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On the Nature of the Magnetic Fields Generated During the Electroweak Phase Transition

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Abstract

In this Letter we reanalyse the question of the origin of magnetic fields during the electroweak phase transition. We show that their formation is intimately connected to some semiclassical configurations of the gauge fields, such as electroweak Z -strings and W -condensates. We describe the formation of these semiclassical configurations during a first order phase transition and argue that they might be generated also in the case of a second order phase transition. We suggest that the instability of electroweak strings does not imply the disappearance of the embedded magnetic field.

I. INTRODUCTION

An essential feature of phase transitions taking place in the early Universe is the breaking of translational invariance. In first order phase transitions translational invariance is broken by the nucleation of bubbles, while in second order phase transitions by the formation of domains where the order parameter is correlated. Although some nontrivial remnant of the breaking of translational invariance may survive, analogously to the case of a ferromagnet below the Curie temperature, this is not generally the case in quantum field theories where a uniform value of the order parameter is energetically preferred. One remarkable exception is represented by topologically stable defects whose interiors do not feel the symmetry breaking and are formed by the Kibble mechanism [1]. Typical examples are local strings or domain walls. However, the topology of the vacuum manifold in the electroweak model does not allow the presence of topologically stable defects and one could wonder whether any trace of the structure present during the electroweak phase transition (EWPT) may remain imprinted and eventually be detectable today. It has been suggested by Vachaspati that the answer to this fundamental question might be positive [2]. He suggested that strong magnetic fields may be produced during the electroweak phase transition as a consequence of nonvanishing spatial gradients of the classical value of the Higgs field. These gradients arise due to the finite correlation length of the two-point correlation function just below the critical temperature. Once magnetic fields are generated they can be imprinted in the highly conductive medium and eventually survive. Recently, Vachaspati's suggestion has been questioned [3]. The electric current due the dynamics of the Higgs field has been computed and showed to be vanishing during the EWPT. It was concluded that long range coherent magnetic fields are not generated by the classical rolling of the Higgs vacuum expectation value during the electroweak phase transition. In ref. [3], however, the contribution to the electric current coming from the dynamics of the gauge fields was not considered since it was assumed that the classical value of these fields was vanishing. We think that such an assumption is not motivated. Indeed, we will show that classical currents of the gauge fields, and hence electromagnetic fields, are generally produced during the EWPT.

It is the purpose of this Letter to reanalyse and possibly clarify the question of

the origin of magnetic fields during the EWPT. We will argue that magnetic fields are indeed formed during the EWPT and that their origin may be interpreted to arise from the appearance of some semiclassical configurations of the gauge fields, such as electroweak Z -strings and W -condensates. The seeds of these configurations are the nonvanishing covariant derivatives of the Higgs field present during the phase transition.

We will describe the formation of semiclassical gauge configurations during a first order EWPT by bubble collisions. This was already analysed by Copeland and Saffin in [4], but here we extend their findings with particular attention to the formation of the magnetic fields. In those cases in which electroweak strings are formed the equilibration of the Higgs phases proceed in some analogy to the $U(1)$ abelian toy model studied in [5, 6] and magnetic fields may be formed as a consequence of such a process.

By making use of some similarities of the electroweak model with the superfluid ^3He system, we will argue that electroweak strings are also expected to be formed if the EWPT is of the second order. Although electroweak strings are unstable, their decay does not imply the disappearance of the embedded magnetic field. This effect may increase the chances for the magnetic field to survive thermal fluctuations.

II. THE IMPORTANCE OF GAUGE FIELD CONFIGURATIONS.

For sake of clarity, we briefly repeat Vachaspati's argument for the generation of magnetic fields during the EWPT [2]. The basic object to analyse is the generalised electromagnetic field tensor given by

$$F_{\mu\nu}^{\text{em}} \equiv -\sin\theta_W \hat{\phi}^a(x) F_{\mu\nu}^a + \cos\theta_W F_{\mu\nu}^Y - i \frac{\sin\theta_W}{g} \frac{4}{\Phi^\dagger\Phi} \left[(D_\mu\Phi)^\dagger D_\nu\Phi - D_\mu\Phi (D_\nu\Phi)^\dagger \right], \quad (1)$$

where

$$\hat{\phi}^a \equiv \frac{\Phi^\dagger \tau^a \Phi}{\Phi^\dagger\Phi}. \quad (2)$$

The definition (1) is inspired by the analogous t'Hooft definition given for the Georgi-Glashow model [7]. The remarkable feature is that it is explicitly gauge invariant and reduces to the standard definition in the presence of a uniform Higgs background. Vachaspati observed that, even if the gauge fields vanish, the second term

in (1) may remain nonzero due to the nonvanishing gradient in the classical value of the Higgs field during the EWPT phase transition. Of course, one can always make a gauge transformation to render the gradients in the Higgs phase vanishing. However, this operation will induce nonvanishing gauge fields and expression (1) remains unchanged. Some ambiguity is already present at this level though. Indeed, as we transfer all the informations about the Higgs gradients into the gauge fields, it is not clear whether and eventually how the electromagnetic fields spring from the dynamics of the the gauge fields and, furthermore, if the dynamics of the Higgs field can be completely decoupled. Some further inspection of the gauge field dynamics is certainly necessary to answer these crucial questions. Using the equations of motions of the field strength tensors $F_{\mu\nu}^a$ and $F_{\mu\nu}^Y$, it is easy to show that

$$\begin{aligned} \partial^\mu F_{\mu\nu}^{\text{em}} = & -\sin\theta_W \left\{ D^\mu \hat{\phi}^a F_{\mu\nu}^a \right. \\ & \left. + \frac{i}{g} \partial^\mu \left[\frac{4}{\Phi^\dagger \Phi} \left((D_\mu \Phi)^\dagger D_\nu \Phi - D_\mu \Phi (D_\nu \Phi)^\dagger \right) \right] \right\} , \end{aligned} \quad (3)$$

where $D_\nu \hat{\phi}^a = \partial_\nu \hat{\phi}^a + g\epsilon^{abc} W_\nu^b \hat{\phi}^c$. A useful exercise in order to clarify the physical nature of the several contributions to the electric current sustaining the magnetic fields is to imagine a region of space where the electroweak symmetry is broken everywhere. Because of gauge invariance, we can fix the unitary gauge for the Higgs field. This implies the reduction $\hat{\phi}^a = -\delta^{a3}$ and amounts to transfer all the physical informations from the Higgs field phases into the gauge fields. In this gauge Eq. (3) reads

$$\begin{aligned} \partial^\mu F_{\mu\nu}^{\text{em}} = & +ie \left[W^{\mu\dagger} (D_\nu W_\mu) - W^\mu (D_\nu W_\mu)^\dagger \right] \\ & - ie \left[W^{\mu\dagger} (D_\mu W_\nu) - W^\mu (D_\mu W_\nu)^\dagger \right] \\ & - ie \partial^\mu \left(W_\mu^\dagger W_\nu - W_\mu W_\nu^\dagger \right) \\ & + 2 \tan\theta_W \partial^\mu \left(Z_\mu \partial_\nu \ln \rho(x) - Z_\nu \partial_\mu \ln \rho(x) \right) . \end{aligned} \quad (4)$$

Here ρ indicates the modulus of the Higgs field. The first two terms on the right-hand side of this equation are the W convective terms and the third term is called the spin term being related to the W anomalous magnetic moment [8]. It is known that these terms can induce an anti-screening of the external magnetic field [8]. These terms are of course classically vanishing in the absence of a W -condensate. As we will show in more details below, the last term in (4) is also related to some possible

semiclassical configurations for the Z -field. Hence, as far as only the gauge sector of the electroweak theory is considered, expression (4) tells us that the currents sustaining classical electromagnetic fields have to reside in nontrivial semiclassical configurations of the gauge fields.

A gauge invariant electric current was computed in [3] and showed to vanish. Clearly, such a result is compatible with (3,4) only in the case semiclassical configurations of the gauge fields are absent. However, as we are going to show, this is generally not the case during the EWPT.

III. BUBBLE COLLISIONS IN $SU(2)$.

To warm up, let us first focus on the case in which the collisions involve two different bubbles carrying a different phase in the pure $SU(2)$ case. In practice we are fixing $\theta_W = 0$ in the Weinberg and Salam model. Here the situation is quite peculiar since no "electromagnetic" field is generated in spite of the presence of gradients in the Higgs field. The Higgs phase is assumed to be uniform across any single bubble. Following ref.[4], we may write the initial Higgs field configuration as a superposition of the two independent bubbles separated by a space distance \mathbf{b}

$$\Phi_{\text{in}}(x) = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ \rho(\mathbf{x}) \end{pmatrix} + \frac{1}{\sqrt{2}} \exp\left(-i\frac{\theta}{2}n^a\tau^a\right) \begin{pmatrix} 0 \\ \rho(\mathbf{x} - \mathbf{b}) \end{pmatrix}. \quad (5)$$

Eq. (5) certainly provides a good description of the real physical situation when the two domains are well separated and the mutual interaction may be neglected. We assume that it holds until the two bubbles collide and fix conventionally $t = 0$ when the collision takes place. The configuration (5) can be recasted in the general form $\Phi_{\text{in}}(\mathbf{x}) = \frac{1}{\sqrt{2}} \exp\left(-i\frac{\tilde{\theta}(\mathbf{x})}{2}n^a\tau^a\right) (0, \tilde{\rho})^T$ [4], where the entire spatial dependence of the phase has been factorized into a new phase $\tilde{\theta}(\mathbf{x})$ (we will omit the tilde from now on). We assume that the gauge fields strength vanish before bubble collision. We also impose that the initial gauge fields W_μ^a and their derivatives are zero at $t = 0$. This condition is of course gauge dependent and should be interpreted as a gauge choice.

In vectorial form $\hat{\phi}^a$ may be written as $\hat{\phi} = \cos\theta \hat{\phi}_0 + \sin\theta \hat{n} \times \hat{\phi}_0 + 2\sin^2\frac{\theta}{2} (\hat{n} \cdot \hat{\phi}_0) \hat{n}$, where $\hat{\phi}_0^T \equiv -(0, 0, 1)$. Note that we are now working in the adjoint representation for the Higgs field. It is straightforward to verify that in the

unitary gauge, $\theta = 0$, $\hat{\phi}$ reduces to $\hat{\phi}_0$. Since the versor \hat{n} associated to the $SU(2)$ gauge rotation does not depend on the space coordinates, we have the freedom to choose \hat{n} to be everywhere perpendicular to $\hat{\phi}_0$. In such a case $\hat{\phi}$ can be always obtained by rotating the unit vector by an angle θ in the plane identified by \hat{n} and $\hat{\phi}_0$. Formally, $\hat{\Phi} = \cos\theta\hat{\phi}_0 + \sin\theta\hat{n} \times \hat{\phi}_0$, which clearly describes a simple $U(1)$ transformation. This already suggests that such particular choice of the relative orientation of \hat{n} and $\hat{\phi}$ the dynamics of the system is determined by an effective $U(1)$ and not by the entire $SU(2)$ gauge group. However, in order to verify this property more properly, we need to investigate the dynamics of the gauge fields. The latter is described by the equation of motion $D^\mu F_{\mu\nu}^a = g|\rho|^2 \epsilon^{abc} D_\nu \hat{\phi}^b \hat{\phi}^c$ which, at $t = 0$, reads

$$\partial^\mu F_{\mu\nu}^a = -g|\rho|^2 \partial_\nu \theta(x) (n^a - n^c \hat{\phi}^a \hat{\phi}^c) . \quad (6)$$

If we now impose the condition $\hat{n} \perp \hat{\phi}_0$, it is straightforward to verify that $\hat{n} \perp \hat{\phi}$. As a result, Eq. (6) reduces to

$$\partial^\mu F_{\mu\nu}^a = -g|\rho|^2 \partial_\nu \theta(x) n^a . \quad (7)$$

As anticipated, only the gauge field component along the direction \hat{n} , namely $A_\mu = n^a W_\mu^a$, does possess some initial dynamics in virtue of the presence of a nonvanishing gradient of the phase between the two domains. In other words, the only field strength possessing some dynamics is the one associated to the $U(1)$ gauge field A_μ . The interaction of critical bubbles during a first order phase transition in a pure $SU(2)$ theory may be effectively described by a simple $U(1)$ gauge group. As noted in ref.[4], this case is of particular interest as it may give rise to the formation of W closed strings during bubble collisions.

To better address the issue of the formation of the "electromagnetic" fields, we make use of t'Hooft definition of the electromagnetic field for a pure $SU(2)$ gauge group $F_{\mu\nu}^{\text{em}} \equiv -\hat{\phi}^a F_{\mu\nu}^a + \frac{1}{g} \hat{\phi}^a D_\mu \hat{\phi}^b D_\nu \hat{\phi}^c \epsilon^{abc}$ [7]. Since we are not considering the full electroweak gauge group structure, it is understood here that $F_{\mu\nu}^{\text{em}}$ is not the conventional electromagnetic field strength. After some algebra one can verify that the condition $\hat{n} \perp \hat{\phi}$ implies $F_{\mu\nu}^a$ identically vanish (more technical details will be given in [9]). Hence, in the absence of stable topological defects such as monopoles, no electromagnetic fields are produced during bubble collision even if the Higgs field

has a nonvanishing gradients. In other words, there are no currents to sustain the "electromagnetic" field. This shows as the presence of nonvanishing gradients in the Higgs field is not a sufficient condition for the generation of electromagnetic fields to take place.

IV. BUBBLE COLLISIONS IN THE ELECTROWEAK THEORY

We now generalise the previous discussion to the gauge group $SU(2)_L \otimes U(1)_Y$ of the electroweak theory. We have to introduce an extra generator, the hypercharge with the relative phase φ . The generalisation of the form (5) is straightforward. The gauge field equations at $t = 0$ are given by

$$\begin{aligned}\partial^\mu F_{\mu\nu}^a &= -\frac{g}{2}\rho^2(x) \left[-n^a \partial_\nu \theta + \hat{\phi}^a \partial_\nu \varphi\right], \\ \partial^\mu F_{\mu\nu}^Y &= -\frac{g'}{2}\rho^2(x) \left[-n^a \hat{\phi}^a \partial_\nu \theta + 2\partial_\nu \varphi\right].\end{aligned}\tag{8}$$

Due to the presence of an extra generator with respect to the pure $SU(2)$ case, the reduction to a simple $U(1)$ is no longer possible, but in some special cases. Different orientations of the versor \hat{n} with respect to $\hat{\phi}_0$ correspond to different physical situations, but in general both W - and Z -configurations are expected to form. Let us briefly address two extreme cases. If \hat{n} is orthogonal to $\hat{\phi}_0$, this implies $\hat{n} \perp \hat{\phi}$. As a consequence, at $t = 0$ we have on the right-hand side of Eq. (8) the sum of two different terms that are perpendicular to each other. This means that at least two independent generators will be involved in the dynamics of the $SU(2)$ gauge fields. Thus, in general we cannot reduce ourselves to an effective $U(1)$. Such a reduction would be possible only imposing the additional assumption $\partial_\nu \varphi = 0$. In such a case the hypercharge field does not evolve and dynamics of the system reduces to that of a pure $SU(2)$. Under such conditions W -strings may be formed [4]. However, even if symmetry may be locally restored, we have shown that no electromagnetic fields are produced in this case.

The case in which \hat{n} is parallel to $\hat{\phi}_0$ is much more interesting. In such a case $\hat{\phi} = \hat{\phi}_0$. We obtain

$$\begin{aligned}\partial^\mu F_{\mu\nu}^3 &= \frac{g}{2}\rho^2(x) (\partial_\nu \theta + \partial_\nu \varphi), \\ \partial^\mu F_{\mu\nu}^Y &= -\frac{g'}{2}\rho^2(x) (\partial_\nu \theta + \partial_\nu \varphi).\end{aligned}\tag{9}$$

No initial evolution for the $a = 1, 2$ components of $F_{\mu\nu}^a$ is present. The equation of motion for the Z -field strength $F_{\mu\nu}^Z = \cos\theta_W F_{\mu\nu}^3 - \sin\theta_W F_{\mu\nu}^Y$ reads

$$\partial^\mu F_{\mu\nu}^Z = \frac{\sqrt{g^2 + g'^2}}{2} \rho^2(x) (\partial_\nu\theta + \partial_\nu\varphi). \quad (10)$$

This equation tells us that a gradient in the phases of the Higgs field gives rise to a nontrivial dynamics of the Z -field with an effective gauge coupling constant $\sqrt{g^2 + g'^2}$. Notice that this takes place even if $\partial_\mu\rho = 0$. Thus, in agreement with [4], we have an effective reduction of the full $SU(2) \otimes U_Y(1)$ gauge structure to an abelian $U(1)$ group, at least at initial time. The equilibration of the phase $(\theta + \varphi)$ can be now treated in analogy to the $U(1)$ toy model studied by Kibble and Vilenkin [5], the role of the $U(1)$ "electromagnetic" field being now played by the Z -field. Fixing an axial gauge for this field, with the z -axis chosen along the line joining the bubble centres, it is easy to show that the only nonvanishing components of $F_{\mu\nu}^Z$ are a longitudinal Z -electric field and a "ring-like" azimuthal Z -magnetic field. The related Z field winds in planes normal to the ring internal axis. An important difference with respect to [5] is that one does not need to require the radial part of the Higgs field to be spatially uniform and constant in time. Indeed, numerical simulations clearly indicate that ρ has a nontrivial evolution during bubble collisions [6, 4]. This is crucial not only for the violation of the geodesic rules, but also for magnetic field generation. Let us take for simplicity $\partial_\mu\theta = 0$. The complete set of equations of motion we may write at finite, though small, times is

$$\begin{aligned} \partial^\mu F_{\mu\nu}^Z &= \frac{g}{2 \cos\theta_W} \rho^2(x) \left(\partial_\nu\varphi + \frac{g}{2 \cos\theta_W} Z_\nu \right), \\ d^\mu d_\mu \left(\rho(x) e^{i\frac{\varphi}{2}} \right) + 2\lambda \left(\rho^2(x) - \frac{1}{2}\eta^2 \right) \rho(x) e^{i\frac{\varphi}{2}} &= 0, \end{aligned} \quad (11)$$

where $d_\mu = \partial_\mu + i\frac{g}{2 \cos\theta_W} Z_\mu$, η is the vacuum expectation value of Φ and λ is the quartic coupling. Note that, in analogy with [5], a gauge invariant phase difference can be introduced by making use of the covariant derivative d_μ . Equations (11) are the Nielsen-Olesen equations of motion [10, 11]. Their solution describes a Z -vortex where $\rho = 0$ at its core. The reader should keep in mind that, as follows from our previous considerations, the geometry of the problem implies that the vortex is closed, forming a ring which axis coincide with the conjunction of bubble centres. What is crucial is that the formation of the magnetic field is always associated to

the appearance of a semiclassical Z -configuration. Indeed, $\partial^\mu F_{\mu\nu}^{\text{em}}$ does not vanish: even rotating away the phase φ

$$\partial^\mu F_{\mu\nu}^{\text{em}} = 2 \tan \theta_W \partial^\mu \left(\tilde{Z}_\mu \partial_\nu \ln \rho(x) - \tilde{Z}_\nu \partial_\mu \ln \rho(x) \right) \quad (12)$$

where now \tilde{Z}_μ is Z -field in the new gauge. What is important is that \tilde{Z}_μ has a nontrivial dynamics. A ring-like magnetic field is formed along the internal axis of the vortex. It is interesting to observe that if closed Z -vortices break into finite segments, *e.g.* due to thermal fluctuations or subsequent bubble collisions, a magnetic flux will emanate from the segment's extremities which will behave as a pair of magnetic monopoles. This effect was already suggested in [12].

Magnetic fields were ignored in [4]. However, their role is crucial for the late evolution of the Z -vortices and the surviving of the $U(1)$ reduction. In fact, as the magnetic field induce a back-reaction on the charged gauge fields, it is clear that the formation of the magnetic field in the core of the Z -string spoil the reduction of the $SU(2) \otimes U(1)_Y$ group to an effective $U(1)$. Together with the restoration of the electroweak symmetry in the core of the string, the magnetic field induces the decay of the Z string into a W -condensate [13]. While electroweak symmetry restoration in the core of the string reduces m_W , the magnetic field via its coupling to the anomalous magnetic moment of the W -field, causes, for $eB > m_W^2$, the formation of a condensate of the W -fields. The presence of a W -condensate gives rise to an electric current which can sustain magnetic fields even after the Z string has disappeared. This may have relevant consequences on the subsequent evolution of magnetic fields and we leave this investigation for future work [9].

It is important to notice that, in the most general case, \hat{n} is neither parallel nor perpendicular to $\hat{\phi}_0$ and we expect the formation of nontrivial W - and Z -configurations [9]. In such a case, one should retain the non-abelian nature of the electroweak theory and no reduction to a simple $U(1)$ abelian group is expected to hold.

We can now wonder what is the strength of the magnetic fields at the end of the EWPT. A partial answer to this question has been recently given in ref. [14] where the formation of ring-like magnetic fields in collisions of bubbles of broken phase in an abelian Higgs model were inspected. Under the assumption that magnetic fields are generated by a process that resembles the Kibble and Vilenkin [5] mechanism, it

was concluded that a magnetic field of the order of $B \simeq 2 \times 10^{20}$ G with a coherence length of about 10^2 GeV^{-1} may be originated. Assuming turbulent enhancement of the field by inverse cascade, a root-mean-square value of the magnetic field $B_{\text{rms}} \simeq 10^{-21}$ G on a comoving scale of 10 Mpc should be present today [14]. Although our previous considerations give some partial support to the scenario advocated in [14] we have to stress, however, that only in some restricted cases it is possible to reduce the dynamics of the system to the dynamics of a simple $U(1)$ abelian group. Furthermore, once Z -vortices are formed the non-abelian nature of the electroweak theory shows due to the back-reaction of the magnetic field on the charged gauge bosons and it is not evident that the same numerical values obtained in [14] will be obtained in the case of the EWPT. This and other issues, *e.g.* how likely is it to form loops, what distribution should we expect and on what length scale, will be addressed in a separate publication [9].

V. MAGNETIC FIELDS FROM A SECOND ORDER TRANSITION.

Let us now briefly address the formation of electromagnetic fields in the case case in which the EWPT is second order. As we argued, electromagnetic fields which are not merely thermal fluctuations can only be formed in the presence of semiclassical gauge field configurations. If the EWPT transition is of the second order, domains where the Higgs field is physically correlated appear near the critical temperature. Although these correlated domains have properties quite different from the bubbles formed during a first order transition, it is however plausible that gauge field configurations can be formed during a second order transition too. The formation of vortices is a common phenomenon in second order phase transitions. In particular, ${}^3\text{He}$ to ${}^3\text{He-A}$ and ${}^3\text{He}$ to ${}^3\text{He-B}$ second order phase transitions are known to give rise to the formation of topological and non-topological vortices via the Kibble mechanism. It is known that non-topological vortices in these systems share many common aspects with the electroweak strings [15]. The use of condensed matter physics experiments to investigate the non-perturbative aspects of particle physics and the formation of defects in the early-Universe is a very modern and active research line (see [16] for a review). We adopt the same point of view to argue that electroweak strings are actually formed during the EWPT if this is second order.

In order to estimate the density of vortices, hence the mean magnetic field, we need to determine the typical size of domains. A very reasonable estimate of the typical minimum size of the domains in the vicinity of the critical temperature is given by the correlation length of the Higgs field computed at the temperature at which thermal equilibrium between the false-vacuum (which is now the symmetric phase $\phi = 0$) and the true-vacuum is no longer attained. In other words, we are interested in the temperature at which thermal fluctuations of the Higgs field inside a given domain of broken symmetry are no longer able to restore the symmetry. This is basically the Ginzburg criterion to determine what is generally called the Ginzburg temperature T_G . A very *rough* estimate of T_G may be obtained just equating the thermal energy $\sim T$ with the energy contained in a domain of size ℓ of broken phase, $E_\ell \simeq \lambda \langle \phi(T) \rangle^4 \ell^3$, where ℓ is typically taken to be the correlation length $\xi(T)$ and λ is the quartic coupling in the Higgs potential. Here, however, we need a more precise determination of the Ginzburg temperature and, in this respect, we will follow the criterion suggested in ref. [17]. Let us imagine that a domain of broken symmetry has been formed in the vicinity of the critical temperature and that the value of the Higgs field inside is of order of $\langle \phi(T) \rangle$. We may model a thermal fluctuation which restores the symmetry inside the domain (*i.e.* the symmetry is unbroken, or $\langle \phi(T) \rangle = 0$, in a sub-region of the domain) by a sub-critical bubble having the following configuration

$$\phi_{\text{ub}}(r) = \langle \phi(T) \rangle \left(1 - e^{-r^2/\ell_{\text{ub}}^2} \right), \quad (13)$$

where ℓ_{ub} is the correlation length in the symmetric phase. The rate per unit volume and unit time of nucleating such a sub-critical bubble of symmetric phase inside a domain of broken phase (with size equal to the correlation length in the broken phase) may be estimated to be

$$\Gamma_{\text{ub}} = \frac{1}{\ell_{\text{b}}^4} e^{-S_3^{\text{ub}}/T}, \quad (14)$$

where ℓ_{b} is the correlation length in the broken phase. S_3^{ub} is the high temperature limit of the Euclidean action computed in correspondence of the configuration given in Eq. (13) and is a complicated function of the parameters present in the Higgs effective potential. A complete expression for S_3^{ub} may be found in ref. [18] and we do not give it here. We would like only to notice that, at fixed T , S_3^{ub}/T increases as

λ increases (and the phase transition becomes very weak first order or second order), rendering the thermal fluctuations less and less efficient as it might be conjectured by making use of the Ginzburg criterion outlined above.

Thermal fluctuations of the unbroken phase inside a domain of broken phase freeze out and cease to be nucleated when the rate Γ_{ub} becomes smaller than H^4 , H being the Hubble expansion rate of the Universe. This happens when $S_3^{\text{ub}}/T \simeq \ln(M_{P\ell}^4/T_c^4) \simeq 160$. We have numerically computed the temperature T_G at which thermal fluctuations freeze out for different values of the parameter λ (or equivalently for different values of the physical Higgs boson mass M_H) and checked that nucleation of regions of unbroken phase inside a domain of broken phase stops at temperatures very close to the critical temperature, $T_G = T_c$ within a few percents. The corresponding size of the domain of broken phase is determined by the correlation length in the broken phase at T_G

$$\frac{1}{\ell(T_G)_b^2} = V''(\langle\phi(T_G)\rangle, T_G) \quad (15)$$

and is weakly dependent on M_H , $\ell_b(T_G) \simeq 11/T_G$ for $M_H = 100$ GeV and $\ell_b(T_G) \simeq 10/T_G$ for $M_H = 200$ GeV.

Using this result and eq. (1), we may estimate a magnetic field of order of $B \sim 4e^{-1} \sin^2 \theta_W \ell_b^2(T_G) \sim 10^{22}$ G, on a correlation length $\ell_b(T_G)$. Notice that this value is about two orders of magnitude smaller than the one suggested by Vachaspati in his original paper [2], the reason being that the correlation scale adopted there was about one order of magnitude larger the one obtained here by detailed balance arguments. The computation of the root-mean-square value of the magnetic field on scales larger than $\ell_b(T_G)$ would require an estimate of the probability of formation of the Z -strings. This and other issues like the stability, the strength and the spatial distribution of the magnetic fields at the end of the EWPT are currently under investigation [9].

We conclude that it is plausible that magnetic fields are produced during the EWPT as a consequence of a nontrivial dynamics of the gauge fields. As classical magnetic fields are odd both under C and CP , it is noticeable that this process give rise to spontaneous breaking of both these symmetries.

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