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On the Electroweak Origin of Cosmological Magnetic Fields

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Abstract

Magnetic fields may have been generated in the electroweak phase transition through spontaneous symmetry breaking or through the subsequent dynamical evolution of semiclassical field configurations. Here I demonstrate explicitly how magnetic fields emerge spontaneously in the phase transition also when no gradients of the Higgs field are present. Using a simple model, I show that no magnetic fields are generated, at least initially, from classical two-bubble collisions in a first-order phase transition. An improved gauge-invariant definition of the electromagnetic field is advocated which is more appropriate in the sense that it never allows electrically neutral fields to serve as sources for the electromagnetic field. In particular, semiclassical configurations of the Z field alone do not generate magnetic fields. The possible generation of magnetic fields in the decay of unstable Z-strings is discussed.

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I. INTRODUCTION

It is known that our galaxy and many other spiral galaxies possess a large-scale correlated magnetic field with strength of the order of 10^{-6} Gauss [1]. In each case the direction of the field seems to accord with the rotation axis of the galaxy, which suggests that it was generated by a dynamo mechanism in which a small seed field of the order of 10^{-21} Gauss or larger was amplified by the turbulent motion of matter during galaxy formation [2]. Various cosmological explanations for such a primordial seed field have been suggested [3–11]. This paper focusses on scenarios in which a strong magnetic field of magnitude 10^{20}-10^{23} Gauss was generated during the electroweak phase transition and was thenceforth diluted by the expansion of the universe to values appropriate for a seed field at the time of onset of galaxy formation.

There have been several models proposed in which the strong magnetic field is produced by the turbulence of the conductive plasma during the phase transition [3,4]. In contrast, I shall restrict myself to mechanisms where the magnetic field would be generated directly from the dynamics of the order parameter (the Higgs field) and from the gauge fields in the process of breaking the electroweak symmetry $SU(2)_{\rm L} \times U(1)_Y$ to $U(1)_{\rm EM}$. Such mechanisms include the spontaneous generation of magnetic fields, collisions of bubbles of broken phase in a first-order phase transition, and the formation and dynamics of non-topological defects. In addition, there are scenarios in which magnetic fields are produced by bound pairs of monopoles in standard and extended electroweak models [6], but I shall not consider them here.

Vachaspati [7] has suggested that strong magnetic fields may emerge spontaneously in the phase transition because the covariant derivatives of the Higgs field in causally disconnected regions must be uncorrelated. The electromagnetic current that produces these fields can receive contributions from gradients of the phases of the Higgs field and charged vector-boson currents or both, depending on which gauge is used. Recently the electromagnetic current from the Higgs field was calculated in Ref. [12] and was found always to be zero. For this reason, it was claimed that no coherent magnetic fields are generated by the rolling Higgs field in the electroweak phase transition. I will show below that these statements are incorrect.

A useful tool in the investigation of magnetic phenomena and magnetogenesis is the gauge-invariant definition of the electromagnetic field tensor introduced in Ref. [7]. It has recently been employed by Grasso and Riotto [8] in the study of semiclassical configurations of the Z and W fields. They discovered a set of puzzling paradoxes in which the electrically neutral Z field appears to act as a source for magnetic fields. In particular, it seemed that a magnetic field would always be present along the internal axis of an electroweak Z-string.

These surprising and counter-intuitive results have prompted me to reexamine the gauge-invariant definition of the electromagnetic field tensor. I find that it is indeed not suited to situations where the magnitude of the Higgs field deviates from its vacuum value. I propose a different definition of this tensor which, in addition to resolving the paradoxes, proves to be a potent calculational tool. For example, it follows immediately that no magnetic field is generated initially from the classical dynamics of the Higgs field in a collision between two bubbles in a first-order electroweak phase transition.

The paper is organized as follows. In section II I describe the problems with the conventional gauge-invariant definition of the electromagnetic field tensor and argue why it should be modified. I then present an improved definition and describe its general properties. In section III I show that the apparent vanishing of the contribution to the electromagnetic current from the Higgs field, shown in Ref. [12], is due to a particular choice of gauge in which any Higgs field is electrically neutral. I go on to demonstrate that in an arbitrary gauge one can always construct electrically charged field directions in the Lie algebra and corresponding charged vector-boson fields. Because the current resulting from these fields is in general non-zero, it will give rise to electromagnetic fields.

In section IV I present an alternative description of the spontaneous generation of magnetic fields where the unitary gauge is imposed. In this gauge there are no angular degrees of freedom of the Higgs field. Instead, the magnetic fields arise from $SU(2)$ and $U(1)$ vector potentials that were already present in the symmetric phase in the form of "pure gauges", which do not contribute to the field tensors and are unphysical, redundant degrees of freedom. As the $SU(2)_{\rm L} \times U(1)_Y$ symmetry breaks, the former pure-gauge vector potentials find themselves having random non-vanishing components along new physical directions which are eigenstates of mass and electric charge. This reinterpretation confirms Vachaspati's original proposal that magnetic fields can be generated spontaneously in the electroweak phase transition [7].

In section V, I show that no magnetic field is generated initially from the classical dynamics of the Higgs field in a collision between two bubbles in a first-order electroweak phase transition. This is shown for arbitrary relative difference and orientation of the $U(1)_Y$ and $SU(2)_{\rm L}$ phases of the two bubbles. It follows that at least three bubbles must collide in order that magnetic fields be created by this mechanism.

In section VI the field configurations of the electroweak Z-string [13] and W-string [14] are investigated, using the redefined electromagnetic field tensor. I verify that they carry neither magnetic fields nor electromagnetic currents. In Ref. [8] it was suggested that magnetic fields may be generated in the decay of electroweak strings. In the case of the Z string, the source of the magnetic field would be charged W fields which are initially present in the decay. By constructing the unstable W mode responsible for the decay, I verify explicitly that a magnetic field is indeed generated.

II. GAUGE-INVARIANT DEFINITION OF THE ELECTROMAGNETIC FIELD

The conventional gauge-invariant definition of the electromagnetic field tensor in the $SU(2) \times U(1)$ Yang-Mills-Higgs system is given by [7]

$$F^{\rm em}_{\mu\nu} \equiv -\sin\theta_{\rm w}\,\hat{\phi}^a(x)F^a_{\mu\nu} + \cos\theta_{\rm w}\,F^Y_{\mu\nu} - i\frac{\sin\theta_{\rm w}}{g}\frac{2}{\Phi^\dagger\Phi}\left[(\mathcal{D}_\mu\Phi)^\dagger\,\mathcal{D}_\nu\Phi - (\mathcal{D}_\nu\Phi)^\dagger\,\mathcal{D}_\mu\Phi\right], \tag{1}$$

where

$$\hat{\phi}^a \equiv \frac{\Phi^\dagger\tau^a\Phi}{\Phi^\dagger\Phi}\ , \qquad \mathcal{D}_\mu = \partial_\mu - i\frac{g}{2}\tau^a W^a_\mu - i\frac{g'}{2}Y^a_\mu \equiv \partial_\mu - i\underline{A}_\mu\ .$$

This definition of $F^{\rm em}_{\mu\nu}$ has the attractive property that, in a "unitary" gauge where $\Phi = (0,\rho)^\top$, $\hat{\phi}^a = -\delta^{a3}$, with ρ real and positive, it reduces to the usual expression $A_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu$ where $A_\mu = \sin\theta_{\rm w}\,W^3_\mu + \cos\theta_{\rm w}\,Y_\mu$. This holds true, however, only when the magnitude ρ is a constant. For a general (positive) $\rho = \rho(x)$, it is easy to show that

$$F^{\rm em}_{\mu\nu} = A_{\mu\nu} - 2\tan\theta_{\rm w}\,(Z_\mu\partial_\nu\ln\rho - Z_\nu\partial_\mu\ln\rho) \qquad \text{(unitary gauge)} \tag{2}$$

with $Z_\mu = \cos\theta_{\rm w}\,W^3_\mu - \sin\theta_{\rm w}\,Y_\mu$.

While such a definition certainly is possible, its physical consequences become highly disturbing when one considers the dynamical equation for $F^{\rm em}_{\mu\nu}$ in this gauge, which takes the form [8]

$$\partial^\mu F^{\rm em}_{\mu\nu} = -ie\left[W^{\mu\dagger}(\mathcal{D}_\mu W_\nu - \mathcal{D}_\nu W_\mu) - (\mathcal{D}_\mu W_\nu - \mathcal{D}_\nu W_\mu)^\dagger\,W^\mu\right] - ie\partial^\mu\left(W^\dagger_\mu W_\nu - W^\dagger_\nu W_\mu\right) - 2\tan\theta_{\rm w}\ \partial^\mu\left(Z_\mu\partial_\nu\ln\rho(x) - Z_\nu\partial_\mu\ln\rho(x)\right) \qquad \text{(unitary gauge)}\ . \tag{3}$$

Here $W^\dagger_\mu$ and $W_\mu \equiv (W^1_\mu - iW^2_\mu)/\sqrt{2}$ are the charged vector bosons, and $\mathcal{D}_\mu W_\nu \equiv (\partial_\mu - igW^3_\mu)W_\nu$.

From Eq. (3) one would infer that an electromagnetic field could be generated from currents involving the fields Z_ν and ρ. From most points of view such a result seems absurd since, in the unitary gauge, Z_ν and ρ are electrically neutral. In fact, the charge operator $(\underline{\mathbf{1}}+\tau^3)/2$ annihilates $(0,\rho)^\top$ and commutes with the Z direction in the Lie algebra, $T_Z \propto \cos^2\theta_{\rm w}\,\tau^3 - \sin^2\theta_{\rm w}\,\underline{\mathbf{1}}$. The fields Z_ν and ρ remain neutral also when ρ is coordinate-dependent because the form of the charge operator can depend only on the choice of gauge. The change from $\rho =$ constant to $\rho = \rho(x)$ does not constitute a change of gauge, since no angular degrees of freedom of the Higgs field are involved.

The definition (1) thus implies that electromagnetic fields can be produced by neutral currents. A more reasonable and practical definition should exclude this possibility.

Through a slight modification of a definition given by 't Hooft [15] for the $SO(3)$ Georgi-Glashow model one obtains an improved gauge-invariant definition of the electromagnetic field tensor,

$$\mathcal{F}^{\rm em}_{\mu\nu} \equiv -\sin\theta_{\rm w}\,\hat{\phi}^a(x)F^a_{\mu\nu} + \cos\theta_{\rm w}\,F^Y_{\mu\nu} + \frac{\sin\theta_{\rm w}}{g}\epsilon^{abc}\hat{\phi}^a(D_\mu\hat{\phi})^b(D_\nu\hat{\phi})^c \ , \tag{4}$$

where $(D_\mu\hat{\phi})^a = \partial_\mu\hat{\phi}^a + g\epsilon^{abc}W^b_\mu\hat{\phi}^c$. This definition depends on the Higgs field only through the unit vector $\hat{\phi}^a$ which is independent of the magnitude $\rho = (\Phi^\dagger\Phi)^{1/2}$. Therefore, the problematic terms in eqs. (2) and (3) involving gradients of ρ will not appear in the unitary gauge, where the field tensor now always reduces to the familiar expression $\mathcal{F}^{\rm em}_{\mu\nu} = A_{\mu\nu}$. An intricate interplay between the first and last term in eq. (4) ensures that the electrically charged $SU(2)$ vector fields cancel (in any gauge), leaving only the neutral component $-\sin\theta_{\rm w}\,\hat{\phi}^a(\partial_\mu W^a_\nu - \partial_\nu W^a_\mu)$. The definition (4) has been proposed earlier by Hindmarsh [16], but has not been applied before in the study of magnetic fields in the electroweak phase transition.

Magnetic fields from magnetic monopoles are also accounted for in definition (4). In fact, it can be shown that the Bianchi identity $\epsilon^{\mu\nu\alpha\beta}\partial_\nu\mathcal{F}^{\rm em}_{\alpha\beta} = 0$ is satisfied everywhere except along world lines around which $\hat{\phi}^a$ takes "hedgehog" configurations [15].

Repeating the calculation done in Ref. [8] for the field tensor of eq. (1), one may derive the equation for the dynamical evolution of the redefined field tensor $\mathcal{F}^{\rm em}_{\mu\nu}$ using the equations of motion for $F^a_{\mu\nu}$ and $F^Y_{\mu\nu}$ and a few Fierz identities. One thus obtains

$$\partial^\mu \mathcal{F}^{\rm em}_{\mu\nu} \equiv -\sin\theta_{\rm w} \left\{ (D^\mu \hat{\phi})^a F^a_{\mu\nu} - \frac{1}{g} \partial^\mu \left[\epsilon^{abc} \hat{\phi}^a (D_\mu \hat{\phi})^b (D_\mu \hat{\phi})^c \right] \right\} \ , \tag{5}$$

where the right-hand side is the electric current.

It should be remarked that no physics is affected by using one definition of the electromagnetic field rather than the other. The choice of definition is, however, important for the interpretation and description of physical processes whenever $\Phi^\dagger\Phi$ is not constant. In this paper, I adopt the modified definition (4) which ensures that no electromagnetic field is generated from electrically neutral sources. Even so, one should remember that there is no exact standard by which definition (1) would be incorrect.

In Ref. [8] it was stated that, because of the last term of eq. (3), the formation of a magnetic field is always associated to the appearance of a semiclassical Z-configuration. As is seen from the above arguments, such a statement depends on the definition of the electromagnetic field. In the view of the modified definition, eq. (4), no magnetic field would accompany the neutral-charge configuration.

III. A NON-VANISHING ELECTROMAGNETIC CURRENT

It was originally suggested by Vachaspati [7] that electromagnetic fields may emerge in the electroweak phase transition through the process of spontaneous symmetry breaking. The principal idea is that as the Higgs field magnitude $\rho = (\Phi^\dagger\Phi)^{1/2}$ becomes non-zero in the phase transition, the covariant derivative $\mathcal{D}_\mu\Phi \equiv (\partial_\mu - i\underline{A}_\mu)\Phi$ cannot remain everywhere zero, because that would imply an inexplicable correlation of phases and gauge fields over distances greater than the causal horizon distance at the time of the phase transition.

In the much simpler case of a global $U(1)$ symmetry (i.e. with the gauge potential A_μ set to zero), an instructive analogy can be made with the phase transition in superfluid He^4 [17]. When such a system is rapidly quenched, the Higgs field $\Phi \in \mathbf{C}$ emerges from the false $\Phi \equiv 0$ ground state attempting to find a new true minimum on the circle $|\Phi| = v$ but is forced to assign values for its phase more quickly than the time it takes information to propagate across the container (given by the speed of "second sound"). Gradients of the phase thus appear and, because the fluid velocity is proportional to the gradient of the phase, a flow is generated.

The analogy with the superfluid has sometimes led to the misinterpretation that magnetic fields in the electroweak phase transition are generated only by gradients of the phases of the Higgs field. Recently, it was claimed [12] that the electromagnetic current resulting from Higgs gradients is always zero, and that for this reason no magnetic field would be produced during the phase transition due to spontaneous symmetry breaking. I will show below that these conclusions were contingent upon making a specific choice of gauge for the vacuum, as well as neglecting the electromagnetic currents from charged vector bosons[1]. In general, as I shall show in section IV, magnetic fields emerge spontaneously also when no gradients of the Higgs field are present.

Let us begin by considering the gauge-*co*variant charge operator proposed in Ref. [12],

$$Q = -\frac{1}{2}\hat{\phi}^a \tau^a + \frac{Y}{2} \ , \quad \hat{\phi}^a = \frac{\Phi^\dagger \tau^a \Phi}{\Phi^\dagger \Phi}, \tag{6}$$

where I define the hypercharge Y of the Higgs doublet to be $+1$. This operator has the property that $Q\Phi = 0$. For this reason, the electromagnetic current from the Higgs field, given by

[1]This was also pointed out in Ref. [8].

$$j^\nu = ie[\Phi^\dagger Q \mathcal{D}^\nu \Phi - (\mathcal{D}^\nu \Phi)^\dagger Q \Phi] \tag{7}$$

was said to vanish.

These results should be reinterpreted as follows: Due to gauge freedom, one may represent the Higgs field of the vacuum state in any "coordinate system" of choice through applying a gauge transformation to $(0, v)^\top$. This would not constitute an active, physical change of the state, but merely a change of basis of the Lie algebra and its representations. In the unitary gauge the vacuum state is represented by

$$\Phi_0 = \begin{pmatrix} 0 \\ v \end{pmatrix} \;, \quad \underline{A}_\mu = \partial_\mu \Lambda(x)\, Q \;, \quad \Lambda \in \mathbf{R} \;, \quad Q = Q_0 \equiv \frac{1}{2}(\underline{\mathbf{1}} + \tau^3) \tag{8}$$

where the $U(1)$ "pure-gauge" form of $\underline{A}_\mu$ is the most general expression for which $\mathcal{D}_\mu \Phi_0$ and the field tensors $F^a_{\mu\nu}$ and $F^Y_{\mu\nu}$ vanish. This vacuum state can be equivalently re-expressed as

$$\Phi_0 = \frac{\Phi}{(\Phi^\dagger \Phi)^{\frac{1}{2}}}\, v, \;\; \Phi(x) \text{ arbitrary}, \quad \underline{A}_\mu = \partial_\mu \Lambda(x)\, Q - i(\partial_\mu V) V^\dagger \tag{9}$$

through a gauge transformation $\Phi_0 \to V \Phi_0$ with $V \in SU(2)$ defined by

$$V = \frac{1}{(\Phi^\dagger \Phi)^{\frac{1}{2}}} \begin{pmatrix} (i\tau^2 \Phi)^* & \Phi \end{pmatrix} \;. \tag{10}$$

Under this transformation, $Q_0 \to Q \equiv V Q_0 V^\dagger$. It can be checked that this definition of Q agrees with eq. (6).

We see that when the charge operator Q is defined covariantly as in eq. (6), Φ is always proportional to the vacuum Higgs field Φ_0 with a real factor. Thus, Φ is always electrically neutral. All angular degrees of freedom of the Higgs field that could give rise to charged currents have been transferred into the vector fields. The end result is a reformulation of the unitary gauge in an arbitrary basis.

In the remainder of this section I shall construct the charged vector fields for an arbitrary choice of Φ in eq. (9) and proceed to show that they give rise to an electromagnetic current which in general is non-zero.

The charged vector-boson fields can be found by determining the $SU(2)\times U(1)$ Lie-algebra eigenstates under the adjoint action of the gauge-covariant charge operator Q. After some algebra and using a series of Fierz identities, one can readily verify that

$$[Q,T_\pm]=\pm T_\pm\ ,\quad [Q,T_3]=[Q,\mathbf{1}]=0, \tag{11}$$

$$[T_3,T_\pm]=\pm T_\pm\ ,\quad [T_+,T_-]=2T_3, \tag{12}$$

where

$$T_+\equiv\frac{(-i\Phi^\dagger\tau^2)^\top\Phi^\dagger}{\Phi^\dagger\Phi}\ ,\quad T_-\equiv\frac{\Phi(i\tau^2\Phi)^\top}{\Phi^\dagger\Phi}={T_+}^\dagger\ ,\quad T_3\equiv-\frac{1}{2}\frac{\Phi^\dagger\tau^a\Phi}{\Phi^\dagger\Phi}\tau^a \tag{13}$$

and $Q=T_3+Y/2$. Thus T_+ and T_- are the generators of the Lie algebra corresponding to charged field directions. Using $T_\pm=T_1\pm iT_2$ we can write

$$\underline{A}_\mu=g\tilde{W}^a_\mu T_a+\frac{g'}{2}Y_\mu\mathbf{1}+\partial_\mu\Lambda(x)\,Q-i(\partial_\mu V)V^\dagger\ , \tag{14}$$

where $\tilde{W}^a_\mu=Y_\mu=0$ corresponds to the vacuum, eq. (9). Under an SU(2) gauge transformation $\Phi\to U\Phi$ the generators T_a, $a=1,2,3,(+,-)$, transform according to the adjoint representation $T_a\to UT_aU^\dagger$, and it can be shown that the fields $\tilde{W}^a_\mu$ are gauge invariant. Furthermore, the field tensor components $\tilde{F}^a_{\mu\nu}=2\mathrm{Tr}(T_a\underline{F}_{\mu\nu})/g$ are invariant under general $SU(2)\times U(1)$ gauge transformations.

The important point is that, in general, there will be charged vector-boson fields $\tilde{W}_\mu\equiv(\tilde{W}^1_\mu-i\tilde{W}^2_\mu)/\sqrt{2}$ and $\tilde{W}^\dagger_\mu$ present regardless of what gauge we choose for the vacuum, corresponding to the components of the Lie algebra along T_+ and T_-. I shall now show that these charged fields give rise to an electromagnetic current and therefore magnetic fields. First, let us evaluate the electromagnetic field tensor. Inserting the components of $\underline{A}_\mu$ and $\underline{F}_{\mu\nu}=\partial_\mu\underline{A}_\nu-\partial_\nu\underline{A}_\mu-i[\underline{A}_\mu,\underline{A}_\nu]$ into eq. (4), one finds after rather lengthy calculations that the derivatives $\partial_\mu T_a$ in the first term cancel against the last term, and we retrieve

$$\mathcal{F}^{\rm em}_{\mu\nu} = \sin\theta_{\rm w}\,(\partial_\mu \tilde{W}^{\,3}_\nu - \partial_\nu \tilde{W}^{\,3}_\mu) + \cos\theta_{\rm w}\, F^Y_{\mu\nu}\ . \tag{15}$$

Turning next to the dynamical equation for $\mathcal{F}^{\rm em}_{\mu\nu}$, eq. (5), insertion and yet more algebra produces

$$\begin{aligned}\partial^\mu \mathcal{F}^{\rm em}_{\mu\nu} &= -ie\left[\tilde{W}^{\mu\dagger}\left(\tilde{\mathcal{D}}_\mu \tilde{W}_\nu - \tilde{\mathcal{D}}_\nu \tilde{W}_\mu\right) - \left(\tilde{\mathcal{D}}_\mu \tilde{W}_\nu - \tilde{\mathcal{D}}_\nu \tilde{W}_\mu\right)^\dagger \tilde{W}^\mu\right] \\ &\quad - ie\partial^\mu\left(\tilde{W}^\dagger_\mu \tilde{W}_\nu - \tilde{W}^\dagger_\nu \tilde{W}_\mu\right)\ .\end{aligned} \tag{16}$$

This is exactly the expression (3) obtained in the unitary gauge, but without the objectionable last term, as was discussed in the previous section.

I have thus established that the treatment of Ref. [12] is equivalent to a treatment in the unitary gauge, where the Higgs field possesses no angular degrees of freedom. These degrees of freedom are absorbed into the vector bosons. However, the current from charged vector bosons was omitted in [12]. In general this current, given by eq. (16), is non-zero and will give rise to electromagnetic fields. In the next section we shall see an example of how this can happen.

IV. SPONTANEOUS GENERATION OF ELECTROMAGNETIC FIELDS

Previous descriptions [7,12] of the spontaneous generation of magnetic fields in the electroweak phase transition have borrowed from the analogy with superfluids in that they attribute the magnetic fields to the presence of gradients of phases of the Higgs field. I present here an alternative description of magnetogenesis where the unitary gauge is imposed. In this gauge, there are no angular degrees of freedom of the Higgs field. Instead, the magnetic fields arise from $SU(2)$ and $U(1)$ vector potentials that were already present in the symmetric phase in the form of “pure gauges” which do not contribute to the field tensors and are unphysical, redundant degrees of freedom. As the $SU(2)_{\rm L} \times U(1)_Y$ symmetry breaks, these former pure-gauge vector potentials find themselves having random non-vanishing components along new physical directions which are the eigenstates of mass and electric charge.

When the symmetry breaks, unstable non-topological defects such as W-strings and Z-strings typically form carrying large fluxes of gauge fields. In the core of these defects the Higgs field Φ goes to zero, at which points the unitary gauge is ill-defined. For now, I shall consider a region of space where such defects are absent. Non-topological defects will be considered in more detail in section VI.

In the symmetric phase, the vacuum state of electroweak model is characterized by $\Phi \equiv 0$, $F^a_{\mu\nu} = F^Y_{\mu\nu} = 0$. Surely, in the high-temperature electroweak plasma there will be fluctuations around the vacuum values, but these fluctuations are expected to have a small correlation length of the order $(2\pi T)^{-1}$, and we are primarily interested in a mechanism that may generate magnetic fields correlated on a larger scale. The macroscopic spatial average of $F^a_{\mu\nu}$ and $F^Y_{\mu\nu}$ on such a scale will also vanish, and therefore the Lie-algebra valued gauge potential must be in the general "pure-gauge" form

$$\underline{A}_\mu = -i(\partial_\mu \Omega)\Omega^\dagger \;\equiv -i(\partial_\mu U(x))U(x)^\dagger + \partial_\mu \chi(x)\,\mathbf{1}\;, \tag{17}$$

where $\Omega \in SU(2)\times U(1)$, $U \in SU(2)$ and $\chi \in \mathbf{R}$. The group element Ω lives on the group manifold $S^3 \times S^1$, the direct product of a three-sphere and a circle, and is completely arbitrary. Because the energy is independent of the space dependence of $\Omega(x)$, there is no reason that Ω should be uniform over space.

Let us now consider the process of symmetry breaking, and for simplicity use the unitary gauge in the broken state. The initial $\underline{A}_\mu$ immediately after the phase transition is given by eq. (17), since a discontinuity in time would give rise to infinite "electric" fields F^a_{0i} or F^Y_{0i}. In general, $\underline{A}_\mu$ will not be aligned with the broken-symmetry vacuum, eq. (8). This would happen only in the special case when Ω is restricted to the embedded circle $\Omega = e^{i\Lambda(x)Q}$ as x covers space. For other choices of Ω, for example $\Omega = e^{i\xi(x)\tau^1}$, it is easy to check that there will be physical, electrically charged W-boson fields present immediately after the phase transition. The ensuing state is a coherent semi-classical field configuration which cannot be constructed from the new vacuum by perturbative means.

Let us now look at a concrete example of how the electromagnetic field is generated. The condition

$$0 = F^3_{\mu\nu} \equiv (\partial_\mu W^3_\nu - \partial_\nu W^3_\mu) + g\,(W^1_\mu W^2_\nu - W^1_\nu W^2_\mu) \tag{18}$$

can be satisfied if both the first term and the second term are non-zero but cancel exactly. The first term enters in the unitary-gauge definition of the electromagnetic field tensor in the broken phase:

$$A_{\mu\nu} = \sin\theta_w\,(\partial_\mu W^3_\nu - \partial_\nu W^3_\mu) + \cos\theta_w F^Y_{\mu\nu}\ , \tag{19}$$

where in our case $F^Y_{\mu\nu} = 0$.

The emerging magnetic field can therefore be traced to a "random" partitioning of fields into the two cancelling terms of eq. (18). In the symmetric phase, these terms had no independent physical meaning, and fields could be moved from one to the other through arbitrary gauge transformations while keeping $F^3_{\mu\nu}$ zero. When the symmetry is broken, the terms take on a new physical meaning. The first term in (18) has components along $A_{\mu\nu}$ as well as along $Z_{\mu\nu} = \partial_\mu Z_\nu - \partial_\nu Z_\mu$. The second term in (18) can be written

$$ig(W^\dagger_\mu W_\nu - W^\dagger_\nu W_\mu) \tag{20}$$

in terms of the charged W fields. It is now apparent that there can be no spontaneous generation of magnetic fields in the electroweak phase transition without the simultaneous generation of charged W-boson currents which act as the only source (in the unitary gauge) for that magnetic field. In fact, the field equation for the electromagnetic field in the unitary gauge, when $F^a_{\mu\nu}=0$, is[2]

$$\partial^\mu A_{\mu\nu} = -ie\partial^\mu(W^\dagger_\mu W_\nu - W^\dagger_\nu W_\mu) \ . \tag{21}$$

The term on the right-hand side of this equation is the magnetization current corresponding to the anomalous magnetic dipole moment of the W boson [18–20]. The initial magnetic field can therefore be viewed as being entirely comprised of magnetization of the vacuum due to W bosons. This state has previously been investigated in the context of the QCD vacuum [21].

Let us now see explicitly how the two terms of eq. (18) obtain non-zero values through the process of a random pure gauge described above. Because $F^a_{\mu\nu} = F^Y_{\mu\nu} = 0$ the initial gauge potential must be given by eq. (17). The most general $SU(2)\times U(1)$ group element Ω can be written

$$\Omega(x) = e^{i\Lambda/2}\begin{pmatrix} e^{i(\Lambda/2-\beta)}\cos\omega & -e^{i(\alpha-\Lambda/2)}\sin\omega \\ e^{i(\Lambda/2-\alpha)}\sin\omega & e^{i(\beta-\Lambda/2)}\cos\omega \end{pmatrix} = e^{i\Lambda/2}U \ , \quad U\in SU(2) \ . \tag{22}$$

The $su(2)$ algebra part of the gauge potential is given by $W^a_\nu\tau^a = -(2i/g)(\partial_\nu U)U^\dagger$. Then the resulting magnetic field can be calculated from

$$\partial_{[\mu} W^a_{\nu]}\tau^a = \frac{2i}{g}\left(\partial_{[\mu}U\right)\left(\partial_{\nu]}U^\dagger\right) \ , \tag{23}$$

where $[\mu\ldots\nu]$ indicates antisymmetrization, and from the trace identity $\mathrm{Tr}\,\tau^a\tau^b = 2\delta^{ab}$. The contribution to eq. (19) becomes

$$\begin{aligned}\partial_\mu W^3_\nu - \partial_\nu W^3_\mu &\equiv -g(W^1_\mu W^2_\nu - W^1_\nu W^2_\mu) \\ &= \frac{2}{g}\sin 2\omega\left(\omega_{[,\mu}\alpha_{,\nu]} + \omega_{[,\mu}\beta_{,\nu]} - \omega_{[,\mu}\Lambda_{,\nu]}\right) \ ,\end{aligned} \tag{24}$$

[2]It should be noted that when the two terms in equation (18) are non-zero, a state with $F^a_{\mu\nu}=0$ does not remain an exact solution in the broken phase because of the mass terms that appear there.

where a comma denotes partial differentiation. Here we see explicitly how the two terms of eq. (18) obtain non-zero values that vary as one changes the pure gauge Ω while keeping $F^3_{\mu\nu} = 0$. Thus, in this description it is a random pure gauge in the symmetric phase that gives rise to a magnetic field in the broken phase. In this sense, the magnetic field was already present in the symmetric phase, but took on a physical meaning only after the symmetry was broken.

By means of a gauge transformation with group element Ω^{-1} we may set the full gauge potential $\underline{A}_\mu$ of eq. (17) to zero. The phases will then reappear in the Higgs field, which becomes

$$\Phi = \rho \begin{pmatrix} e^{i\alpha} \sin\omega \\ e^{-i\beta} \cos\omega \end{pmatrix} . \tag{25}$$

The phase Λ does not appear here because the broken vacuum still has the electromagnetic $U(1)$ symmetry.

Therefore, as long as $F^a_{\mu\nu}$ can be considered to vanish, one can give two *equivalent* descriptions of magnetogenesis in two different gauges. (1) In a gauge where all vector potentials are identically zero, the magnetic field arises spontaneously from the angular degrees of freedom of the Higgs field. (2) In the unitary gauge, with $\Phi = (0, \rho)^\top$, the initial magnetic field is the result of $SU(2)\times U(1)$ pure-gauge vector-potential remnants of the symmetric phase whose associated field tensor finds itself with a non-zero projection along the electromagnetic field after symmetry breaking.

There are several reasons to prefer the second gauge. First the purely esthetical ones: In the first gauge, it seems a bit silly that there should be an electromagnetic field when all vector potentials are identically zero. This field is given by the last term of eq. (4). Moreover, there is no simple global definition of electric charge in this gauge, while in the second gauge the constant operator $Q = (\underline{\mathbf{1}} + \tau^3)/2$ defines simple charge eigenstates for all fields.

More importantly, the equivalence holds only as long as $F^a_{\mu\nu} = 0$. As soon as the symmetry breaks, mass terms appear for the charged W bosons and for the Z field, and the fields will start evolving into states with non-zero $F^a_{\mu\nu}$. Vector-field degrees of freedom of this type can no longer be transferred into the Higgs field by a gauge transformation. Even in the simplest case of a $U(1)$ symmetry, only the longitudinal degree of freedom of the vector field can be exchanged with a phase of the Higgs field, while the transverse degrees of freedom are unaffected by gauge transformations. The vector fields thus contain more dynamical degrees of freedom than does the Higgs field. Therefore, treating the issue of generation of magnetic fields in the unitary gauge from the point of view of the vector-boson fields is more appropriate.

It is an interesting problem to investigate the subsequent evolution of the initial vector fields. It has been shown that magnetic fields generated in a second-order phase transition are stable to thermal fluctuations in the period immediately following the transition [22]. Presumably, the mass terms in the Hamiltonian will cause W and Z fields to taper off faster than the massless electromagnetic field. In this context one should also take into account the effects of the highly conductive plasma [23]. In a perfect conductor the magnetic flux lines are known to be frozen into the plasma [24], while a finite conductivity σ leads to the suppression of magnetic fields on length scales smaller than $\sim (4\pi\sigma H)^{-1/2}$ [25] with H being the Hubble constant. Because the early universe is a good conductor [2,10,25,26], large-scale correlated magnetic fields may be preserved until late times. In order for this scenario to work, magnetohydrodynamic inverse-cascade mechanisms [9] are needed [27] to transfer magnetic energy from small to large scales. As the temperature decreases, the currents are transferred by particle decays and will be carried increasingly by less massive charged particles for which the Boltzmann factor remains unsuppressed; first quarks and then, below the quark-gluon transition, electrons.

V. NO MAGNETIC FIELDS FROM BUBBLE COLLISIONS

Let us now consider the possibility of forming magnetic fields in the collisions of two bubbles of broken vacuum in a first-order electroweak phase transition. Such collisions were investigated in Refs. [28,8] for some special cases. Using the same model as those references, I shall show here that no magnetic field is generated for arbitrary relative difference and orientation of the $U(1)_Y$ and $SU(2)_{\rm L}$ phases of the two bubbles.

For initial times, the Higgs field configurations of two bubbles whose centers are separated by a vector $\boldsymbol{b}$ are, respectively,

$$\Phi^1_i(\boldsymbol{x}) = \begin{pmatrix} 0 \\ \rho_1(\boldsymbol{x}) \end{pmatrix} \quad \text{and} \quad \Phi^2_i(\boldsymbol{x}) = \exp\left[i\frac{\theta_0}{2}n^a\tau^a + i\varphi_0\right] \begin{pmatrix} 0 \\ \rho_1(\boldsymbol{x}-\boldsymbol{b}) \end{pmatrix} , \tag{26}$$

where $\hat{n} = (n^1, n^2, n^3)$ is a constant unit vector. The $U(1)$ and $SU(2)$ phases and orientations within each bubble have equilibrated to constant values. Because $n^a\tau^a$ is the only $SU(2)$ Lie-algebra direction involved, one may write the initial complete Higgs field as [28]

$$\Phi_i(\boldsymbol{x}) = \exp\left[i\frac{\theta(\boldsymbol{x})}{2}n^a\tau^a + i\varphi(\boldsymbol{x})\right] \begin{pmatrix} 0 \\ \rho(\boldsymbol{x}) \end{pmatrix} , \tag{27}$$

Furthermore, the authors of [28,8] have assumed that all gauge potentials and their derivatives vanish initially. As we learned in the preceding section, one is free to choose such a gauge when the field tensors $F^a_{\mu\nu}$ and $F^Y_{\mu\nu}$ also vanish. Such an ansatz is self-consistent because the sources in the equations for the field tensors are at least linear in the gauge potentials.

Proceeding as the references, we assume that the above expressions are valid until the two bubbles collide. One may easily evaluate $\hat{\phi}^a$ which may be written as $\hat{\phi} = \cos\theta\,\hat{\phi}_0 + \sin\theta\,\hat{n}\times\hat{\phi}_0 + 2\sin^2\frac{\theta}{2}\,(\hat{n}\cdot\hat{\phi}_0)\hat{n}$ where $\hat{\phi}_0 = (0,0,-1)^\top$. Then $\partial_\mu\hat{\phi}^a$ takes the particularly simple form $\partial_\mu \hat{\phi} = \partial_\mu \theta \, \hat{n} \times \hat{\phi}$. The last term of the electromagnetic field $\mathcal{F}^{\rm em}_{\mu\nu}$ (eq. (4)) thus vanishes, and since $F^a_{\mu\nu}$ and $F^Y_{\mu\nu}$ are zero, the electromagnetic field vanishes. Similarly, the electromagnetic current (5) vanishes.

It is instructive to check this result by transforming the Higgs field into the unitary gauge, using the group element $\Omega = e^{-i\varphi}U$, $U = \exp[-in^a\tau^a\theta/2]$. This gives rise to pure-gauge vector fields of the form (17). It follows easily from eq. (23) or (24) that their contribution to the magnetic field is zero. From the latter of these equations it is apparent that the phases of our Ω are rather special, and that there in general would be a magnetic field. The absence of a magnetic field can be traced directly to the fact that the unit vector $\hat{n}$ is a constant. In Ref. [28] it was proven that the Higgs field in any two-bubble collision can be written in the form (27) for constant n^a. We thus conclude that no magnetic field is generated from the classical evolution of the Higgs field in an electroweak two-bubble collision. The simplest case where n^a cannot be set to a constant occurs for a three-bubble collision.

It should be noted that the present result is not in conflict with previous results for the abelian $U(1)$ model [29]. The $U(1)$ vector field in that model is massive and the corresponding field strength is generated as a result of the coupling of the $U(1)$ field to the Higgs field. In contrast, the electromagnetic $U(1)$ field in the broken electroweak theory is distinguished as that direction of the Lie algebra that does *not* couple to the Higgs field.

The generation of magnetic fields in two-bubble collisions is not excluded if one relaxes the assumption that the gauge potentials are zero initially. Magnetic fields may then emerge spontaneously within each bubble by the mechanism described in the previous section. The above treatment is also probably too simplistic to describe what takes place after the bubbles have collided. When the presence of the plasma is taken into account, other processes may lead to the creation of magnetic fields. In particular, magnetic fields may stem from the motion of dipole charge layers that develop on bubble walls because of the baryon asymmetry [4]. It is also possible that bubble collisions give rise to field configurations which indirectly produce magnetic fields. This will be investigated as part of the following section.

VI. MAGNETIC FIELDS FROM NON-TOPOLOGICAL DEFECTS

It was recently suggested by Grasso and Riotto [8] that magnetic fields may arise from non-topological defects formed in the electroweak phase transition, such as Z-strings [13] and W-strings [14]. These are string-like embedded vortex solutions of the electroweak theory characterized by the winding of a phase of the Higgs field around a core where the Higgs field goes to zero. The core encloses a flux quantum of one of the gauge-field components which attains considerable field strength, since the characteristic width is given by the inverse vector-boson mass. In a $U(1)$ model, these defects are topologically stable, but in the electroweak theory the phase can unwind by slipping over the simply connected vacuum manifold, and the defect decays to the vacuum.

Saffin and Copeland [28] have shown that W-string and Z-string configurations may be generated during bubble collisions in the $SU(2)_{\mathrm{L}} \times U(1)_Y$ theory. In terms of the notation of the previous section, this occurs in the two cases when the unit vector $\hat{n}$ is perpendicular or parallel to $\hat{\phi}_0$, respectively. In these cases, the effective symmetry group of the problem reduces to $U(1)$, for which vortex production in bubble collisions has been studied earlier [29]. In simulations the strings form as circular loops along the circle of intersection of the two bubbles, with the axis of the loop coinciding with the line through the two bubble centers.

There are three important questions that need be answered in connection with the possible generation of magnetic fields from non-topological defects.

1. Do the defects themselves carry magnetic fields?
2. Do the defects contain electrically charged fields which could produce electromagnetic currents?
3. Are electromagnetic fields generated when these unstable defects decay?

I shall defer the last question to the end of this section and begin instead to address the first two questions. For a reasonable set of definitions, and in the absence of magnetic monopoles, they should be equivalent.

In defiance of such expectations, some surprising results were recently obtained in Ref. [8]. The results seemed to indicate that a magnetic field would always be present along the internal axis of a Z-string, which is known to contain only neutral fields. This interpretation was based on the conventional gauge-invariant definition of the electromagnetic field tensor, eq. (1), which led to the inclusion of the last term of eqs. (2), (3) in the unitary gauge.

As we have learned in section II, there exist alternative definitions of the electromagnetic field tensor which coincide only when the magnitude of the Higgs field is constant. I have argued that the definitions of the field tensor and electric current given in eqs. (4) and (5) are more appropriate, in that $\mathcal{F}^{\rm em}_{\mu\nu}$ always reduces to $A_{\mu\nu}$ in the unitary gauge and electrically neutral fields never serve as sources for the electromagnetic field. Indeed, with the new definitions everything becomes perfectly consistent with naive expectations. In order to illustrate this, let us investigate the field configurations for the Z- and W-strings in some detail. They can be written in the form

$$\underline{A}^Z_\varphi = \frac{mv(r)}{r}\begin{pmatrix}\cos 2\theta_{\rm w} & 0 \\ 0 & -1\end{pmatrix} \quad , \quad \Phi^Z = \rho(r)\begin{pmatrix}0 \\ e^{im\varphi}\end{pmatrix} \tag{28}$$

and

$$\underline{A}_{\varphi}^{W} = \frac{m\tilde{v}(r)}{r} \begin{pmatrix} 0 & e^{i\delta} \\ e^{-i\delta} & 0 \end{pmatrix} \ , \quad \Phi^{W} = \tilde{\rho}(r) \begin{pmatrix} ie^{i\delta} \sin m\varphi \\ \cos m\varphi \end{pmatrix} \ , \tag{29}$$

where r, φ are cylindrical coordinates, δ is an arbitrary real number labeling a family of gauge-equivalent W vortex solutions, and m is the integer winding number. Because of its particular phase singularity at $r = 0$, there is no non-singular expression for the W vortex in a unitary gauge [30].

For the Z-string configuration, we obtain $\hat{\phi}^a = -\delta^{a3}$, and thus the last term of eq. (4) vanishes. The first two terms combine to give $\sin\theta_{\rm w}\,\partial_{[\mu}W^3_{\nu]} + \cos\theta_{\rm w}\,F^Y_{\mu\nu} = 0$ and so $\mathcal{F}^{\rm em}_{\mu\nu}$ vanishes. With the electromagnetic current, eq. (5), we find that $(D^\mu\hat{\phi})^3 = F^1_{\mu\nu} = F^2_{\mu\nu} = 0$, and the last term is just a derivative of the term we previously found to be zero, so there is no electromagnetic current.

Next, let us investigate the W-string solution. It is convenient to recognize that it is of the form $\underline{A}_\varphi = mn^a\tau^a\tilde{v}(r)/r$ and $\Phi = \exp[im\varphi n^a\tau^a](0, \tilde{\rho}(r))^\top$ for $\hat{n} = (\cos\delta, -\sin\delta, 0)$. Using the method of the previous section, we find $\hat{\phi} = \cos(2m\varphi)\,\hat{\phi}_0 + \sin(2m\varphi)\,\hat{n}\times\hat{\phi}_0 + 2\sin^2(m\varphi)\,(\hat{n}\cdot\hat{\phi}_0)\hat{n}$ where $\hat{\phi}_0 = (0, 0, -1)^\top$. The only non-zero field-tensor components are $F^a_{r\varphi} = [m\tilde{v}'(r)/r]n^a$. Because $n^a\hat{\phi}^a \equiv n^a\hat{\phi}^a_0 = 0$, we have that the term $\hat{\phi}^a F^a_{r\varphi} = 0$ in eq. (4) vanishes. In the last term of this equation, one of the factors is $\partial\hat{\phi}^b/\partial r = 0$. Thus $\mathcal{F}^{\rm em}_{r\varphi}$ vanishes.

The issue of whether there is an electromagnetic current is more interesting in the case of the W-string, since its gauge fields involve charged fields W^1_φ and W^2_φ. On the other hand, also the phases of the Higgs field are charged, as compared with the unitary-gauge vacuum. We find the last term of the current (5) to be zero as before. Since $\partial\hat{\phi}^a/\partial r = 0$ and there are no radial components of $\underline{A}_\varphi$, only the r-component of the current may be non-vanishing. We now make use of the relation $\partial_\varphi\hat{\phi} = 2m\hat{n}\times\hat{\phi}$ and can write $(D_\varphi\hat{\phi})^a = 2m[(1+v(r))/r]\hat{n}\times\hat{\phi}$. This is perpendicular to $\hat{n}$, and so the term

$(D_\varphi \hat{\phi})^a F^a_{\varphi r}$ vanishes, and there is no electromagnetic current.

Although this section has so far only confirmed what was expected, it has served as a nice illustration of the properties and applicability of the new definition of the electromagnetic field tensor $\mathcal{F}^{\rm em}_{\mu\nu}$. We have established that it works and that it gives results that are reasonable in cases where the conventional definition appears to lead to absurdities.

Finally, I shall discuss the suggestion made in Ref. [8] that magnetic fields may be generated in the decay of Z-strings. It is well-known that the unstable Z-string decays initially through charged W-boson fields [31, 30] . The idea is that these W fields form a "condensate" which then in turn would act as a source of magnetic fields. One extremely important caveat is that the presence of W fields is highly transient, as the Z-string is known to decay to a vacuum configuration [32]. It is conceivable, however, that the large conductivity of the plasma may cause the magnetic field lines to freeze into the fluid so that it remains preserved at later times.

The instability of the Z-string is a result of the occurence in the energy of a term

$$ig \cos\theta_{\rm w}\, Z_{12}(W_1^\dagger W_2 - W_2^\dagger W_1) \tag{30}$$

which couples the field strength $Z_{12} = \partial_1 Z_2 - \partial_2 Z_1$ with the magnetic dipole moment of the W boson. The energy is lowered through a suitable alignment of this magnetic moment, corresponding to $W_1 = -iW_2 \equiv W$. The instability is greatest at the center of the vortex, where Z_{12} is largest and where the W mass term is reduced by the vanishing of the Higgs field. Let us make the simplified assumption that Z_{12} is approximately uniform in the core of the vortex. This is actually justified if the Higgs-boson mass is considerably larger than the Z-boson mass. In such a case, the unstable modes of the W field are well known [18, 19]. The mode that peaks in the center of the vortex is given by

$$W(r) = W(0) \exp(-\frac{1}{4} g C r^2) \ , \tag{31}$$

where $C = \cos\theta_{\rm w}\, Z_{12}$. For this mode, it is easy to check that $F^1_{ij} = F^2_{ij} = 0$. This is in fact true for any unstable mode [18, 19]. Neglecting back reactions on the Higgs field, we still have $\hat{\phi}^a = -\delta^{a3}$. The last term of eq. (4) evaluates to $2e|W|^2$ which cancels against parts of the first term, leaving $\mathcal{F}^{\rm em}_{ij} = A_{ij}$ as usual. In the current eq. (5) something more interesting happens. Since $(D_i\hat{\phi})^3 = 0$, we are left only with the last term, and the equation for the magnetic field can be written

$$\partial_i \left(\mathcal{F}^{\rm em}_{12} - 2e|W|^2\right) = 0 \; . \tag{32}$$

The (non-uniform) magnetic field $\mathcal{F}^{\rm em}_{12}$ is thus entirely comprised of the magnetization from the W bosons. It is apparent that the W bosons initially present in the decay of the Z-string do indeed generate a magnetic field.

VII. CONCLUSIONS

The main results of this paper are as follows: I have established that magnetic fields are indeed generated classically from Higgs and gauge fields in the electroweak phase transition through the mere process of spontaneous symmetry breaking, as was originally suggested by Vachaspati [7]. Reformulating the problem in the unitary gauge, I have explicitly constructed the magnetic field thus generated. Previous claims that no such magnetic field is produced were based on an investigation in which charged Higgs currents were excluded by the choice of gauge, and currents from charged vector bosons were neglected.

Moreover, I have shown that no magnetic field results initially from the classical evolution of the Higgs field in a collision of two bubbles in a first-order electroweak phase transition. This was shown for arbitrary relative difference and orientation of the $U(1)_Y$ and $SU(2)_{\rm L}$ phases. The reason is that only one constant direction in the Lie algebra is involved. At least three bubbles must collide in order that magnetic fields be created by this mechanism.

Furthermore, I have pointed out that the notion of an electromagnetic field tensor is ambiguous whenever the magnitude of the Higgs field is not constant. With the conventional gauge-invariant definition, eq. (1), electrically neutral currents may give rise to electromagnetic fields. In particular, magnetic fields may be present inside electrically neutral configurations such as the Z-string. In order to remedy this, I have proposed a different gauge-invariant definition of the electromagnetic field, eq. (4), which ensures that no electromagnetic fields are generated from neutral sources and which coincides with the other definition for constant Higgs-field magnitude.

The issue of the definition of the electromagnetic field tensor is important for the interpretation and description of physical phenomena, but should have no bearing on the physics, as the various fields evolve independently of how we interpret them. One particular example concerns the simultaneous collision of multiple similar-sized bubbles at the time of percolation, after which the Higgs magnitude is expected to fluctuate violently [33]. In the presence of Z fields one would then conclude from eq. (3), which follows from definition (1), that electromagnetic fields are created from the gradients of this magnitude. In such a context it is important to realize that no unequivocal statement can be made about the presence of electromagnetic fields unless the evolution of all fields is traced to a later time when the Higgs magnitude has assumed a uniform value. Nevertheless, if one makes the reasonable assumption that the Higgs field relaxes to a uniform value without exciting any new dynamics in the angular degrees of freedom, the new definition (4) has the property that it predicts the same magnetic field during the fluctuating stage as it does after the fluctuations of the Higgs magnitude have subsided.

Finally, I have verified that a magnetic field is produced in the initial decay of the Z string, as was suggested in Ref. [8]. Although such a field is transient in the pure

Yang-Mills-Higgs model, it is conceivable that it may survive until later times due to the high conductivity of the plasma in the early universe.

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