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Abstract

In supersymmetric theories the charged sleptons of different generations may oscillate amongst themselves while they decay. In the case of three generation oscillations, superpartner production at the LHC and NLC may lead to an observable CP-violating signal $N(e^+\mu^-) - N(\mu^+e^-)$. This signal is proportional to a CP-violating invariant of the slepton mass matrix, $\widetilde{J}$, which is not constrained by searches for the electric dipole moment of the electron. The sensitivity of the LHC and NLC to this signal is highly dependent on superpartner masses, but $\widetilde{J}$ may be probed to a level of 10^{-3}. Observation of the CP-violating signal would imply a definite structure for the slepton mass matrices and have strong implications for models of flavor and SUSY breaking.

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I. INTRODUCTION

Supersymmetry (SUSY) may provide a solution to the gauge hierarchy problem and is one of the most attractive candidates for physics beyond the Standard Model (SM). If SUSY is discovered, the next experimental task will be to determine the SUSY parameters as accurately as possible. The precision measurements of $\sin^2 \theta_W$ at LEP and SLD have already provided us with indirect evidence for supersymmetric unification at high energy scales. A precise knowledge of superpartner properties could further test the idea of supersymmetric unification, and may reveal additional aspects of the theory at high energies, perhaps even up to the Planck scale.

With so much at stake, much work has been done on the feasibility of determining SUSY parameters at future colliders[1]. These studies typically exploit precision measurements of kinematic distributions and cross sections. At the Large Hadron Collider (LHC)[2], where many, if not all, of the superpartners are likely to be produced, recent studies in the minimal supergravity framework have demonstrated the possibility of precise determinations of some superpartner mass differences and branching fractions[3, 4]. At the proposed Next Linear Collider (NLC)[5], numerous studies of the gaugino/Higgsino[6, 7], slepton[6, 8, 9, 10], squark[10, 11], and Higgs[12] sectors have shown that if superpartners are kinematically accessible, highly model-independent measurements of their masses and properties may be made, and underlying SUSY parameters may be determined[13, 14].

A complementary approach, however, is to look for SUSY-mediated rare phenomena that violate some of the (approximate) conservation laws of the SM. The discovery of these phenomena would be tremendously exciting, providing essential information about the structure of the theory that could not be obtained from the studies mentioned above. In addition, we will see that these phenomena may be highly sensitive probes of superpartner mass patterns. This approach is therefore particularly well-suited to hadron colliders, where precision spectroscopy may be very difficult.

In this paper, we explore the phenomenon of CP violation arising from SUSY-mediated violations of lepton flavor conservation. Lepton flavor, although conserved in the SM, is typically violated in any supersymmetric extension of the SM, since the scalar partners of the leptons must be given mass, and the scalar mass matrices are generally not diagonal in the same basis as the fermion masses. For instance, in the superfield basis where the lepton Yukawa coupling λ_E is diagonal, the left- and right-handed slepton masses may be written as

$$\tilde{e}_{L\alpha}^* m_{L\alpha\beta}^2 \tilde{e}_{L\beta} + \tilde{e}_{R\alpha}^* m_{R\alpha\beta}^2 \tilde{e}_{R\beta} \ , \tag{1}$$

where α and β are generational indices. The scalar mass matrices $m_{L,R}^2$ are diagonalized by unitary matrices $W_{L,R}$ through

$$m_{L,R}^2 = W_{L,R}^\dagger m_{DL,R}^2 W_{L,R} \ , \tag{2}$$

where $m_{DL,R}^2$ are diagonal mass matrices. Together with their analogues in the quark sector, the W matrices are new flavor mixing matrices analogous to the CKM matrix. If we work in the mass eigenstate basis for all fields, the W matrices appear in neutralino and chargino vertices. For the neutralinos, these vertices are given by the interactions

$$\tilde{e}_{Li} W^*_{Li\alpha} \overline{e_{L\alpha}} \tilde{\chi}^0 + \tilde{e}^*_{Li} W_{Li\alpha} \overline{\tilde{\chi}^0} e_{L\alpha} + \tilde{e}_{Ri} W^*_{Ri\alpha} \overline{e_{R\alpha}} \tilde{\chi}^0 + \tilde{e}^*_{Ri} W_{Ri\alpha} \overline{\tilde{\chi}^0} e_{R\alpha} ~, \tag{3}$$

where the Latin and Greek subscripts are generational indices for scalars and fermions, respectively.

Non-trivial W matrices violate lepton flavor conservation and may give sizeable contributions to rates for rare processes such as $\mu \to e\gamma$. For nearly degenerate sleptons, however, these contributions are suppressed by $\Delta m/m$ through the superGIM mechanism; in the limit of exact degeneracy, the W matrices can be rotated to the unit matrix, and the gaugino interactions conserve lepton flavor. In fact, if the mixing between the first two generation sleptons is not very small, a high degree of degeneracy is *required* in order to evade bounds from $\mu \to e\gamma$. This is an example of the supersymmetric flavor-changing problem.

In Ref. [15] we considered high energy probes of the W matrices through the flavor-violating decays of sleptons at the NLC. This flavor-violating signal is only suppressed by the superGIM mechanism when $\Delta m \lesssim \Gamma$, and the suppression factor is $\Delta m/\Gamma$, where Γ is the slepton decay width. As Γ/m is typically $10^{-2} - 10^{-3}$, there is a large range of parameters with $\Gamma \lesssim \Delta m \lesssim m$ in which the rates for rare low energy processes, such as $\mu \to e\gamma$, are suppressed, but the high energy collider signal is not. We considered the case of $\tilde{e}_R - \tilde{\mu}_R$ mixing, and found that the direct collider probe of W was considerably more powerful than current bounds from $\mu \to e\gamma$. In this paper, we will extend this analysis by considering the potential of colliders to probe the phases of the W matrices through CP-violating asymmetries in lepton flavor violation. We will consider both the LHC and the NLC, and will find that both colliders may be sensitive to small CP-violating invariants.

In addition, we will find that any observation of a CP asymmetry implies a particular structure for the slepton mass matrices, shedding light on the connection between SUSY breaking and the origin of flavor. It is well-known that the supersymmetric flavor-changing problem can be elegantly solved. The two dominant prototypical schemes may be characterized as follows:

- SUSY breaking is mediated to squarks and sleptons by flavor-blind gauge interactions at a scale well below the Planck scale. In this case the sfermions of a given charge are degenerate, and the W matrices can be rotated to the unit matrix. There are no interesting flavor signals in this scheme, other than the scalar spectrum itself.

- Flavor physics of both fermion and scalar sectors is controlled by a spontaneously broken flavor symmetry. In this case the experimental signatures of the supersymmetric flavor sector depend on the flavor symmetry group. Since the flavor symmetry controls the quark and lepton masses and mixings, it is frequently the case that the W matrices have entries that are comparable to the corresponding CKM matrix elements, and that the scalars of a given charge of the lightest two generations are closely degenerate. It may also be that the large interactions which give the top quark its mass also give a large splitting between the scalar of the third generation and the other two scalars. In this case there are many exciting prospects for discovering supersymmetric flavor physics in rare processes [16].

In the simplest versions of both these cases, the CP-violating signals discussed in this paper will typically be too small to be detected. However, extra interactions may be present in a

more general theory and there is a general framework in which these CP-violating signals are large:

- The dynamics that generates the slepton masses — whether from supergravity, gauge, or other interactions — is flavor-blind, leading to degenerate scalars. However, the presence of flavor-violating scalar mass perturbations can lead, via degenerate perturbation theory, to large mixing angles in W and small scalar non-degeneracies, precisely the features that lead (as we will show) to large CP asymmetries at colliders.

The outline for the rest of the paper is as follows. In Sec. II we give a rephase-invariant description of CP violation in the slepton mass matrices. In Sec. III we compute the CP asymmetries in lepton flavor-violating events at colliders in two different ways, first directly from field theory and then using the more transparent but also more heuristic language of flavor oscillations. In Sec. IV we determine the conditions for observable CP violation. Sec. V contains simple analytic estimates of the CP-violating signals at the LHC and the NLC, and Sec. VI gives the results of the numerical calculations and discusses the experimental possibilities for detecting this signal. In Secs. VII and VIII, we consider constraints on the size of the signal coming from the electron electric dipole moment (EDM) and $\mu \to e\gamma$, respectively. We discuss the implications of observing a CP-violating signal for models of scalar mass matrices in Sec. IX and draw our conclusions in Sec. X.

II. A REPHASE INVARIANT DESCRIPTION OF SLEPTON CP VIOLATION

In this section, we count the number of relevant CP-violating phases in the $W_{L,R}$ matrices of Eq. (3). These phases allow a variety of CP-violating phenomena which we describe in terms of a well-motivated and complete set of rephase invariant quantities that are analogous to the Jarlskog invariant of the CKM matrix. Neglecting the left-right scalar mass terms, which are suppressed by lepton Yukawa couplings, slepton mixing may be completely described by the matrices W_L and W_R of Eq. (3). There are then two 3×3 matrices with a combined $9 + 9 = 18$ degrees of freedom. Any physical quantity must be invariant under rephasings of the slepton and lepton fields that leave the lepton masses real:

$$W_{Li\alpha} \to e^{i\phi_L^i} W_{Li\alpha} e^{i\theta^\alpha} \ , \quad W_{Ri\alpha} \to e^{i\phi_R^i} W_{Ri\alpha} e^{i\theta^\alpha} \ . \tag{4}$$

Thus 9 independent phases may be removed. However, one overall rephasing has no effect, and so we can remove 8 degrees of freedom, leaving 10 parameters to describe $W_{L,R}$. Of these, $3 + 3$ are real rotations and 4 are CP-violating phases.

We now find a complete set of CP violation rephase invariants, beginning with the invariants that can be built out of only one of the two W matrices. As in the SM, there is only one CP-violating phase associated with a single W matrix, and the corresponding rephase invariant is

$$Im \left[W_{i\alpha} W_{i\beta}^* W_{j\alpha}^* W_{j\beta} \right] \equiv \tilde{J} \sum_{k\gamma} \epsilon_{ijk} \epsilon_{\alpha\beta\gamma} \ . \tag{5}$$

$\tilde{J}_L$ and $\tilde{J}_R$ are the supersymmetric analogues to the Jarlskog invariant[17], and the antisymmetric tensors ε_{ijk} and $\varepsilon_{\alpha\beta\gamma}$ simply determine the relative sign. In the next section, we

4

will see that the CP asymmetries in left- and right-handed slepton decays at colliders are directly proportional to $\widetilde{J}_{L,R}$. The two remaining phase degrees of freedom must involve both W_L and W_R. In fact, the dominant contributions to the electron EDM d_e from phases in the W matrices are given by one-loop diagrams with internal sleptons and gauginos involving left-right scalar mass insertions proportional to m_μ or m_τ.[1] These contributions therefore require non-trivial flavor mixing for both left- and right-handed sleptons. If there is only 12 mixing, d_e is proportional to $m_\mu Im[W_{L21}W_{L22}^* W_{R21}^* W_{R22}]$. On the other hand, if there is mixing with the third generation but the first two generation sleptons are exactly degenerate (or if there is only 13 mixing), d_e is proportional to $m_\tau Im[W_{L31}W_{L33}^* W_{R31}^* W_{R33}]$. Since these two CP-violating quantities are clearly independent and physically meaningful, we choose the remaining two CP-violating invariants to be

$$\widetilde{K}_{12} = Im\left[W_{L21}W_{L22}^* W_{R21}^* W_{R22}\right] \ , \quad \widetilde{K}_{13} = Im\left[W_{L31}W_{L33}^* W_{R31}^* W_{R33}\right] \ . \tag{6}$$

Every CP-violating quantity will then be a linear combination of the four independent invariants $\widetilde{J}_{L,R}$ and $\widetilde{K}_{12,13}$.

It is also instructive to understand the CP-violating phases of the theory in the superfield basis where the lepton masses are diagonal but the slepton mass matrices may have off-diagonal components. The only phases are then in the 12, 23, and 31 components of the slepton mass matrices. All physical quantities should be invariant under lepton superfield rephasings $m_{L,R_{\alpha\beta}}^2 \rightarrow e^{-i\theta^\alpha} m_{L,R_{\alpha\beta}}^2 e^{i\theta^\beta}$. If we concentrate on rephase invariants built from a single mass matrix, the only possibility is $Im[m_{12}^2 m_{23}^2 m_{31}^2]$. Note that this invariant vanishes unless there is full 3 generation mixing. It is also easy to see that if two of the slepton mass eigenvalues are degenerate, this CP invariant vanishes. For instance, if the first two generation sleptons are degenerate at m_0^2, we have $m_{\alpha\beta}^2 = \sum_i W_{i\alpha}^* W_{i\beta} m_i^2 = W_{3\alpha}^* W_{3\beta}(m_3^2 - m_0^2)$ for $\alpha \neq \beta$, where m_i are the physical slepton masses, and we have used the unitarity of W, which implies $\sum_i W_{i\alpha}^* W_{i\beta} = 0$ for $\alpha \neq \beta$. Then, $Im[m_{12}^2 m_{23}^2 m_{31}^2] = Im[|W_{31}W_{32}W_{33}|^2(m_3^2 - m_0^2)^3] = 0$. It is possible to show that (see Appendix A)

$$Im\left[m_{12}^2 m_{23}^2 m_{31}^2\right] = \widetilde{J}(m_2^2 - m_1^2)(m_3^2 - m_2^2)(m_1^2 - m_3^2) \ . \tag{7}$$

The invariants $\widetilde{K}_{12,13}$ are similarly related to $Im[m_{L12}^{2\,*} m_{R12}^2]$ and $Im[m_{L13}^{2\,*} m_{R13}^2]$. If we only have mixing between the first two generations, then

$$Im\left[m_{L12}^{2\,*} m_{R12}^2\right] = \widetilde{K}_{12}(m_{L1}^2 - m_{L2}^2)(m_{R1}^2 - m_{R2}^2) \ , \tag{8}$$

whereas if the first two generation sleptons are exactly degenerate,

$$Im\left[m_{L13}^{2\,*} m_{R13}^2\right] = \widetilde{K}_{13}(m_{L3}^2 - m_{L0}^2)(m_{R3}^2 - m_{R0}^2) \ , \tag{9}$$

where m_{L0}^2 and m_{R0}^2 are the common scalar masses of the first two generation left- and right-handed sleptons.

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[1] Additional contributions may arise from phases in the A, μ and gaugino mass parameters. For the following discussion, we set these to zero. The justification for this, and the the relevance of these phases to this study, will be discussed at the end of this section.

Although not needed in the rest of this paper, the CP-violating phases in the squark sector may be similarly counted and classified. Again treating the left-right scalar mass mixings as perturbations (an approximation that may be invalid for the third generation squarks), an analysis similar to the one above finds that there are 10 independent phases in the squark mass matrices.[2] These also have rephase invariant descriptions analogous to the $\widetilde{J}$ and $\widetilde{K}$ invariants defined above. In the squark sector there are analogous mixing matrices W_{U_L}, W_{U_R}, W_{D_L}, and W_{D_R}. Together with V_{CKM}, four of them are independent (W_{D_L} is fixed by W_{U_L} and the quark and squark masses). From these four matrices, we may form 4 $\widetilde{J}$ invariants (including the Jarlskog invariant of the CKM matrix), and 2 $\widetilde{K}$ invariants from each of the 3 pairs of matrices $\{W_{U_L}, W_{U_R}\}$, $\{W_{D_L}, W_{D_R}\}$, and $\{W_{U_L}, W_{D_L}\}$. These 10 quantities form a complete set.

So far we have discussed only the phases in the W mixing matrices. Of course, in addition to these phases, there are other possible sources of supersymmetric CP violation. The usual SUSY CP problem arises from relative phases between the A and μ parameters and the gaugino masses. In this study, we will examine CP-violating asymmetries in lepton flavor-violating events at colliders. These will be seen to be direct measures of the CP-violating phases in the W matrices, and are insensitive to phases in the A, μ and gaugino mass parameters for the following reasons. First, the μ parameter and gaugino masses do not violate lepton flavor, and while their phases may cause changes in quantities like the total production cross sections, they do not contribute to flavor-violating asymmetries. Second, while non-universal A parameters do violate lepton flavor and may contribute to the CP-violating collider signals through left-right scalar mass terms, these terms are suppressed by lepton masses. Unless the left- and right-handed sleptons are very nearly degenerate, the induced left-right mixing is too small to significantly modify the collider CP-violating asymmetry.

In contrast, the phases in the A and μ terms and the gaugino masses may give important contributions to d_e. For this reason, if a non-zero electron EDM is discovered, it may arise from either these terms or the W matrices, and does not necessarily imply a lower bound on the collider signals discussed here. On the other hand, a bound on d_e does constrain possible contributions from the W matrices (in the absence of unexpected cancellations), and therefore also places an upper bound on our collider signals. In general, the electron EDM is a complicated function of both the phases in W and the other phases in the A, μ and gaugino mass parameters. However, as we do not know what these other phases are, we calculate the bounds on W phases from the electron EDM by assuming that the other phases vanish. The electron EDM is then proportional to the invariants $\widetilde{K}_{12,13}$ as noted above. We will see in Sec. VII that the collider signals are observable even if the current limit on d_e improves by a factor of 10.

To summarize, we have found a set of 4 rephase invariants that characterize CP violation in the SUSY lepton sector: $\widetilde{J}_L$ and $\widetilde{J}_R$ determine the sizes of CP-violating signals at colliders, while $\widetilde{K}_{12}$ and $\widetilde{K}_{13}$ determine the size of the electron EDM. We now consider possible collider probes of $\widetilde{J}_{L,R}$.

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[2]The counting for only left-handed quarks and squarks is considered in Ref. [18].

III. SLEPTON CP ASYMMETRIES

In this section, we derive results for CP asymmetries from slepton production at colliders. Our signal will be asymmetries between flavor-violating final states, such as $e^+\mu^-$ and μ^+e^-. We will require results for single slepton production, the dominant production mechanism at hadron colliders where, for example, sleptons are produced in gluino cascades, and also for correlated slepton pair production, which is most relevant at e^+e^- colliders. We begin with a derivation in field theory, in which various subtleties will be noted and our approximations explicitly stated. With these approximations, the results for single slepton production will be familiar from other contexts, such as B physics, and we will highlight these similarities. However, the case of pair production is more complicated, and warrants careful treatment. We conclude this section by rederiving these results in the simpler, but more heuristic, language of flavor oscillations.

A. Field theory derivation

We begin with processes involving single slepton production. Such processes are most relevant for hadron colliders, where a single slepton may be produced in a gluino or squark cascade decay. The general form for such a process is $f_1 f_2 \to e_\alpha^+ X \tilde{e}_i^- \to e_\alpha^+ X e_\beta^- Y$, where the subscripts i, α and β are generational indices.[3] The initial state partons are denoted by f_1 and f_2, and X and Y are m- and n-body final states, respectively. In the simplest case, sleptons decay directly to LSPs, so $n = 1$ and $Y = \tilde{\chi}_1^0$. For nearly massless f_1 and f_2,

$$
\begin{aligned}
\sigma_{\alpha\beta} &\equiv \sigma(f_1 f_2 \to e_\alpha^+ X e_\beta^- Y) \\
&= \int \frac{(2\pi)^4}{2\hat{s}} |\mathcal{M}_{\alpha\beta}|^2 \, d\Phi_{m+n+2}(P; p_{e_\alpha^+}, p_{X_1}, \ldots, p_{X_m}, p_{e_\beta^-}, p_{Y_1}, \ldots, p_{Y_n}) \,,
\end{aligned} \tag{10}
$$

where $P = p_{f_1} + p_{f_2}$, $\hat{s} = P^2$, and the phase space factor and its decomposition are

$$
\begin{aligned}
d\Phi_l(P; p_1, \ldots, p_l) &\equiv \delta^4\left(P - \sum_{i=1}^l p_i\right) \prod_{i=1}^l \frac{d^3 p_i}{(2\pi)^3 2E_i} \\
&= d\Phi_{j+1}(P; q, p_1, \ldots, p_j) d\Phi_{l-j}(q; p_{j+1}, \ldots, p_l)(2\pi)^3 dq^2 \,,
\end{aligned} \tag{11}
$$

where $q^2 = (\sum_{i=j+1}^l p_i)^2$. The amplitude $\mathcal{M}_{\alpha\beta}$ may be written as

$$
\mathcal{M}_{\alpha\beta} = \sum_i \mathcal{M}_P W_{i\alpha} \frac{i}{q^2 - m_i^2 + im\Gamma} W_{i\beta}^* \mathcal{M}_D \,, \tag{12}
$$

where the sum is over all slepton generations that may be produced on-shell. $\mathcal{M}_P$ is the amplitude for $f_1 f_2 \to e^+ X \tilde{e}^-$ in the absence of flavor violation, where e here represents

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[3]In fact, for sleptons produced in decays of Majorana particles, the same final state also arises from another process, $f_1 f_2 \to e_\beta^- X \tilde{e}_i^+ \to e_\beta^- X e_\alpha^+ Y$, which must be included. A modification of the following analysis is necessary, but leads to results identical to those encapsulated in Eqs. (17) and (21). For notational simplicity, we ignore this complication.

any one of the available lepton flavors, and $\mathcal{M}_D$ is the amplitude for decay $\tilde{e}^- \to e^- Y$, also in the absence of flavor violation. We assume that the generational dependence of these amplitudes, for example, through variations in slepton mass or dependence on Yukawa couplings, is negligible. We also neglect generational differences in the widths $m\Gamma$.

With this amplitude, and substituting the phase space decomposition of Eq. (11) with $l = m + n + 2$ and $j = m + 1$,

$$\sigma_{\alpha\beta} = \int \frac{(2\pi)^4}{2\hat{s}} |\mathcal{M}_P|^2 \, d\Phi_{m+2} (2\pi)^3 |\mathcal{M}_D|^2 \, d\Phi_{n+1} \sum_{ij} W_{i\alpha} W_{i\beta}^* W_{j\alpha}^* W_{j\beta} A_{ij}(q^2) dq^2 \ , \qquad (13)$$

where

$$\begin{aligned}
A_{ij}(q^2) &= \frac{i}{q^2 - m_i^2 + im\Gamma} \frac{-i}{q^2 - m_j^2 - im\Gamma} \\
&= \frac{1}{q^2 - \bar{m}_{ij}^2 - \frac{1}{2}\Delta m_{ij}^2 + im\Gamma} \frac{1}{q^2 - \bar{m}_{ij}^2 + \frac{1}{2}\Delta m_{ij}^2 - im\Gamma} \ ,
\end{aligned} \qquad (14)$$

and we have defined $\bar{m}_{ij}^2 = (m_i^2 + m_j^2)/2$ and $\Delta m_{ij}^2 = (m_i^2 - m_j^2)/2 \approx 2m\Delta m_{ij}$, with $\Delta m_{ij} = m_i - m_j$. For $\Delta m_{ij}^2, m\Gamma \ll \bar{m}_{ij}^2$, we may use the approximation

$$\begin{aligned}
\frac{1}{(z - a + ib)(z + a - ib)} &= \frac{1}{2(a - ib)} \left[\frac{z - a - ib}{(z - a)^2 + b^2} - \frac{z + a + ib}{(z + a)^2 + b^2} \right] \\
&\approx \frac{1}{2(a - ib)} \left[i\pi\delta(z - a) + i\pi\delta(z + a) \right] \\
&\approx \frac{1}{1 + i(a/b)} \frac{\pi}{b} \delta(z)
\end{aligned} \qquad (15)$$

to write $A_{ij}(q^2) \approx A_{ij} \frac{\pi}{m\Gamma} \delta(q^2 - \bar{m}_{ij}^2)$. Substituting this form of $A_{ij}(q^2)$ into Eq. (13) yields

$$\begin{aligned}
\sigma_{\alpha\beta} &= \int \sum_{ij} W_{i\alpha} W_{i\beta}^* W_{j\alpha}^* W_{j\beta} A_{ij} \delta(q^2 - \bar{m}_{ij}^2) dq^2 \\
&\times \left[\frac{(2\pi)^4}{2\hat{s}} |\mathcal{M}_P|^2 \, d\Phi_{m+2} \right] \left[\frac{1}{\Gamma} \frac{(2\pi)^4}{2m} |\mathcal{M}_D|^2 \, d\Phi_{n+1} \right] \ ,
\end{aligned} \qquad (16)$$

and so the general form for the flavor-violating cross section is

$$\begin{aligned}
\sigma_{\alpha\beta} &= S_{\alpha\beta} \sigma_0 \\
S_{\alpha\beta} &= \sum_{ij} W_{i\alpha} W_{i\beta}^* W_{j\alpha}^* W_{j\beta} A_{ij} \\
A_{ij} &= \frac{1}{1 + ix_{ij}} \ ,
\end{aligned} \qquad (17)$$

where we define

$$x_{ij} \equiv \Delta m_{ij}/\Gamma \ , \qquad (18)$$

in analogy to the variables x_d and x_s in B physics. The cross section $\sigma_0 = \sigma(f_1 f_2 \to e^+ X \tilde{e}^-) B(\tilde{e}^- \to e^- Y)$ is the analogous cross section in the absence of flavor violation, where

again e here represents any one of the available lepton flavors. The sum is to be taken over all slepton generations that may be produced on-shell. Note that $S_{\alpha\beta}$ is real, since $A_{ij} = A_{ji}^*$. If the mass splittings are much larger than the widths, x_{ij} is large and $A_{ij} \approx 0$ for $i \neq j$, and so only terms with $i = j$ contribute. However, for $x_{ij} \lesssim 1$, interference terms play an important role.

The analysis above assumes that slepton flavor violation arises from single slepton production. We now consider the case of correlated slepton pair production. Such production may occur at hadron colliders when sleptons are created in Drell-Yan production, but typically, these cross sections are too small to be of interest in this study. At e^+e^- colliders, however, the dominant slepton production mechanism is slepton pair production through s-channel photon and Z processes and t-channel neutralino exchange. These processes are then $e^+e^- \to \tilde{e}_i^+\tilde{e}_j^- \to e_\alpha^+Xe_\beta^-Y$, where X and Y are again m- and n-body final states, respectively.[4] For this process, the most general amplitude is

$$
\mathcal{M}_{\alpha\beta}^{\text{pair}} = \sum_i \mathcal{M}_P^s \frac{i}{p^2 - m_i^2 + im\Gamma} W_{i\alpha} \mathcal{M}_D^+ \frac{i}{q^2 - m_i^2 + im\Gamma} W_{i\beta}^* \mathcal{M}_D^-
$$
$$
+ \sum_{jk} \mathcal{M}_P^t W_{j\alpha} \frac{i}{q^2 - m_j^2 + im\Gamma} W_{j1}^* \mathcal{M}_D^+ W_{k1} \frac{i}{q^2 - m_k^2 + im\Gamma} W_{k\beta}^* \mathcal{M}_D^- \ , \tag{19}
$$

where $\mathcal{M}_P^s$ and $\mathcal{M}_P^t$ are the s- and t-channel production amplitudes for $e^+e^- \to \tilde{e}^+\tilde{e}^-$, and $\mathcal{M}_D^+$ and $\mathcal{M}_D^-$ are the decay amplitudes for $\tilde{e}^+ \to e^+X$ and $\tilde{e}^- \to e^-Y$. The cross section is then

$$
\sigma_{\alpha\beta}^{\text{pair}} = \int \frac{(2\pi)^4}{2\hat{s}} \left[|\mathcal{M}_P^s|^2 \sum_{il} W_{i\alpha} W_{i\beta}^* W_{l\alpha}^* W_{l\beta} A_{il}(p^2) A_{il}(q^2) \right.
$$
$$
+ \left(\mathcal{M}_P^s \mathcal{M}_P^{t*} \sum_{imn} W_{i\alpha} W_{i\beta}^* W_{m\alpha}^* W_{m1} W_{n1}^* W_{n\beta} A_{im}(p^2) A_{in}(q^2) + \text{h.c.} \right)
$$
$$
\left. + |\mathcal{M}_P^t|^2 \sum_{jkmn} W_{j\alpha} W_{j1}^* W_{k1} W_{k\beta}^* W_{m\alpha}^* W_{m1} W_{n1}^* W_{n\beta} A_{jm}(p^2) A_{kn}(q^2) \right]
$$
$$
\times dp^2 dq^2 d\Phi_2 (2\pi)^3 \left| \mathcal{M}_D^+ \right|^2 d\Phi_{m+1} (2\pi)^3 \left| \mathcal{M}_D^- \right|^2 d\Phi_{n+1} \ . \tag{20}
$$

Letting σ_0^{ss}, σ_0^{st}, and σ_0^{tt} denote the products of $B(\tilde{e}^+ \to e^+X)B(\tilde{e}^- \to e^-Y)$ and the flavor-conserving "cross sections" corresponding to production amplitudes $|\mathcal{M}_P^s|^2$, $\mathcal{M}_P^s\mathcal{M}_P^{t*}$, and $|\mathcal{M}_P^t|^2$, respectively, we find

$$
\sigma_{\alpha\beta}^{\text{pair}} = S_{\alpha\beta}^{ss}\sigma_0^{ss} + \left(S_{\alpha\beta}^{st}\sigma_0^{st} + \text{h.c.} \right) + S_{\alpha\beta}^{tt}\sigma_0^{tt}
$$
$$
S_{\alpha\beta}^{ss} = \sum_{ij} W_{i\alpha} W_{i\beta}^* W_{j\alpha}^* W_{j\beta} A_{ij}^2
$$
$$
S_{\alpha\beta}^{st} = \sum_{ijk} W_{i\alpha} W_{i\beta}^* W_{j\alpha}^* W_{j1} W_{k1}^* W_{k\beta} A_{ij} A_{ik}
$$
$$
S_{\alpha\beta}^{tt} = S_{\alpha 1} S_{1\beta} \ . \tag{21}
$$

---

[4]For simplicity, throughout this section, we consider only $\tilde{l}_L\tilde{l}_L$ or $\tilde{l}_R\tilde{l}_R$ production. Associated production of $\tilde{l}_L\tilde{l}_R$ is also possible through t-channel neutralino exchange.

$S^{tt}_{\alpha\beta}$ decomposes into a product of single slepton factors $S_{\alpha\beta}$, as expected for uncorrelated production from two vertices.

The general expressions for $\sigma_{\alpha\beta}$ and $\sigma^{\text{pair}}_{\alpha\beta}$ given in Eqs. (17) and (21) may be applied to a number of interesting scenarios. In particular, let us consider the case where

$$W = \begin{pmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix} \, , \tag{22}$$

and both $\tilde{e}$ and $\tilde{\mu}$ may be produced on-shell. Substituting this W into Eq. (17) and letting

$$\chi_{ij} \equiv \frac{x^2_{ij}}{2(1 + x^2_{ij})} \, , \tag{23}$$

in analogy to the parameters χ_d and χ_s in B physics, we find

$$S_{12} = 4\chi_{12} \sin^2\theta \cos^2\theta \, , \tag{24}$$

reproducing the 2 generation flavor-violating result given in Ref. [15]. The pair production case is more complicated, but the final results are

$$S^{ss}_{12} = 4\chi_{12}(3 - 4\chi_{12}) \sin^2\theta \cos^2\theta \tag{25}$$

$$Re\left[S^{st}_{12}\right] = 2\chi_{12}(3 - 4\chi_{12}) \sin^2\theta \cos^2\theta \tag{26}$$

$$S^{tt}_{12} = S_{11}S_{12} = S_{12}(1 - S_{12}) \, . \tag{27}$$

These quantities are unchanged if $\tilde{\tau}$ pairs may also be produced.

Finally, we derive expressions for CP-violating asymmetries. In the presence of CP violation, the cross sections $\sigma_{\alpha\beta} = \sigma(f_1 f_2 \to e^+_\alpha X e^-_\beta Y)$ and $\sigma_{\beta\alpha} = \sigma(f_1 f_2 \to e^+_\beta X e^-_\alpha Y)$ are no longer equal. For the case of single slepton production, it is easy to show that, following the analysis above,

$$\Delta_{\alpha\beta} \equiv S_{\alpha\beta} - S_{\beta\alpha} = \frac{\sigma_{\alpha\beta} - \sigma_{\beta\alpha}}{\sigma_0} = -4 \sum_{i<j} Im\left[W_{i\alpha} W^*_{i\beta} W^*_{j\alpha} W_{j\beta}\right] Im\left[A_{ij}\right] \, . \tag{28}$$

Again, the sum is over sleptons that may be produced on-shell. We therefore reproduce the familiar result that CP violation requires the presence of at least two amplitudes that differ both in their CP-odd ("weak") phases and their CP-even ("strong") phases. The W-dependent part may be written as

$$Im\left[W_{i\alpha} W^*_{i\beta} W^*_{j\alpha} W_{j\beta}\right] = \tilde{J} \sum_{k\gamma} \varepsilon_{ijk} \varepsilon_{\alpha\beta\gamma} \equiv \tilde{J}^{ij}_{\alpha\beta} \, , \tag{29}$$

where $\tilde{J}$ is the supersymmetric analogue to the Jarlskog invariant introduced in Sec. II. All single slepton CP-violating quantities are given in terms of $\tilde{J}$, the single CP-violating invariant that may be formed from one 3×3 slepton mixing matrix. The CP-violating single slepton cross section asymmetry is then

$$\Delta_{\alpha\beta} = -4 \sum_{i<j} \tilde{J}^{ij}_{\alpha\beta} Im[A_{ij}] = 4 \sum_{i<j} \tilde{J}^{ij}_{\alpha\beta} \frac{x_{ij}}{1 + x^2_{ij}} \, . \tag{30}$$

For all $\alpha \neq \beta$, the $\Delta_{\alpha\beta}$ are equal up to a sign, and so we see that there is only one CP-violating quantity, and the $e\mu$, $\mu\tau$, and τe asymmetries are all measurements of the same underlying CP-violating parameter.

For pair production, the asymmetry is

$$\sigma_{\alpha\beta}^{\text{pair}} - \sigma_{\beta\alpha}^{\text{pair}} = \Delta_{\alpha\beta}^{ss}\sigma_0^{ss} + \left(\Delta_{\alpha\beta}^{st}\sigma_0^{st} + \text{h.c.}\right) + \Delta_{\alpha\beta}^{tt}\sigma_0^{tt} \,, \tag{31}$$

where

$$\Delta_{\alpha\beta}^{ss} \equiv S_{\alpha\beta}^{ss} - S_{\beta\alpha}^{ss} = -4\sum_{i<j}\widetilde{J}_{\alpha\beta}^{ij}Im[A_{ij}^2] \tag{32}$$

$$\Delta_{\alpha\beta}^{st} \equiv S_{\alpha\beta}^{st} - S_{\beta\alpha}^{st} = 2i\sum_{ijk}Im[W_{i\alpha}W_{i\beta}^*W_{j\alpha}^*W_{j1}W_{k1}^*W_{k\beta}]A_{ij}A_{ik} \tag{33}$$

$$\Delta_{\alpha\beta}^{tt} \equiv S_{\alpha\beta}^{tt} - S_{\beta\alpha}^{tt} = S_{\alpha1}S_{1\beta} - S_{\beta1}S_{1\alpha} = \Delta_{1\beta}S_{1\alpha} - \Delta_{1\alpha}S_{1\beta} \,. \tag{34}$$

The pair asymmetry must also be proportional to $\widetilde{J}$, and it is clear from the expressions above that $\Delta_{\alpha\beta}^{ss}$ and $\Delta_{\alpha\beta}^{tt}$ are. For $\Delta_{\alpha\beta}^{st}$, this dependence may also be made manifest, as is shown in Appendix B.

B. Oscillation derivation

In this section, we show how the above results derived directly from field theory can be reproduced using the more familiar language of flavor oscillations. Suppose that at time $t = 0$, we produce e_α^+ in association with a *gauge* eigenstate slepton $\tilde{e}_\alpha^-$. We denote this initial state by $|\psi(0)\rangle = |\alpha^-\rangle$. This state then oscillates, and the amplitude for the slepton to decay into e_β^- is

$$A_{\alpha^-\to\beta^-}(t) = \langle\beta^-|e^{-imt}e^{-\Gamma t/2}|\alpha^-\rangle = \sum_i W_{i\alpha}W_{i\beta}^*e^{-im_i t}e^{-\Gamma t/2} \,, \tag{35}$$

where the sum is over all generations.[5] The corresponding amplitude for anti-sleptons, $A_{\alpha^+\to\beta^+}(t)$, is the same with $W \leftrightarrow W^*$. The probability that e_β^- is produced between t and $t + dt$ is $\Gamma dt|A_{\alpha^-\to\beta^-}(t)|^2$, so the total probability of producing an $e_\alpha^+e_\beta^-$ final state is (throughout we use $\int_0^\infty \Gamma dt e^{-i(m_i-m_j)t}e^{-\Gamma t} = \Gamma/(\Gamma + i(m_i - m_j)) \equiv A_{ij}$)

$$P_{\alpha^-\to\beta^-} = \int_0^\infty \Gamma dt|A_{\alpha^-\to\beta^-}(t)|^2 = \sum_{ij}W_{i\alpha}W_{i\beta}^*W_{j\alpha}^*W_{j\beta}A_{ij} \,, \tag{36}$$

which reproduces Eq. (17) in the previous section.

---

[5]The oscillation picture is applicable only if all the sleptons are nearly degenerate. Otherwise, the amplitude for producing the initial gauge eigenstate slepton is ill-defined. For instance, suppose that the gauge eigenstate is a superposition of a mass eigenstate beneath production threshold and a mass eigenstate above threshold. The amplitude for producing the gauge eigenstate slepton is clearly not well-defined in this case.

Turning now to the case where correlated sleptons are produced from s- and t-channel diagrams, the initial state is

$$|\psi(0)\rangle = \mathcal{M}_P^s \sum_\gamma |\gamma^+, \gamma^-\rangle + \mathcal{M}_P^t |1^+, 1^-\rangle \ , \tag{37}$$

where $\mathcal{M}_P^s$ and $\mathcal{M}_P^t$ are the s- and t-channel amplitudes, respectively. The amplitude for producing e_α^+ at time t_1 and e_β^- at time t_2 is

$$A_{\alpha^+\beta^-}^{\text{pair}}(t_1, t_2) = \mathcal{M}_P^s \sum_\gamma A_{\gamma^+\rightarrow\alpha^+}(t_1) A_{\gamma^-\rightarrow\beta^-}(t_2) + \mathcal{M}_P^t A_{1^+\rightarrow\alpha^+}(t_1) A_{1^-\rightarrow\beta^-}(t_2) \ . \tag{38}$$

However,

$$\sum_\gamma A_{\gamma^+\rightarrow\alpha^+}(t_1) A_{\gamma^-\rightarrow\beta^-}(t_2) = \sum_{\gamma ij} W_{i\gamma}^* W_{i\alpha} W_{j\gamma} W_{j\beta}^* e^{-im_i t_1} e^{-\Gamma t_1/2} e^{-im_j t_2} e^{-\Gamma t_2/2}$$

$$= \sum_i W_{i\alpha} W_{i\beta}^* e^{-im_i t_1} e^{-\Gamma t_1/2} e^{-im_i t_2} e^{-\Gamma t_2/2} \ , \tag{39}$$

so

$$A_{\alpha^+\beta^-}^{\text{pair}}(t_1, t_2) = \mathcal{M}_P^s \sum_i W_{i\alpha} W_{i\beta}^* e^{-im_i t_1} e^{-\Gamma t_1/2} e^{-im_i t_2} e^{-\Gamma t_2/2}$$

$$+ \mathcal{M}_P^t A_{1^+\rightarrow\alpha^+}(t_1) A_{1^-\rightarrow\beta^-}(t_2) \ . \tag{40}$$

Therefore, the total probability of producing $e_\alpha^+ e_\beta^-$ is

$$P_{\alpha^+\beta^-}^{\text{pair}} = \int_0^\infty \Gamma dt_1 \Gamma dt_2 |A_{\alpha^+\beta^-}^{\text{pair}}(t_1, t_2)|^2$$

$$= |\mathcal{M}_P^s|^2 \sum_{ij} W_{i\alpha} W_{i\beta}^* W_{j\alpha}^* W_{j\beta} A_{ij}^2$$

$$+ \left(\mathcal{M}_P^s \mathcal{M}_P^{t*} \sum_{ijk} W_{i\alpha} W_{i\beta}^* W_{j1} W_{j\alpha}^* W_{k1}^* W_{k\beta} A_{ij} A_{ik} + \text{h.c.} \right)$$

$$+ |\mathcal{M}_P^t|^2 P_{1^+\rightarrow\alpha^+} P_{1^-\rightarrow\beta^-} \ , \tag{41}$$

in agreement with Eq. (21) of the previous section.

IV. SCENARIOS WITH OBSERVABLE CP VIOLATION

In the previous section, we derived formulas for CP-violating cross sections, which depended on the slepton mixing angles and mass splittings. In this section, we examine these expressions and determine what conditions must be met to yield promising CP-violating signals. We will find that CP violation in the lepton sector may be large in two scenarios. These two scenarios will be used in the following sections to determine the typical reaches in parameter space of collider experiments.

The CP-violating difference in cross sections was derived in Sec. III A for both single and correlated pair slepton production. Here we will analyze the simpler case of single slepton production, as the conclusions are based on general arguments that apply in both

cases. Recall that the CP-violating difference in cross sections in the case of single slepton production is given by

$$\Delta_{\alpha\beta} = \frac{\sigma_{\alpha\beta} - \sigma_{\beta\alpha}}{\sigma_0} = 4 \sum_{i<j} \tilde{J}^{ij}_{\alpha\beta} \frac{x_{ij}}{1 + x_{ij}^2} \ . \tag{42}$$

In the typical case where all three slepton generations are produced,

$$\Delta_{e\mu} = \Delta_{\mu\tau} = \Delta_{\tau e} = 4\tilde{J} \left(\frac{x_{12}}{1 + x_{12}^2} + \frac{x_{23}}{1 + x_{23}^2} + \frac{x_{31}}{1 + x_{31}^2} \right) \ . \tag{43}$$

For a significant CP-violating signal, both $\tilde{J}$ and the kinematic part depending on the mass splittings $x_{ij} \equiv \Delta m_{ij}/\Gamma$ must be fairly large.

Concentrating first on $\tilde{J}$, we recall that in the parametrization

$$W = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} \ , \tag{44}$$

where $s_{ij} = \sin\theta_{ij}$ and $c_{ij} = \cos\theta_{ij}$[19, 20], the supersymmetric Jarlskog invariant is $\tilde{J} = s_{12}s_{13}s_{23}c_{12}c_{13}^2 c_{23} \sin\delta$, with maximal value $\frac{1}{6\sqrt{3}} \approx 0.096$. Large CP violation therefore requires large three generation mixing and a large CP-violating angle. The CP-violating signal for single slepton production is completely determined by $\tilde{J}$. However, to evaluate backgrounds, and to compute the CP-violating signal at the NLC from correlated slepton pair production, we need to know the individual angles θ_{ij} of the W matrix, rather than just the combination $\tilde{J}$. For the numerical calculations of the following sections, we choose

$$\theta \equiv \theta_{12} = \theta_{23} = \theta_{13} \ . \tag{45}$$

In this parametrization, $\tilde{J} = \sin^3\theta \cos^4\theta \sin\delta$, and for $\sin\theta = \sqrt{\frac{3}{7}}$ ($\sin 2\theta = \frac{4\sqrt{3}}{7} \approx 0.99$), $\tilde{J}$ attains its maximal value $\frac{48\sqrt{3}}{343\sqrt{7}} \approx 0.092$. This simplifying assumption therefore captures nearly all of the available range of $\tilde{J}$.

Turning now to the kinematic factor, it is clear that it may be large only if at least one x_{ij} is near 1, since each term is maximal for $x_{ij} = 1$, and drops off for larger and smaller x_{ij}. In addition, since $x_{12} + x_{23} + x_{31} = 0$, the signal is suppressed if any pair of sleptons is highly degenerate, in accord with Eq. (7). There are therefore two possible scenarios with large CP violation: either (I) one x_{ij} is $\mathcal{O}(1)$ and the other two are much larger than 1, or (II) all three x_{ij} are roughly $\mathcal{O}(1)$. As will be discussed in Sec. VIII, the three sleptons must have some degree of degeneracy ($x_{ij} \lesssim 10 - 100$) to ensure that the large mixing angles of W do not induce too large a branching ratio for $\mu \to e\gamma$. For concreteness, we assume that $m_1 > m_2 > m_3$ and in each scenario we parametrize the mass splittings in terms of a single variable x:

$$\text{Scenario I}: \ x \equiv x_{12} \ , \quad 0.1 \le x \le 10 \ , \quad x_{23} = 10 \tag{46}$$

$$\text{Scenario II}: \ x \equiv x_{12} = x_{23} \ , \quad 0.1 \le x \le 10 \ . \tag{47}$$

With these parametrizations, in both Scenarios I and II the signal and background cross sections depend on only three parameters: x, θ, and δ.

In summary, we can identify the necessary features of the slepton mass matrices if a CP-violating slepton oscillating signal is to be visible at future colliders:

1. The W matrices must have large CP- and flavor-violating angles.

2. The two sleptons of highest degeneracy should have Δm of order Γ, giving x of order unity.

3. All three sleptons must have some degree of degeneracy, so that $x_{ij} \lesssim 10 - 100$.

There conditions may be satisfied in two regions of parameter space, which we denote Scenarios I and II.

V. ANALYTIC ESTIMATE OF THE LHC AND NLC SIGNALS

CP violation in the slepton sector may be detected at colliders by considering a sample of dilepton events and observing a statistically significant asymmetry

$$
A \equiv \frac{S}{B} = \frac{N_{e_\alpha^+ e_\beta^-} - N_{e_\beta^+ e_\alpha^-}}{N_{e_\alpha^+ e_\beta^-} + N_{e_\beta^+ e_\alpha^-}} \; , \tag{48}
$$

where α and β are generational indices. The numerator is the CP-violating signal. The denominator, the "background," includes all events that pass the cuts, and includes contributions from the flavor-changing slepton events themselves, as well as other backgrounds. To maximize the statistical significance of the asymmetry, we must isolate a large and pure sample of slepton events.

In the following section, we will present precise calculations of the signal and backgrounds at the LHC and NLC. In this section, however, we give rough order-of-magnitude estimates, with a view to gaining a general understanding of what ranges of flavor parameters may be probed. The validity of these estimates is justified for certain SUSY parameters by the detailed analysis in the following section.

For optimal mass splittings, the signal is roughly $S \approx \sigma_S \varepsilon \widetilde{J} L$. L is the total integrated luminosity, and σ_S is the production cross section of the relevant superparticles — squarks and gluinos for hadron colliders, and sleptons for lepton colliders (see below). The efficiency ε is the fraction of such superparticle events that end up in the final sample of slepton events (for any one of the available slepton generations) and therefore includes all branching ratios and kinematic cut efficiencies. Denoting the background by $B = \sigma_B L$, we find that a 3σ signal requiring $S > 3\sqrt{B}$ implies that a CP-violating signal may be observed for

$$
\widetilde{J} \gtrsim \frac{3}{\sigma_S \varepsilon} \sqrt{\frac{\sigma_B}{L}} \; , \tag{49}
$$

where the cross sections are in fb, and L is in fb^{-1}.

At the LHC with $\sqrt{s} = 14$ TeV, the most promising source of sleptons is in cascade decays $(\tilde{g} \to) \tilde{q}_L \to \tilde{\chi}_2^0 \to \tilde{l} \to \tilde{\chi}_1^0$. Such cascades may be prominent when the charginos and neutralinos are gaugino-like, and $m_{\tilde{l}} < m_{\tilde{\chi}_2^0}$. The efficiency is typically $\varepsilon \sim 1\%$. The background is dominated by flavor-changing slepton events and gluino/squark pair production leading to two leptonically-decaying charginos. Assuming $L = 100$ fb^{-1} and $m_{\tilde{g}}, m_{\tilde{q}} \approx 300$ (700) GeV, the cross sections of gluino/squark pair production and the background we may

14

expect are $\sigma_S \sim 1000\,(10)$ pb and $\sigma_B \sim 1000\,(10)$ fb, and from Eq. (49) we find that a 3σ signal requires $\widetilde{J} \gtrsim 10^{-3}\,(10^{-2})$.

At the LHC the efficiency ε is a product of many factors. Of the total strong production cross section, only events containing $\tilde{q}_L$ may produce sleptons, as right-handed squarks decay directly to $q\tilde{\chi}_1^0$. In contrast, $B(\tilde{q}_L \to q\tilde{\chi}_2^0(q\tilde{\chi}_1^+)) = 1/3\,(2/3)$, since $B(\tilde{q}_L \to q\tilde{\chi}_1^0)$ is suppressed relative to these by hypercharge couplings. Decays to charginos are also not useful, as the lepton flavor-changing information is carried off in neutrinos.[6] Another reduction in efficiency arises in the branching fraction $B(\tilde{\chi}_2^0 \to e_\alpha^+ e_\beta^- \tilde{\chi}_1^0)$. Assuming for simplicity that the produced sleptons are left-handed,

$$B(\tilde{\chi}_2^0 \to e_\alpha^+ e_\beta^- \tilde{\chi}_1^0) = \frac{S_{\alpha\beta}}{2N} \; , \tag{50}$$

where $N = 3$ is the number of generations of sleptons to which $\tilde{\chi}_2^0$ can decay, and the factor of $1/2$ accounts for the fact that we include only the decays to charged sleptons. Finally, there is the efficiency of kinematic cuts, which is typically $\gtrsim 10\%$.

At the NLC sleptons are produced directly in pairs. The signal is therefore not degraded by branching ratios, and typical efficiencies for the kinematic cuts are $\varepsilon \approx 30\%$. For correlated pair production, the signal is no longer simply proportional to $\widetilde{J}$. However, for a rough estimate, we can again apply the above analysis. Assuming 200 GeV sleptons are produced at a $\sqrt{s} = 500$ GeV collider with $L = 50$ fb^{-1}, the signal cross section is $\sigma_S \sim 300$ fb. The background is largely flavor-violating slepton events and so varies with $\widetilde{J}$; in the regions of $\widetilde{J}$ we can probe, $\sigma_B \sim 10$ fb. A 3σ signal requires $\widetilde{J} \gtrsim 10^{-2}$. However, for $L = 500$ fb^{-1}, we expect to be able to probe to the $\widetilde{J} \sim \mathcal{O}(10^{-3})$ level.

In this paper, we will concentrate on signals at the LHC and NLC, because, as seen from the above rough estimates, these machines offer the possibility of probing $\widetilde{J}$ to the level of 10^{-3}, well below the maximum of value of $\frac{1}{6\sqrt{3}}$. However, before analyzing these colliders' capabilities in detail, it is worthwhile to consider whether such signals may be seen at LEP II or the Tevatron. At LEP II with $\sqrt{s} = 190$ GeV, for slepton masses of 80 GeV, just above the current bound, σ_S may be ~ 500 fb, depending on the neutralino masses. Systems of cuts for flavor-conserving slepton events[23] may be adopted for flavor-violating events[15] and typically give values of $\varepsilon \approx 40\% - 60\%$ and $\sigma_B \approx 10 - 100$ fb. Assuming a total integrated luminosity of 500 pb^{-1}, we found in Ref. [15] that lepton flavor violation could indeed be observed for large mixing angles. Substituting these ranges of quantities into Eq. (49), we find the requirement $\widetilde{J} \gtrsim 0.04 - 0.2$; an observable CP-violating signal may therefore be possible for near-maximal $\widetilde{J}$. At Run II and future upgrades of the Tevatron with $\sqrt{s} = 2$ TeV, if squark and gluino masses are in the region of 200 GeV, the total strong SUSY production cross section is $\sigma_S \sim 10$ pb. Given $L \sim 2 - 30$ fb^{-1}, the number of signal events is roughly only an order of magnitude below the number of events at the LHC with 700 GeV gluinos and squarks. It is therefore possible that flavor-violating slepton signals could be observed, and possibly even CP-violating asymmetries could be seen for

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[6]For the case that $\tilde{\chi}_1^\pm$ can decay to all three generations of sleptons, the branching fraction for decays $\tilde{\chi}_1^- \to e_\alpha^- \tilde{\chi}_1^0$ and $\tilde{\chi}_1^+ \to e_\alpha^+ \tilde{\chi}_1^0$ are both proportional to $\sum_\beta S_{\alpha\beta} = 1$, and so do not contribute to a CP asymmetry.

near-maximal $\tilde{J}$. However, such possibilities depend crucially on many factors, such as the effectiveness of cuts to isolate the slepton signal from $t\bar{t}$ and other backgrounds, and will not be explored in detail here.

VI. EXPERIMENTAL POSSIBILITIES FOR THE LHC AND NLC

In this section, we will consider the possibilities for detecting slepton CP violation in two experimental environments: the LHC at CERN and the NLC, a proposed high energy linear e^+e^- collider. The sensitivity of these two colliders to slepton CP violation is dependent on many of the underlying SUSY parameters. These parameters set the masses and branching fractions that determine the number of sleptons produced. In addition, they fix the rates and kinematic distributions of the many background processes, which determine the extent to which a pure slepton signal may be isolated. A comprehensive analysis would require a scan of parameter space and optimization of cuts for each parameter set and is beyond the scope of this study.

Instead, to gain an understanding of what sensitivities are typically achievable and what aspects of the sparticle spectrum are most important, we will limit our analysis by considering a particular set of SUSY parameters for each of the two colliders. This choice will determine our signal and background rates, and will also allow us to use systems of cuts that have been developed previously. After presenting the results, we will also discuss the implications for our analysis of variations away from these parameters.

A. LHC

The LHC is a pp collider with $\sqrt{s} = 14$ TeV and luminosity $\mathcal{L} \sim 10$–100 fb^{-1}/yr. At hadron colliders, sleptons may be produced (1) directly in pairs through Drell-Yan processes, (2) singly in cascade decays of electroweak gauginos that are produced in pairs through Drell-Yan processes, and (3) singly in squark and gluino cascade decays. While, for certain parameters, sleptons may be discovered through the first two production mechanisms[24], the event rates are in general too low for the precision studies considered here. We therefore examine the third possibility. To do so, we begin by considering a point in parameter space where a promising number of sleptons are produced in gluino and squark cascades. This point, analyzed in Refs. [3, 4], is given by minimal supergravity boundary conditions[7]

$$m_0 = 100 \text{ GeV}, \ m_{1/2} = 300 \text{ GeV}, \ A_0 = 300 \text{ GeV}, \ \tan\beta = 2.1, \ \mu > 0 \, , \qquad (51)$$

with resulting weak scale SUSY parameters

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[7]Of course, strictly in minimal supergravity, all the sleptons are exactly degenerate at M_{Pl} and lepton flavor is conserved. However, given that all the sleptons must be nearly degenerate to produce a CP-violating signal, we simply use the minimal supergravity case analyzed in Refs. [3, 4] to obtain the relevant superpartner mass spectrum and branching ratios, which are unaffected by the small slepton non-degeneracies and mixing angles required for our CP-violating signal.

$$M_1 = 126 \text{ GeV}, \ M_2 = 252 \text{ GeV}, \ M_3 = 752 \text{ GeV}, \ \mu = 479 \text{ GeV} . \tag{52}$$

The more relevant masses, cross sections, and branching fractions are given in Tables I, II, and III. At this parameter point, a large number of sleptons are produced through the cascades $(\tilde{g} \to) \tilde{q}_L \to \tilde{\chi}_2^0 \to \tilde{l}_R \to \tilde{\chi}_1^0$. The large rate results from the fact that in minimal supergravity, $\tilde{\chi}_2^0 \approx \tilde{W}^3$, and $\tilde{\chi}_1^0 \approx \tilde{B}$, and so decays $\tilde{q}_L \to \tilde{\chi}_1^0$, though favored by phase space over $\tilde{q}_L \to \tilde{\chi}_2^0$, are highly suppressed by hypercharge couplings. Note that left-handed sleptons are heavier than $\tilde{\chi}_2^0$ and so are almost never produced.

Cuts to isolate such slepton events are presented in Ref. [4]. As noted below Eq. (30), the CP-violating cross section differences $\Delta_{\alpha\beta}$ in the $e\mu$, $\mu\tau$, and τe channels are all identical. Here we will consider the subset of slepton events with $e^{\pm}\mu^{\mp}$ final states and try to measure the asymmetry in this event sample. Other asymmetries, for example, in $e^{\pm}\pi^{\mp}$ events, where the pion results from τ decay, also could be used. These asymmetries suffer from τ branching fractions, and require that hadronic jets not be misidentified as τ decay products. They are, however, free of backgrounds from $\tilde{e}$ and $\tilde{\mu}$ production, whereas the $e^{\pm}\mu^{\mp}$ asymmetry suffers from $\tilde{\tau} \to \tau \to l$ backgrounds, as discussed below. Of course, as all of these cross section differences $\Delta_{\alpha\beta}$ are predicted to be equal, they could ultimately be combined to yield the most powerful measurement of slepton CP violation.

To quantify the statistical significance, we divide the signal and background into two parts and define

$$A \equiv \frac{S}{B} = \frac{S^{\tilde{l}} + S^0}{B^{\tilde{l}} + B^0} , \tag{53}$$

where

$$S^{\tilde{l}} = N^{\tilde{l}}_{e^+_\alpha e^-_\beta} - N^{\tilde{l}}_{e^+_\beta e^-_\alpha} , \qquad S^0 = N^0_{e^+_\alpha e^-_\beta} - N^0_{e^+_\beta e^-_\alpha} , \tag{54}$$

$$B^{\tilde{l}} = N^{\tilde{l}}_{e^+_\alpha e^-_\beta} + N^{\tilde{l}}_{e^+_\beta e^-_\alpha} , \qquad B^0 = N^0_{e^+_\alpha e^-_\beta} + N^0_{e^+_\beta e^-_\alpha} . \tag{55}$$

The superscripts "$\tilde{l}$" and "0" denote events arising from slepton production and events from other sources, respectively. The statistical uncertainty in A is given by $\sigma_A^2 = (1 - A^{0\,2})/B$, where $A^0 = S^0/B$ is the asymmetry in the absence of slepton CP violation. If there is no asymmetry from other sources, $A^0 = 0$, and an $N\sigma$ signal requires simply $|S|/\sqrt{B} > N$.

For hadron colliders, ignoring possible differences in l^+ and l^- detection efficiencies, the source of the CP-violating numerator of Eq. (53) is entirely slepton events, and so $S^0 = 0$. Since we are looking at $e^{\pm}\mu^{\mp}$ events, these slepton events include events involving leptonically-decaying τ leptons. The total signal is therefore

$$S = S^{\tilde{l}} = (S_{12} - S_{21})\sigma_0 \varepsilon_{ll} L + B_\tau (S_{13} - S_{31} - S_{23} + S_{32})\sigma_0 \varepsilon_{l\tau} L , \tag{56}$$

where $l = e, \mu$, $B_\tau \equiv B(\tau \to l\nu\bar{\nu}) \approx 18\%$, and L is the integrated luminosity. The efficiencies for accepting ll and $l\tau$ slepton events are denoted by ε_{ll} and $\varepsilon_{l\tau}$, and these include all branching fractions, kinematic cut efficiencies, and detector acceptances. Note that the contributions from τ decays always reduce the signal S, since $S_{13} - S_{31} - S_{23} + S_{32} = -\Delta_{31} - \Delta_{23} = -2\Delta_{12} = -2(S_{12} - S_{21})$; these processes are therefore more dangerous than other backgrounds, which only dilute the asymmetry by increasing the background B.

17

The backgrounds include slepton events, as well as additional processes, and are given by

$$B^{\tilde{l}} = (\sigma_{ll} + \sigma_{l\tau} + \sigma_{\tau\tau})L \tag{57}$$

$$B^0 = (\sigma_{\text{SUSY}} + \sigma_{\text{SM}})L \ , \tag{58}$$

where

$$\sigma_{ll} = (S_{12} + S_{21})\sigma_0\varepsilon_{ll} \tag{59}$$

$$\sigma_{l\tau} = B_\tau(S_{13} + S_{31} + S_{23} + S_{32})\sigma_0\varepsilon_{l\tau} \tag{60}$$

$$\sigma_{\tau\tau} = 2B_\tau^2 S_{33}\sigma_0\varepsilon_{\tau\tau} \tag{61}$$

$$\sigma_{\text{SUSY}} = \sigma_{\tilde{\chi}_1^\pm \tilde{\chi}_1^\pm}\varepsilon_{\tilde{\chi}_1^\pm \tilde{\chi}_1^\pm} \tag{62}$$

$$\sigma_{\text{SM}} = \sigma_{tt}\varepsilon_{tt} + \sigma_{WW}\varepsilon_{WW} \ . \tag{63}$$

For each process, $\sigma\varepsilon$ denotes the cross section into opposite sign, unlike flavor dilepton events, again including all branching ratios, cuts, and detector efficiencies. The cross section σ_{ll} is the irreducible background resulting directly from slepton flavor violation, and $\sigma_{l\tau}$ ($\sigma_{\tau\tau}$) is the cross section of $e^\pm\mu^\mp$ events resulting from slepton production with one (two) leptonically-decaying τ lepton(s). Other than slepton events, the leading SUSY background process is events in which both gluino/squark cascade decays result in charginos, which then both decay leptonically. These cascades are of the form $(\tilde{g} \to)\tilde{q}_L \to \tilde{\chi}_1^\pm \to l$, and the cross section is denoted by σ_{SUSY} above. The leading SM backgrounds are $t\bar{t}$ and WW production, which together form σ_{SM}.

The number of opposite sign dilepton events passing all cuts has been plotted in Ref. [4]. From these results, one may determine the cross sections from the various sources after all cuts, being careful to remember that opposite sign, *like* flavor dilepton events resulting from $\tilde{\chi}_1^\pm \tilde{\chi}_1^\pm$, $t\bar{t}$, and WW events are included in the plot, but are not to be included in our background. For the sample of events in the range $0 < M_{ll} < 110$ GeV $= M_{ll}^{\text{max}}$, where $M_{ll}^{\text{max}} = m_{\tilde{\chi}_2^0} - m_{\tilde{\chi}_1^0}$ is the maximal value allowed in slepton events, $\sigma_0\varepsilon_{ll}, \sigma_0\varepsilon_{l\tau}, \sigma_0\varepsilon_{\tau\tau} \approx 80$ fb, $\sigma_{\tilde{\chi}_1^\pm \tilde{\chi}_1^\pm}\varepsilon_{\tilde{\chi}_1^\pm \tilde{\chi}_1^\pm} \approx 6$ fb, and $\sigma_{tt}\varepsilon_{tt} + \sigma_{WW}\varepsilon_{WW} \approx 2$ fb. Note that, although the chargino background depends on a number of additional SUSY parameters, its size may be determined by considering *same* sign dilepton events. In addition, since dileptons in all of the backgrounds have dilepton masses M_{ll} that may extend beyond $m_{\tilde{\chi}_2^0} - m_{\tilde{\chi}_1^0}$, the size of the backgrounds may be estimated by extrapolating from this region.

Given these results, for each of the scenarios specified above, the signal and background are completely determined by the parameters $x = \Delta m/\Gamma$, θ, and δ. In Fig. 1, we plot contours of constant CP-violating cross section S/L for Scenario I in the $(\sin 2\theta, \Delta m/\Gamma)$ plane for fixed $\sin\delta = 1$. For the point we are considering, $\Gamma(\tilde{l}_R \to l\tilde{\chi}_1^0) = 0.13$ GeV $= 8.1 \times 10^{-4}m$. We see that there may be a large CP-violating signal for $\Delta m \sim \mathcal{O}(\Gamma)$. In Fig. 2, we incorporate the effects of background and plot 3σ discovery contours in the same plane with $\sin\delta = 1$. The wide contour is the discovery limit for the point we have considered, given an integrated luminosity of 100 fb^{-1}, the luminosity expected for one detector in one year at high luminosity. (The other contours will be described below.) The behavior of the contours very near $\sin 2\theta = 1$ is an artifact of our parametrization; as noted in Sec. IV, $\tilde{J}$ reaches its maximum at $\sin 2\theta \approx 0.99$. The overall shape of the contour may be very roughly

understood by neglecting variations in the background and considering the behavior of the signal S. For $\Delta m \gg \Gamma$ ($x_{12} \gg 1$),

$$S \sim \Delta_{12} = 4 \sum_{i<j} \tilde{J}_{12}^{ij} \frac{x_{ij}}{1+x_{ij}^2} \approx 4\tilde{J} \frac{x_{12}}{1+x_{12}^2} \approx 4\tilde{J}/x_{12} \sim \theta^3/\Delta m \ , \tag{64}$$

whereas for $\Delta m \ll \Gamma$ ($x_{12} \ll 1$),

$$S \sim \Delta_{12} \approx 4\tilde{J} \frac{x_{12}}{1+x_{12}^2} \approx 4\tilde{J}x_{12} \sim \theta^3 \Delta m \ . \tag{65}$$

For $\Delta m \approx \Gamma$, the reach in $\sin 2\theta$ is maximal, and $\sin \theta = 0.23$ and a supersymmetric Jarlskog invariant $\tilde{J}$ as low as 0.01 may be probed. We note that, while CP violation requires general three generation mixing, large CP-violating effects do not require all three generations to be degenerate to within Γ.

If such a scenario were actually realized in nature, these results could be improved with optimized cuts. For example, for slepton events, M_{ll} peaks near its maximum, and so by considering only events with 80 GeV $< M_{ll} < 110$ GeV, the signal to background ratio is improved. Such a cut is also effective in removing the $\tau \to l$ backgrounds, and would lead to improvements in the results.

Far greater variations in these results arise if different SUSY parameters are considered. An obvious dependence is on the gluino and squark masses. For the example considered, $m_{\tilde{g}} = 767$ GeV, and $m_{\tilde{q}_L} = 662$ GeV. For lower masses, the cross sections for squark and gluino production increase rapidly, and, if the relevant branching fractions and cut efficiencies are not greatly altered, the sample of slepton events increases dramatically. Conversely, as the gluino and squark masses increase, the results deteriorate. To give an indication of the dependence of our results on these masses, we have also plotted 3σ discovery contours in Fig. 2 for scenarios with values of $m_{\tilde{g}} = m_{\tilde{q}}$ as labeled. For these contours, we make the naive assumption that the signal and background cross sections scale with $\sigma(\tilde{g}\tilde{g}) + \sigma(\tilde{q}\tilde{q}) + \sigma(\tilde{g}\tilde{q})$ relative to our prototype point. With this assumption, we see that in the favorable case of a light gluino with mass 300 GeV, a supersymmetric Jarlskog invariant as low as $\tilde{J} = 10^{-3}$ may be probed. Of course, branching fractions and efficiencies are also highly dependent on the various SUSY parameters, as will be discussed below, and in fact one generally expects the difficulty of isolating SUSY signals above SM background to increase as the superpartner masses decrease. These contours do, however, give an indication of the strong dependence on gluino and squark masses, and show that far stronger probes of $\tilde{J}$ may be possible if these masses are significantly lower.

In Fig. 3, we have plotted the analogous contours for Scenario II, the case of three generation near degeneracy. The reach in $\sin 2\theta$ is virtually unchanged, as is the behavior for $\Delta m \gg \Gamma$. For $\Delta m \ll \Gamma$,

$$S \sim \Delta_{12} = 4 \sum_{i<j} \tilde{J}_{12}^{ij} \frac{x_{ij}}{1+x_{ij}^2} = 4\tilde{J} \left[\frac{x_{12}}{1+x_{12}^2} + \frac{x_{23}}{1+x_{23}^2} + \frac{x_{31}}{1+x_{31}^2} \right] \sim \theta^3 x_{12}^3 \sim \theta^3 (\Delta m)^3 \ , \tag{66}$$

where the term linear in Δm vanishes since $x_{12} + x_{23} + x_{31} = 0$. For Scenario II, therefore, the signal is more strongly suppressed for small Δm than in Scenario I.

We have seen that the possibility of probing small CP-violating parameters at the LHC exists. However, as noted above, the size of the CP-violating signal is governed by a number

of factors, which we now discuss in greater detail. First, as is obvious from the figures, given a sample of slepton events, CP violation is only significant if Δm is within an order of magnitude or so of Γ. Classes of models in which this arises naturally will be discussed below in Sec. IX, and we therefore defer comments on the likelihood of this possibility.

In addition, the size of the slepton event sample is governed by several conditions on the sparticle masses and decay patterns. For this analysis, the strength of the LHC lies in the fact that leptonic events, which stand out above backgrounds, are produced with strong interaction rates. The power of this analysis therefore depends crucially on the number of on-shell sleptons produced in the decays of gluinos and squarks. We may discuss each step in the cascade $(\tilde{g} \rightarrow) \tilde{q}_L \rightarrow \tilde{\chi}_2^0 \rightarrow \tilde{l}$. To initiate this decay chain, it is of course important that there be large cross sections for gluinos and squarks; the strong dependence on their masses was discussed above. If we are in the gaugino region, where the lightest two neutralinos are gaugino-like, slepton events arise from $\tilde{q}_L$, as decays $(\tilde{g} \rightarrow) \tilde{q}_R \rightarrow \tilde{\chi}_1^0$ bypass sleptons. This event rate could be much larger if, for example, $m_{\tilde{q}_R} > m_{\tilde{g}} > m_{\tilde{q}_L}$, as then the branching fraction of $\tilde{g} \rightarrow \tilde{q}_L$ would be nearly 100%. In the gaugino region, it is also essential that $m_{\tilde{l}} < m_{\tilde{\chi}_2^0}$.[8] Possible decay modes of $\tilde{\chi}_2^0$ are $\tilde{\chi}_2^0 \rightarrow h\tilde{\chi}_1^0, Z\tilde{\chi}_1^0, \tilde{l}_R l, \tilde{l}_L l$. Neglecting obvious phase space considerations, if the decay $\tilde{\chi}_2^0 \rightarrow \tilde{l}_L$ is open, it will dominate, as all other decay modes are suppressed by mixing angles in the gaugino region. The slepton CP violation would then measure $\tilde{J}_L$ with very little contamination from $\tilde{J}_R$, and with large statistics from $B(\tilde{\chi}_2^0 \rightarrow \tilde{l}_L) \approx 100\%$. If the $\tilde{l}_L$ decay mode is closed, but the $\tilde{l}_R$ mode is open, we may still measure $\tilde{J}_R$. However, this measurement is greatly degraded if the decay $\tilde{\chi}_2^0 \rightarrow h$ is open, as this is less suppressed by mixing angles in the gaugino region.

In the minimal supergravity example considered above, the total cross section for gluino and squark pair production was 15.3 pb, and the slepton cross section in the final event sample was 240 fb. Thus, including branching ratio and kinematic cut efficiencies, only 1.6% of the total strongly interacting sparticle production was available for use in our analysis. In that example, $\tilde{\chi}_2^0 \rightarrow \tilde{l}_L$ decays were closed and decays $\tilde{\chi}_2^0 \rightarrow h$ were open, and so $B(\tilde{\chi}_2^0 \rightarrow \tilde{l}_R)$ was only 36%. In a more favorable scenario in which either $m_{\tilde{l}_L} < m_{\tilde{\chi}_2^0}$ or $m_{\tilde{l}_R} < m_{\tilde{\chi}_2^0} < m_h$, the branching ratio to sleptons could be nearly 3 times larger. Clearly the results of the previous analysis would be noticeably more powerful in such a scenario. For comparison, we present in Fig. 4 the results of a scenario in which the event sample is increased relative to Fig. 3 by a factor of 10. Such an improvement could come from improved branching ratios, the optimized cuts discussed above, the combination of all lepton asymmetries, or by increasing the assumed integrated luminosity to include multi-year event samples and both detectors.

Of course, the scenario we considered was already optimistic in the sense that sleptons were in fact produced in gluino and squark cascades. For $m_{\tilde{l}} < m_{\tilde{\chi}_2^0}$, it is likely that some probe of slepton CP violation will be possible, although the sensitivity depends on the many issues discussed above. However, in the gaugino region, if $m_{\tilde{l}} > m_{\tilde{\chi}_2^0}$, such an analysis

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[8]If the parameters did not lie in the gaugino region, that is, if $|\mu|$ were not much greater than $M_{1,2}$, the heavier two neutralinos would also have significant gaugino components, making decays $\tilde{g}, \tilde{q} \rightarrow \tilde{\chi}_{3,4}^0 \rightarrow \tilde{l}$ possible, and this requirement would not be necessary.

will be extremely difficult. In this case, there is likely to be little opportunity for high precision studies of sleptons at the LHC. Sleptons may be produced in large numbers at the NLC, however, and this production is direct, that is, independent of various cascades and branching ratios. We therefore now turn to a scenario in which slepton CP violation is undetectable at the LHC and consider the sensitivity of similar probes at the NLC.

B. NLC

At e^+e^- colliders, sleptons are dominantly produced directly through correlated pair production. Relative to the situation at hadron colliders, slepton production therefore is not as heavily influenced by details of the sparticle spectrum and branching ratios. We will see that, if slepton pairs are kinematically accessible, e^+e^- colliders offer a robust opportunity to measure slepton CP violation. Sleptons pairs $\tilde{l}_L\tilde{l}_L$ and $\tilde{l}_R\tilde{l}_R$ may be produced through s-channel photon and Z diagrams, and also through t-channel neutralino exchange. CP asymmetries in $\tilde{l}_L\tilde{l}_L$ and $\tilde{l}_R\tilde{l}_R$ events measure $\widetilde{J}_L$ and $\widetilde{J}_R$, respectively.[9] In this subsection, we will study the prospects for e^+e^- colliders by considering the experimental setting of the NLC, a proposed high energy linear e^+e^- collider, with maximum center-of-mass energy $\sqrt{s} = 0.5 - 1.5$ TeV and luminosity $\mathcal{L} \sim 50 - 100$ fb^{-1}/yr. At such a collider, highly polarized e^- beams are expected to be available.

For the NLC, we study a scenario specified by weak-scale SUSY parameters

$$m_{\tilde{l}_R} = 200 \text{ GeV}, \ m_{\tilde{l}_L} = 350 \text{ GeV}, \ M_2 = 2M_1 = 190 \text{ GeV},$$
$$\tan\beta = 2, \quad \mu = -400 \text{ GeV} . \tag{67}$$

In this scenario, the lightest two neutralinos are gaugino-like and the decay $\tilde{\chi}_2^0 \to \tilde{l}$ is forbidden. Thus, sleptons are not produced in gluino and squark cascades at the LHC, and precision studies of sleptons there are likely to be very difficult.

With the underlying SUSY parameters above and $\sqrt{s} = 500$ GeV, right-handed sleptons are pair-produced and decay through $\tilde{l}_R \to l\tilde{\chi}_1^0$. Decays $\tilde{l}_R \to \nu_l\tilde{\chi}_1^\pm, l\tilde{\chi}_2^0$ are also allowed, but are highly suppressed by phase space and mixing angles, since the chargino and neutralino are SU(2) gaugino-like. Note that both $\tilde{l}_L\tilde{l}_R$ and $\tilde{l}_L\tilde{l}_L$ production are inaccessible at $\sqrt{s} = 500$ GeV. Thus, the effects of CP violation in the $\tilde{l}_L$ sector are completely removed, and all slepton CP violation is a measure of $\widetilde{J}_R$. Of course, in any fortunate scenario in which both left- and right-handed sleptons may be produced, the beam energy may always be tuned to eliminate production of either $\tilde{l}_L$ or $\tilde{l}_R$, with possibly significant loss in slepton cross section. If flavor and CP violation in the $\tilde{l}_R$ sector is well-understood, one could then increase the beam energy and attempt to measure the additional parameters that enter with the production of left-handed sleptons.

Cuts have been studied in Ref. [8] for slepton flavor-conserving signals. We adopt those cuts here for the case of slepton flavor-violating signals. The efficiency of these cuts for the

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[9]Associated production of $\tilde{l}_L\tilde{l}_R$ through t-channel neutralino exchange is also possible. This is uncorrelated production of sleptons at separate vertices, and so the CP asymmetry in this process is a linear combination of $\widetilde{J}_L$ and $\widetilde{J}_R$.

signal is approximately 30%.[10] The dominant SM backgrounds are WW, $e\nu W$, and $eeWW$. With unpolarized beams, the SM background cross section is 4.8 fb. However, as in our previous study[15], we may exploit the advantages of right-polarized beams, which, in this case, doubles our signal cross section and reduces the SM background to 2.6 fb for 90% beam polarization.

We must also consider the additional SUSY backgrounds. For this beam energy, $\tilde{\chi}_1^0 \tilde{\chi}_2^0$ and $\tilde{\chi}_2^0 \tilde{\chi}_2^0$ production are kinematically accessible, but these are suppressed to negligible levels by mixing angles in the gaugino region. Chargino pair production is also allowed, and may produce dilepton events with large acoplanarity and missing p_T. However, the chargino background is reduced by the branching ratio $B(\tilde{\chi}_1^{\pm} \to l\nu\tilde{\chi}_1^0)$. In addition, the e_R^- beam polarization strongly suppresses chargino production, as the charginos are almost purely gaugino. For 90% beam polarization, the chargino background is suppressed to a low level relative to the leading SM backgrounds, and may be safely ignored in the following analysis.[11]

The expressions for $S^{\tilde{l}}$ and $B^{\tilde{l}}$ are as in the previous subsection, with the exception that the correlated pair expressions are to be used. For the other background processes,

$$S^0 = 0.6 \text{ fb} \times L \tag{68}$$

$$B^0 = 2.6 \text{ fb} \times L \ . \tag{69}$$

In contrast to the LHC case, S^0 is not zero — the beam polarization induces an asymmetry in the SM background, since $e^+ e_R^- \to e^- \nu W^+$ is allowed, but $e^+ e_R^- \to e^+ \nu W^-$ is forbidden. It is therefore important that the beam polarization and SM asymmetry be well estimated.

Contours of constant CP-violating cross section S/L for Scenario I are plotted in Fig. 5 in the $(\sin 2\theta, \Delta m/\Gamma)$ plane for fixed $\sin\delta = 1$. The CP asymmetry is maximal for $\Delta m \approx \Gamma$, and for $\tilde{J} \approx 10^{-2}$, the difference can be as large as 100 events per year. In Figs. 6 and 7 the discovery reach at the NLC for various integrated luminosities is plotted for Scenarios I and II. Relative to the LHC, the contour shapes are more difficult to understand, as the expressions for correlated pair production are complicated, but the essential behavior is similar to that of the LHC case. The maximal reach in $\sin 2\theta$ is for $\Delta m \sim \Gamma = 0.58$ GeV $= 2.9 \times 10^{-3} m$, and values of $\tilde{J} \approx 10^{-3} - 10^{-2}$ may be probed, depending on the integrated luminosities.

For large integrated luminosities, the NLC may therefore also probe small CP-violating parameters. Of course, for SUSY parameters ideally suited to the LHC, where gluinos and squarks are light and decay frequently to sleptons, the sensitivity at the NLC is not competitive with that at the LHC. This stems from the fact that the slepton production mechanism at the NLC is through weak interactions, whereas that at the LHC is through strong interactions. The strength of the NLC, however, is that the analysis does not rely

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[10]The 30% efficiency is for $\tilde{e}$ and $\tilde{\mu}$ events from Ref. [8]. For the purposes of calculating background, we will assume this efficiency also for events involving $\tilde{\tau}$.

[11]If left-handed sleptons are to be studied, chargino pair background cannot be removed with beam polarization. However, for $m_{\tilde{\chi}_1^{\pm}} > m_{\tilde{l}_L}$, the beam energy could be tuned to eliminate chargino pair production.

on favorable gluino and squark masses and decay patterns. For example, if gluino and squark masses are very large, or sleptons are simply not produced in squark and gluino cascade decays, slepton studies may be extremely difficult at the LHC, but would still be possible at the NLC. In addition, in some instances, the NLC provides the possibility of disentangling left- and right-handed slepton flavor and CP violation by gradually raising the beam energy. The NLC therefore provides a robust probe of slepton CP violation, for the most part requiring only that slepton pairs be kinematically accessible.

VII. CONSTRAINTS FROM THE ELECTRON EDM

We have seen in the previous sections that two conditions are required for a large CP-violating signal in the oscillation of sleptons: all the mixing angles in the W matrix should be large (so that $\tilde{J}$ is large), and at least two of the sleptons should be degenerate to nearly their widths. In this section, we discuss the contribution to the electron EDM coming from the CP-violating phases in the W matrices. As already mentioned in the discussion of CP-violating rephase invariants, the dominant contributions to the electron EDM due to the phases in W require non-trivial mixing in both the left and right slepton sectors, and are thus probing the invariants $\widetilde{K}_{12,13}$, which are completely distinct from the invariants $\tilde{J}_{L,R}$ probed at colliders. Nevertheless, we wish to show that in the case when all CP-violating phases contributing to the $\tilde{J}$'s and $\widetilde{K}$'s are comparable, and even if the bound on the electron EDM improves by a factor of 10, a large CP-violating signal can still be visible at the LHC and NLC. This is due to the fact that, for sleptons degenerate enough for visible collider CP violation, the constraint on the $\widetilde{K}$ from d_e places almost no restriction on the size of the mixing angles, and so does not limit the size of the collider CP-violating signal.

Consider the contribution to d_e coming from $\widetilde{K}_{12,13}$. For simplicity, we compute the contribution from each of them separately, $i.e.$, we first assume only two generation mixing and compute the contribution coming from $\widetilde{K}_{12}$, then assume that the first two generation sleptons are exactly degenerate and compute the contribution from $\widetilde{K}_{13}$. In the following we assume that the lightest neutralino is primarily $\tilde{B}$ with mass M_1. The contribution from $\widetilde{K}_{12}$ is then

$$\frac{d_e^{12}}{e} = -\widetilde{K}_{12} \times \frac{\alpha_1}{4\pi} \frac{m_\mu(A - \mu \tan\beta)}{M_1^3} \left(\frac{\Delta m_{12}^2}{\bar{m}_{12}^2}\right)_L \left(\frac{\Delta m_{12}^2}{\bar{m}_{12}^2}\right)_R f(x_L, x_R) , \qquad (70)$$

where $\Delta m_{12}^2 = m_1^2 - m_2^2$, $\bar{m}_{12}^2 = (m_1^2 + m_2^2)/2$, $x = \bar{m}_{12}^2/M_1^2$, and

$$f(x_L, x_R) = x_L x_R \frac{\partial^2}{\partial x_L \partial x_R} \left(\frac{g(x_L) - g(x_R)}{x_L - x_R}\right) , \quad g(x) = \frac{x^2 - 2x \ln x - 1}{2(x-1)^3} . \qquad (71)$$

Numerically, $f(1,1) = -1/30$. The contribution from $\widetilde{K}_{13}$ is enhanced by m_τ/m_μ relative to the one from $\widetilde{K}_{12}$:

$$\frac{d_e^3}{e} = -\widetilde{K}_{13} \times \frac{\alpha_1}{4\pi} \frac{m_\tau(A - \mu \tan\beta)}{M_1^3} \left(\frac{\Delta m_{12-3}^2}{\bar{m}_{12-3}^2}\right)_L \left(\frac{\Delta m_{12-3}^2}{\bar{m}_{12-3}^2}\right)_R f(x_L, x_R) , \qquad (72)$$

with $\Delta m_{12-3}^2 = m_3^2 - m_0^2$, $\bar{m}_{12-3}^2 = (m_3^2 + m_0^2)/2$, where m_0 is the common mass for the first two generations, and here $x = \bar{m}_{12-3}^2/M_1^2$. (Actually, since all three generations need to

be approximately degenerate, the slepton masses in the definition of x can be taken as the average of all the scalar masses.)

We know that in order to have a large signal from our direct probe of the $\widetilde{J}$, we need the sleptons degenerate to within widths, so we expect $\Delta m^2/m^2 \sim 10^{-3} - 10^{-2}$. Requiring the induced EDM to be less than the current experimental bound of $4 \times 10^{-27} e$ cm [21] by a factor of 10 yields the bounds

$$\left|\widetilde{K}_{12}\right| < 3 \times \left[\frac{M_1}{200 \text{ GeV}}\right]^2 \left|\frac{M_1}{A - \mu \tan \beta}\right| \left|\left[\frac{f(1,1)}{f(x_L, x_R)}\right]\right| \left|\frac{10^{-2}}{\left(\frac{\Delta m_{12}^2}{m_{12}^2}\right)_L}\right| \left|\frac{10^{-2}}{\left(\frac{\Delta m_{12}^2}{m_{12}^2}\right)_R}\right|$$

$$\left|\widetilde{K}_{13}\right| < 0.2 \times \left[\frac{M_1}{200 \text{ GeV}}\right]^2 \left|\frac{M_1}{A - \mu \tan \beta}\right| \left|\left[\frac{f(1,1)}{f(x_L, x_R)}\right]\right| \left|\frac{10^{-2}}{\left(\frac{\Delta m_{12-3}^2}{m_{12-3}^2}\right)_L}\right| \left|\frac{10^{-2}}{\left(\frac{\Delta m_{12-3}^2}{m_{12-3}^2}\right)_R}\right| . \quad (73)$$

These constraints do not place significant restrictions on the sizes of the mixing angles (unless $\tan \beta$ is known to be large) and hence do not affect the reach for our signal from the $\widetilde{J}$. Of course, if an electron EDM is discovered, we cannot determine whether it is due to the $\widetilde{K}$; it could dominantly be due to a relative phase between the $\tilde{B}$ mass and the A parameter, for instance. Thus, a non-zero d_e does not necessarily imply non-zero $\widetilde{K}_{12,13}$. However, the CP asymmetries between lepton flavor-violating events we discussed in the previous section must be due to $\widetilde{J}_{L,R}$, and seeing these asymmetries means that at least one of the $\widetilde{J}_{L,R}$ is non-zero.

VIII. CONSTRAINTS FROM $\mu \to e\gamma$

The large mixing angles in the W matrices needed for a large CP-violating signal may give dangerously large contributions to the rate for $\mu \to e\gamma$. We know, however, that a large CP-violating signal requires at least two of the sleptons to be degenerate to within widths. In this section, we show that as long as the third generation sleptons also have small splittings from the other two generations, the slepton degeneracy is sufficient to suppress the $\mu \to e\gamma$ amplitude to harmless levels, even if there are large mixing angles in the W matrices.

There are many contributions to $\mu \to e\gamma$, but they fall into two classes: those which require flavor mixing for both left- and right-handed sleptons, and those which require only left- or right-handed mixing. The most dangerous case is when there is mixing with the third generation for both left- and right-handed scalars; insertion of the left-right mass for the stau gives a contribution to the $\mu \to e\gamma$ amplitude proportional to m_τ, while all other contributions are at most proportional to m_μ. If we assume for simplicity that the first two generation sleptons are exactly degenerate and keep only the $\tilde{B}$ intermediate gaugino, then with the same notation as in Eq. (72) the branching ratio for $\mu \to e\gamma$ relative to the current experimental bound of 4.9×10^{-11}[22] is

$$\frac{B(\mu \to e\gamma)}{4.9 \times 10^{-11}} = \left[\frac{200 \text{ GeV}}{M_1}\right]^4 \left|\frac{A - \mu \tan \beta}{M_1}\right|^2 \left[\frac{f(x_L, x_R)}{f(1,1)}\right]^2$$

24

$$\times \left| \frac{\left(W_{32} W_{33}^* \frac{\Delta m_{12-3}^2}{m_{12-3}^2} \right)_L}{5 \times 10^{-2}} \right|^2 \left| \frac{\left(W_{31}^* W_{33} \frac{\Delta m_{12-3}^2}{m_{12-3}^2} \right)_R}{5 \times 10^{-2}} \right|^2 , \tag{74}$$

plus a similar contribution with $L \to R$. We see that, if all the mixing angles $W_{L,R}$ are large, we cannot allow $\mathcal{O}(1)$ splittings between the third generation and first two generations for both left- and right-handed sleptons. If, however, the splittings are less than $\sim 10^{-1} - 10^{-2}$ as in our Scenario I and II, it does not constrain the size of the mixing angles.

We now turn to contributions needing either only left-handed or only right-handed mixing. There are again many diagrams. Let us consider only the contribution from the one involving the left-right mass insertion for the $\tilde{\mu}$. Since it is proportional to $A - \mu \tan \beta$, this contribution begins to dominate for moderately large $\tan \beta$ and is otherwise comparable to the other contributions. We find that

$$\frac{B(\mu \to e\gamma)}{4.9 \times 10^{-11}} = \left[\frac{200 \text{ GeV}}{M_1} \right]^4 \left| \frac{A - \mu \tan \beta}{M_1} \right|^2 \left[\frac{h(x_L, x_R)}{h(1,1)} \right]^2$$
$$\times \left| \frac{\left(W_{32} W_{31}^* \frac{\Delta m_{12-3}^2}{m_{12-3}^2} + W_{22} W_{21}^* \frac{\Delta m_{12}^2}{m_{12}^2} \right)_L}{2 \times 10^{-2}} \right|^2 . \tag{75}$$

with a similar contribution from $L \to R$, and where

$$h(x_L, x_R) = x_L \frac{\partial}{\partial x_L} \frac{g(x_L) - g(x_R)}{x_L - x_R} , \tag{76}$$

with $h(1,1) = 1/20$.

From the above we see that even if only pure L or R mixing contributions are considered, if all the mixing angles are large, no $\mathcal{O}(1)$ splittings are allowed between any pair of sleptons. On the other hand, if (as in our Scenario II for large CP violation) all the sleptons are degenerate to within their widths (so that $\Delta m^2/m^2 \sim 10^{-3} - 10^{-2}$), the contributions to $\mu \to e\gamma$ are already suppressed enough even for maximal mixing angles. Even if only the first two generation sleptons are degenerate within widths, and the third is split off by a factor of 10 compared to the width (as in our Scenario I), the bounds from $\mu \to e\gamma$ can still be met with moderately small products of mixing angles $W_{32} W_{31}^* \sim 1/10$, which does not place a strong constraint on the size of $\tilde{J}$. For large $W_{22} W_{21}^*$ (since its size is not restricted if $\Delta m_{12} \sim \Gamma$) and $\sin \delta$, $\tilde{J}$ can still be as large as a few $\times 10^{-2}$, well within the reach of the experiments.

IX. IMPLICATIONS FOR DISTINGUISHING MODELS FOR SCALAR MASSES

From the discussion in the previous sections, we see that an observable CP-violating signal requires the following conditions. First, the slepton mixing angles and CP-violating phase should be quite large to give a sufficiently large $\tilde{J}$, and at least two generations of sleptons should be degenerate to about their widths to maximize the signal. In addition, constraints from the flavor changing process $\mu \to e\gamma$ require that all three generations of

25

sleptons should be quite degenerate if all mixing angles are large. These considerations strongly suggest a particular pattern for the slepton mass matrix. Since the sleptons must be nearly degenerate, their mass matrix must be very nearly proportional to the identity. However, since all the mixing angles must be large and the mass splittings must be small, the slepton mass matrix must deviate from the identity by small perturbations to all the entries. The mass matrix must therefore have the form

$$m_{ij}^2 = m_0^2 \left(\delta_{ij} + \epsilon_{ij} \right) \ , \tag{77}$$

where the ϵ_{ij} are of order Γ/m, and are of comparable size for all i, j. What sort of models can give rise to this sort of mass matrix? The degeneracy among different generations of sleptons can originate from dynamics, flavor symmetry, or both. In the following, we discuss how this sort of slepton mass matrix can arise in these different cases.

First, let us consider a case where no particular flavor symmetry is present, and the sleptons are degenerate because the mediation of SUSY breaking to the ordinary fields is flavor-blind. Models with SUSY breaking mediated by gauge interactions are examples of this class[25]. In this case, there are two important scales relevant to our study: Λ_S, the messenger scale of SUSY breaking, where SUSY breaking is first transmitted to the ordinary sector, and Λ_D, the scale where extra non-universal and flavor-violating interactions decouple. If $\Lambda_D > \Lambda_S$, the flavor-blind mediation of SUSY breaking will guarantee the degeneracy of sleptons, and hence render the slepton mixing matrices $W_{L,R}$ trivial. On the other hand, suppose $\Lambda_D \lesssim \Lambda_S$, and the sleptons have flavor-violating interactions $h_{iI\alpha} l_i \bar{F}_I \phi_\alpha$, where $h_{iI\alpha}$ are Yukawa couplings, l_i is the generation i slepton, and $\bar{F}$ and ϕ are extra fields decoupling at Λ_D. The slepton masses then receive extra non-universal, flavor-violating corrections

$$\Delta m_{ij}^2 = -\frac{1}{8\pi^2} \sum_{I\alpha} h_{iI\alpha} h_{jI\alpha}^* \left(m_0^2 + m_{\phi_\alpha}^2 + m_{\bar{F}_I}^2 \right) \left(\ln \frac{\Lambda_S}{\Lambda_D} + c_{\text{TH}} \right) \ , \tag{78}$$

where c_{TH} represents the threshold corrections. If the couplings $h_{iI\alpha}$ are complex and $\mathcal{O}(1)$ (since there is no flavor symmetry to restrict their sizes), the mixing matrix that diagonalizes the slepton mass matrix will have large mixing angles and a $\mathcal{O}(1)$ CP-violating phase. If the logarithm is not large, then the splittings among different generations are small, naturally of the order $\Delta m/m \sim h^2/16\pi^2 \sim 10^{-2} - 10^{-4}$ for $h \sim 0.1 - 1$, which is comparable to the slepton width $\Gamma/m \sim g_1^2/8\pi \times$ phase space factor, as required for an observable CP-violating signal. The closeness of Λ_D and Λ_S could be accidental, or may arise because they have a common origin.[12] Another possibility is that the dominant contribution to the scalar masses is flavor-blind, say, from gauge mediated SUSY breaking, but the fundamental SUSY breaking vacuum energy F^2 is large enough ($F \sim 10^{18} - 10^{20}$ GeV^2) that the supergravity contribution to the scalar mass squared $\sim (F/M_{\text{Pl}})^2 \sim (1 - 10$ GeV$)^2$ is about $10^{-4} - 10^{-2}$ of the gauge-mediated contribution. If the supergravity mediation does not obey any flavor symmetry, we expect comparable contributions to all elements of the scalar mass matrices,

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[12]For instance, they may both be triggered by the SUSY breaking. Such a framework is discussed in Ref. [26] within the context of flavor symmetries.

again leading to the small non-degeneracies and large mixing angles needed for the CP-violating signal.

In the case when there is a flavor symmetry that is broken at a scale Λ_F and gives rise to the observed pattern of fermion masses, the degeneracy among different generation sleptons can either come from the dynamics or be ensured by the flavor symmetry itself (for certain non-Abelian flavor symmetries). Beneath the flavor symmetry breaking scale Λ_F, extra flavor-violating interactions can be present down to Λ_D if not all the fields decouple at Λ_F. We may still get the radiative corrections to the slepton masses given in Eq. (78) for $\Lambda_D \lesssim \Lambda_S$, but some of the couplings $h_{iI\alpha}$ will be small because of the flavor symmetry. In particular, we do not expect that both l_i and l_j, for $i \neq j$, can have $\mathcal{O}(1)$ couplings to the same fields $\bar{F}, \phi$ if l_i and l_j transform differently under the flavor symmetry. The off-diagonal elements of the mass matrix generated from radiative corrections will then be suppressed. The splitting generated in the diagonal elements may be less suppressed or unsuppressed because the different generations may have different couplings, if, for example, the flavor symmetry is Abelian. However, if $\Lambda_F < \Lambda_S$, there will be additional corrections to the scalar masses from the flavor symmetry breaking effects. These corrections come from flavor symmetric operators like

$$m_S^2 \frac{\phi^{i\dagger}}{M} l_i^\dagger \frac{\phi^j}{M} l_j \; , \tag{79}$$

where the ϕ fields are SM singlets that carry flavor ("flavons"), and with which the sleptons combine to form flavor singlets. These non-renormalizable operators may be present in the fundamental theory as M_{Pl} suppressed operators (in which case $M = M_{\text{Pl}}$), or can arise in the Froggatt-Nielsen mechanism[27] when the heavy Froggatt-Nielsen fields are integrated out (in which case $M = M_{\text{FN}}$). When the flavon fields ϕ acquire nonzero vacuum expectation values, breaking the flavor symmetry, the operators of Eq. (79) become mass terms for the sleptons. They can contribute to both the off-diagonal elements and the splitting in the diagonal elements, depending on the details of particular models. The (spontaneously broken) flavor symmetry is expected to explain the light fermion masses: small Yukawa couplings are understood as powers in the small parameters $\langle\phi\rangle/M$. In many theories, the parameters $\langle\phi\rangle/M$ are of the order $10^{-1} - 10^{-2}$ in order to generate the observed fermion mass hierarchy. The slepton mass splitting induced by Eq. (79) could be close to their widths for suitable powers of ϕ in these operators. In addition, large CP-violating phases in the slepton mixing matrices are expected to be generated in this case since similar operators also give rise to the CP-violating phase in the CKM matrix. Thus, in theories in which spontaneously broken flavor symmetries are used to explain the fermion mass hierarchy, the necessary conditions for generating a large CP-violating signal may also arise, as long as the scale of symmetry breaking is beneath the scale at which SUSY breaking is transmitted to the sfermions.

X. CONCLUSIONS

In this paper, we have studied the possibility of probing CP violation in the supersymmetric lepton mixing matrices $W_{L,R}$ which diagonalize the left- and right-handed slepton mass matrices in the basis where the charged lepton masses are diagonal. There are four

independent CP-violating phases associated with the slepton mass matrices, and we gave them a rephase invariant description in terms of the four quantities $\widetilde{J}_{L,R}$ and $\widetilde{K}_{12,13}$. While the electron EDM depends on the $\widetilde{K}$, the CP-violating asymmetries in the oscillations and decays of sleptons at colliders depend on $\widetilde{J}$, and are not directly constrained from the bounds on the electron EDM. The CP-violating effects associated with a single $\widetilde{J}$ disappear both as $\widetilde{J}$ goes to zero as well as when any two slepton masses become degenerate. We found that three conditions are necessary to produce a large collider CP asymmetry in lepton flavor-violating events:

1. The W matrices must have large CP and flavor-violating angles, leading to $\widetilde{J} > 10^{-3}$.

2. The two sleptons of highest degeneracy should have Δm within an order of magnitude of Γ.

3. All three sleptons must have some degree of degeneracy, $i.e.$, $\Delta m/m \ll 1$.

The last requirement is necessary to ensure that the large flavor mixing angles of W do not violate bounds from $\mu \to e\gamma$. It is interesting that the observation of a sizeable CP-violating signal tells us so much about the structure of the slepton mass matrix in a region where the sleptons are so degenerate that the limits from rare processes and EDMs are irrelevant. This is because we are probing an oscillation phenomenon, and the superGIM suppression of the amplitude due to near slepton degeneracy occurs only for $\Delta m/\Gamma \ll 1$, rather than $\Delta m/m \ll 1$ as in the low energy rare processes.

We showed that the experimental probe of $\widetilde{J}$ at both the LHC and NLC can be powerful. Our signal at the LHC is from strong production of squarks and gluinos, followed by cascade decays to the second neutralino, which in turn decays into slepton and lepton, with the slepton subsequently oscillating and decaying into the lightest neutralino and lepton. If such cascade decays are realized in nature, we find that the LHC can probe significant regions in the $(\widetilde{J}, \Delta m/\Gamma)$ plane. The reach depends sensitively on the strong production cross section and hence on the squark and gluino masses, but we find that $\widetilde{J} > 10^{-3}$ and $\Delta m/\Gamma$ between $1/10$ and 10 can be probed at the LHC. If slepton CP violation is undetectable at the LHC, a robust probe is provided by the NLC, where sleptons are produced in great numbers independent of the various cascades and branching ratios required at the LHC. We find that the NLC has a comparable reach in the $(\widetilde{J}, \Delta m/\Gamma)$ plane for high integrated luminosities. In this paper, we have concentrated on signals at the LHC and NLC, because these machines offer the possibility of probing $\widetilde{J}$ to the level of 10^{-3} or perhaps even further, well below the maximum of value of $\frac{1}{6\sqrt{3}}$. We also noted, however, that if slepton masses are near their current bound and $\widetilde{J}$ is near maximal, slepton CP violation may also be observable at LEP II. Similarly, if squarks and gluinos are very light, a possibility of detecting slepton CP violation may exist at Run II and future upgrades of the Tevatron.

We finally considered the implications of an observable CP-violating signal for models of the scalar masses. If a CP-violating asymmetry is observed, we saw that the mass splittings must be non-zero and $\sim \Gamma$. It should be noted that the sensitivity of the CP-violating signal to small but non-zero mass splittings is extraordinary. In the examples we gave, mass splittings as small as ~ 50 MeV could be distinguished from zero; such precision for slepton mass differences is typically unobtainable through conventional considerations of kinematic distributions. We saw that an observable CP-violating signal implies further that the scalar

masses are dominantly universal, but have extra, non-hierarchical contributions to all elements of the mass matrix that are $\sim \Gamma/m \sim 10^{-2} - 10^{-4}$ of the universal mass. Such a structure for scalar mass matrices does not typically arise in the simplest models for solving the supersymmetric flavor-changing problem: it is absent in theories with gauge-mediated SUSY breaking and in theories where the scalars are kept degenerate (or aligned with the fermions) by spontaneously broken flavor symmetries. However, there is an important framework in a more general theory in which the CP-violating signals are large. In this class of models, the dynamics dominantly responsible for generating scalar masses is flavor-blind, leading to nearly degenerate slepton masses. However, the presence of flavor-violating perturbations beneath the scale of the communication of SUSY breaking, or of sub-dominant, flavor-violating transmission of SUSY breaking, can give non-hierarchical, small contributions to all the elements of the slepton mass matrices needed for a large CP-violating signal. Observation of a CP-violating signal gives critical information on the relationship between the nature of the communication of SUSY breaking and the presence of new flavor-violating interactions at high energies.

ACKNOWLEDGMENTS

APPENDIX A: RELATING $Im(m_{12}^2 m_{23}^2 m_{31}^2)$ AND $\widetilde{J}$

In this appendix, we derive the relationship between the CP-violating invariants $\widetilde{J}$ and $Im[m_{12}^2 m_{23}^2 m_{31}^2]$ given in Eq. (7). Using $m_{\alpha\beta}^2 = \sum_i W_{i\alpha}^* W_{i\beta} m_i^2$, we have

$$Im\left[m_{12}^2 m_{23}^2 m_{31}^2\right] = \sum_{ijk} Im\left[W_{i1}^* W_{i2} W_{j2}^* W_{j3} W_{k3}^* W_{k1}\right] m_i^2 m_j^2 m_k^2 . \tag{A1}$$

However, using the unitarity of W as $W_{i1}^* W_{k1} = \delta_{ik} - W_{i2}^* W_{k2} - W_{i3}^* W_{k3}$, the imaginary part in the above becomes manifestly proportional to $\widetilde{J}$. Since we know that the LHS vanishes if any two of the m_i^2 are identical and the RHS is cubic in the m_i^2, all the m_i^2 dependence must be proportional to $(m_2^2 - m_1^2)(m_3^2 - m_2^2)(m_1^2 - m_3^2)$. It is easy to show that the constant of proportionality is 1, and Eq. (7) is established.

APPENDIX B: s-, t-INTERFERENCE

The CP-violating asymmetries at colliders must be linearly dependent on $\widetilde{J}$. For the case of pair production, the σ_0^{st} piece may be demonstrated to have this form by breaking the sum into 5 pieces:

$$\sum_{i=j=k} \; : \; \sum_i Im[W_{i\alpha}W_{i\beta}^*W_{i\alpha}^*W_{i1}W_{i1}^*W_{i\beta}]A_{ii}A_{ii} = 0 \tag{B1}$$

$$\sum_{i=j\neq k} \; : \; \sum_{i\neq k} Im[W_{i\alpha}W_{i\beta}^*W_{i\alpha}^*W_{i1}W_{k1}^*W_{k\beta}]A_{ii}A_{ik} = \sum_{i\neq k}|W_{i\alpha}|^2\,\widetilde{J}_{1\beta}^{ik}A_{ik} \tag{B2}$$

$$\sum_{i=k\neq j} \; : \; \sum_{i\neq j} Im[W_{i\alpha}W_{i\beta}^*W_{j\alpha}^*W_{j1}W_{i1}^*W_{i\beta}]A_{ij}A_{ii} = \sum_{i\neq j}|W_{i\beta}|^2\,\widetilde{J}_{\alpha 1}^{ij}A_{ij} \tag{B3}$$

$$\sum_{i\neq j=k} \; : \; \sum_{i\neq j} Im[W_{i\alpha}W_{i\beta}^*W_{j\alpha}^*W_{j1}W_{j1}^*W_{j\beta}]A_{ij}A_{ij} = \sum_{i\neq j}|W_{j1}|^2\,\widetilde{J}_{\alpha\beta}^{ij}A_{ij}^2 \tag{B4}$$

$$\sum_{i\neq j\neq k} \; : \; \sum_{i\neq j\neq k} Im[W_{i\alpha}W_{i\beta}^*W_{j\alpha}^*W_{j1}W_{k1}^*W_{k\beta}]A_{ij}A_{ik}$$

$$= \sum_{i\neq j\neq k} Im\left[(\delta_{\alpha\beta} - W_{j\alpha}W_{j\beta}^* - W_{k\alpha}W_{k\beta}^*)W_{j\alpha}^*W_{j1}W_{k1}^*W_{k\beta}\right]A_{ij}A_{ik}$$

$$= \sum_{i\neq j\neq k}\left[-|W_{j\alpha}|^2\,\widetilde{J}_{1\beta}^{jk}A_{ij}A_{ik} - |W_{k\beta}|^2\,\widetilde{J}_{1\alpha}^{jk}A_{ij}A_{ik}\right] \;, \tag{B5}$$

where the unitarity of W has been used to simplify the final term. We thus see that $\Delta_{\alpha\beta}^{st}$ may be written in a form explicitly proportional to $\widetilde{J}$ as

$$\Delta_{\alpha\beta}^{st} = 2i\sum_{i\neq j}\left[|W_{i\alpha}|^2\,\widetilde{J}_{1\beta}^{ij}A_{ij} - |W_{i\beta}|^2\,\widetilde{J}_{1\alpha}^{ij}A_{ij} + |W_{j1}|^2\,\widetilde{J}_{\alpha\beta}^{ij}A_{ij}^2\right]$$

$$-2i\sum_{i\neq j\neq k}\left[|W_{j\alpha}|^2\,\widetilde{J}_{1\beta}^{jk}A_{ij}A_{ik} - |W_{j\beta}|^2\,\widetilde{J}_{1\alpha}^{jk}A_{ij}A_{ik}\right] \;. \tag{B6}$$

The sum is over all on-shell slepton generations, and so the last sum is relevant only when all three generations may be produced.

TABLE I. Particle masses (in GeV) at the LHC analysis point, with $q = u, d, c, s$ and $l = e, \mu, \tau$.

	Mass		Mass		Mass		Mass
$\tilde{g}$	767	$\tilde{b}_2$	662	$\tilde{\chi}_2^0$	231	$\tilde{l}_L$	239
$\tilde{q}_L$	688	$\tilde{b}_1$	635	$\tilde{\chi}_1^\pm$	230	$\tilde{\nu}_l$	230
$\tilde{q}_R$	662	$\tilde{t}_2$	717	$\tilde{\chi}_1^0$	121	$\tilde{l}_R$	157
		$\tilde{t}_1$	498			h	100

TABLE II. Cross sections σ (in fb) at the LHC analysis point, with $q = u, d, c, s$.

	σ		σ		σ
$\tilde{g}\tilde{g}$	1751	$\tilde{q}\tilde{q}$	2379	$\tilde{b}\tilde{\bar{b}}$	297
$\tilde{g}\tilde{q}, \tilde{g}\tilde{\bar{q}}$	8306	$\tilde{q}\tilde{\bar{q}}$	2820	$\tilde{t}\tilde{\bar{t}}$	701

TABLE III. Branching fractions B (in percent) at the LHC analysis point, with $q = u, d, c, s$ and $l = e, \mu, \tau$.

	B		B		B
$\tilde{g} \to \tilde{q}_L \bar{q}$	31	$\tilde{q}_L \to q \tilde{\chi}_2^0$	32	$\tilde{\chi}_2^0 \to \tilde{l}_R l$	36
$\tilde{g} \to \tilde{q}_R \bar{q}$	31	$\tilde{q}_L \to q \tilde{\chi}_1^\pm$	65	$\tilde{\chi}_2^0 \to h \tilde{\chi}_1^0$	63
$\tilde{g} \to \tilde{b}_1 \bar{b}$	10	$\tilde{q}_L \to q \tilde{\chi}_1^0$	3	$\tilde{\chi}_2^0 \to Z \tilde{\chi}_1^0$	1
$\tilde{g} \to \tilde{b}_2 \bar{b}$	14	$\tilde{q}_R \to q \tilde{\chi}_1^0$	99	$\tilde{\chi}_1^\pm \to W \tilde{\chi}_1^0$	98
$\tilde{g} \to \tilde{t}_1 \bar{t}$	14			$\tilde{l}_R \to l \tilde{\chi}_1^0$	100

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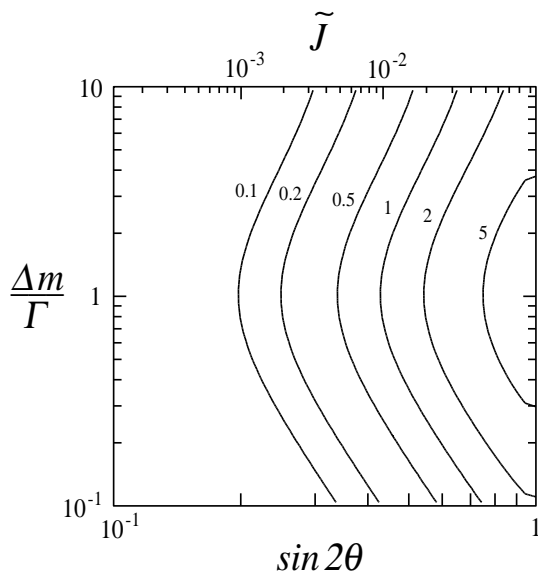
FIG. 1. Constant contours (in fb) of the CP-violating cross section S/L for Scenario I at the LHC. The signal is maximal for $\Delta m \sim \Gamma = 0.13$ GeV $= 8.1 \times 10^{-4}m$. The signal S is directly proportional to $\widetilde{J}$. For the $\sin 2\theta$ axis the CP-violating phase is fixed to $\sin \delta = 1$.
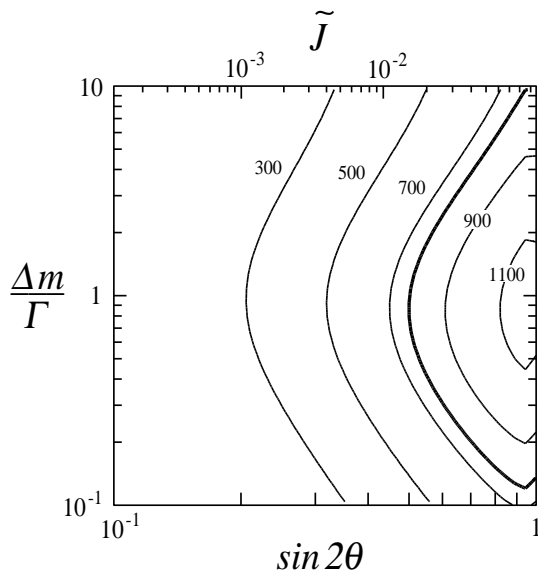

FIG. 2. 3σ slepton CP violation discovery contours at the LHC with an integrated luminosity of 100 fb^{-1} in Scenario I (two generation degeneracy). The CP-violating phase is fixed to $\sin \delta = 1$. The wide contour is for the parameters discussed in the text, where $m_{\tilde{g}} = 767$ GeV, and $\Gamma = 0.13$ GeV $= 8.1 \times 10^{-4}m$. The other contours give an indication of possible discovery reaches for scenarios with $m_{\tilde{g}} = m_{\tilde{q}}$ shown (in GeV), under the naive assumption that all cross sections scale simply as the gluino and squark cross sections. See full discussion in the text.
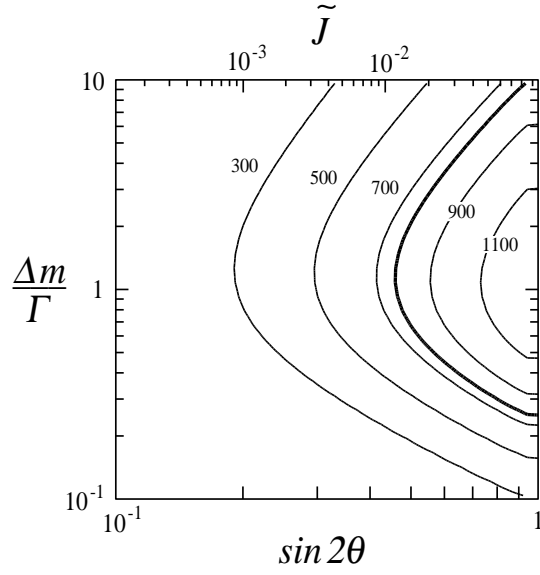
34

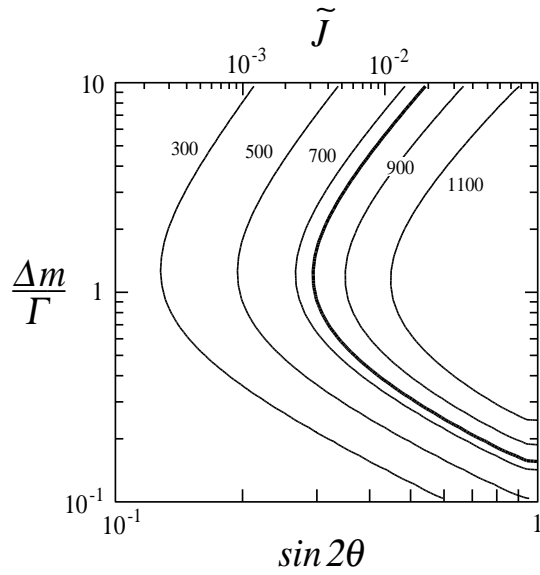FIG. 3. Same as for Fig. 2, but for Scenario II (three generation degeneracy).

FIG. 4. Same as for Fig. 3, but for 10 times improved statistics. Possible sources of such an improvement include more favorable branching ratios, the combination of all lepton asymmetries, and an increase in the assumed integrated luminosity to include multi-year event samples and both detectors.
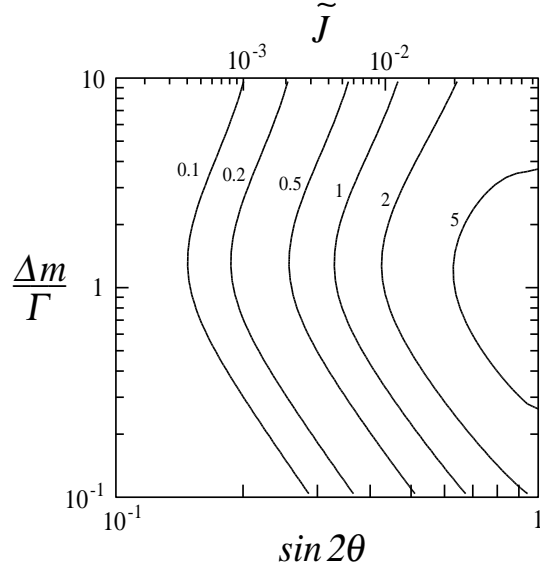
FIG. 5. Constant contours (in fb) of the CP-violating cross section S/L for Scenario I at the NLC. The signal is maximal for $\Delta m \sim \Gamma = 0.58$ GeV $= 2.9 \times 10^{-3} m$. The CP-violating phase is fixed to $\sin \delta = 1$.
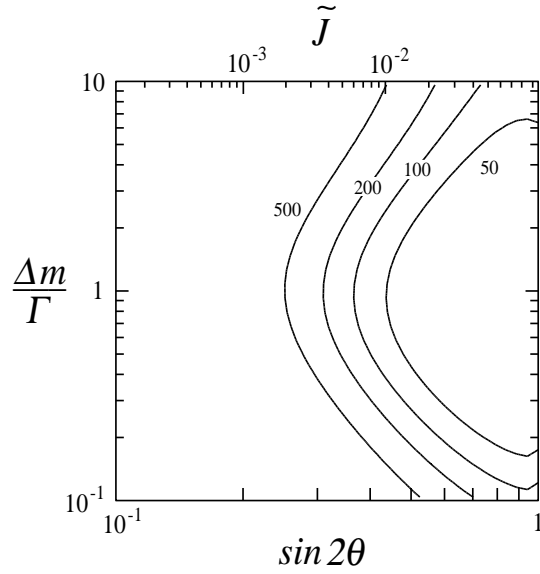

FIG. 6. 3σ slepton CP violation discovery contours at the NLC with integrated luminosity given (in fb^{-1}) for Scenario I (two generation degeneracy). The CP-violating phase is fixed to $\sin \delta = 1$. The SUSY parameters are as given in the text, with $\Gamma = 0.58$ GeV $= 2.9 \times 10^{-3} m$.
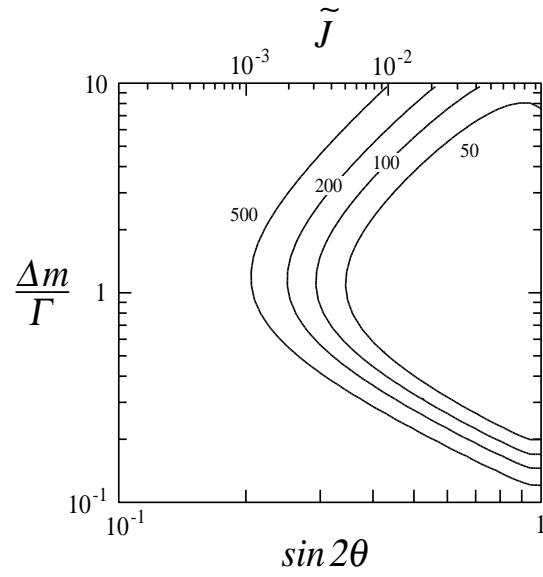
FIG. 7. Same as for Fig. 6, but for Scenario II (three generation degeneracy).