which is of course smaller than the value obtained in the absence of cosmic strings. Moreover, cosmic strings contribute to the angular spectrum an amount of order of 75% in the simplest version of D -term inflation [148], which might render the angular spectrum, when both cosmic strings and inflation contributions are summed up, too smooth to be in agreement with present day observations [7, 8].

Thus, even though cosmic strings produced at the end of D -term inflation may play a fundamental role in the production of the baryon asymmetry [43], all the previous considerations and, above all, the fact that the value of $\sqrt{\xi}$ is further reduced with respect to the case in which cosmic strings are not present, would appear to exacerbate the problem of reconciling the value of $\sqrt{\xi}$ suggested by COBE with the value inspired by weakly coupled string theory when cosmic strings are present. One has to remember, however, the condition to produce cosmic strings is $\Pi_1(\mathcal{M}) \neq \mathbf{1}$ and therefore consider the structure of the *whole* potential, *i.e.* all the F -terms and all the D -terms. When this is done, it turns out

that, depending on the specific models, some or all of the (global and local) cosmic strings may disappear. In general there can be models with anomalous $U(1)$ that have just global cosmic strings, just local cosmic strings, both global and local strings or, more important, no cosmic strings at all [50, 51]. The latter case is certainly the most preferable case since the presence of cosmic strings renders the problem of reconciling the COBE normalized low value of ξ with the one suggested by string theory even worse.

In the case in which the Fayet-Iliopoulos D -term is present in the theory from the very beginning because of anomaly-free $U(1)$ symmetry and not due to some underlying string theory, the value $\sqrt{\xi} \sim 10^{15}$ GeV is very natural and is not in conflict with the presence of cosmic strings. The only shortcoming seems to be a too smooth angular spectrum because cosmic strings may provide most contribution to the angular spectrum. If this problem is taken seriously and one wants to avoid the presence of cosmic strings, a natural solution to it is to assume that the $U(1)$ gauge group is broken before the onset of inflation so that no cosmic strings will be produced when ϕ_- rolls down to its ground state. This may be easily achieved by introducing a pair of vector-like (under $U(1)$) fields Ψ and $\bar{\Psi}$ and two gauge singlets X and σ with a superpotential of the form

$$W = X \left(\kappa \bar{\Psi} \Psi - M^2 \right) + \beta \sigma \bar{\Psi} \Phi_+ + \lambda S \Phi_+ \Phi_-, \quad (488)$$

where M is some high energy scale, presumably the grand unified scale. It is easy to show that the scalar components of the two-vector superfields acquire vacuum expectation values $\langle \psi \rangle = \langle \bar{\psi} \rangle = M$, and $\langle X \rangle = \langle \sigma \rangle = 0$ which leave supersymmetry unbroken and D -term inflation unaffected. In this example, cosmic strings are produced prior to the onset of inflation and subsequently diluted.

9.5 A GUT model of D -term inflation

A D -term inflationary scenario may be constructed within the framework of concrete supersymmetric Grand Unified Theories (GUT's) where realistic fermion masses are predicted and the doublet-triplet splitting problem is naturally solved by the pseudo-Goldstone boson mechanism in $SU(6)$ [90]. The presence of the D -term is essential in order to generate vacuum expectation values and therefore simplify the structure of the superpotential. As a by-product, the model has a built-in inflationary trajectory in the field space along which all F -terms are vanishing and only the associated $U(1)$ D -term is nonzero. In this case, the COBE-normalized scale $\sqrt{\xi} \sim 10^{16}$ GeV appears more natural to accept since the the GUT scale is of the same order of magnitude, even though it must be put in by hand along with two similar mass scales M and M' .

This model gives a four-component inflaton (Section 4), instead of the usual one-component inflaton. Its predictions depend on the initial conditions as well as on the potential, but for a significant range of initial conditions they will be the same as for the other D -term inflation models. A problem is that the field values while cosmological scales leave the horizon are of order M_P , making it questionable if the field theory is really under control.

The model is based on the $SU(6)$ supersymmetric GUT with one adjoint Higgs Σ and a number of fundamental Higgses $H_A, \bar{H}^A, H'_A, \bar{H}'^A$. Each of these fundamentals transforms

as a doublet of a certain custodial $SU(2)_c$ symmetry that is required to solve the hierarchy problem. The index $A = 1, 2$ is the $SU(2)_c$ -index. The H_A, \bar{H}^A carry unit charges opposite to ξ and are the ones that compensate $U(1)$ D -term in the present Universe. The superpotential reads

$$W = c\text{Tr}\Sigma^3 + (\alpha\Sigma + aX + M)H_A\bar{H}'^A + (\alpha'\Sigma + a'X + M')H'_A\bar{H}^A. \quad (489)$$

Minimizing both the D - and the F -terms we get the following supersymmetric vacuum which leaves the Standard Model $SU(3)_C \otimes SU(2)_L \otimes U(1)$ as unbroken gauge symmetry

$$\begin{aligned} H_{Ai} &= \bar{H}^{Ai} = \delta_{A1}\delta_{i1}\sqrt{\frac{\xi}{2}}, & H'_A &= \bar{H}^{A'} = 0, \\ \Sigma &= \frac{aM' - a'M}{a'\alpha - \alpha'a}\text{diag}(1, 1, 1, -1, -1, -1), & X &= -\frac{\alpha M' - \alpha' M}{a'\alpha - \alpha'a}. \end{aligned} \quad (490)$$

Here $i, k = 1, 2, \dots, 6$ are $SU(6)$ indexes. The role of the Σ vacuum expectation value is crucial since it leaves the unbroken $SU(3)_C \otimes SU(3)_L \otimes U(1)_Y$ symmetry, consequently it can cancel masses of all upper three or lower three components of the fundamentals. The fundamental vevs are $SU(5)$ symmetric, so that the intersection gives the unbroken standard model symmetry group.

In this vacuum the electroweak Higgs doublets from $H_2, \bar{H}^2, H'_2, \bar{H}'^2$ are massless. This is an effect of custodial $SU(2)_c$ symmetry. Indeed, since H_1 and \bar{H}^1 break one of the $SU(3)$ subgroups to $SU(2)_L$, their electroweak doublet components become eaten up Goldstone multiplets and cannot get masses from the superpotential due to the Goldstone theorem. This forces the vevs of Σ and X to exactly cancel their mass terms and those of $H_2, \bar{H}^2, H'_2, \bar{H}'^2$ due to the custodial symmetry. This solves the doublet-triplet splitting problem in a natural way [89].

Quarks and leptons of each generation are placed in a minimal anomaly free set of $SU(6)$ group: 15-plet plus two $\bar{6}_A$ -plets per family. We assume that $\bar{6}_A$ form a doublet under $SU(2)_c$ so that $A = 1, 2$ is identified as $SU(2)_c$ index. The fermion masses are then generated through the couplings ($SU(6)$ and family indices are suppressed) $\bar{H}^A \cdot 15 \cdot \bar{6}_A + \epsilon^{AB} \frac{H_A \cdot H_B}{M_\xi} 15 \cdot 15$, where M_ξ has to be understood as the mass of order $\sqrt{\xi}$ of integrated-out heavy states. When the large vevs of H_1 and \bar{H}^1 are inserted, the additional, vectorlike under $SU(5)$ -subgroup, states: 5-s from 15-s and $\bar{5}$ -s from $\bar{6}_1$, become heavy and decouple. Low energy couplings are just the usual $SU(5)$ -invariant Yukawa interactions of the light doublets from H_2 and \bar{H}^2 with the usual quarks and leptons.

The relevant branch for inflation in the field space is represented by the $SU(6)$ D - and F -flat trajectory parameterized by the invariant $\text{Tr}\Sigma^2$. This corresponds to an arbitrary expectation value along the component

$$\Sigma = \text{diag}(1, 1, 1, -1, -1, -1) \frac{S}{\sqrt{6}}. \quad (491)$$

The key point here is that above component has no self-interaction (*i.e.* $\text{Tr}\Sigma^3 = 0$) and appears in the superpotential linearly. At the generic point of this moduli space the gauge $SU(6)$ symmetry is broken to $SU(3) \otimes SU(3) \otimes U(1)$. All gauge-non singlet Higgs fields

are getting masses $\mathcal{O}(S)$ and therefore, for large values of S , $S \gg \sqrt{\xi}$, they decouple. Part of them gets eaten up by the massive gauge superfields. These are the components of Σ transforming as $(3, \bar{3})$ and $(\bar{3}, 3)$ under the unbroken subgroup. All other Higgs fields get large masses from the superpotential. The massless degrees of freedom along the branch are therefore: two singlets S and X , the massless $SU(3) \otimes SU(3) \otimes U(1)$ super- Yang-Mills multiplet and the massless matter superfields.

By integrating out the heavy superfields, we can write down an effective low energy superpotential by simply using holomorphy and symmetry arguments. This superpotential, as well as all gauge $SU(6)$ D -terms, is vanishing. Were not for the $U(1)$ -gauge symmetry, the branch parameterized by S , would simply correspond to a SUSY preserving flat vacuum direction remaining flat to all orders in perturbation theory. The D -term, however, lifts this flat direction, taking an asymptotically constant value for arbitrarily large S at the tree-level. This is because all Higgs fields with charges opposite to ξ gain large masses and decouple, and ξ can not be compensated any more (notice that heavy fields decouple in pairs with opposite charges and therefore $\text{Tr} \mathbf{Q}$ over the remaining low energy fields is not changed). As a result, the branch of interest is represented by two massless degrees of freedom X and S whose vevs set the mass scale for the heavy particles, and a constant tree level vacuum energy density $V_{\text{tree}} = \frac{g^2}{2} \langle D^2 \rangle = \frac{g^2}{2} \xi^2$ which is responsible for inflation.

This result can be easily rederived by explicit solution of the equations of motion along the inflationary branch. For doing this, we can explicitly minimize all D - and F - terms subject to large values of S and X . The relevant part of the potential is

$$V = |F_{H'_A}|^2 + |F_{\bar{H}'_A}|^2 + \frac{g^2}{2} D^2, \quad (492)$$

since the remaining F - and D - terms are automatically vanishing as long as all other gauge-non singlet Higgses are zero. We would need to include them only if the minima of the potential (492) (subject to $S, X \gg \xi$) were incompatible with such an assumption. However for the branch of our interest this turns out to be not the case.

As with the simpler models that we considered earlier, the negatively charged fields that might drive V_D can acquire positive masses-squared from the F term. These fields come purely from the H, \bar{H}', H', \bar{H} superfields.⁹² These are the fragments $(1, 3), (1, \bar{3})$ and $(3, 1), (\bar{3}, 1)$ of the H, \bar{H}' with masses-squared

$$\phi_+^2 \pm g^2 \xi \quad (493)$$

and

$$\phi_-^2 \pm g^2 \xi, \quad (494)$$

where

$$\phi_{\pm} \equiv \pm \alpha S / \sqrt{6} + aX + M, \quad (495)$$

and the analogous fragments of the H', \bar{H} with masses-squared

$$\phi_+^{\prime 2} \pm g^2 \xi \quad (496)$$

⁹²All other states either have vanishing charge (these are X, Σ and the gauge fields), or have no inflaton dependent mass but positive charge (these are matter fields).

and

$$\phi_{\pm}^{\prime 2} \pm g^2 \xi, \quad (497)$$

where

$$\phi'_{\pm} \equiv \pm \alpha' S / \sqrt{6} + aX' + M', \quad (498)$$

For each of these four cases there are eight pairs of charged fields.

When ϕ_{\pm}^2 and $\phi'_{\pm}{}^2$ are both bigger than $g^2 \xi$, there is inflation. Including the loop correction the potential is

$$V_{\text{inf}} = \frac{g^2}{2} \xi^2 \left[1 + \frac{3g^2}{16\pi^2} \ln \left(|\phi_+|^2 |\phi_-|^2 |\phi'_+|^2 |\phi'_-|^2 \right) \right]. \quad (499)$$

(To obtain this expression, we added the four contributions given by Eq. (461), with $C = 8$ for each of them.) This potential is a function of four real fields, namely the real and imaginary parts of S and X . As discussed in Section 4, there will in general be a family of non-equivalent inflationary trajectories. We are dealing with a four-component inflaton, and the predictions depend in general on the initial conditions. However, for a significant range of initial conditions, the inflaton trajectory after the observable Universe leaves the horizon will be roughly a straight line pointing towards the origin, in the space of the fields. If ϕ is the canonically-normalized field along the trajectory, the inflaton potential is then given by Eq. (461) with $C = 96$ (except for an insignificant change in V_0 coming from the constant ratio of ϕ^8 and the argument of the log).

From the estimate Eq. (243), one sees that in this case, when the observable Universe leaves the horizon, ϕ is at least of order M_{P} and maybe of order $10M_{\text{P}}$. One needs the former case to have any chance of keeping the field theory under control.

Notice that in the usual hybrid inflationary scenarios inflation is terminated by the rolling down of a Higgs field coupled to the inflaton and consequent phase transition with symmetry breaking. Whenever the vacuum manifold has a non-trivial homotopy, the topological defects will form much in the same way as in the conventional thermal phase transition. Thus, the straightforward generalization of the hybrid scenario in the GUT context would result in the post-inflationary formation of the unwanted magnetic monopoles. In the model proposed in [90] this disaster never happens, since the inflaton field is the GUT Higgs itself. The GUT symmetry is broken both during and after inflation and the monopoles (even if present at the early stages) get inevitably inflated away. The unbroken symmetry group along the inflationary branch is $G_{\text{inf}} = SU(3) \otimes SU(3) \otimes U(1) \otimes SU(2) \otimes U(1)$ ⁹³ which gets broken to $G_{\text{postinf}} = SU(3) \otimes SU(2) \otimes U(1) \otimes U(1)$ modulo the electroweak phase transition (extra $U(1)$ -factor is global). Since $\pi_2(G_{\text{inf}}/G_{\text{postinf}}) = 0$ no monopoles are formed⁹⁴.

The model described above demonstrates that D -term inflation may satisfy a sort of "minimal principle" [86] which requires that any successful inflationary scenario should naturally arise from models which are entirely motivated by particle physics considerations and should not involve (usually complicated and *ad hoc*) sectors on top of the existing structures.

⁹³If the gauge $U(1)$ is a stringy anomalous $U(1)$, it will be broken by the dilaton even if all other charged fields vanish. In this case the unbroken symmetry has to be understood as a global one.

⁹⁴Other ways of solving the monopole problem exist in previous papers [286, 188, 223, 210].

10 Conclusion

In the face of increasingly accurate observations of the cosmic microwave background anisotropy and of the galaxy distribution, slow-roll inflation seems to provide the only known origin for structure in the Universe. In this review we have seen how to build models of inflation, and test them against observation.

What is the point of such an exercise? To address this question, one needs to understand what is meant by a model of inflation. One can think of a model as something analogous to a building. It has an outer shell, which is visible to the casual observer, but hopefully also something inside.

The shell is a specification of the form inflationary potential. In a single-field model the potential depends only on the inflaton field, while in a hybrid model it depends on one or more additional fields. Observation, notably through the spectral index of the density perturbation, can discriminate sharply between different shells. Most, and perhaps all, of the present zoo of shells will be rejected by observations in the next ten to fifteen years, culminating with the Planck satellite that will give an essentially complete measurement of the cmb anisotropy. One can imagine that eventually just one basic form for the shell is singled out by the community, which by virtue of its intrinsic beauty and its accurate description of the observations is likely to be the one chosen by Nature. Then, in a sense, there will be a consensus about the origin of all structure in the Universe. One will have arrived at the rather boring conclusion, that it probably comes from a certain scalar field potential!

Things are very different when we come to consider the interior of the shell. Here, one recognizes that the inflationary potential is part of an the extension of the Standard Model, that is supposed to describe the fundamental interactions at the level of field theory. The field theory description is, hopefully, an approximation to some more fundamental theory like weakly coupled string theory or Horava-Witten M-theory. Although different interiors generally have different shells, that is not inevitable as we have seen in more than one example.

At this point, inflation model-building becomes part of the enterprise that has occupied the particle physics community for more than two decades. That is, to find the extension of the Standard Model that has been chosen by Nature.⁹⁵ Because there is so little guidance from observation, this enterprise has been driven by theoretical considerations to an extent that is unprecedented in the history of science. In particular, the rich structure of supersymmetry is almost always assumed because it seems to be the only way of avoiding a certain type of extreme fine-tuning. In the foreseeable future we shall find out whether supersymmetry and other theoretical structures have been chosen by Nature, and therefore whether pure thought has successfully pulled so far ahead of observation. Whether positive or negative, this resolution will surely be a permanent landmark in the history of the human intellect.

⁹⁵This is the usual viewpoint but one can vary it. Maybe there is only one mathematically consistent theory that gives anything resembling physical reality, in which case we have in principle little need of observation. Maybe the usual assumption that there are many possible theories is correct, but many or all of them have been realized by Nature in different parts of the universe, that may or may not be connected with the homogeneous Universe around us. These variations make no difference for the present purpose.

Assuming that current ideas are basically correct, one still has to ask to what extent it will ever be possible to discriminate between different fundamental theories. Observation by itself provides, so far, only a few numbers relevant to this purpose, together with some upper and lower limits. Among them are the parameters of the Standard Model and, if one accepts the increasingly strong evidence, one or two numbers relating to the neutrino masses. There is also strong evidence for non-baryonic dark matter, which probably has to be in the form of one or more as-yet undiscovered particle species. And finally, coming to the concern of this review, there is the magnitude of the spectrum of the primeval density perturbation, measured on the scales explored by COBE.

Among the quantities with crucial upper or lower limits one might mention on the particle physics side the Higgs masses, neutrino masses and mixing angles, the proton lifetime and the electric dipole moment of the neutron. As we have seen, one should add to these the limit on the departure from scale invariance represented by the result $|1 - n| < 0.2$, and the upper limit of order 50% on the relative contribution of gravitational waves to the spectrum of the cmb anisotropy.

These lists are incomplete but they serve to explain the role of inflation. It will add to the precious collection of numbers and limits, that guide us in a search for what lies beyond the Standard Model. Possibly there will even be a non-trivial function, $n(k)$, that requires explanation.

Analogously with the situation concerning the outer shell of a model of inflation, the hope is that the community will eventually be able to agree that some model of the fundamental interactions is likely to be the one that Nature has chosen, by virtue of its intrinsic beauty and accurate agreement with the few numbers provided by observation. Because the numbers are few, this would hardly be possible at the level of a field theory, but it might be possible at the level of something like string theory where there are essentially no free parameters and everything is dictated by group theoretic and topological considerations.

With this perspective, let us look at some of the models of inflation that are presently under consideration.

As we have discussed at length, supersymmetry is both a blessing and a curse for inflation model-building. It is a blessing, primarily because it allows one to understand the existence of scalar fields. As a bonus, it can practically eliminate the quartic term in the inflaton potential, which would normally spoil inflation. It is a curse, because in a generic supergravity theory all scalar fields have masses that are too big to support inflation. Let us recall ways of handling this problem.

According to supergravity, the potential is the sum of an F -term and a D -term. In most models the F -term dominates and we consider them first. With an F -term of generic form, the inflaton mass is too big. One can suppose that it is suppressed by an accidental cancellation, but one can instead invoke a non-generic form, which guarantees the suppression. Such a form can emerge from weakly coupled heterotic string theory, though probably not from Horava-Witten M-theory. Alternatively, one can suppose that while the inflaton mass is indeed unsuppressed at the Planck scale, quantum corrections drive it to a small value at lower scales so as to permit inflation after all. At the present time this ‘running mass’ model looks quite attractive.

A different strategy is to suppose that a Fayet-Illiopoulos D -term dominates, with the charged fields driven to zero. These models have received a lot of attention because at least

in the simplest versions they have two remarkable features. One is that supergravity corrections to the inflaton mass are absent. The other is that there is an accurate prediction for the spectral index, $n = 0.96$ to 0.98 which will eventually be testable. Further investigation, though, has revealed a serious problem. In contrast with the F -term models, the inflaton field value has to be at least of order M_{P} . As a result, one has gained control of the inflaton mass, only to be in danger of losing it for the quartic and higher terms of the potential. In string theory there are two additional problems. One is the existence of fields which are liable to drive the D -term to zero. The other is that the predicted magnitude of the cmb anisotropy is far higher than the COBE measurement. It is fair to say that D -term inflation is under considerable pressure at the moment.

The predictions of different models for the spectral index n , and for its scale-dependence, are summarised in the table on page 80. Remarkably, the eventual accuracy $\Delta n \sim 0.01$ offered by the Planck satellite is just what one might have specified in order to distinguish between various models, or at least between their various shells. At the most extravagant, one might have asked for $\Delta n \sim 10^{-3}$.

In summary, observation will discriminate strongly between models of inflation during the next ten or fifteen years. By the end of that period, there may be a consensus about the form of the inflationary potential, and at a deeper level we may have learned something valuable about the nature of the fundamental interactions beyond the Standard Model. We shall also have confirmed, or practically rejected, the remarkable hypothesis that inflation is responsible for structure in the Universe.

Postscript

At the final proof-reading, observation is beginning to pin down the cosmological parameters, and therefore the spectral index. A preliminary estimate [R. Bond, Pritzker Symposium, <http://www-astro-theory.fnal.gov/Personal/psw/talks/bond/bond.03.gif>.] is $|n-1| < 0.05$. Looking at Table 1, this would rule out a potential of the form $V = V_0(1 - c\phi^3)$, and almost rule out one of the form $V = V_0(1 - c\phi^4)$. The latter case is practically equivalent to the form chosen for the first viable model of inflation, Eq. 186, so that form is almost ruled out as well. For a number of other forms of the potential (Table 2) the preliminary result for n places a non-trivial lower limit on N , the number of e -folds of inflation occurring after cosmological scales leave the horizon. It seems that we are already entering the promised land, the golden age of cosmology!

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