Fermi National Accelerator Laboratory

Maximizing Spin Correlations in
Top Quark Pair Production at the Tevatron

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Abstract

A comparison is made between the off-diagonal and helicity spin bases for top quark pair production at the FNAL Tevatron. In the off-diagonal basis, 92% of the top quark pairs are in the spin configuration up-down plus down-up, whereas in the helicity basis only 70% are left-right plus right-left. The off-diagonal basis maximizes the spin asymmetry and hence the measured angular correlations between the decay products, which are more than twice as big in this basis as compared to the helicity basis. In addition, for the process $q\bar{q} \rightarrow t\bar{t}$, we give a very simple analytic expression for the matrix element squared which includes *all* spin correlations between the production and subsequent decay of the top quarks.

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Since the discovery [1] of the top quark in 1995, a number of authors [2, 3, 4] have revisited the question of spin correlations in top quark pair production at the FNAL Tevatron. (References to the extensive literature of earlier works can be found in these recent papers.) With its high mass of about 175 GeV, the top quark decays before it hadronizes [5]. Thus, the decay products of a top quark produced in a definite spin state will have characteristic angular correlations. The reason for these new studies is the realization that the number of like-spin and unlike-spin top quark pairs can be made significantly different by an appropriate choice of spin basis. Both Mahlon and Parke [2] and Stelzer and Willenbrock [3] discussed the spin asymmetry using the helicity basis for the Tevatron. However, the top quarks produced at the Tevatron are not ultra-relativistic; therefore, the helicity basis is probably not the optimal basis for these studies at this machine. Consequently, Ref. [2] went on to consider the beamline basis, which is more suitable for spin studies near threshold, and found larger effects at the Tevatron in this basis. The question remained, however: what is the optimal spin basis for these correlation studies? We now have an answer to this question.

Recently Parke and Shadmi [6] studied the spin correlations for the process $e^+e^- \to t\bar{t}$ and showed that you can choose a spin basis in which the like spin components, up-up (UU) and down-down (DD), identically vanish. They have called this spin basis the off-diagonal basis. In a footnote these authors briefly applied this result to $q\bar{q} \to t\bar{t}$, which is the dominant top quark production process at the Tevatron. The purpose of this letter is to study the $q\bar{q} \to t\bar{t}$ process in more detail, comparing the off-diagonal basis to the more traditional helicity basis. Although the type of analysis described in Ref. [6] can also be applied to the gg initial state, the dominant $t\bar{t}$ production mechanism at the LHC, we find that no significant improvements over the helicity basis are possible at the LHC. At the Tevatron[1] however, the process $q\bar{q} \to t\bar{t}$ dominates, accounting for approximately 88% of

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[1]Throughout this paper, we take the center of mass energy for the $p\bar{p}$ collisions to be 2.0 TeV. We

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the total $t\bar{t}$ cross section. Thus, we will concentrate our analytic discussions on that process. Our numerical results, however, include both the $q\bar{q}$ and gg initial states.

In this letter, we present the production density matrix for $q\bar{q} \to t\bar{t}$ in the off-diagonal basis and contrast it with the known result for the helicity basis. Then we obtain a surprisingly simple and compact expression for the production and decay of a pair of top quarks from a quark-antiquark initial state which includes *all* of the correlations among the particles. We also show that the interference terms for the off-diagonal basis are substantially smaller than for the helicity basis at the Tevatron. Finally, we apply this basis to spin correlation studies at the Tevatron, where we find that in the off-diagonal basis, 92% of the $t\bar{t}$ pairs produced at the Tevatron have unlike spins. This represents a significant improvement over the helicity basis, in which only 70% of the $t\bar{t}$ pairs have unlike helicities. As a result, the correlations obtained in the off-diagonal basis are more than twice as big as those in the helicity basis.

In Fig. 1 the spatial part of the top quark momenta $(t, \bar{t})$ and spin vectors $(s, \bar{s})$ are given for the process $q\bar{q} \to t\bar{t}$ in the zero momentum frame (ZMF) of the incoming quarks. What was shown in Ref. [6] is that the spin projections of the top-antitop pair in $q\bar{q} \to t\bar{t}$ are purely up-down and down-up if the spin vectors make an angle ψ with respect to the beam axis. This angle is given by

$$\tan\psi = \frac{\beta^2 s_{qt} c_{qt}}{1 - \beta^2 s_{qt}^2}, \tag{1}$$

where β is the speed of the top quarks in the ZMF.[2] Throughout this paper we use the notation s_{ij} and c_{ij} to denote the sine and cosine of the angle between the momenta of particles i and j in the ZMF: hence, c_{qt} is the cosine of the top quark scattering angle (often denoted by $\cos\theta^*$). Note that near threshold the spin vectors are aligned along the beam

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use a top quark mass of 175 GeV, W boson mass of 80 GeV, and employ the MRS(R1) structure functions [7] evaluated at the scale $Q^2 = m_W^2$.

[2]The angle ψ is related to the angle α introduced by Tsai [8] for $e^+e^- \to \tau^+\tau^-$.

direction and that at very high energies they are aligned along the direction of the top and antitop momenta.

For the off-diagonal basis, the production density matrix averaged over the color and spin of the incoming quarks is given by[3]

$$\sum \mathcal{M}_{\lambda\bar{\lambda}}\mathcal{M}^*_{\lambda'\bar{\lambda}'} = \frac{g_s^4}{9}\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 2-\beta^2 s_{qt}^2 & \beta^2 s_{qt}^2 & 0 \\ 0 & \beta^2 s_{qt}^2 & 2-\beta^2 s_{qt}^2 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}. \tag{2}$$

The matrix element for the production of a t quark with spin λ and a $\bar{t}$ quark with spin $\bar{\lambda}$ is $\mathcal{M}_{\lambda\bar{\lambda}}$, and g_s is the strong coupling constant. The ordering of the columns and rows in (2) is (UU,UD,DU,DD). This production density matrix is very simple because in the off-diagonal basis the amplitudes with like spins, UU and DD, vanish identically. Also note that the non-diagonal terms have an explicit factor of β^2.

In contrast, the same production density matrix in terms of the helicity basis reads [9]

$$\sum \mathcal{M}_{\lambda\bar{\lambda}}\mathcal{M}^*_{\lambda'\bar{\lambda}'} = \frac{g_s^4}{9}\begin{bmatrix} s_{qt}^2/\gamma^2 & -s_{qt}c_{qt}/\gamma & s_{qt}c_{qt}/\gamma & s_{qt}^2/\gamma^2 \\ -s_{qt}c_{qt}/\gamma & 2-s_{qt}^2 & s_{qt}^2 & -s_{qt}c_{qt}/\gamma \\ s_{qt}c_{qt}/\gamma & s_{qt}^2 & 2-s_{qt}^2 & s_{qt}c_{qt}/\gamma \\ s_{qt}^2/\gamma^2 & -s_{qt}c_{qt}/\gamma & s_{qt}c_{qt}/\gamma & s_{qt}^2/\gamma^2 \end{bmatrix}, \tag{3}$$

where $\gamma = (1-\beta^2)^{-1/2}$ is the usual Lorentz boost factor. The columns and rows of this matrix have been ordered (RR,RL,LR,LL), with obvious notations for right and left helicities.

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[3]The phases of the non-diagonal terms of the production density matrix are dependent on many of our conventions, which we have chosen to make the phases as simple as possible. We take the 31 plane to be the scattering plane with the top quark direction the 1 axis. We use the conventions of [2] with the spinor products singular along the minus 2 direction and $\langle k+|q-\rangle \equiv \left\{[q_3(k_0+k_2) - k_3(q_0+q_2)] + i[k_1(q_0+q_2) - q_1(k_0+k_2)]\right\}\left\{(k_0+k_2)(q_0+q_2)\right\}^{-1/2}$ for positive energy light-like vectors k and q.

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In the off-diagonal spin basis, the non-diagonal terms of the production density matrix are at least a factor of β^2 times smaller than for the helicity basis. For the Tevatron at 2 TeV this corresponds to a factor of typically 0.3 to 0.4. Furthermore, the terms lying on the edges of the production density matrix in the helicity basis are not very strongly suppressed compared to the diagonal terms, as γ is only 1.2 to 1.3 at these values of β^2. These properties of the two matrices translate into smaller interference terms in the off-diagonal basis once the decays are included.

Using either of the above production density matrices and the corresponding decay density matrices, the total matrix element squared for the production and decay process[4] $q\bar{q} \to t\bar{t} \to W^+ b \ W^- \bar{b} \to \bar{e}\nu b \ \mu\bar{\nu}\bar{b}$ averaged over the initial quark's color and spin and summed over the final colors and spins is given by

$$\sum |\mathcal{M}|^2 =$$
$$\frac{g_s^4}{9} \mathcal{T}\bar{\mathcal{T}} \left\{ (2 - \beta^2 s_{qt}^2) - \frac{(1 - c_{\bar{e}q} c_{\mu\bar{q}}) - \beta(c_{\mu\bar{t}} + c_{\bar{e}t}) + \beta c_{qt}(c_{\bar{e}q} + c_{\mu\bar{q}}) + \frac{1}{2}\beta^2 s_{qt}^2 (1 - c_{\bar{e}\mu})}{\gamma^2 (1 - \beta c_{\bar{e}t})(1 - \beta c_{\mu\bar{t}})} \right\}. \quad (4)$$

The factor $\mathcal{T}$ comes from the decay of the top quark ($t \to W^+ b \to \bar{e}\nu_e b$):

$$\mathcal{T} = \frac{g_W^4}{4 m_t^2 \Gamma_t^2} \left(m_t^2 - 2\bar{e} \cdot \nu \right) \frac{m_t^2 (1 - \hat{c}_{\bar{e}b}^2) + (2\bar{e} \cdot \nu)(1 + \hat{c}_{\bar{e}b})^2}{(2\bar{e} \cdot \nu - m_W^2)^2 + (m_W \Gamma_W)^2}, \quad (5)$$

where $\hat{c}_{\bar{e}b}$ is the cosine of angle between $\bar{e}$ and b in the $W^+(= \bar{e} + \nu)$ rest frame, $2\bar{e} \cdot \nu$ is the invariant mass of the positron and neutrino, (m_t, Γ_t) and (m_W, Γ_W) are the masses and widths of the top quark and W-boson respectively, and g_W is the weak coupling constant. Apart from the factor $(m_t \Gamma_t)^{-2}$, which comes from the top quark propagator, $\mathcal{T}$ is just the matrix element squared for unpolarized top quark decay. Likewise, $\bar{\mathcal{T}}$ is from the antitop decay ($\bar{t} \to W^- \bar{b} \to \mu\bar{\nu}_\mu\bar{b}$):

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[4]We have assumed that the decays of both W-bosons are leptonic. To change one or both of the W-boson decays to a hadronic decay, replace the charged lepton with a down type quark and the neutrino with an up type quark. This will preserve all correlations.

$$\bar{\mathcal{T}} = \frac{g_W^4}{4m_t^2\Gamma_t^2}\left(m_t^2 - 2\mu\cdot\bar{\nu}\right)\frac{m_t^2(1 - \hat{c}_{\mu\bar{b}}^2) + (2\mu\cdot\bar{\nu})(1 + \hat{c}_{\mu\bar{b}})^2}{(2\mu\cdot\bar{\nu} - m_W^2)^2 + (m_W\Gamma_W)^2}, \tag{6}$$

where $\hat{c}_{\mu\bar{b}}$ is the cosine of angle between μ and $\bar{b}$ in the $W^-(=\mu+\bar{\nu})$ rest frame, and $2\mu\cdot\bar{\nu}$ is the invariant mass of the muon and anti-neutrino. Eq. (4) agrees with the results of Kleiss and Stirling[10] for on mass shell top quarks and massless b-quarks[5] independent of whether or not the W-bosons are on or off mass shell.

If we did not include the spin correlations between production and decay, the total matrix element squared would be simply $g_s^4\mathcal{T}\bar{\mathcal{T}}(2 - \beta^2 s_{qt}^2)/9$. Thus, all of the correlations between the production and decay of the top quarks are contained in the second term inside the braces of Eq. (4). It is noteworthy that of the six final state particles, only the directions of the two charged leptons are required in addition to the directions of the t and $\bar{t}$ to fully specify these correlations. For the up-down spin configuration, the preferred emission directions for the charged leptons are $(t + ms)/2$ for the positron and $(\bar{t} + m\bar{s})/2$ for the muon: for the down-up configuration they are $(t - ms)/2$ and $(\bar{t} - m\bar{s})/2$ (see Fig. 1). These light-like vectors make an angle ω with respect to the beam axis given by

$$\sin\omega = \beta s_{qt}, \tag{7}$$

and their energy components are $\gamma m_t(1 \pm \beta c_{qt}\sec\omega)/2$.

In the off-diagonal spin basis the interference terms (i.e. those terms in Eq. (4) coming from the non-diagonal pieces of the production density matrix) are

$$\mathcal{I}_o = \frac{g_s^4\mathcal{T}\bar{\mathcal{T}}}{9\gamma^2(1 - \beta c_{\bar{e}t})(1 - \beta c_{\mu\bar{t}})}\frac{\beta^2}{2}\Big[(c_{\bar{e}t} - c_{qt}c_{\bar{e}q} - \beta s_{qt}^2)(c_{\mu\bar{t}} - c_{qt}c_{\mu\bar{q}} - \beta s_{qt}^2)/(1 - \beta^2 s_{qt}^2)$$
$$+ (c_{\bar{e}t} - c_{qt}c_{\bar{e}q})(c_{\mu\bar{t}} - c_{qt}c_{\mu\bar{q}}) + s_{qt}^2(c_{\bar{e}\mu} + c_{\bar{e}q}c_{\mu\bar{q}})\Big], \tag{8}$$

whereas for the helicity basis the interference terms are

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[5]Inclusion of the finite mass effects for the b-quarks would result in straightforward but messy modifications to Eqs. (5) and (6). The size of these effects is typically less than 1%.

$$\mathcal{I}_h = \frac{g_s^4 \mathcal{T}\bar{\mathcal{T}}}{9\gamma^2(1-\beta c_{\bar{e}t})(1-\beta c_{\mu\bar{t}})}\Big[(\beta c_{qt}-c_{\bar{e}q})(\beta c_{qt}-c_{\mu\bar{q}})$$
$$-c_{qt}^2(\beta-c_{\bar{e}t})(\beta-c_{\mu\bar{t}}) + \tfrac{1}{2}\beta^2 s_{qt}^2(c_{\bar{e}\mu}+c_{\bar{e}t}c_{\mu\bar{t}})\Big]. \tag{9}$$

Here we see again that the interference terms are a factor of β^2 smaller for the off-diagonal spin basis than the helicity basis. This point may be illustrated by plotting the distribution in $\hat{\mathcal{I}} \equiv \mathcal{I}/\sum|\mathcal{M}|^2$: for each phase space point we compute the value of the interference term and divide by the total matrix element squared at that point. Because the total matrix element squared can range from 0 (maximal destructive interference) to $2\mathcal{I}$ (maximal constructive interference), the variable $\hat{\mathcal{I}}$ must lie in the interval $(-\infty, \frac{1}{2}]$. The resulting differential distributions for both bases at the Tevatron is shown in Fig. 2. In the off-diagonal basis, $d\sigma/d\hat{\mathcal{I}}$ resembles a sharp spike: in fact, 90% of the cross section comes from points where $|\hat{\mathcal{I}}| < 0.15$. In contrast, only about half of the cross section comes from this region in the helicity basis. To enclose 90% of the cross section in the helicity basis, we must expand the range to $|\hat{\mathcal{I}}| < 0.35$. Clearly, the interference terms are much less important in the off-diagonal basis than in the helicity basis. Thus, the off-diagonal basis provides a far superior description of the $q\bar{q} \to t\bar{t}$ process at this accelerator.

In Fig. 3 we show the breakdown of the total $t\bar{t}$ cross section into like- and unlike-spin pairs using the off-diagonal basis versus the $t\bar{t}$ invariant mass for the Tevatron. We find that 92% of the $t\bar{t}$ pairs produced have unlike spins (UD+DU) in this basis. In contrast, only 70% of the $t\bar{t}$ pairs have unlike helicities.[6] Note that we have chosen spin combinations which are insensitive to which beam donated the quark in $q\bar{q} \to t\bar{t}$.

To observe the resulting correlations between the decay products of the top and the antitop, we proceed as described in Ref. [2]. Suppose that the ith decay product of the top quark is emitted at an angle θ_i with respect to the top spin axis in the top rest frame,

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[6]The value of 67% unlike helicities we quoted in Ref. [2] is based on an older set of structure functions, which contained a somewhat larger gluon component. The distributions we consider are not significantly affected by the choice of structure functions.

and that the $\bar{i}$th decay product of the antitop is emitted at an angle $\bar{\theta}_{\bar{i}}$ with respect to the antitop spin axis in the antitop rest frame. We tag the top quark in a particular event as having spin up if $\alpha_i \cos \theta_i > 0$, and as having spin down otherwise. The angular distribution of the $\bar{i}$th decay product in this situation is

$$\frac{1}{\sigma_{\text{TOT}}} \frac{d\sigma}{d(\cos \bar{\theta}_{\bar{i}})} = \frac{1}{2} \Big[1 + \tfrac{1}{2}(1 - 2P_\times) \, \alpha_i \alpha_{\bar{i}} \cos \bar{\theta}_{\bar{i}} \Big], \tag{10}$$

where $P_\times$ is the fractional purity of the unlike-spin component of the sample of $t\bar{t}$ events. The α_i's ($\alpha_{\bar{i}}$'s) are the correlation coefficients from the decay distribution of a polarized top (antitop) [11] and take on the values $\alpha_{\bar{e}} = \alpha_{\bar{d}} = 1$, $\alpha_\nu = \alpha_u = -0.31$, and $\alpha_b = -0.41$ for the decay of a 175 GeV spin-up top quark. The α_i's for spin-down top quarks have opposite sign. The correlation coefficients for a spin up (spin down) antitop are the same as for a spin down (spin up) top quark.

Because the W boson has primarily hadronic decays, it is useful to have a method to probabilistically determine which of the two jets in such a decay was initiated by the down-type quark. As explained in Ref. [2], the jet which lies closest to the b-quark direction as viewed in the W rest frame is most likely the d-type quark. The probability that this identification is correct is $P_d = \frac{1}{4}(2m_t^2 + 7m_W^2)/(m_t^2 + 2m_W^2)$, and equals 0.61. The effective correlation coefficient for this "d"-type quark is given by $\alpha_{\text{``}d\text{''}} = P_d \alpha_d + (1 - P_d)\alpha_{\bar{u}}$, or about -0.49.

Since the coefficient of $\cos \bar{\theta}_{\bar{i}}$ governs the size of the observable correlations, it is desirable to make it as large as possible. Thus, we should choose to work in the off-diagonal basis where $1 - 2P_\times$ is -0.84 instead of the helicity basis where $1 - 2P_\times$ is only -0.39. The other two factors, α_i and $\alpha_{\bar{i}}$ depend on which top decay product is used to tag the top quark spin, and which antitop decay product angular distribution is generated. The largest correlations are clearly between the two charged leptons. For the same pair of decay products, the correlations are more than twice as large in the off-diagonal basis as in the helicity basis.

In Fig. 4 we show results of a first-pass Monte Carlo study of the correlations at the parton level without any hadronization or jet energy smearing effects included. We required

all final state particles to satisfy the cuts $p_T > 15$ GeV, $|\eta| < 2$. Plotted in this figure are the angular distributions for various antitop decay products for samples tagged as containing spin up or spin down top quarks. In the absence of correlations, the two curves in each section of the figure would lie on top of each other. Even in the presence of the cuts, the off-diagonal basis still produces correlations more than twice as large as those in the helicity basis.

One advantage of the off-diagonal basis which is readily apparent from Fig. 4 is that the cuts affect the spin-up and spin-down samples in a symmetric manner: *i.e.* the two curves remain reflections of each other about the point $\cos\bar\theta = 0$. Consequently, there is no systematic bias introduced between the two data sets when using this basis. The same is not true in the helicity basis: our cuts are slightly more likely to exclude left-handed helicity top quarks than right-handed, and the two angular distributions acquire different shapes [2].

In conclusion, we have shown that the off-diagonal basis of Parke and Shadmi enjoys several advantages over the more traditional helicity basis when applied to a study of angular correlations in top quark pair production at the FNAL Tevatron. Firstly, the vanishing of the amplitude for the production of like spin top pairs from quark-antiquark annihilation leads to a total cross section consisting of 92% unlike spin pairs in the off-diagonal basis. This is significantly better than the 70% unlike helicity pairs obtained in the helicity basis. Secondly, most of the non-diagonal terms in the production density matrix vanish in the off-diagonal basis. Those non-diagonal terms which are not zero are suppressed by a factor of β^2. No such simplicity exists in this matrix in the helicity basis. As a result, the contributions attributed to interference terms in the off-diagonal basis are far less important than those in the helicity basis. Thirdly, the larger production asymmetry of the off-diagonal basis translates into angular correlations among the t and $\bar t$ decay products which are more than double those in the helicity basis. And, finally, the kinds of experimental cuts which are typically imposed on collider data do not introduce a systematic bias between spin up and spin down top quarks in the off-diagonal basis. The same is not true in the helicity basis. These advantages make the off-diagonal basis the basis of choice for $t\bar t$ correlation studies at

the Tevatron.

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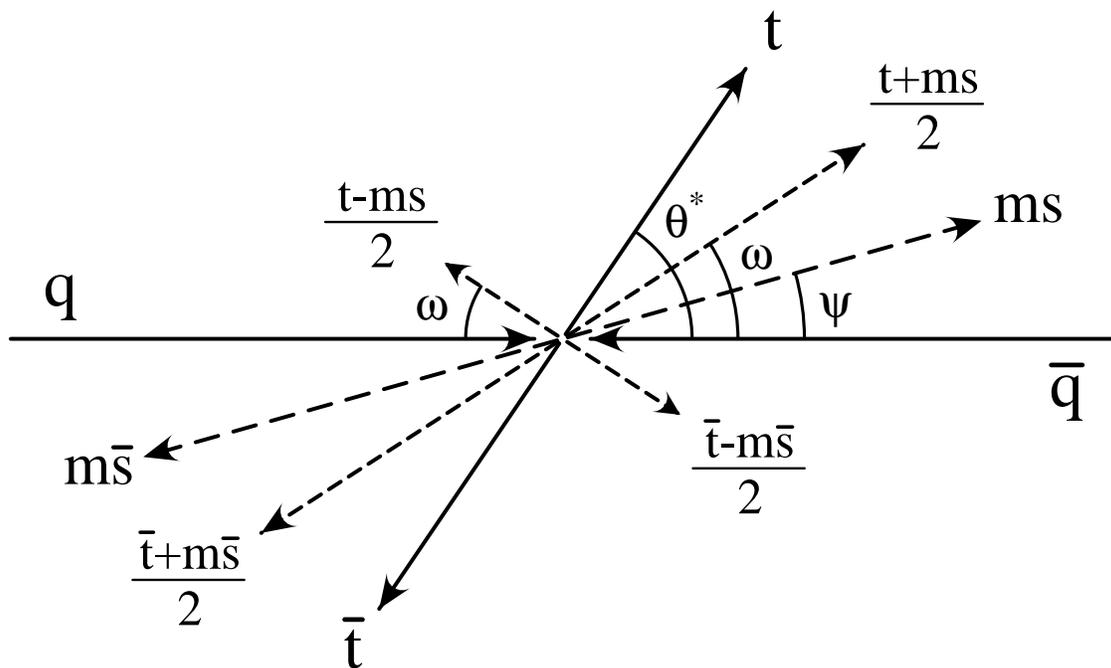[11] M. Jeżabek and J.H. Kühn, Phys. Lett. **B329**, 317 (1994).

FIG. 1. The relevant angles and vectors in the zero momentum frame of the initial $q\bar{q}$ pair for the off-diagonal basis of Parke and Shadmi. The top quark is produced at an angle θ^* with respect to the beam axis ($\cos\theta^* \equiv c_{qt}$). The spin vector s makes an angle ψ (given by Eq. (1)) with respect to the beam axis. The vectors $(t \pm ms)/2$, where m is the top quark mass, indicate the preferred emission directions for the charged lepton or down-type quark from the decaying W^+ (see Eq. (7)). The vectors describing the antitop lie back-to-back with the corresponding top quark vectors.
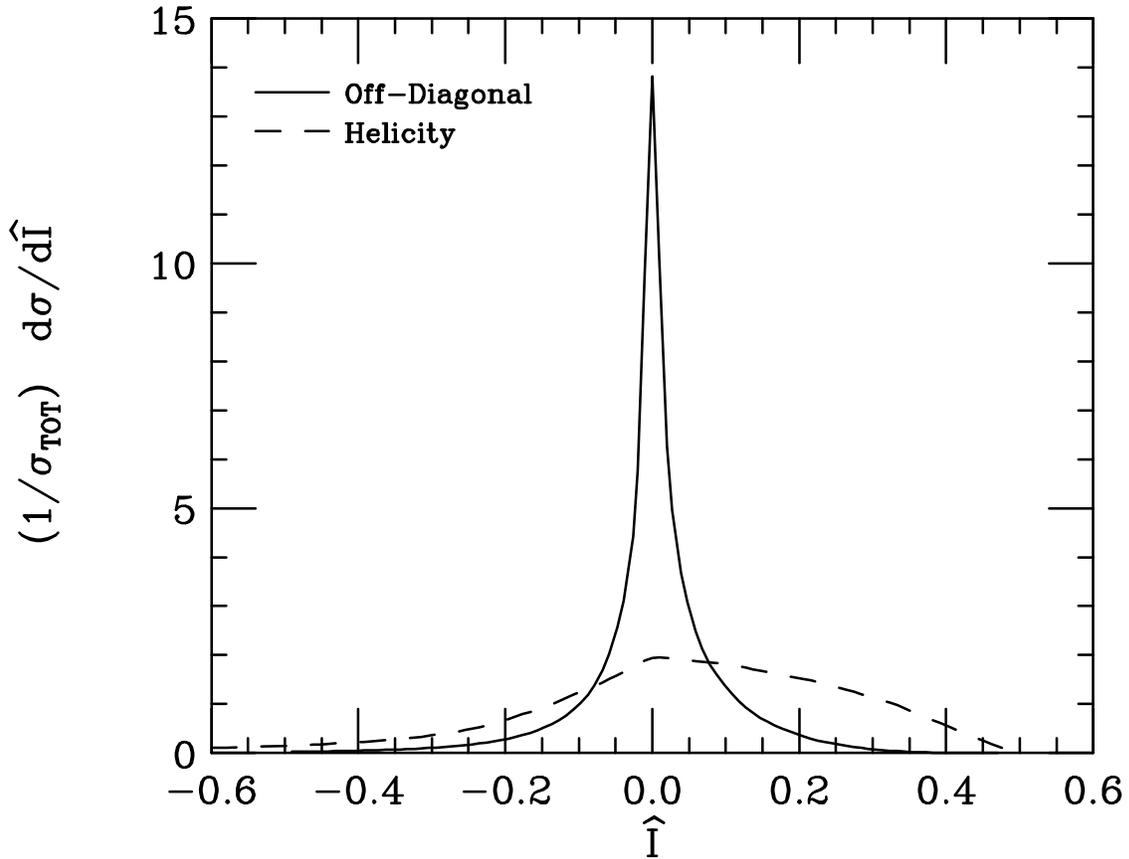
FIG. 2. The relative importance of the interference terms in the off-diagonal and helicity bases in $q\bar{q} \to t\bar{t}$ for the Tevatron at $\sqrt{s} = 2$ TeV. Plotted is the differential distribution in $\hat{\mathcal{I}} \equiv \mathcal{I}/\sum |\mathcal{M}|^2$, the value of the interference term (Eq. (8) or (9)) normalized to the square of the total matrix element (Eq. (4)). In the off-diagonal basis, 90% of the cross section comes from phase space points where $|\hat{\mathcal{I}}| < 0.15$, whereas in the helicity basis only 50% of the cross section comes from this region.
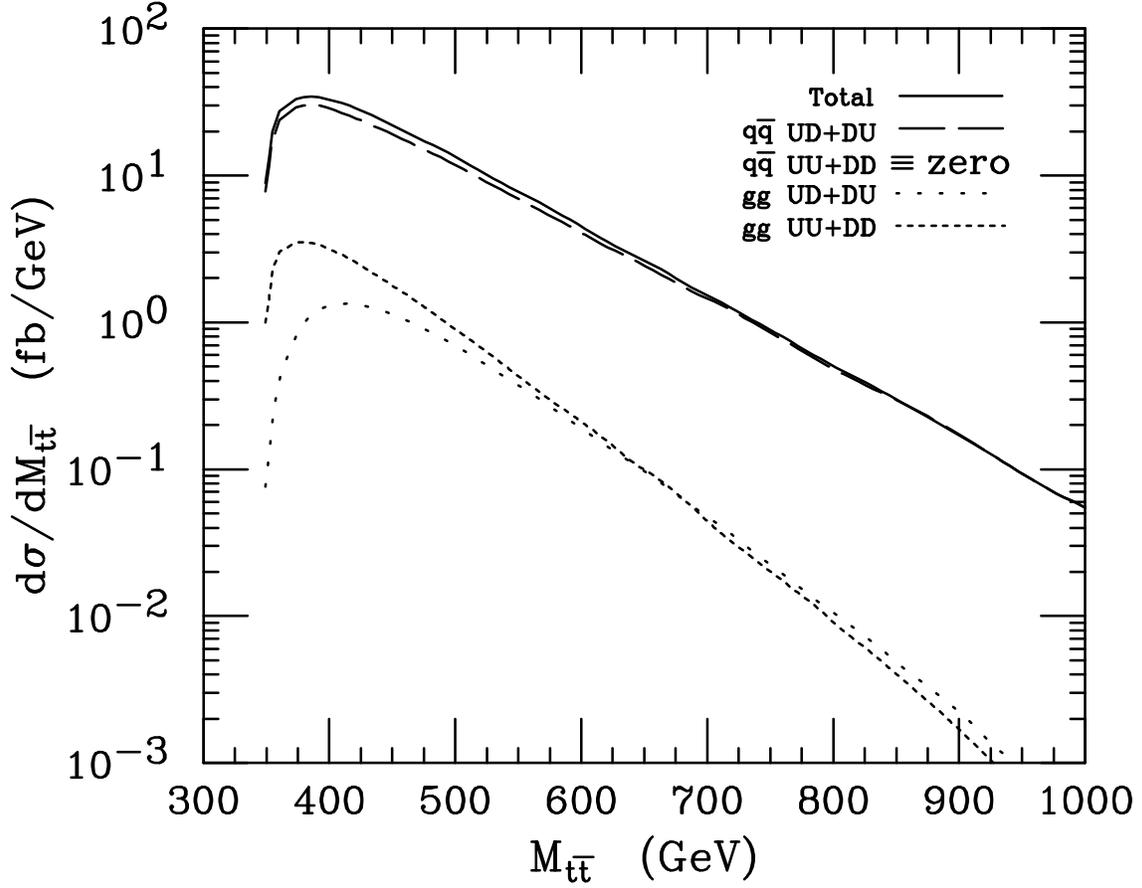
FIG. 3. Differential cross section for $t\bar{t}$ production as a function of the $t\bar{t}$ invariant mass $M_{t\bar{t}}$ for the Tevatron with center-of-mass energy 2.0 TeV, decomposed into UD+DU and UU+DD spins of the $t\bar{t}$ pair using the off-diagonal basis for both $q\bar{q}$ and gg components.
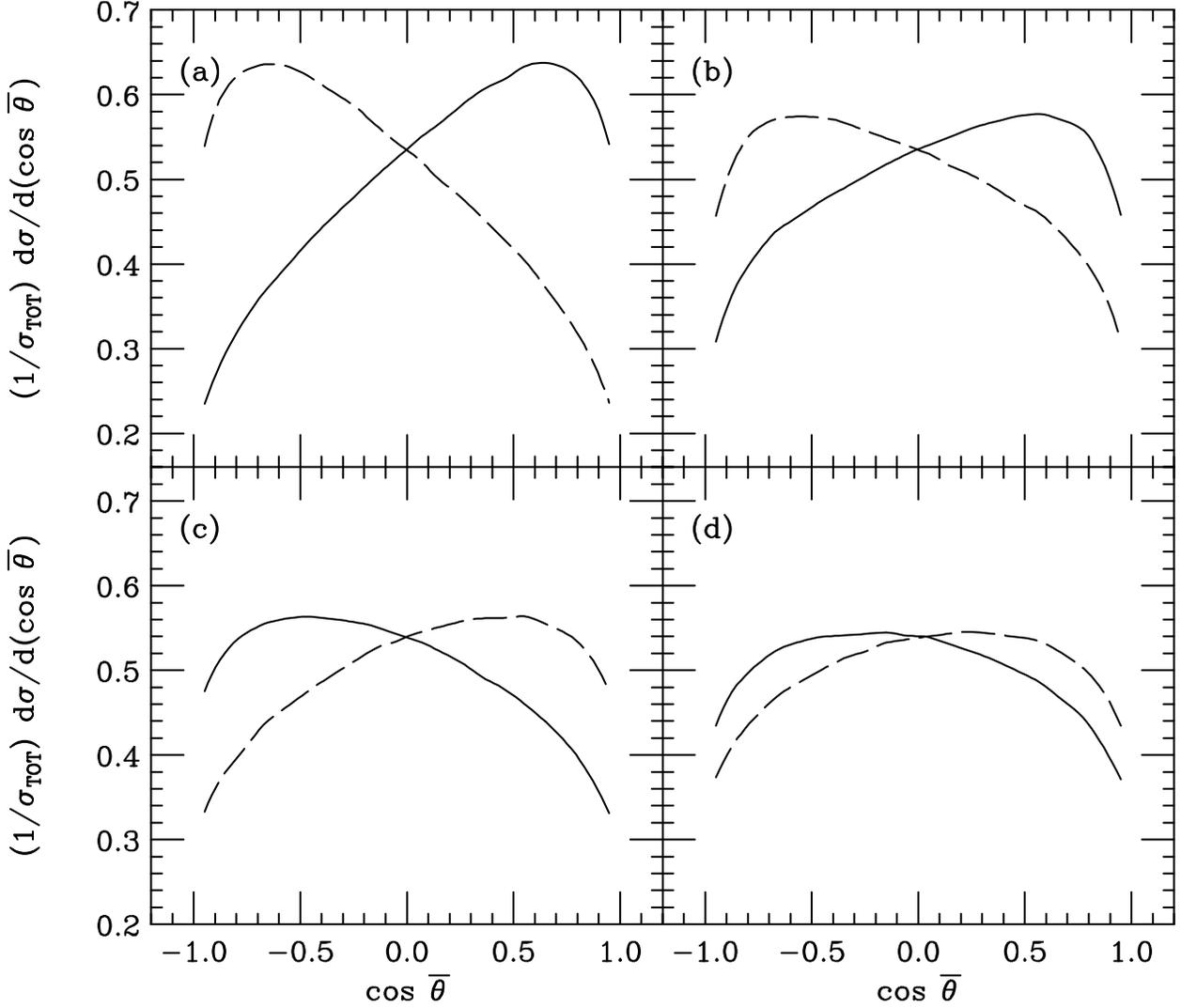
FIG. 4.   Angular correlations in Tevatron $t\bar{t}$ events at $\sqrt{s} = 2$ TeV using the off-diagonal basis. The data in each plot are divided into spin-"up" (solid) and spin-"down" (dashed) top quark components, determined by using the charged lepton from the t decay in (a)–(c), and the b-quark in (d). Plotted are the angular distributions with respect to the $\bar{t}$ spin axis in the $\bar{t}$ rest frame for the following $\bar{t}$ decay products: (a) the charged lepton, (b) the "d"-type quark, (c) and (d) the $\bar{b}$-quark.